Title: Twistors, integrability, and 4d Chern-Simons theory

Speakers: Roland Bittleston

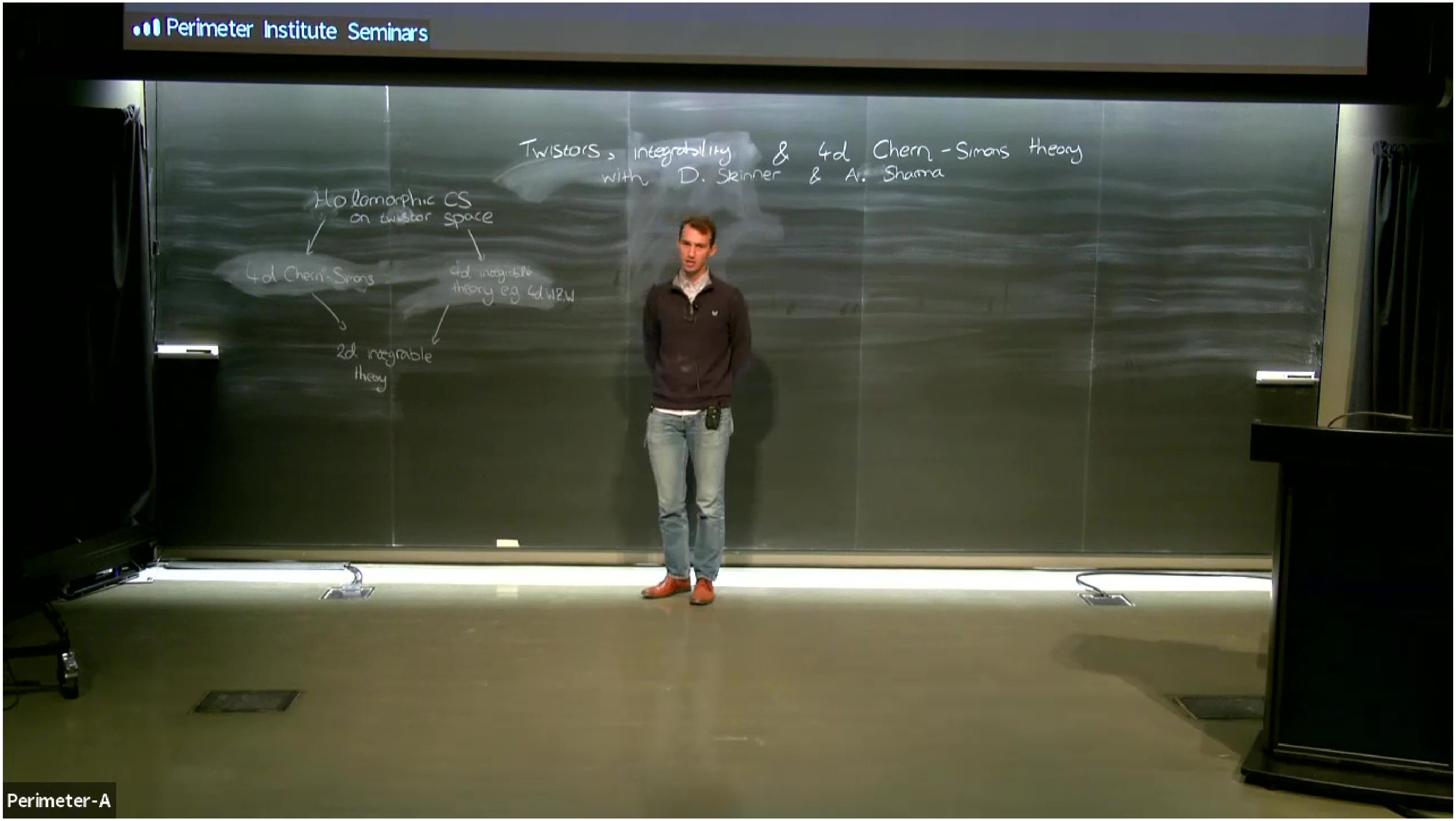
Series: Mathematical Physics

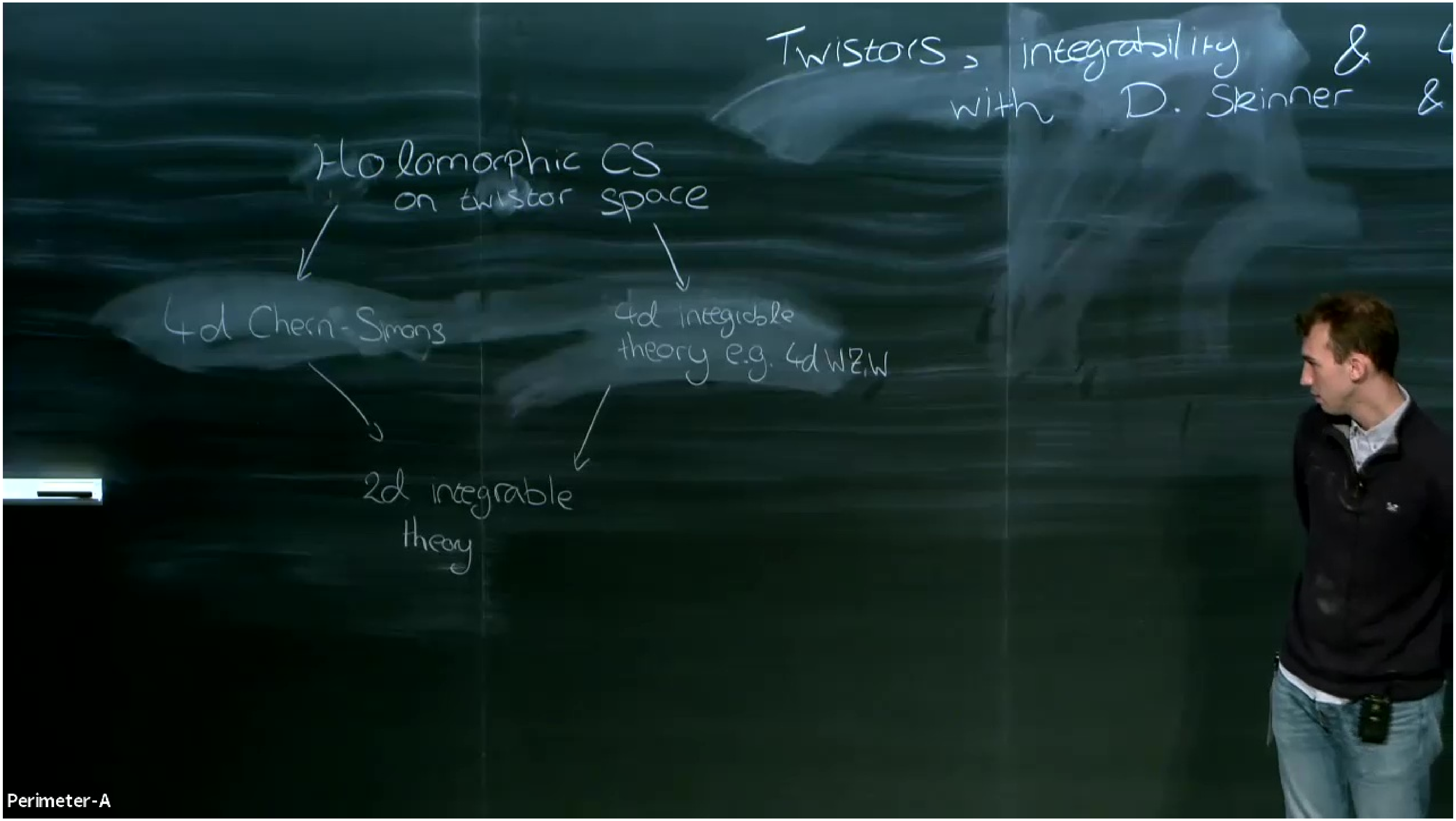
Date: October 29, 2021 - 1:30 PM

URL: https://pirsa.org/21100050

Abstract: I will connect approaches to classical integrable systems via 4d Chern-Simons theory and via symmetry reductions of the anti-self-dual Yang-Mills equations. In particular, I will consider holomorphic Chern-Simons theory on twistor space, defined using a range of meromorphic (3,0)-forms. On shell these are, in most cases, found to agree with actions for anti-self-dual Yang-Mills theory on space-time. Under symmetry reduction, these space-time actions yield actions for 2d integrable systems. On the other hand, performing the symmetry reduction directly on twistor space reduces the holomorphic Chern-Simons action to 4d Chern-Simons theory.

Zoom Link: https://pitp.zoom.us/j/99193672959?pwd=RUJ3N3h2V3RFK3ZNVVVCK1E3bXJ2Zz09


Perimeter Institute Seminars
Twistors, integrability & 4d Chern-Simons theory
with D. Skinner & A. Sharma
Holomorphic CS
on twistor space
4d Chern-Simons
4d integrable
theory eg 4d WZW
2d integrable
theory
Perimeter-A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integrability & 4d Chern-Simons theory

with D. Skinner & A. Sharma

$$\mathcal{V} = \Sigma_1 \times \mathbb{CP}^1$$

$$\omega = \frac{(z-z_1)(z-z_2)}{z^2}\, dz$$

$$S[A] \propto \int_{\mathcal{V}} \omega \wedge CS A$$

$$A^{1,0}_{\Sigma_1} \sim \frac{1}{z-z_1}, \quad A^{0,1}_{\Sigma_1} \sim \frac{1}{z-z_2}$$

$$S_{PCM}[\sigma] = \frac{z_1 - z_2}{i z} \int \mathrm{tr}\, J \wedge * J + \frac{z_1 + z_2}{3} \int_{\Sigma_1 \times [0,1]} \mathrm{tr}\, \tilde{J}^3$$

$$A = A_{\Sigma_1} + \bar{A}_{\mathbb{CP}^1}$$

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$M = \mathbb{R}^4$, $A \in \Omega^1(M, \mathfrak{g})$

$F = - * F$

* $\mathbb{R}$ → Bogomolny
* $\mathbb{R}^2$ → PCM
* $\mathbb{R}^2 \times S^1$ → finite Toda
* $\mathbb{R}^2 \times \mathbb{Z}_N$ → affine Toda

integrability & 4d Chern-Simons theory

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- $M = \mathbb{R}^4 \cong \mathbb{C}^2$

$$F^{2,0} = F^{0,2} = \kappa \wedge F^{1,1} = 0$$

$$A^{1,0} = 0 \qquad A^{0,1} = -\bar{\partial}\sigma\, \sigma^{-1}$$

$$\kappa \wedge \partial(\bar{\partial}\sigma\sigma^{-1}) = 0$$

$$S_{4dWZW} = \frac{1}{2}\int_M \mathrm{tr}\, J \wedge * J + \frac{1}{3}\int_{M\times[0,1]} \kappa\, \mathrm{tr}\, \tilde{J}^3$$

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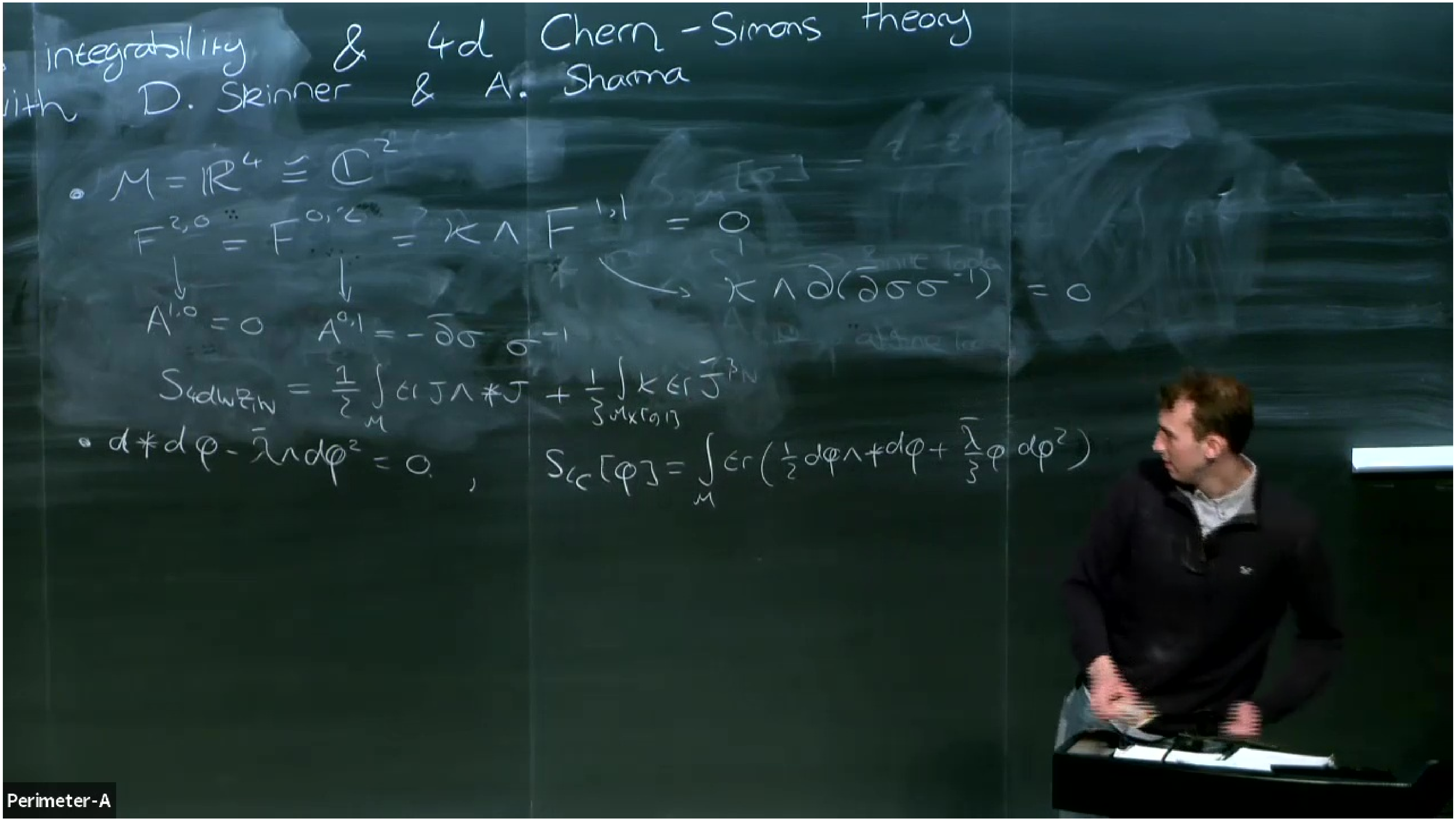
integrability & 4d Chern - Simons theory
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• $M = \mathbb{R}^4 \cong \mathbb{C}^2$
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$S_{4dWZW} = \frac{1}{2}\int_M \mathrm{tr}\, J \wedge * J + \frac{1}{3}\int_{M\times[0,1]} \kappa\, \mathrm{tr}\, \tilde{J}^3$
• $d * d\varphi - \bar\lambda \wedge d\varphi^2 = 0$, $S_{C}[\varphi] = \int_M \mathrm{tr}\left(\frac{1}{2} d\varphi \wedge * d\varphi + \frac{\bar\lambda}{3}\varphi\, d\varphi^2\right)$
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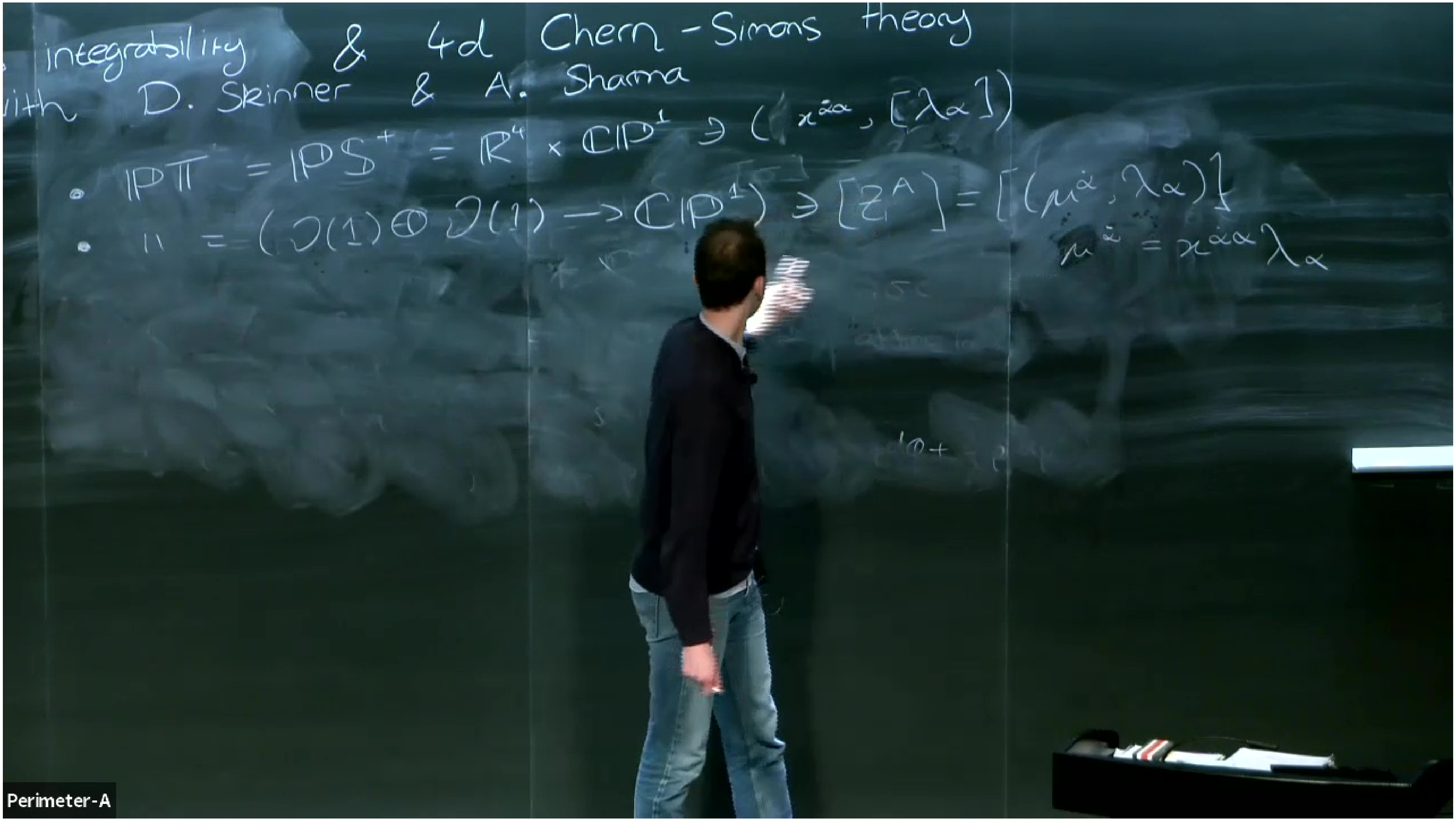
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• $\mathbb{PT} = \mathbb{PS}^+ = \mathbb{R}^4 \times \mathbb{CP}^1 \ni (x^{\dot{\alpha}\alpha}, [\lambda_\alpha])$
• $" = (\mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1) \ni [Z^A] = [(\mu^{\dot{\alpha}}, \lambda_\alpha)]$
$\mu^{\dot{\alpha}} = x^{\dot{\alpha}\alpha} \lambda_\alpha$
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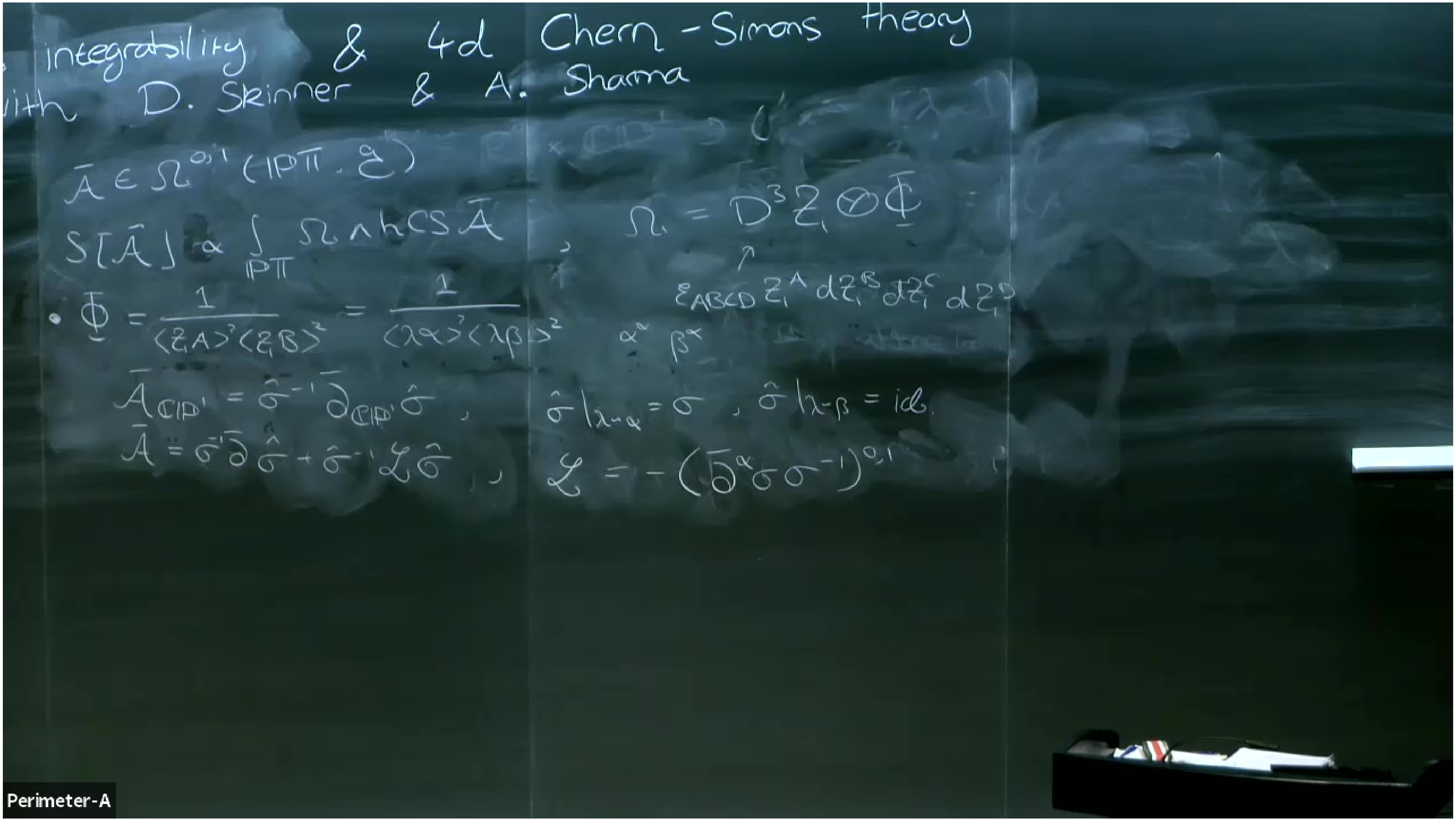

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$\bar{A} \in \Omega^{0,1}(\mathbb{PT}, \mathfrak{g})$
$S[\bar{A}] \propto \int_{\mathbb{PT}} \Omega \wedge hCS\,\bar{A}$, $\Omega = D^3Z \otimes \Phi$
$\varepsilon_{ABCD} Z^A dZ^B dZ^C dZ^D$
$\Phi = \frac{1}{\langle Z A\rangle^2 \langle Z B\rangle^2} = \frac{1}{\langle\lambda\alpha\rangle^2\langle\lambda\beta\rangle^2}$ α^α β^α
$\bar{A}_{\mathbb{CP}^1} = \hat\sigma^{-1}\bar\partial_{\mathbb{CP}^1}\hat\sigma$, $\hat\sigma|_{\lambda=\alpha} = \sigma$, $\hat\sigma|_{\lambda=\beta} = id$
$\bar{A} = \hat\sigma^{-1}\bar\partial\hat\sigma + \hat\sigma^{-1}\mathcal{L}\hat\sigma$, $\mathcal{L} = -(\bar\partial^\alpha\sigma\sigma^{-1})^{0,1}$
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- $\Phi = \frac{1}{\langle \xi, A\rangle^4} \longrightarrow$ light cone action
- $S = \int \Omega_h \wedge \left(h \bar{\partial} h + \frac{h}{3}\{h,h\}\right) \longleftarrow \frac{1}{2}\partial_{\mu\dot\alpha} \wedge \partial_{\mu\dot\alpha}$

  $h \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))$
- $\frac{1}{\langle \xi, A\rangle^4 \langle \xi, B\rangle^4} \longrightarrow 1^{st}$ Plebanski / heavenly eq^n
- $\frac{1}{\langle \xi, A\rangle^8}$ $\quad 2^{nd}$

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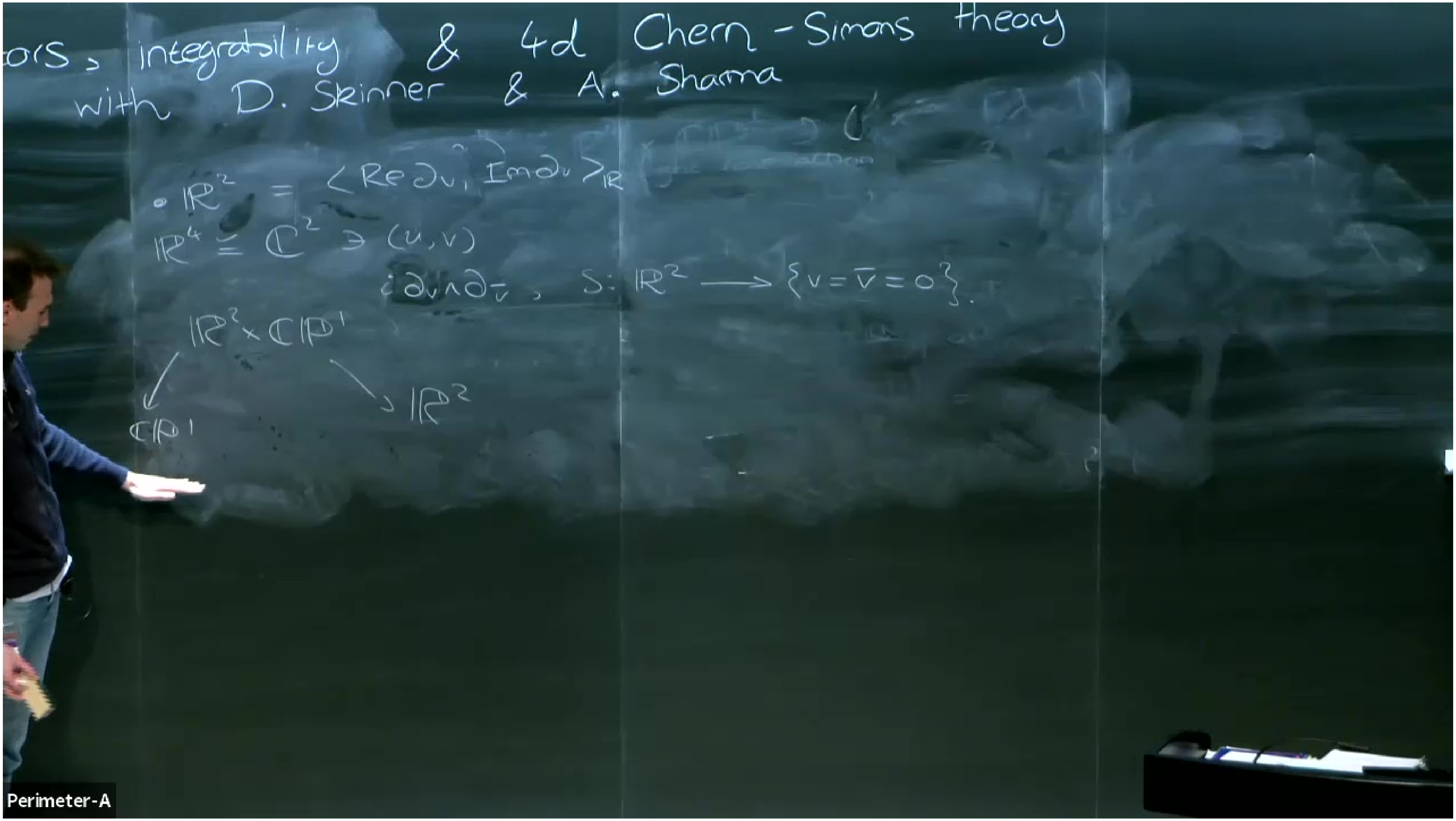
• $\mathbb{R}^2 = \langle \mathrm{Re}\,\partial_v, \mathrm{Im}\,\partial_v \rangle_{\mathbb{R}}$

$\mathbb{R}^4 \cong \mathbb{C}^2 \ni (u,v)$

$\gamma^\alpha \qquad i\,\partial_v \wedge \partial_{\bar v}\,, \quad S: \mathbb{R}^2 \longrightarrow \{v = \bar v = 0\}.$

W

$i\,\partial_v \wedge \partial_{\bar v} \lrcorner\, \chi_\alpha = \frac{i(z_1+z_2)}{z_1-z_2}$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$\mathbb{R}^2 \times \mathbb{CP}^1$
$\mathbb{CP}^1$
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$$\omega = \iota \lrcorner \Omega = i\frac{(z-z_1)(z-z_2)}{(z_1-z_2)z^2}\,dz$$

$$A = S^*\bar A - \frac{z-z_2}{z-z_1}\,du\,S^*\iota_{\partial_v}\bar A + \frac{z-z_1}{z-z_2}\,d\bar u\,S^*\iota_{\partial_{\bar v}}\bar A$$

W

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