Title: Topological aspects of quantum cellular automata in one dimension

Speakers: Zongping Gong

Series: Perimeter Institute Quantum Discussions

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Abstract: Quantum cellular automata (QCA) are unitary transformations that preserve locality. In one dimension, QCA are known to be fully characterized by a topological chiral index that takes on arbitrary rational numbers [1]. QCA with nonzero indices are anomalous, in the sense that they are not finite-depth quantum circuits of local unitaries, yet they can appear as the edge dynamics of two-dimensional chiral Floquet topological phases [2].

In this seminar, I will focus on the topological aspects of one-dimensional QCA. First, I will talk about how the topological classification of QCA will be enriched by finite unitary symmetries [3]. On top of the cohomology character that applies equally to topological states, I will introduce a new class of topological numbers termed symmetry-protected indices. The latter, which include the chiral index as a special case, are genuinely dynamical topological invariants without state counterparts [4].

In the second part, I will show that the chiral index lower bounds the operator entanglement of QCA [5]. This rigorous bound enforces a linear growth of operator entanglement in the Floquet dynamics governed by nontrivial QCA, ruling out the possibility of many-body localization. In fact, this result gives a rigorous proof to a conjecture in Ref. [2]. Finally, I will present a generalized entanglement membrane theory that captures the large-scale (hydrodynamic) behaviors of typical (chaotic) QCA [6].

References:
[1] D. Gross, V. Nesme, H. Vogts, and R. F. Werner, Commun. Math. Phys. 310, 419 (2012).
[2] H. C. Po, L. Fidkowski, T. Morimoto, A. C. Potter, and A. Vishwanath, Phys. Rev. X 6, 041070 (2016).
[3] Z. Gong, C. Sünderhauf, N. Schuch, and J. I. Cirac, Phys. Rev. Lett. 124, 100402 (2020).
[4] Z. Gong and T. Guaita, arXiv:2106.05044.
[5] Z. Gong, L. Piroli, and J. I. Cirac, Phys. Rev. Lett. 126, 160601 (2021).
[6] Z. Gong, A. Nahum, and L. Piroli, arXiv:2109.07408.

Topological Aspects of Quantum Cellular Automata in One Dimension

Zongping Gong

Max-Planck-Institut für Quantenoptik

MPQ

ZG, C. Sünderhauf, N. Schuch, and J. I. Cirac, PRL **124**, 100402 (2020)

ZG and T. Guaita, arXiv:2106.05044

ZG, L. Piroli, and J. I. Cirac, PRL **126**, 160601 (2021)

ZG, A. Nahum, and L. Piroli, arXiv:2109.07408

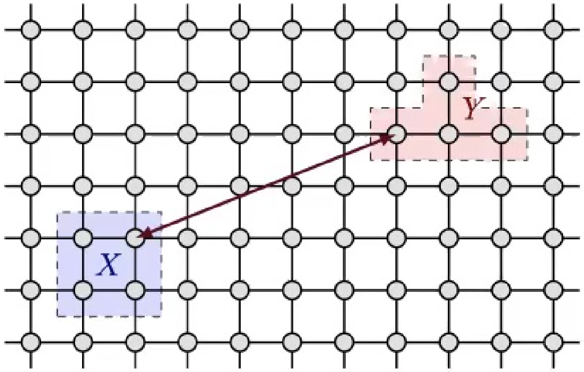
Quantum cellular automata

- Lieb-Robinson bound

"Soft" light cone from locality:

$$\|[O_X(t), O_Y]\| \leq Ce^{-\kappa[\text{dist}(X,Y)-vt]}$$

E. H. Lieb and D. W. Robinson, CMP **28**, 251 (1972)

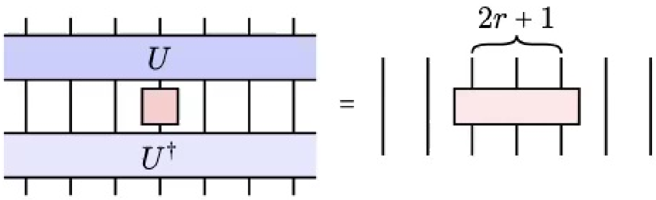


- Definition of quantum cellular automata (QCA)

Unitary that ***strictly*** preserves locality:

$$O_{\bar{A}} = UO_AU^\dagger$$

B. Schumacher and R. F. Werner, arXiv:quant-ph/0405174

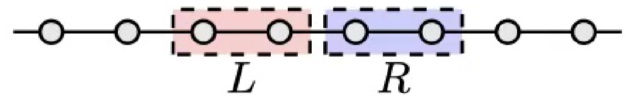


In 1D, QCA = matrix-product unitary (MPU) J. I. Cirac *et al.*, JSM (2017) 083105

Index for 1D QCA

- Definition of the (chiral) index D. Gross *et al.*, Commun. Math. Phys. **310**, 419 (2012)

$$\mathrm{ind} = \log \frac{\eta(\mathcal{A}_L^U, \mathcal{A}_R)}{\eta(\mathcal{A}_L, \mathcal{A}_R^U)} \in \log \mathbb{Q}^+$$



$$\eta(\mathcal{A}_L^U, \mathcal{A}_R) = \frac{\sqrt{d_L d_R}}{d_\Lambda} \sqrt{\sum_{i,j=1}^{d_L} \sum_{m,n=1}^{d_R} |\mathrm{Tr}_\Lambda [U e_{ij}^{L\dagger} U^\dagger e_{mn}^R]|^2} \qquad e_{ij} = |i\rangle\langle j|$$

Finite-depth quantum circuits are of zero index

Nontrivial example: right translation T on a qudit lattice

$$L = x, R = x+1$$

$$\eta(\mathcal{A}_x^T, \mathcal{A}_{x+1}) = \eta(\mathcal{A}_{x+1}, \mathcal{A}_{x+1}) = d \text{ (maximal overlap)} \quad \Rightarrow \boxed{\mathrm{ind} = \log d}$$

$$\eta(\mathcal{A}_x, \mathcal{A}_{x+1}^T) = \eta(\mathcal{A}_x, \mathcal{A}_{x+2}) = 1 \text{ (zero overlap)}$$

Part I

Classification of 1D QCA with finite unitary symmetries

Collaborators: Christoph Sünderhauf, Norbert Schuch,

J. Ignacio Cirac

Phys. Rev. Lett. **124**, 100402 (2020)

Floquet SPT phases (review)

- A heuristic classification

D. V. Else and C. Nayak, PRB **93**, 201103(R) (2016)

Künneth formula

$$H^{d+1}(\mathbb{Z} \times G, \mathrm{U}(1)) = \underbrace{H^{d+1}(G, \mathrm{U}(1))}_{\text{bulk}} \times \underbrace{H^{d}(G, \mathrm{U}(1))}_{\text{edge}}$$

$\mathbb{Z}$: Time-translation symmetry U_{F}

bulk: Classification of static d-dim. SPT phases

edge: Classification of static (d-1)-dim. SPT phases

NOT complete! At least in d=2 dim. H. C. Po *et al.*, PRX **6**, 041070 (2016)

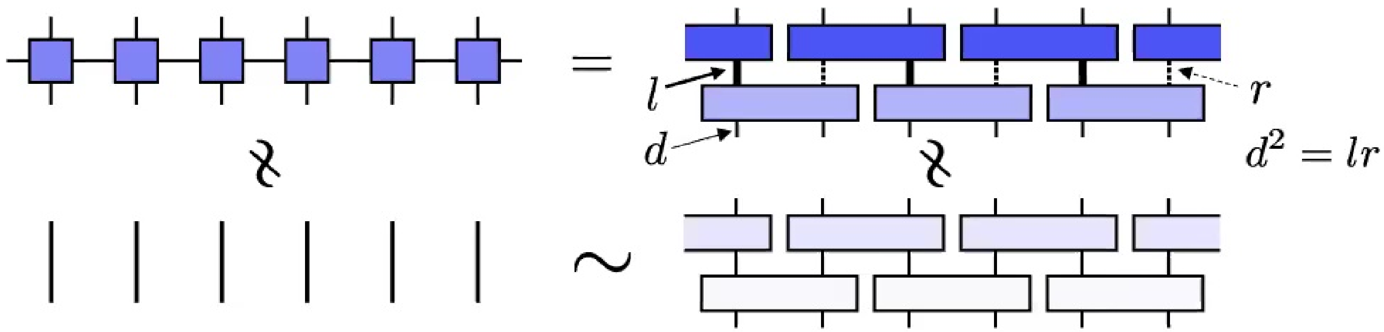
∵ 1D edges, which are locality-preserving unitaries well described by ***MPUs***, are characterized by the ***chiral index even without symmetry protection***

D. Gross *et al.*, Commun. Math. Phys. **310**, 419 (2012)
J. I. Cirac *et al.*, JSM (2017) 083105

Question: What is the classification (beyond $H^2(G,\mathrm{U}(1))$) of matrix-product unitaries with symmetries?

MPU & MPS w/o symmetry (review)

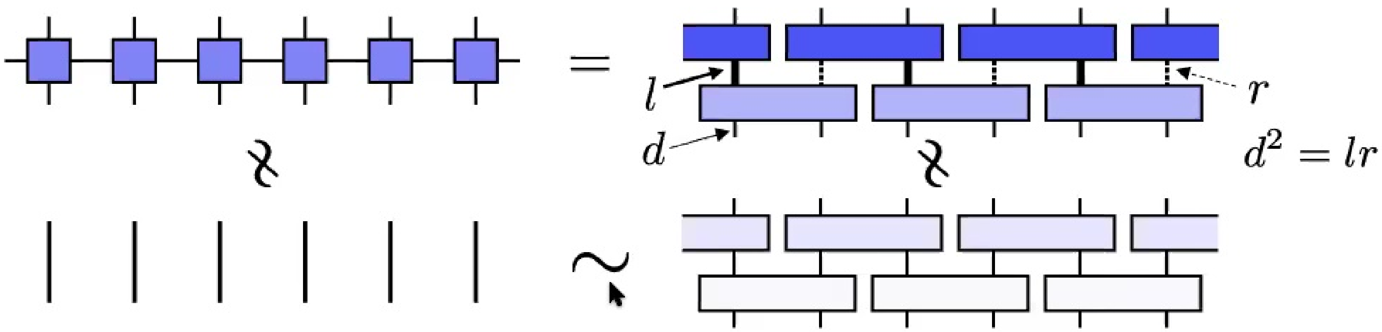
- MPU is ***not*** always trivial J. I. Cirac *et al.*, JSM (2017) 083105



Chiral index: $\mathrm{ind} = \frac{1}{2}\log\frac{r}{l} = \log\frac{r}{d} \in \log\mathbb{Q}^+$

MPU & MPS w/o symmetry (review)

- MPU is ***not*** always trivial J. I. Cirac *et al.*, JSM (2017) 083105



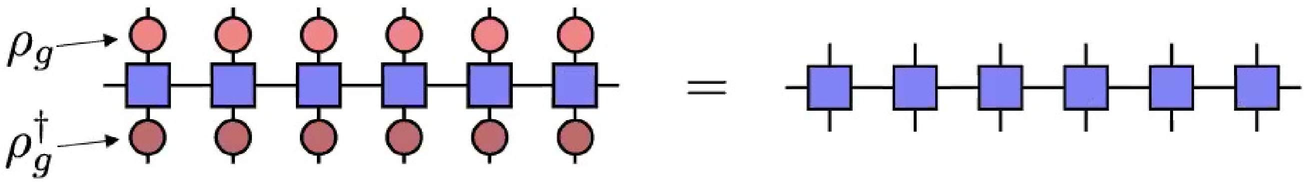
Chiral index: $\mathrm{ind} = \frac{1}{2}\log\frac{r}{l} = \log\frac{r}{d} \in \log\mathbb{Q}^+$

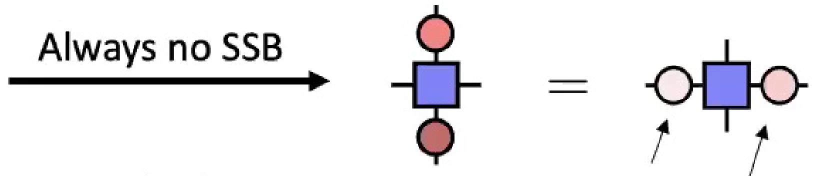
Cf. MPS is always trivial X. Chen, Z.-C. Gu, and X.-G. Wen, PRB **83**, 035107 (2011)

$$|\Psi\rangle \quad \sim \quad |\psi\rangle^{\otimes L}$$

Cohomology classes for MPUs

- Analogy to MPSs





$$\sum_{i',j'} [\rho_g]_{ii'} [\rho_g^*]_{jj'} \mathcal{U}_{i'j'} = z_g^\dagger \mathcal{U}_{ij} z_g$$

$$z_g z_h = \omega_{g,h} z_{gh}$$

$$[\omega] \in H^2(G, \mathrm{U}(1))$$

Cohomology classes for MPUs

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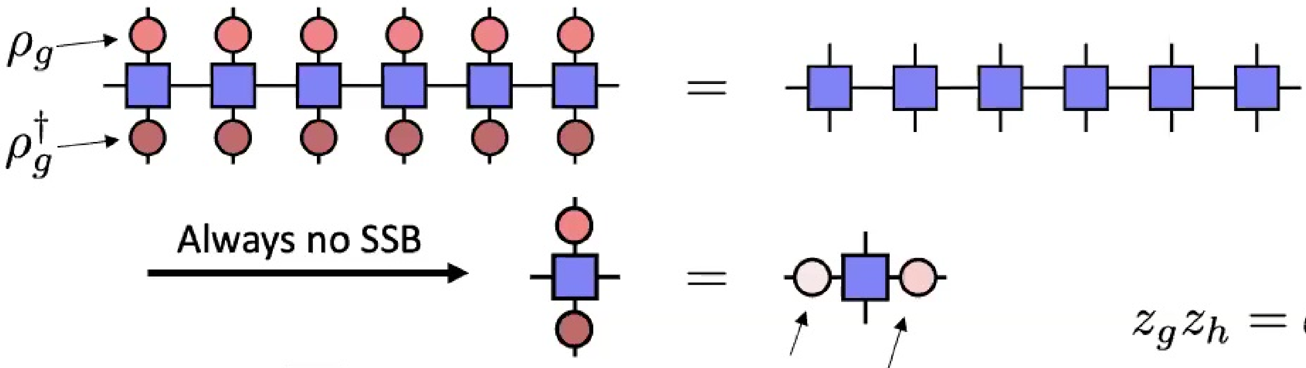

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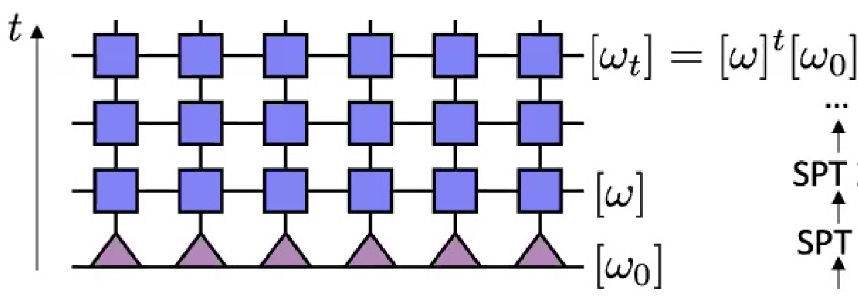
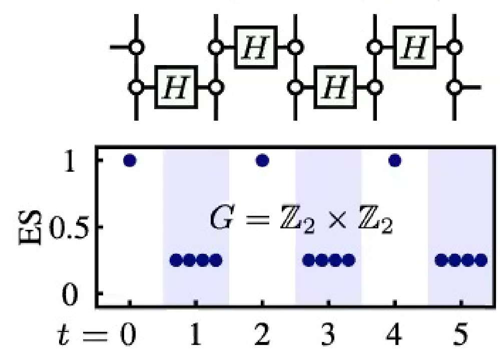
$$[\omega] \in H^2(G, \mathrm{U}(1))$$

- SPT discrete time-crystalline oscillation

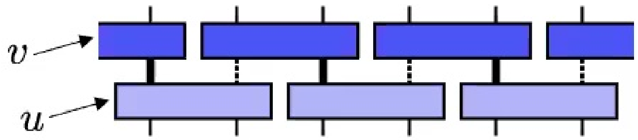
A. C. Potter and T. Morimoto, PRB **95**, 155126 (2017)

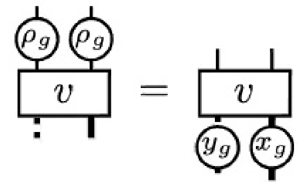


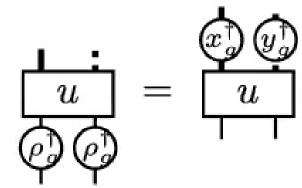


Beyond cohomology

- Symmetry action on the bilayer-unitary form





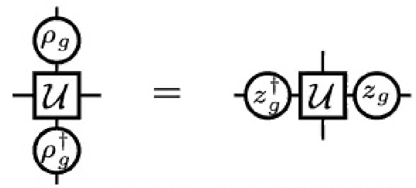


Gauge transformation: J. I. Cirac *et al.*, JSM (2017) 083105

$$v \to v(y \otimes x)$$
$$u \to (x^\dagger \otimes y^\dagger)u$$

$$x_g x_h = \omega_{g,h} x_{gh}$$
$$y_g y_h = \omega_{g,h}^{-1} y_{gh}$$

cf. $z_g z_h = \omega_{g,h} z_{gh}$



- Symmetry-protected index (SPI)

cf. chiral index: $\mathrm{ind} = \frac{1}{2}\log\frac{r}{l} = \log\frac{r}{d}$

$$= \frac{1}{2}\log\frac{\mathrm{Tr}\ y_e}{\mathrm{Tr}\ x_e} = \log\frac{\mathrm{Tr}\ y_e}{\mathrm{Tr}\ \rho_e}$$

Cyclotomic field:

$$\mathbb{Q}(\omega_n \equiv e^{\frac{2\pi i}{n}}) \equiv \{\textstyle\sum_{j=0}^{n-1} q_j \omega_n^j : q_j \in \mathbb{Q}\}$$

If $\chi_g \equiv \mathrm{Tr}\ \rho_g \neq 0$, $\mathrm{ind}_g = \frac{1}{2}\log\left|\frac{\mathrm{Tr}\ y_g}{\mathrm{Tr}\ x_g}\right| = \log\left|\frac{\mathrm{Tr}\ y_g}{\mathrm{Tr}\ \rho_g}\right| \in \log|\mathbb{Q}(\omega_{d_g})/\{0\}|$

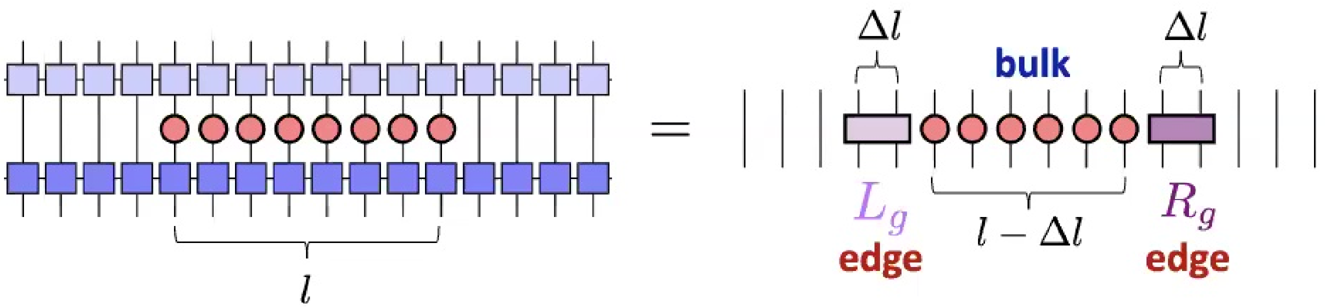
$$d_g \equiv \min\{d \in \mathbb{Z}^+ : g^d = e\}$$

Interpreting the symmetry-protected index

- **Evolved symmetry-string operator**

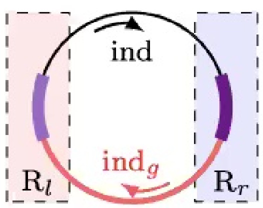
  cf. edge modes in equilibrium SPT phases

$$U^\dagger \rho_g^{\otimes l} U = L_g \otimes \rho_g^{\otimes (l-\Delta l)} \otimes R_g$$



$$\mathrm{ind}_g - \mathrm{ind} = \frac{1}{2}\log\left|\frac{\mathrm{Tr}\ L_g}{\mathrm{Tr}\ R_g}\right|$$ – ***Measure of asymmetry***

- Symmetry-charge-pump picture



State-like vs. Genuinely dynamical

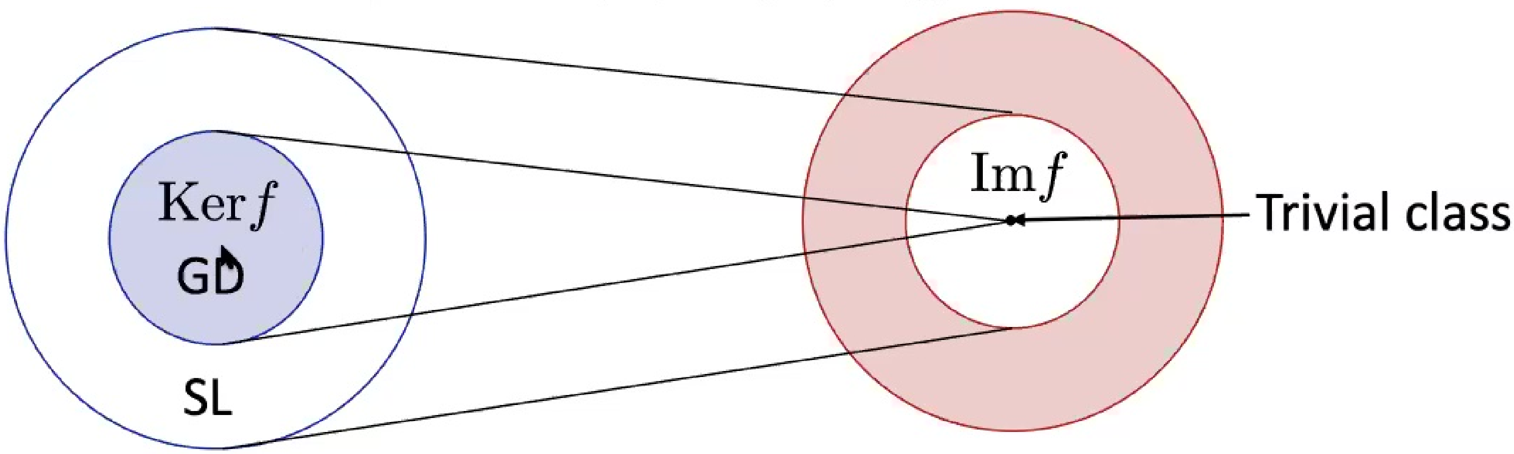
- Two types of topological invariants

Cohomology class: trivial -> SPT – ***state-like (SL)***

SPI / index: trivial -> trivial – ***genuinely dynamical (GD)***

- Homomorphism from QCA to short-range states

$$f : K_U \to K_\Psi, [U] \to [U|\Psi_0\rangle]$$



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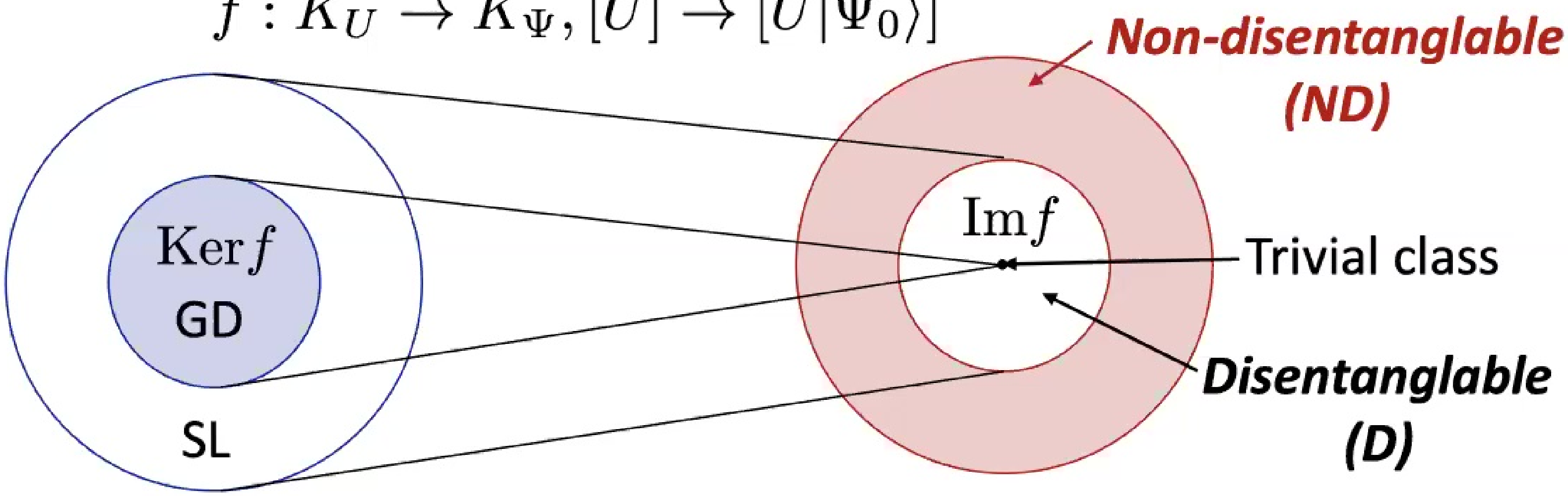
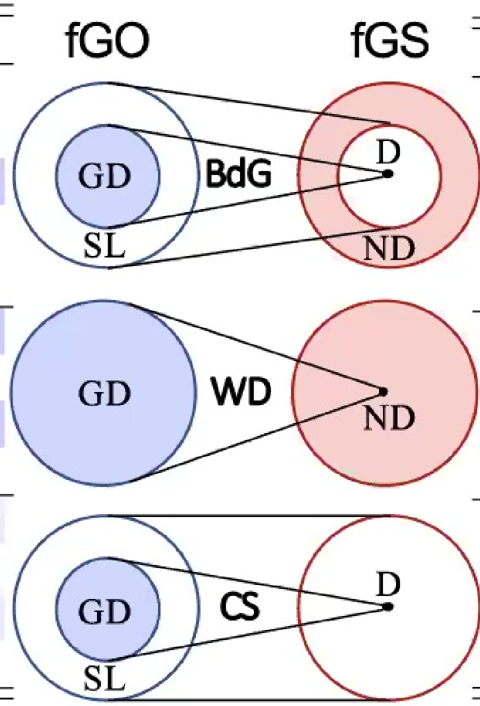


Refining the periodic table

- Map from fermionic Gaussian QCA to fermionic Gaussian states with fundamental symmetries

Cf. A. Schnyder *et al.*, PRB **78**, 195125 (2008)

d	0	1	2	3
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
DIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$
C	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
CI	0	$\mathbb{Z}$	0	$\mathbb{Z}$
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
AII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
AIII	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
BDI	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0
CII	0	$2\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$



0	1	2	3
$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
0	0	$2\mathbb{Z}$	0
0	0	0	$2\mathbb{Z}$
$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\mathbb{Z}$	0	0	0
$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$

ZG and T. Guaita, arXiv: 2106.05044

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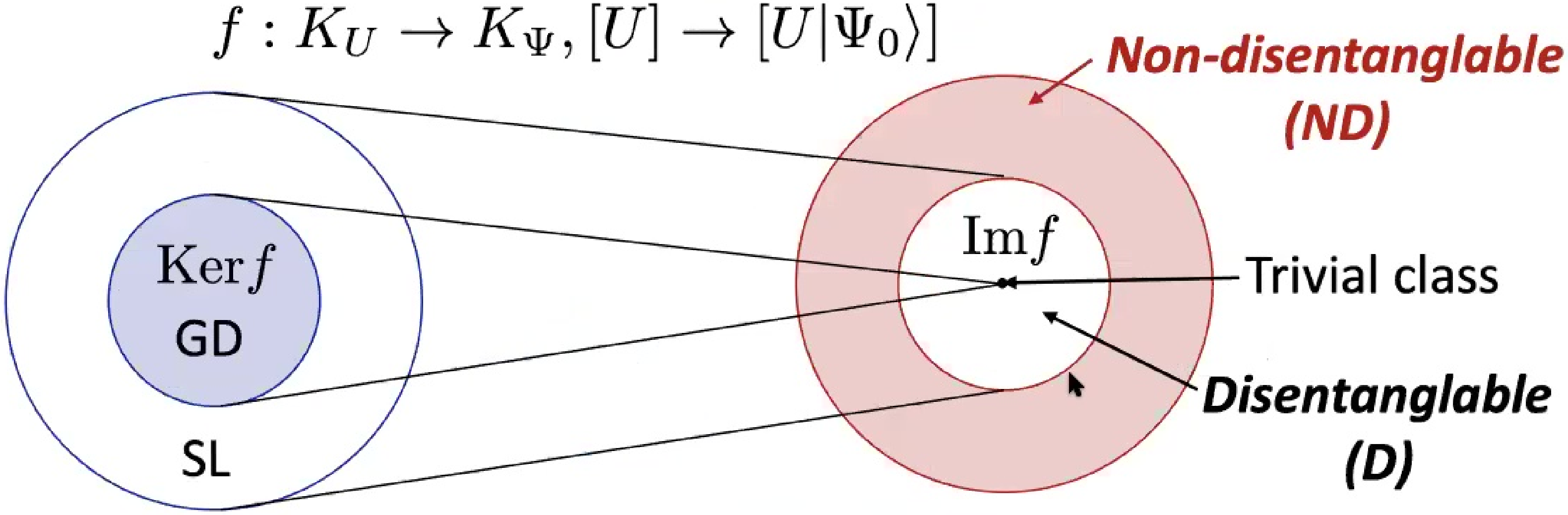

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Cf. A. Schnyder *et al.*, PRB **78**, 195125 (2008)

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CI	0	$\mathbb{Z}$	0	$\mathbb{Z}$
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AIII	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
BDI	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0
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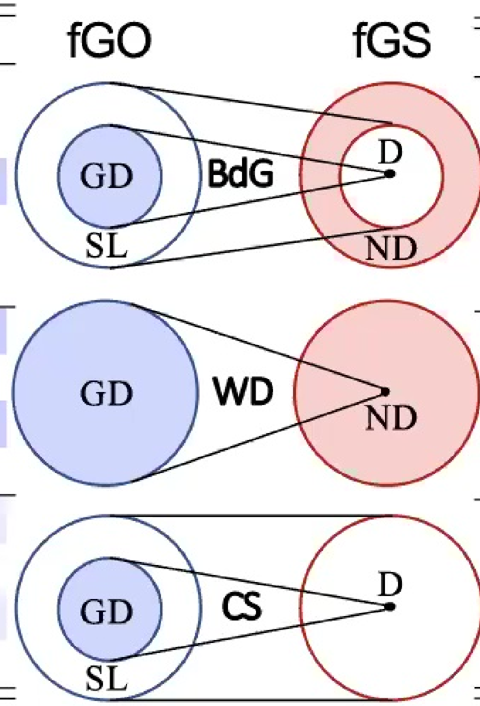
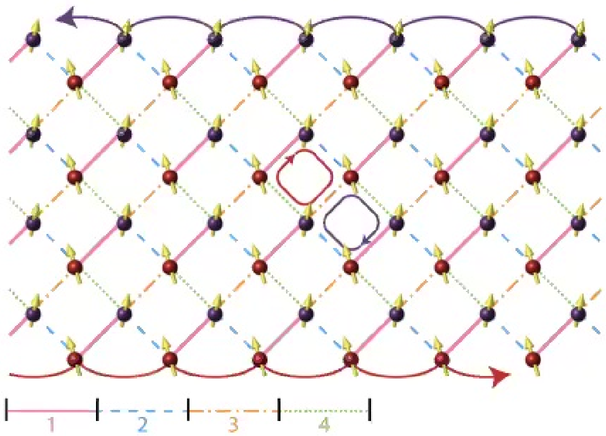
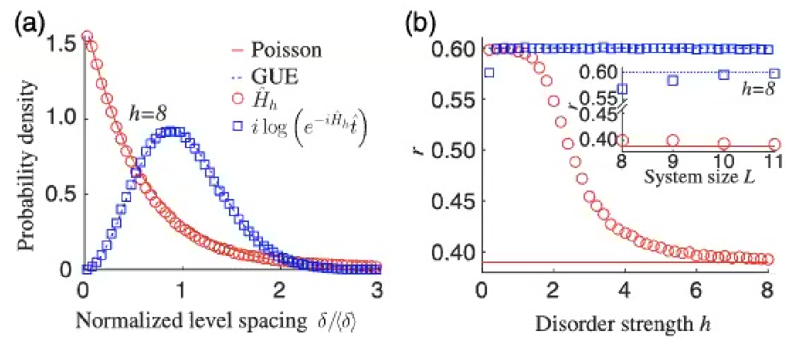


0	1	2	3
$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
0	0	$2\mathbb{Z}$	0
0	0	0	$2\mathbb{Z}$
$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\mathbb{Z}$	0	0	0
$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$

ZG and T. Guaita, arXiv: 2106.05044

Motivation





H. C. Po *et al.*, PRX **6**, 041070 (2016)

2D Chiral Floquet MBL Phases 1D Edge cannot be MBL

Locality + MBL ⟶ QCA

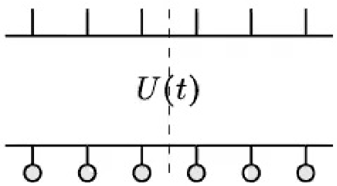
Question: Can we rigorously rule out MBL?

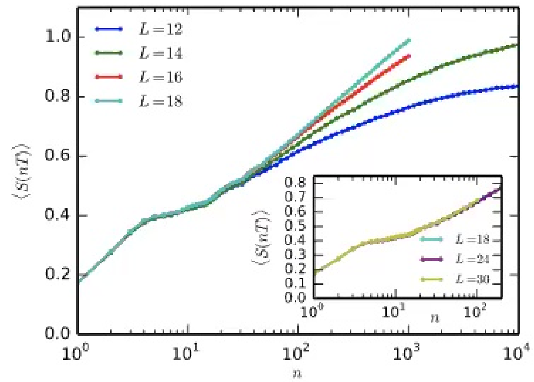
Entanglement growth

MBL

Translation

State entanglement



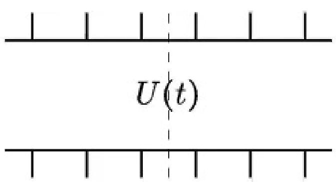


P. Ponte *et al.*, PRL **114**, 140401 (2015)

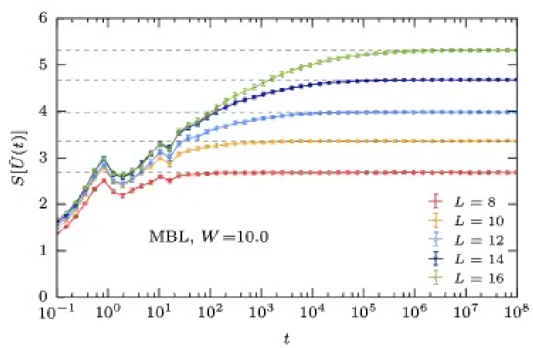
$$|\Psi_{\text{prod}}\rangle \to |\Psi'_{\text{prod}}\rangle$$

$$S(t) = 0$$

Operator entanglement



P. Zanardi, PRA **63**, 040304(R) (2001)



T. Zhou and D. J. Luitz, PRB **95**, 094206 (2017)

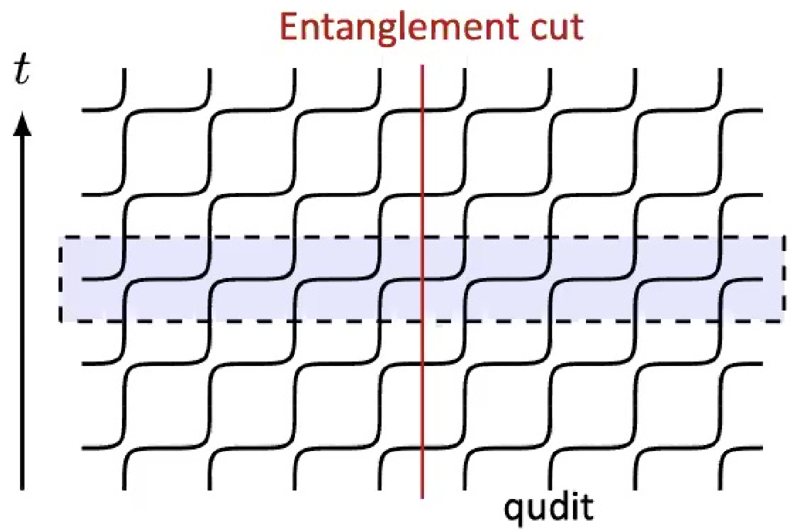
?

Conjecture (@2020 MPHQ interview)

- Idea:

$$\mathrm{ind} = \log d$$

$$S_{\mathrm{OEE}} = t \log d$$



Right translation

Not MBLizable

H. C. Po *et al*., PRX **6**, 041070 (2016)

Operator entanglement bounded by the chiral index?

- Impact: Rigorous result on topology & thermalization

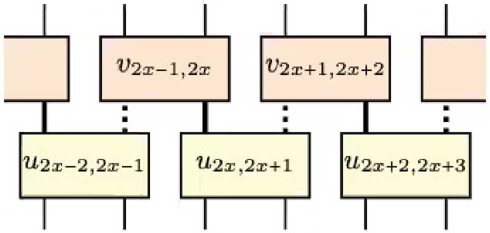
  Lower bound on chaos (cf. MSS bound)

J. Maldacena, S. H. Shenker, and D. Stanford, JHEP 2016, 106

Setup and the main result

- Index of a QCA

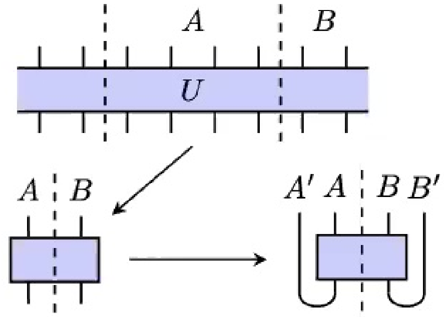
$$\text{ind} = \log \frac{d'_{2x}}{d_{2x}} = \log \frac{d_{2x+1}}{d'_{2x+1}} \in \log \mathbb{Q}^+$$



- Operator entanglement

$$|U\rangle \equiv (U \otimes \mathbb{I})|I\rangle, \quad |I\rangle \equiv d^{-N/2}(\textstyle\sum_{j=1}^{d} |jj\rangle)^{\otimes N}$$

$$S^{(\alpha)}_{AA'} \equiv \tfrac{1}{1-\alpha} \log \mathrm{Tr}\rho^{\alpha}_{AA'}$$



- Lower bound on chaos

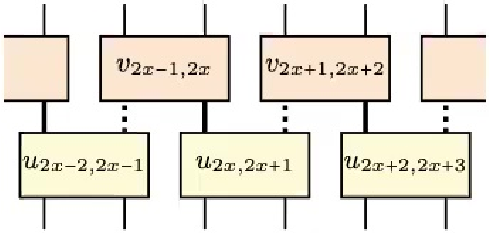
$$S^{(\alpha)}_{AA'} \geq 2|\text{ind}|$$

If $\min\{|A|, N-|A|\} \geq 2r$

Setup and the main result

- Index of a QCA

$$\text{ind} = \log \frac{d'_{2x}}{d_{2x}} = \log \frac{d_{2x+1}}{d'_{2x+1}} \in \log \mathbb{Q}^+$$



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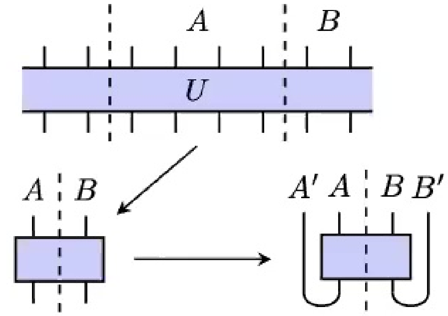
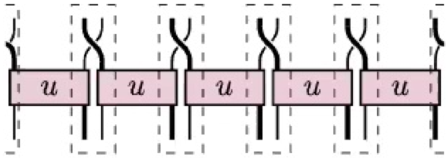


- Lower bound on chaos

$$S^{(\alpha)}_{AA'} \geq 2|\text{ind}|$$

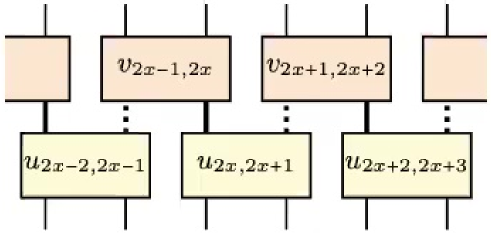
If $\min\{|A|, N - |A|\} \geq 2r$

Tight for
(i) $|\text{ind}| \in \log \mathbb{Z}^+$
(ii) $\alpha = \infty$



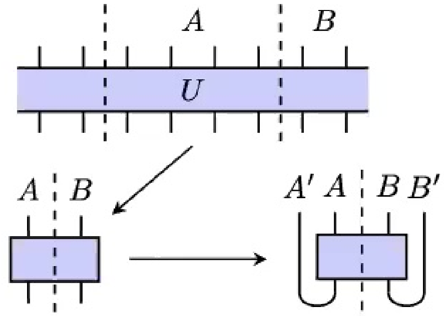
Setup and the main result

- Index of a QCA



$$\mathrm{ind} = \log \frac{d'_{2x}}{d_{2x}} = \log \frac{d_{2x+1}}{d'_{2x+1}} \in \log \mathbb{Q}^+$$

- Operator entanglement



$$|U\rangle \equiv (U \otimes \mathbb{I})|I\rangle, \quad |I\rangle \equiv d^{-N/2}(\textstyle\sum_{j=1}^{d} |jj\rangle)^{\otimes N}$$

$$S^{(\alpha)}_{AA'} \equiv \tfrac{1}{1-\alpha} \log \mathrm{Tr}\rho^{\alpha}_{AA'}$$

- Lower bound on chaos

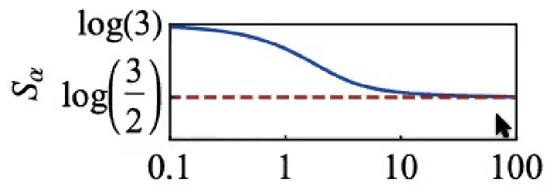
$$S^{(\alpha)}_{AA'} \geq 2|\mathrm{ind}|$$

If $\min\{|A|, N-|A|\} \geq 2r$

Tight for

(i) $|\mathrm{ind}| \in \log \mathbb{Z}^+$

(ii) $\alpha = \infty$



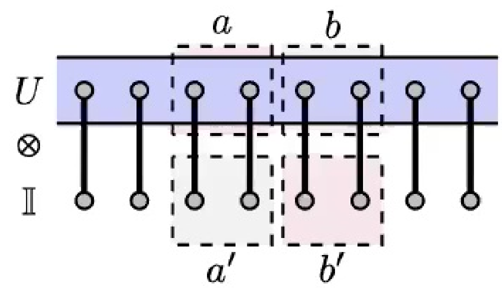
Proof of the main result

- **Entropy formula of the index**

$$\mathrm{ind} = \frac{1}{2}(S_{ab'}^{(\alpha)} - S_{a'b}^{(\alpha)})$$

$$S_{aba'b'} \geq |S_{ab'} - S_{a'b}| = 2|\mathrm{ind}|$$

$$\Rightarrow S_{aba'b'}^{(\alpha \leq 1)} \geq 2|\mathrm{ind}|$$

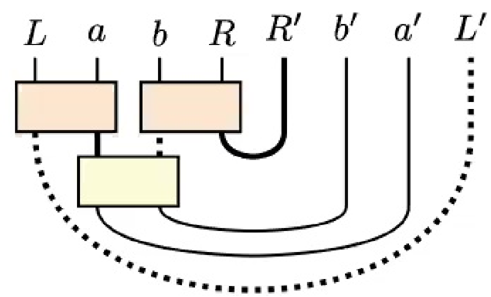


- **Weak subadditivity** W. van Dam and P. Hayden, arXiv:quant-ph/0204093

$$S_{aba'b'}^{(\alpha)} = S_{LL'}^{(\alpha)} + S_{RR'}^{(\alpha)}$$

$$S_{LL'}^{(\alpha)} \geq \max\{S_L^{(\alpha)} - S_{L'}^{(0)}, S_{L'}^{(\alpha)} - S_L^{(0)}\} = |\mathrm{ind}|$$

$$S_{RR'}^{(\alpha)} \geq \max\{S_R^{(\alpha)} - S_{R'}^{(0)}, S_{R'}^{(\alpha)} - S_R^{(0)}\} = |\mathrm{ind}|$$



Stability against exponential tails

$$U = \hat{\mathrm{T}} e^{-i \int_0^T dt \sum_j h_j(t)} U_{\mathrm{QCA}}$$

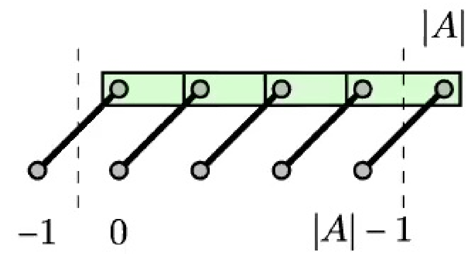
Local and bounded $h \equiv \max_{j,t} \|h_j(t)\|$

- Example of violating $S^{(\alpha)} \geq 2\,\mathrm{ind}$

$$H(t) = h\mathbb{S}^{[j_t, j_t+1]}, \quad j_t = \lfloor t|A|/T \rfloor$$

$$S^{(\infty)} = 2\log d - \log[1 + (d^2 - 1)\epsilon]$$

$$\epsilon = \sin^{2|A|}(hT/|A|) \sim e^{-\mathcal{O}(|A| \log |A|)}$$



- General proof for $S^{(\alpha)} > 2\,\mathrm{ind} - e^{-\mathcal{O}(|A| \log |A|)}$

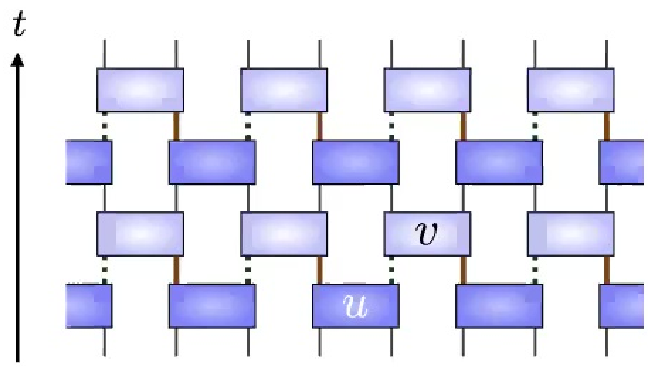
Step 1 – Approximate Hamiltonian evolution by quantum circuit

T. J. Osborne, PRL **97**, 157202 (2006)

Step 2 – Optimizing the (time-dependent) Lieb-Robinson bound

M. B. Hastings, arXiv:1008.5137; ZG *et al.*, PRA 101, 052122 (2020)

Exactly solvable chaotic anomalous dynamics: Random QCA



i.i.d Haar random

$$u : \mathbb{C}^d \otimes \mathbb{C}^d \to \mathbb{C}^p \otimes \mathbb{C}^q$$
$$v : \mathbb{C}^q \otimes \mathbb{C}^p \to \mathbb{C}^d \otimes \mathbb{C}^d$$
$$\Rightarrow \text{ind} = \frac{1}{2} \log \frac{q}{p}$$

Cf. Random quantum circuits ($p = q = d$)

$$v_B = \frac{d^2-1}{d^2+1}$$

$$v_E = \log_d(\frac{d^2+1}{2d})$$

A. Nahum, S. Vijay, and J. Haah, PRX **8**, 021014 (2018)

Butterfly velocity:

$$v_L = \frac{p^2q^2-q(p+q)+1}{p^2q^2-1}, v_R = \frac{p^2q^2-p(p+q)+1}{p^2q^2-1}$$

State- & Operator-entanglement velcotiy:

$$v_E = \log_{d^2}[\frac{(pq+1)^2}{2d(p+q)}], v_E^{(\mathrm{o})} = \log_{d^2}[\frac{(pq+1)^2(pq-1)}{d(\sqrt{p(q^2-1)}+\sqrt{q(p^2-1)})^2}]$$

Tripartite-information velocity: $v_{\text{tri}} = v_E^{(\mathrm{o})} - \frac{|\text{ind}|}{\log(pq)}$

ZG, A. Nahum, and L. Piroli, arXiv:2109.07408

Entanglement-membrane theory

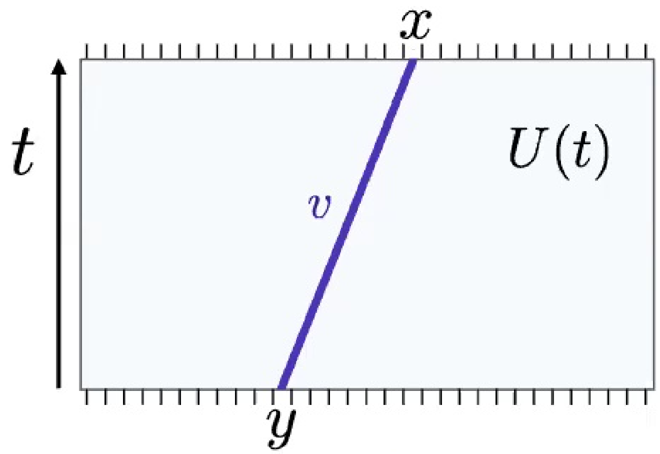
- Hydrodynamic equation (zero index)

$$\frac{\partial S}{\partial t} = s_{\text{eq}}\Gamma\left(\frac{\partial S}{\partial x}\right)$$

$S(x,t)$: Entanglement entropy of subsystem $(-\infty, x]$ at time t

Formal solution:

$$S(x,t) = \min_y \left(t s_{\text{eq}}\mathcal{E}\left(\frac{x-y}{t}\right) + S(y,0)\right) \qquad \mathcal{E}(v) = \max_s \left(\Gamma(s) + \frac{vs}{s_{\text{eq}}}\right)$$



C. Jonay, D. A. Huse, and A. Nahum, arXiv:1803.00089

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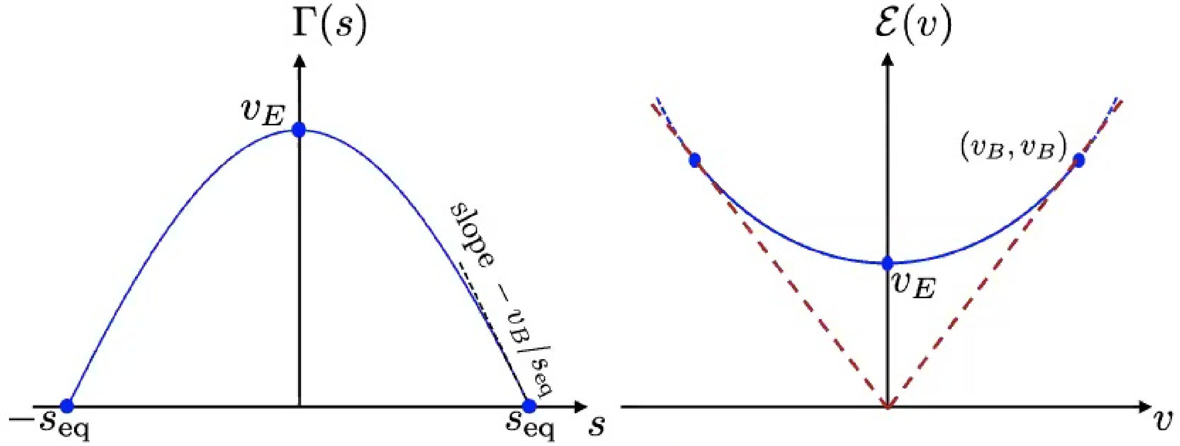


C. Jonay, D. A. Huse, and A. Nahum, arXiv:1803.00089

Entanglement-membrane theory

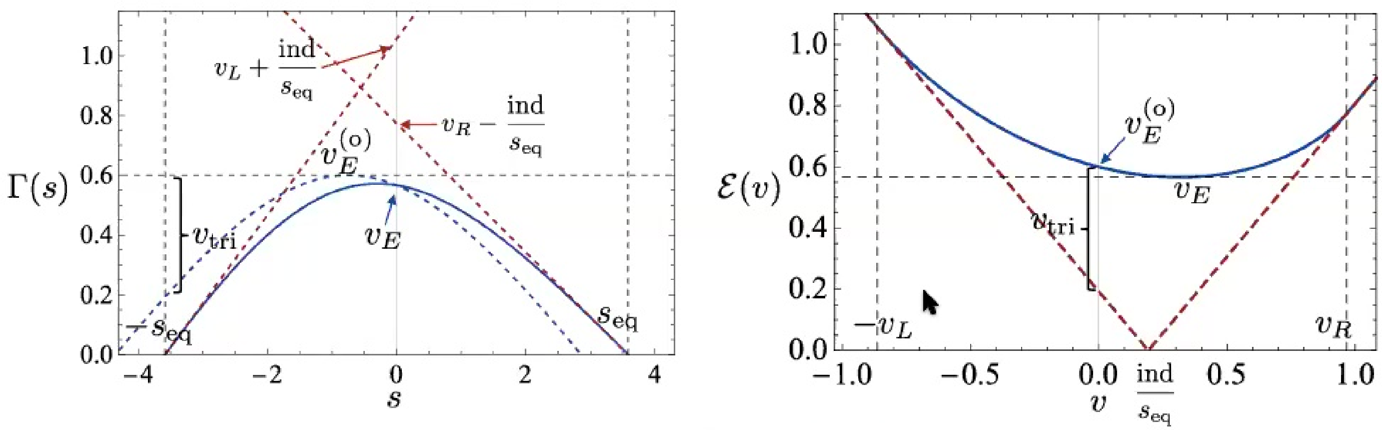
- Hydrodynamic equation (nonzero index)

$$\frac{\partial S}{\partial t} + \boxed{\frac{\text{ind}}{s_{\text{eq}}} \frac{\partial S}{\partial x}} = s_{\text{eq}} \Gamma \left(\frac{\partial S}{\partial x} \right)$$

Index appears as a background velocity

Formal solution:

$$S(x,t) = \min_y \left(t s_{\text{eq}} \mathcal{E} \left(\frac{x-y}{t} \right) + S(y,0) \right) \qquad \mathcal{E}(v) = \max_s \left(\Gamma(s) + \frac{vs}{s_{\text{eq}}} - \boxed{\frac{\text{ind}}{s_{\text{eq}}} \frac{s}{s_{\text{eq}}}} \right)$$



ZG, A. Nahum, and L. Piroli, arXiv:2109.07408

Summary

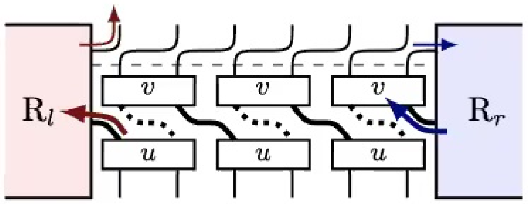
- Classification of symmetric 1D QCA

Cohomology + SPI

State-like vs. genuinely dynamical

arXiv:2106.05044

Part I



PRL **124**, 100402 (2020)

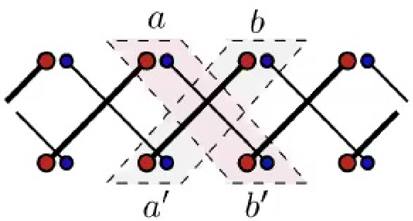
- Impact on entanglement dynamics

$$S^{(\alpha)} \geq 2|\mathrm{ind}|$$

Generalized entanglement-membrane theory

arXiv:2109.07408

Part II



PRL **126**, 160601 (2021)

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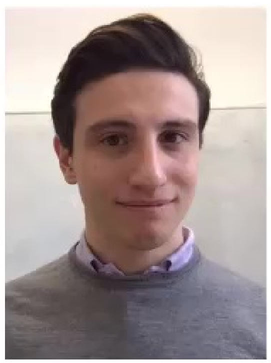
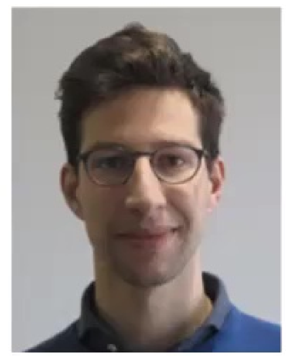


Lorenzo

Christoph



Tommaso

Outlook

Part I

- Complete classification in 1D
- Nontrivial QCA in higher dimensions Cf. J. Haah, L. Fidkowski, and M. B. Hastings, arXiv:1812.01625
- Homomorphism for interacting systems Cf. T. D. Ellison and L. Fidkowski, PRX **9**, 011016 (2019)

Part II

- Tighter bound for finite α
- Impact of SPI
- Interplay between (genuinely dynaimcal) topology and conservation law or/and measurement

Cf. T. Rakovszky, F. Pollmann, and C. W. von Keyserlingk, PRX **8**, 031058 (2018); A. Lavasani, Y. Alavirad, and M. Barkeshli, Nat. Phys. **17**, 342 (2021)