Title: Aspects of ghost-free nonlocal field theories

Speakers: Luca Buoninfante

Series: Quantum Gravity

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Abstract: In this talk I introduce nonlocal (infinite derivative) field theories. First of all, I discuss how and which principles of quantum field theory are affected when higher-order derivative operators are taken into account in a Lagrangian. In particular, I focus on the issue of unitarity and on how to make higher-derivative theories healthy by means of non-polynomial differential operators. I extend the treatment to the gravity sector and consider nonlocal theories whose graviton propagators are ghost-free, and explore the possibility of regularizing singularities. Next, I discuss some recent progress in proving perturbative unitarity for a very general class of nonlocal field theories. Finally, I will make some remarks on nonlocality and quantum gravity.

Aspects of ghost-free nonlocal field theories

Luca Buoninfante

In collaboration with
A.S. Koshelev, G. Lambiase, J. Marto,
A. Mazumdar, M. Yamaguchi,...

Quantum Gravity Seminar
Perimeter Institute @Zoom
28th October 2021

東京工業大学
Tokyo Institute of Technology

JSPS

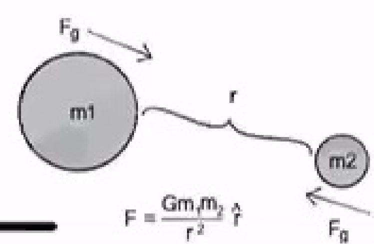
Introduction & motivations

- Einstein's general relativity (GR) has been tested to very high precision in the IR regime;

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\mathcal{R}$$

- Despite the great succes of GR, there are still open problems that make the theory incomplete in the UV regime, e.g. Blackhole and Cosmological singularities, non-renormalizability ['t hooft & Veltmann (1974); Goroff & Sagnotti (1985)]

- **To what extent is GR valid in the UV?**
- The inverse-square law of Newton's potential has been tested **only** up to $\sim 10\,\mu m$ with torsion balance experiments.



4th order gravity

- The 4th order gravitational action quadratic in the curvature is power-counting renormalizable:

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(\mathcal{R} + \alpha\mathcal{R}^2 + \beta\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\right)$$

- Unitarity is violated at the tree-level:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G}\,h_{\mu\nu}$$

$$\Pi(k) = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}^0}{k^2+m_0^2} - \frac{\mathcal{P}^2}{k^2+m_2^2},$$

$$m_0 := (3\alpha+\beta)^{-1/2}$$
$$m_2 := \left(-\tfrac{1}{2}\beta\right)^{-1/2}$$

Spin-2 ghost degree of freedom

- **Conflict: Unitarity VS Renormalizability!**

[Stelle, 1977, PRD]

4th order gravity

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$\beta = 0$

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Spin-2 ghost degree of freedom

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[Stelle, 1977, PRD]

Unitarity VS Renormalizability

- Einstein's GR is **unitary but non-renormalizable**, while 4th order quadratic gravity is power-counting **renormalizable but non-unitary!**

Several (recent) attempts:

- Asymptotically safe gravity [Reuter, Eichhorn, Saueressig, Platania, Knorr......]
- 4th order gravity with Fakeons [Anselmi & Piva 2017+]
- 4th order gravity with unstable ghosts [Donoghue, Menezes, Salvio, Strumia...]
- Lee-Wick gravity theories [Modesto & Shapiro 2016+; Anselmi & Piva 2017+]
- **Nonlocal gravity theories** [Born, Pais, Yukawa, Efimov, Krasnikov, Kuz'min, Moffat, Woodard, Tomboulis, Dragovich, Aref'eva, Volovich, Koshelev, Siegel, Biswas, Mazumdar, Modesto, Frolov, Zelnikov, Rachwal, Starobinsky, Kumar, Tokareva, Boos,......]

Ghosts

- 4-derivative theory (-+++):

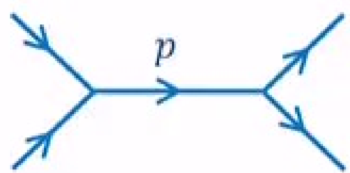
$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{m^2}\right)\phi - V(\phi) \quad \Longrightarrow \quad i\Pi(p) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}$$

GHOST!

- Optical theorem:

$$S^{+}S = 1, \qquad S = 1 + iT \qquad \Longrightarrow \qquad 2Im\{T\} = T^{+}T \quad ("\geq 0")$$

- Tree-level amplitude:



$$Im\{T\} = \pi\theta(p^0)[\delta(p^2) - \delta(p^2 + m^2)]$$

It can be negative: **violation of unitarity!**

22:45 Thu Oct 28

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Beyond 4-derivative theories

- 4-derivative theory (-+++):

GHOST!

$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{m^2}\right)\phi - V(\phi) \Longrightarrow \quad i\Pi(p) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}$$

- Generalized higher-derivative theory:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)(\Box - m^2)\phi \quad \Longrightarrow \quad i\Pi(p) = \frac{1}{F(-p^2)}\frac{1}{p^2 + m^2}$$

- **Question:** Is there any higher-derivative operator $F(-p^2)$ such that the propagator is ghost-free? **YES!**

- **Nonlocality can help us!**

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Local VS Nonlocal

- Local (polynomial) Lagrangians:

$n < \infty$

$$\mathcal{L}_L \equiv \mathcal{L}_L(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi)$$

- Nonlocal (non-polynomial) Lagrangians:

$$\mathcal{L}_{NL} \equiv \mathcal{L}_{NL}\left(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi, \dots, e^{\Box}\phi, \ln(\Box)\,\phi, \frac{1}{\Box}\phi, \dots\right)$$

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e.g. string field theory and p-adic string

[Witten, Freund, Zwiebach, Aref'eva, Volovich, Dragovich, Koshelev, Sen, Siegel,....]

22:47 Thu Oct 28

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Generalized higher-derivative Lagrangian

- Scalar field Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi F(\Box)\phi - V(\phi),$$

Entire function

(good IR limit $F(\Box) \to -\Box + m^2$)

- Weierstrass' theorem:

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,$$

- $\gamma(\Box)$ is another entire function.

- N is the number of zeroes m_i^2; r_i is the multiplicity of each zero

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- Weierstrass' theorem:

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,$$

- Propagator:

$$i\Pi(-p^2) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2} \prod_{i=2}^{N} \frac{1}{\left(p^2 + m_i^2\right)^{r_i}}$$

Generalized higher-derivative Lagrangians

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}$$

- $N = 1,\ r_i = 1,\ \gamma(\Box) \neq 0$

$\Longrightarrow$ infinite-derivative theory with one real zero

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2)$$

- Propagator:

$$i\Pi(p) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}$$

Nonlocal Lagrangian

- Nonlocal scalar field:

$$\mathcal{L} = \frac{1}{2}\phi e^{-\gamma(\Box/M_s^2)}(\Box - m^2)\phi - V(\phi),$$

$$\gamma(\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} \gamma_n \left(\frac{\Box}{M_s^2}\right)^n$$

Entire function: No extra poles!

- Ghost-free propagator:

$$\Pi(p) = \frac{e^{\gamma(-p^2/M_s^2)}}{p^2 + m^2}$$

Perturbative unitarity (optical theorem and Cutkosky rules)
[Pius & Sen 2015; Briscese & Modesto 2018; Chin & Tomboulis 2018]

Causality violation at microscopic scales (acausal Green functions and local commutativity violation)
[Tomboulis 2015; LB, Lambiase, Mazumdar 2018]

Generalized quadratic action

- Locality, causality, unitarity, renormalizability, positive norms, positive energies… too many requirements?
- Beyond fourth-order derivatives? Diffeomorphism invariance allows more…
- Generalized quadratic gravitational action, parity-invariant and torsion-free:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\,\sqrt{-g}\left(RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}F_3(\Box)R^{\mu\nu\rho\sigma}\right)$$

$$F_i(\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} f_{i,n}\left(\frac{\Box}{M_s^2}\right)^n, \qquad [N=\infty \;\; \textbf{Nonlocal}]$$

Nonlocal Lagrangian

- Nonlocal scalar field:

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Perturbative unitarity (optical theorem and Cutkosky rules)
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Causality violation at microscopic scales (acausal Green functions and local commutativity violation)
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Microcausality violation

- **NO time-ordered propagator:**

$$e^{-\gamma(\Box)}(\Box - m^2)\Pi(x) = i\delta^{(4)}(x),$$

$$\boxed{\Pi(x) = \Pi_c(x) + \Pi_{nc}(x),}$$

$$\Pi_c(x) = \langle T\{\phi(x)\phi(0)\}\rangle$$

$$\Pi_{nc}(x) = i\sum_{q=1}^{\infty}\frac{i^{q-1}}{q!}\partial_{x^0}^{(q-1)}[W^{(q)}(x) - W^{(q)}(-x)]$$

$$W^{(q)}(x) = \int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}\theta(k^0)\delta(k^2+m^2)\frac{\partial^{(q)}e^{-\gamma(-k^2)}}{\partial k^{0(q)}}$$

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$$F_i(\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} f_{i,n}\left(\frac{\Box}{M_s^2}\right)^n, \qquad [N=\infty \quad \textbf{Nonlocal}]$$

Generalized quadratic action

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- By using the identity

$$R_{\mu\nu\rho\sigma}\Box^n R^{\mu\nu\rho\sigma} = 4R_{\mu\nu}\Box^n R^{\mu\nu} - R\Box^n R + \mathcal{O}(R^3) + tot.div.$$

- We can write

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\sqrt{-g}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + \mathcal{O}(R^3)\right)$$

Generalized quadratic action

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$$\mathcal{O}(R^3) \sim \mathcal{O}(h^3)$$

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Generalized graviton propagator

- Up to quadratic order around Minkowski the relevant part of the action is:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\,\sqrt{-g}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}\right)$$

- Gauge independent part of the graviton propagator:

$$\boxed{\Pi_{\mu\nu\rho\sigma}(k) = \frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{f(k)k^2} + \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{(f(k)-3g(k))k^2}}$$

$$f(\Box) = 1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box,$$

$$g(\Box) = 1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$

Ghost-free higher derivative gravity

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- **Ghost-freeness condition:**

$$f(k) = e^{\gamma_1(k)}, \qquad f(k) - 3g(k) = e^{\gamma_2(k)}(k^2 + m_0^2),$$

Entire functions

[Biswas et al., PRL 2012]

Ghost-free higher derivative gravity

- Nonlocal higher-derivative gravity theories can be ghost-free; also known as Infinite Derivative Gravity (IDG)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(\mathcal{R} + G_{\mu\nu}F(\Box)\mathcal{R}^{\mu\nu}\right), \qquad \mathcal{F}_2(\Box) = -2\mathcal{F}_1(\Box) \equiv F(\Box) = \frac{f(\Box)-1}{\Box}$$

- Ghost-free propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{1}{f(k)}\Pi^{GR}_{\mu\nu\rho\sigma}(k) = \frac{1}{f(k)}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2} - \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2k^2}\right)$$

$$f(\Box) = e^{-\gamma(\Box/M_s^2)}$$

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- Non-local higher derivative theories can be ghost-free:

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- Ghost-free propagator:

$$\Pi_{\mu\nu\rho\sigma}(k)=e^{\gamma(-k^2/M_s^2)}\Pi^{GR}_{\mu\nu\rho\sigma}(k)=e^{\gamma(-k^2/M_s^2)}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2}-\frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2k^2}\right)$$

- Entire function, e.g.

$$\boxed{e^{-\gamma(\Box/M_s^2)}=e^{-\Box/M_s^2}}$$

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$$e^{-\gamma(\Box/M_s^2)} = e^{-\Box/M_s^2}$$

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$$\Pi_{\mu\nu\rho\sigma} = \frac{P^2_{\mu\nu\rho\sigma}}{k^2} - \frac{1}{2}\frac{P^0_{s,\mu\nu\rho\sigma}}{k^2}$$

$$+\alpha\left[P^1, P^0_s\right]$$

$p^2 \quad +2 \; +1 \; 0 \; -1 \; -2$

$$h_{\mu\nu} \in 2 \oplus 1 \oplus 0 \oplus 0$$

Ghost-free higher derivative gravity

- Linearized metric for a static point-like source:

$$ds^2 = -(1+2\phi(r))dt^2 + (1-2\phi(r))(dr^2 + r^2 d\Omega^2)$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2\phi(\vec{r}) = 4\pi G m\,\delta^{(3)}(\vec{r}) \Longrightarrow \phi(r) = -\frac{Gm}{r}Erf\left(\frac{M_s r}{2}\right)$$

IR

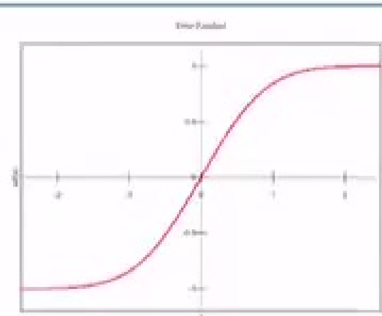
$$\phi(r) \sim -\frac{Gm}{r}$$

UV

$$\phi(r) \sim -\frac{GmM_s}{\sqrt{\pi}} < \infty$$

Singularity-free!

$$Erf(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$



[Tseytlin, 1995; Biswas et al., 2006; Biswas et al., 2012]

Ghost-free higher derivative gravity

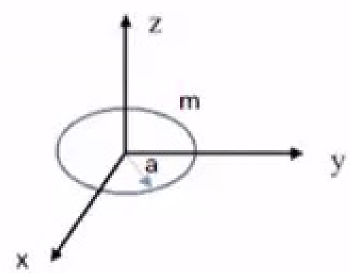
- Linearized metric for a rotating ring source in IDG:

$$ds^2 = -\big(1 + 2\phi(r)\big)dt^2 + 2\vec{h}\cdot d\vec{r}dt + \big(1 - 2\phi(r)\big)(dr^2 + r^2 d\Omega^2)$$

- Stress-energy tensor:

$$T_{00} = m\delta(z)\frac{\delta(x^2 + y^2 - a^2)}{\pi}, \quad T_{0i} = T_{00}v_i,$$

$$v_x = -y\omega, \;\; v_y = x\omega, \;\; v_z = 0$$



- Differential equations:

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2\phi(\vec{r}) = 4Gm\delta(z)\delta(x^2 + y^2 - a^2),$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2 h_{0x}(\vec{r}) = -16Gm\omega y\delta(z)\delta(x^2 + y^2 - a^2),$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2 h_{0y}(\vec{r}) = 16Gm\omega x\delta(z)\delta(x^2 + y^2 - a^2)$$

[LB, et al. PRD]

23:10 Thu Oct 28

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Enlarging the class of ghost-free operators

- Scalar field Lagrangian:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi),$$

Entire function

- Weierstrass theorem:

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} (\Box - m_i^2)^{r_i}$$

- $N = 1,\ r_i = 1$ (ghost-free):

$$F(\Box) = e^{-\gamma(\Box)}(\Box - m^2)$$

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- Weierstrass theorem:

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} (\Box - m_i^2)^{r_i}$$

- $N > 1$ (unhealthy real pole):

$$F(\Box) = e^{-\gamma(\Box)} (\Box - m_1^2)(\Box - m_2^2)$$

Ghost!

Enlarging the class of ghost-free operators

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}$$

- $N = 3,\ r_i = 1,\ \gamma(\Box) \neq 0$ [local case $\gamma(\Box) = 0$: Lee & Wick; Modesto & Shapiro 2016+; Anselmi & Piva 2017+]

$\Longrightarrow$ infinite-derivative theory with a pair of complex conjugate zeroes

$$F(\Box) = \frac{i}{M^4} e^{-\gamma(\Box)} (-\Box + m^2)(\Box + iM^2)(\Box - iM^2)$$
$$= e^{-\gamma(\Box)} (-\Box + m^2) \left(1 + \frac{\Box^2}{M^4}\right)$$

- Propagator:

$$i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4} \left[\frac{1}{p^2 + m^2 - i\epsilon} + \frac{p^2 + m^2}{p^4 + M^4} \right]$$

Nonlocal Lee-Wick

- Scalar field Lagrangian:

$$\mathcal{L} = \frac{1}{2}\phi e^{-\gamma(\Box)}(\Box - m^2)\left(1 + \frac{\Box^2}{M^4}\right)\phi - V(\phi),$$

- Propagator:

$$i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4}\left[\frac{1}{p^2+m^2} + \frac{p^2+m^2}{p^4+M^4}\right]$$

- Tree-level unitarity:

$$Im\left\{\frac{e^{\gamma(-p^2)}}{p^2+m^2-i\epsilon}\right\} + e^{\gamma(-p^2)} \underbrace{Im\left\{\frac{b}{p^2+iM^2} + \frac{b^*}{p^2-iM^2}\right\}}_{=0} = e^{\gamma(m^2)}\pi\delta^{(4)}(p^2+m^2) > 0$$

23:12 Thu Oct 28

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Nonlocal gravity with complex conjugate poles

- Nonlocal gravitational action:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R}-G_{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu}+G_{\mu\nu}e^{-\gamma(\Box)}\frac{\Box^2-M^4}{M^4\Box}\mathcal{R}^{\mu\nu}\right)$$

- Nonlocal graviton propagator:

$$i\Pi_{\mu\nu\rho\sigma}(p)=\frac{e^{\gamma(-p^2)}M^4}{p^4+M^4}\Pi^{GR}_{\mu\nu\rho\sigma}(p)=\frac{e^{\gamma(-p^2)}M^4}{p^4+M^4}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{p^2}-\frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2p^2}\right)$$

- Ghost-free graviton propagator! Poles: massless spin-2 graviton pole + 1 pair of Lee-Wick poles

Perturbative unitarity

$$S^{+}S = 1, \qquad S = 1 + iT \qquad \Longrightarrow \qquad i(T^{+} - T) = T^{+}T$$

$$i[\langle b|T^{+}|a\rangle - \langle b|T|a\rangle] = \sum_{n} \langle b|T^{+}|n\rangle\langle n|T|a\rangle$$

$$\langle b|T|a\rangle = (2\pi)^4 \delta^{(4)}(P_b - P_a)\langle b|\mathcal{M}|a\rangle$$

$$i[\langle b|\mathcal{M}^{+}|a\rangle - \langle b|\mathcal{M}|a\rangle]$$
$$= \sum_{\{n\}} \prod_{l=1}^{n} \int \frac{d^3k_l}{(2\pi)^3} \frac{1}{2\omega_l} (2\pi)^4 \, \delta^{(4)}\left(P_a - \sum_{l=1}^{n} k_l\right) \langle b|\mathcal{M}^{+}|\{k_l\}\rangle\langle\{k_l\}|\mathcal{M}|a\rangle$$

23:14 Thu Oct 28

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Perturbative unitarity

$$S^+S = 1, \qquad S = 1 + iT \qquad \Longrightarrow \qquad i(T^+ - T) = T^+T$$

$$i[\langle b|T^+|a\rangle - \langle b|T|a\rangle] = \sum_n \langle b|T^+|n\rangle\langle n|T|a\rangle$$

$$\langle b|T|a\rangle = (2\pi)^4\delta^{(4)}(P_b - P_a)\langle b|\mathcal{M}|a\rangle$$

LHS

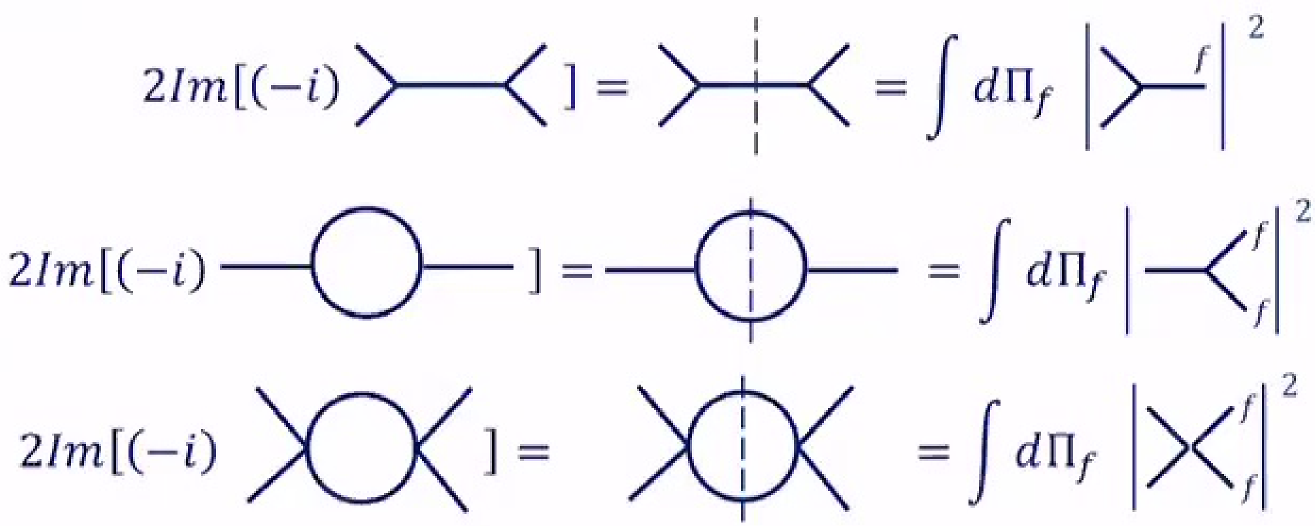
$$i[\langle b|\mathcal{M}^+|a\rangle - \langle b|\mathcal{M}|a\rangle]$$

$$= \sum_{\{n\}}\prod_{l=1}^{n}\int \frac{d^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}\,(2\pi)^4\,\delta^{(4)}\left(P_a - \sum_{l=1}^{n}k_l\right)\langle b|\mathcal{M}^+|\{k_l\}\rangle\langle\{k_l\}|\mathcal{M}|a\rangle$$

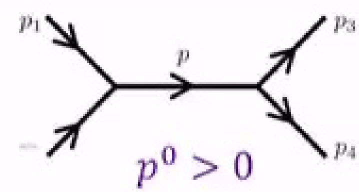
RHS

Perturbative unitarity

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



Tree-level: nonlocality + real poles



$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2), \qquad i\Pi(p) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}, \qquad \gamma(m^2) = 0$$

$$LHS = i[\langle p_3, p_4|\mathcal{M}^+|p_1, p_2\rangle - \langle p_3, p_4|\mathcal{M}|p_1, p_2\rangle]$$

$$= i\lambda^2 e^{\gamma(-p^2)}\left[\frac{1}{p^2 + m^2 + i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}\right]$$

$$= 2\pi\lambda^2\theta(p^0)\delta(p^2 + m^2)$$

23:16 Thu Oct 28 37%

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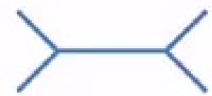
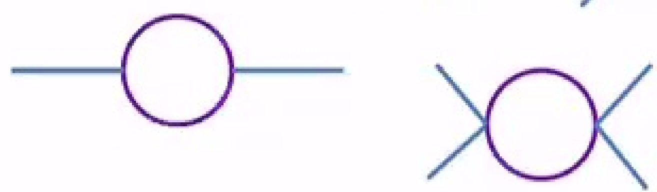
Perturbative unitarity

- So far, we have shown tree-level unitarity



- What about loops?



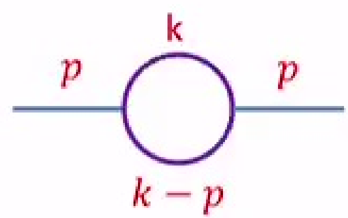
- Unitarity of nonlocal theories with standard poles
  [Sen & Pius 2015; Carone 2017; Briscese & Modesto 2018; Chin & Tomboulis 2018; Koshelev & Tokareva 2021]

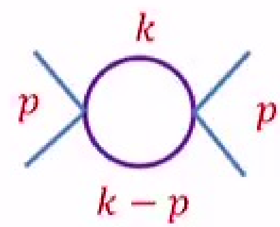
- Unitarity of nonlocal theories with complex conjugate poles
  [LB, Yamaguchi – arXiv:21XX.XXXXX]

- We only consider one-loop bubble diagrams

$$\mathcal{L}=\frac{1}{2}\phi F(\Box)\phi-\frac{\lambda}{3!}\phi^3-\frac{g}{4!}\phi^4+\kappa\,\phi\psi^2+\cdots$$

Bubble diagram: local 2-derivative case





$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2-i\epsilon} \frac{1}{(k-p)^2+m^2-i\epsilon}$$

$$Q_1 = -\omega_k + i\epsilon$$

$$Q_2 = p^0 - \omega_{k-p} + i\epsilon$$

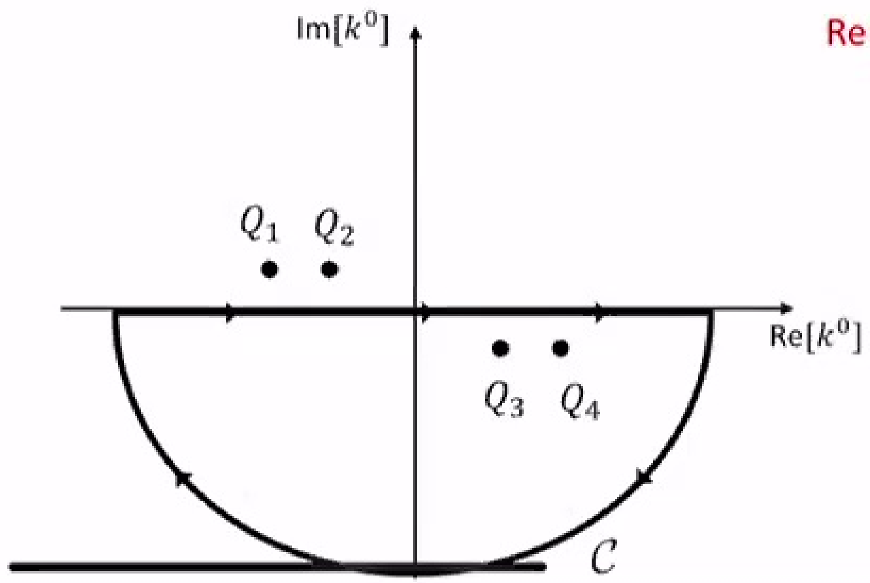
$$Q_3 = \omega_k - i\epsilon$$

$$Q_4 = p^0 + \omega_{k-p} - i\epsilon$$

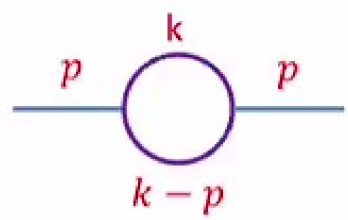
$$\omega_k = \sqrt{\vec{k}^2+m^2}$$

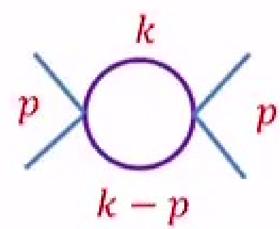
$$\omega_{k-p} = \sqrt{(\vec{k}-\vec{p})^2+m^2}$$

$\mathrm{Re}(p^0) > 0$



Bubble diagram: nonlocal with real poles

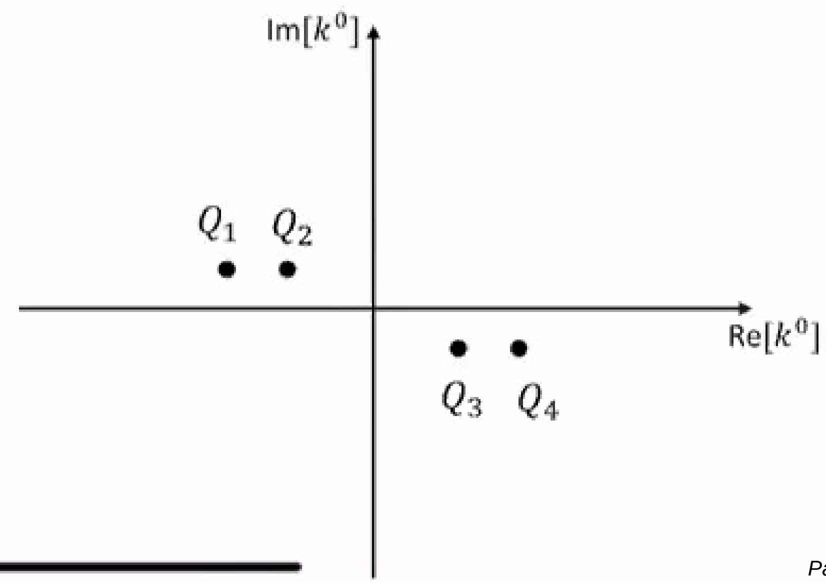




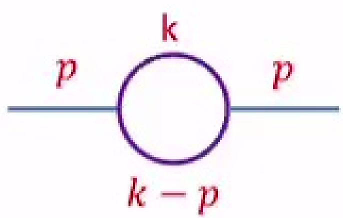
$\mathrm{Re}(p^0) > 0$

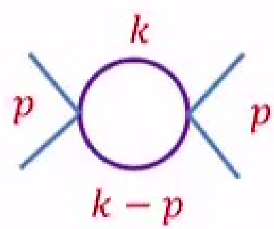
$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

- Liouville's theorem implies the presence of singularities at infinity!



Bubble diagram: nonlocal with real poles





$\text{Re}(p^0) > 0$

$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

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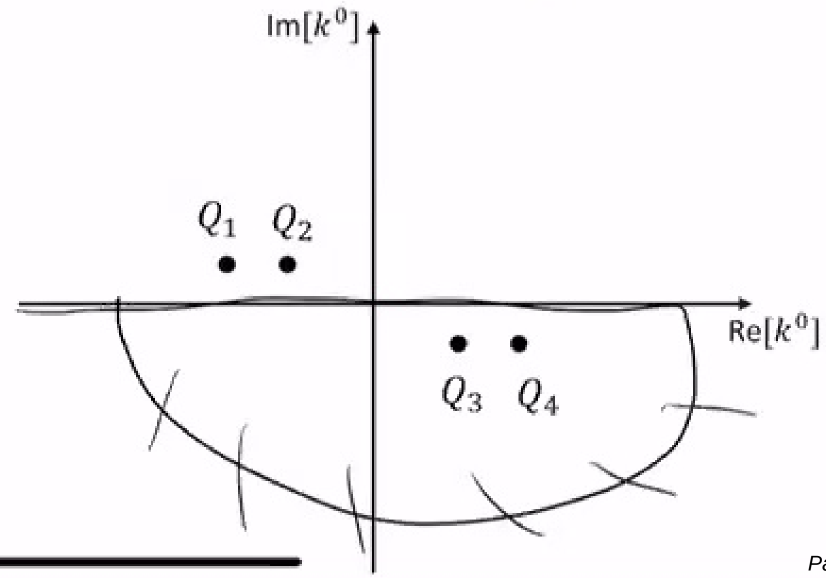
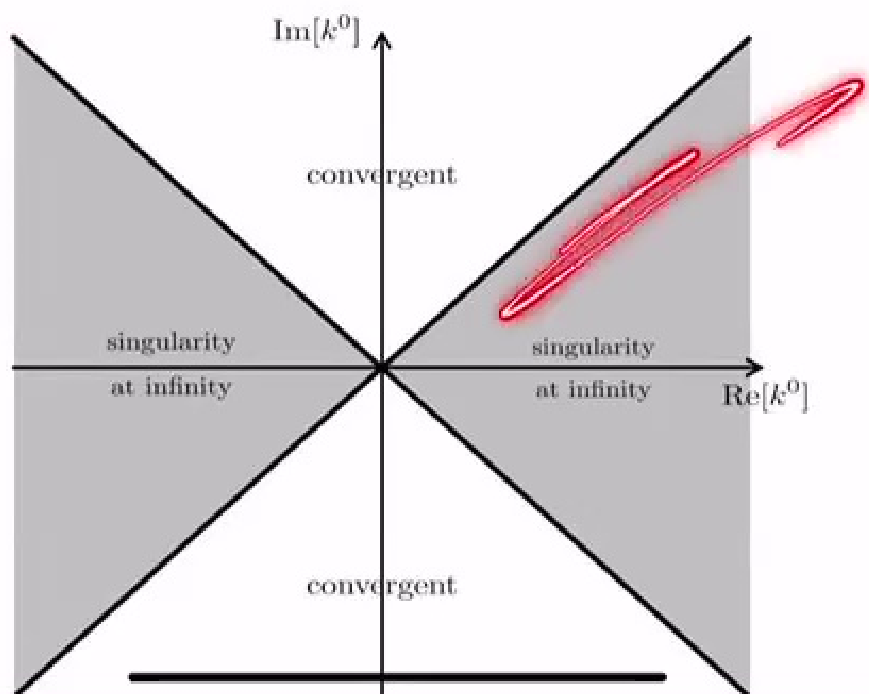


Singularities at infinity

- Example of singularity at infinity: $\gamma(-k^2) = -k^2 = (k^0)^2 - \vec{k}^2$

$$k^0 = \kappa e^{i\vartheta}, \qquad \kappa \in \mathbb{R}_+$$

$$e^{-k^2} = e^{(k^0)^2 - \vec{k}^2} = e^{-\vec{k}^2} e^{i\,\kappa^2 \sin 2\vartheta} e^{\kappa^2 \cos 2\vartheta}$$



23:19 Thu Oct 28

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Contour prescription

- In local two-derivative theories starting from Minkowski is equivalent to starting from Euclidean
- In nonlocal theories starting from Minkowski is NOT well-defined because of divergencies at infinity

---

- Define the contour $\mathcal{C}$ to be the imaginary k^0 –axis
- Complexify internal and external energies: $k^0 \in \mathbb{C}, \quad p^0 \in \mathbb{C}$
- To avoid poles and pinchings deform the contour in finite-distance region of the complex plane by keeping the ends fixed at $\pm i\infty$
- Analitically continue external energies to real values

- Nonlocal theories + real poles [Sen & Pius 2015]
- Nonlocal theories + complex conjugate poles [LB, Yamaguchi - arXiv:21XX.XXXXX]

23:20 Thu Oct 28

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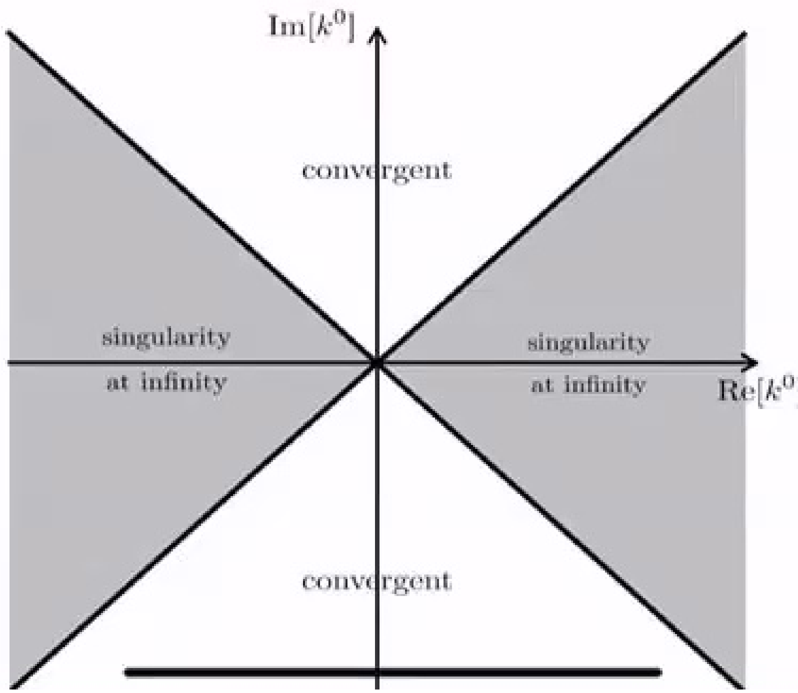

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Bubble diagram: nonlocal with real poles

$\mathrm{Re}(p^0) > 0$

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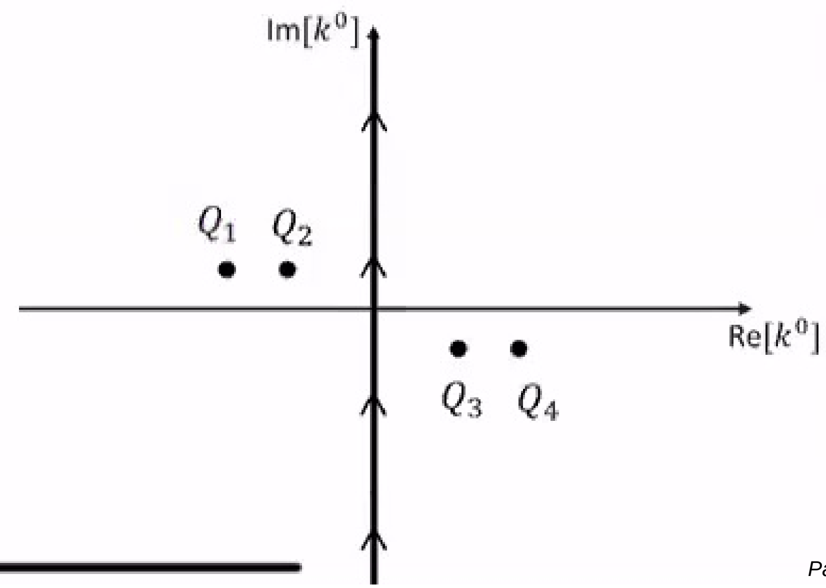
- Pinching:

$$Q_2 = Q_3 \quad \Leftrightarrow \quad p^0 = \omega_k + \omega_{k-p}$$



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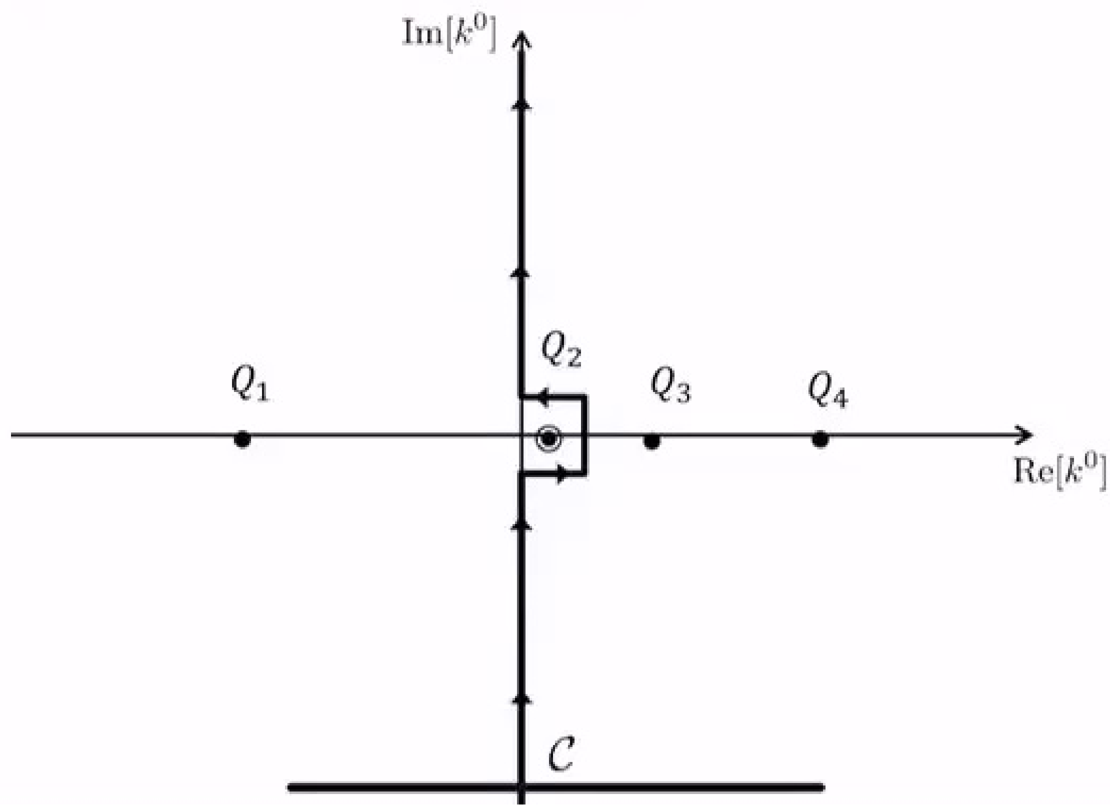
Bubble diagram: nonlocal with real poles

- Region I:

$\text{Re}(p^0) > 0$

$$Re[Q_2] < Re[Q_3] \quad \Leftrightarrow \quad p^0 < \omega_k + \omega_{k-p}$$

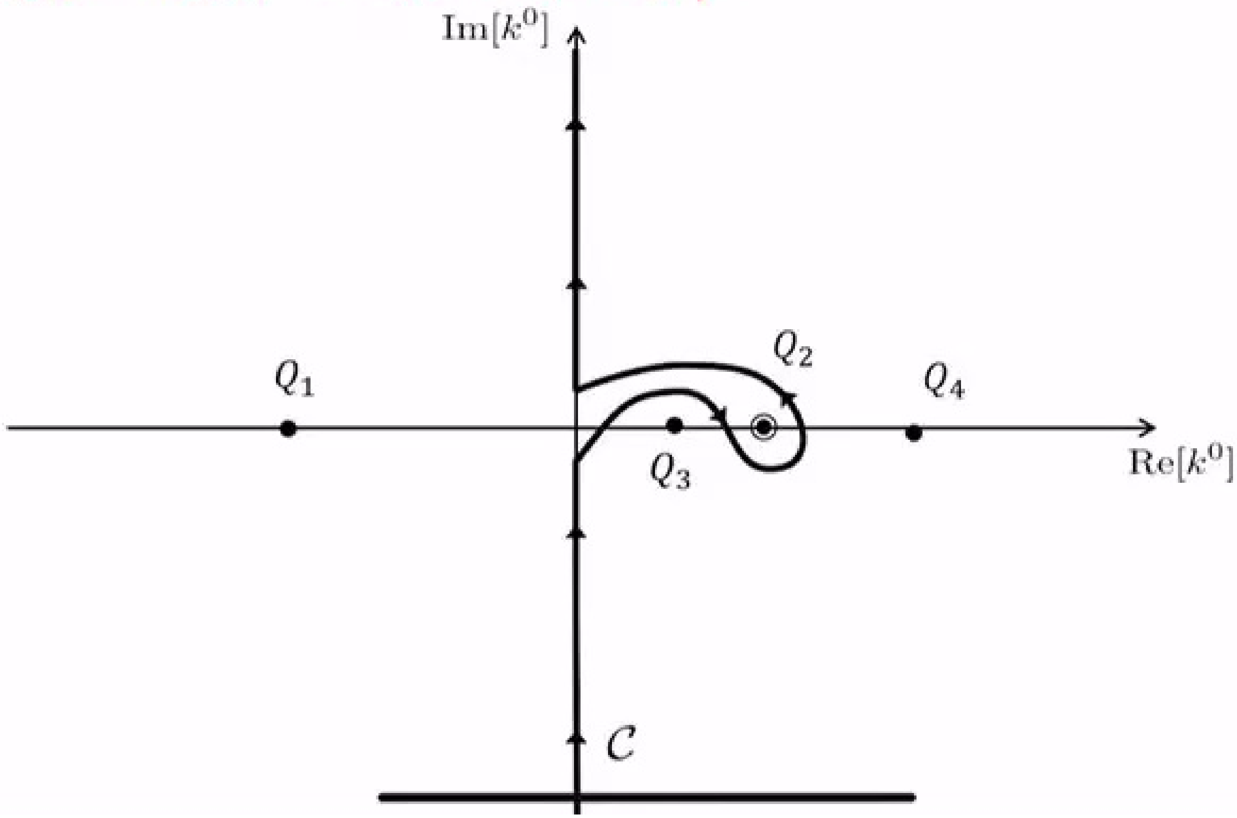


Bubble diagram: nonlocal with real poles

- Region II:

$\mathrm{Re}(p^0) > 0$

$$Re[Q_2] > Re[Q_3] \quad \Leftrightarrow \quad p^0 > \omega_k + \omega_{k-p}$$



Bubble diagram: nonlocal with real poles

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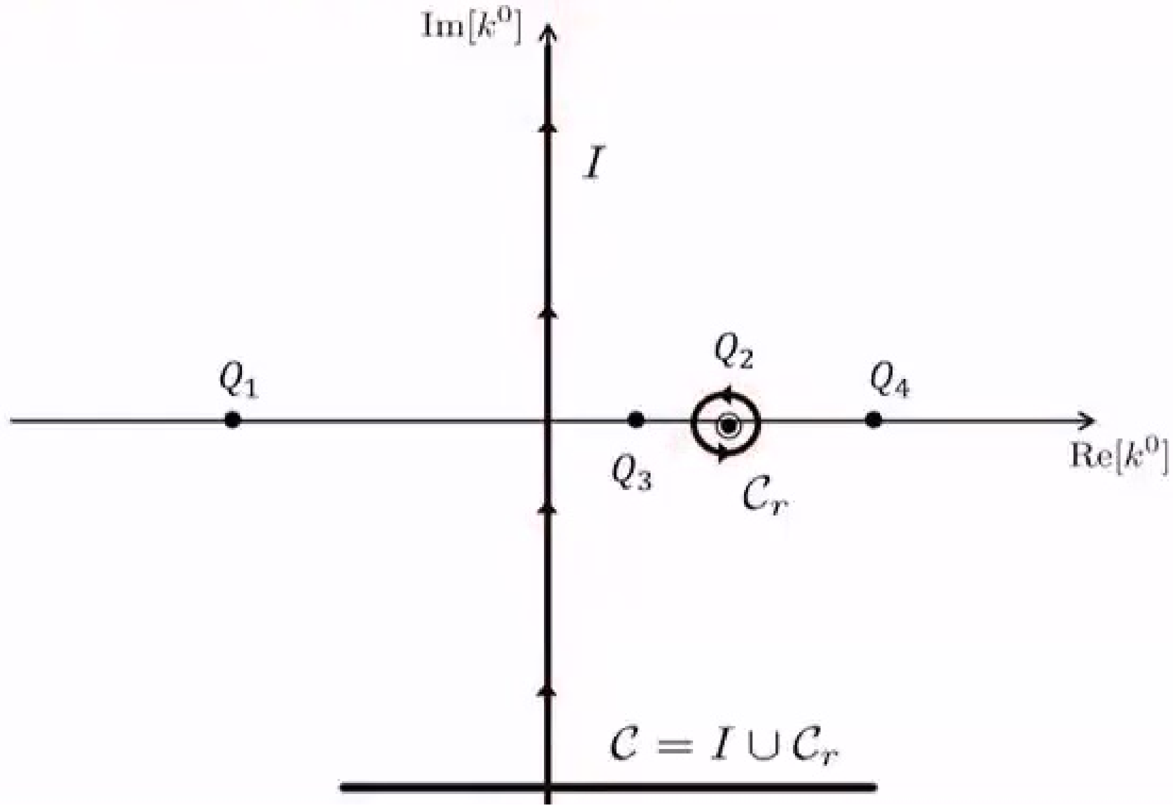
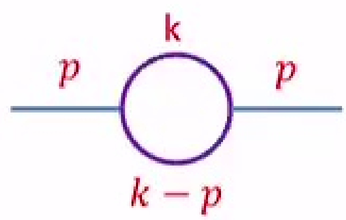
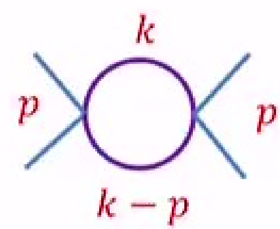


Bubble diagram: nonlocal with real poles





$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) = (-i)\lambda^2 \int_{I\cup\mathcal{C}_r} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

$$= \mathcal{M}_I(p) + \mathcal{M}_{\mathcal{C}_r}(p)$$

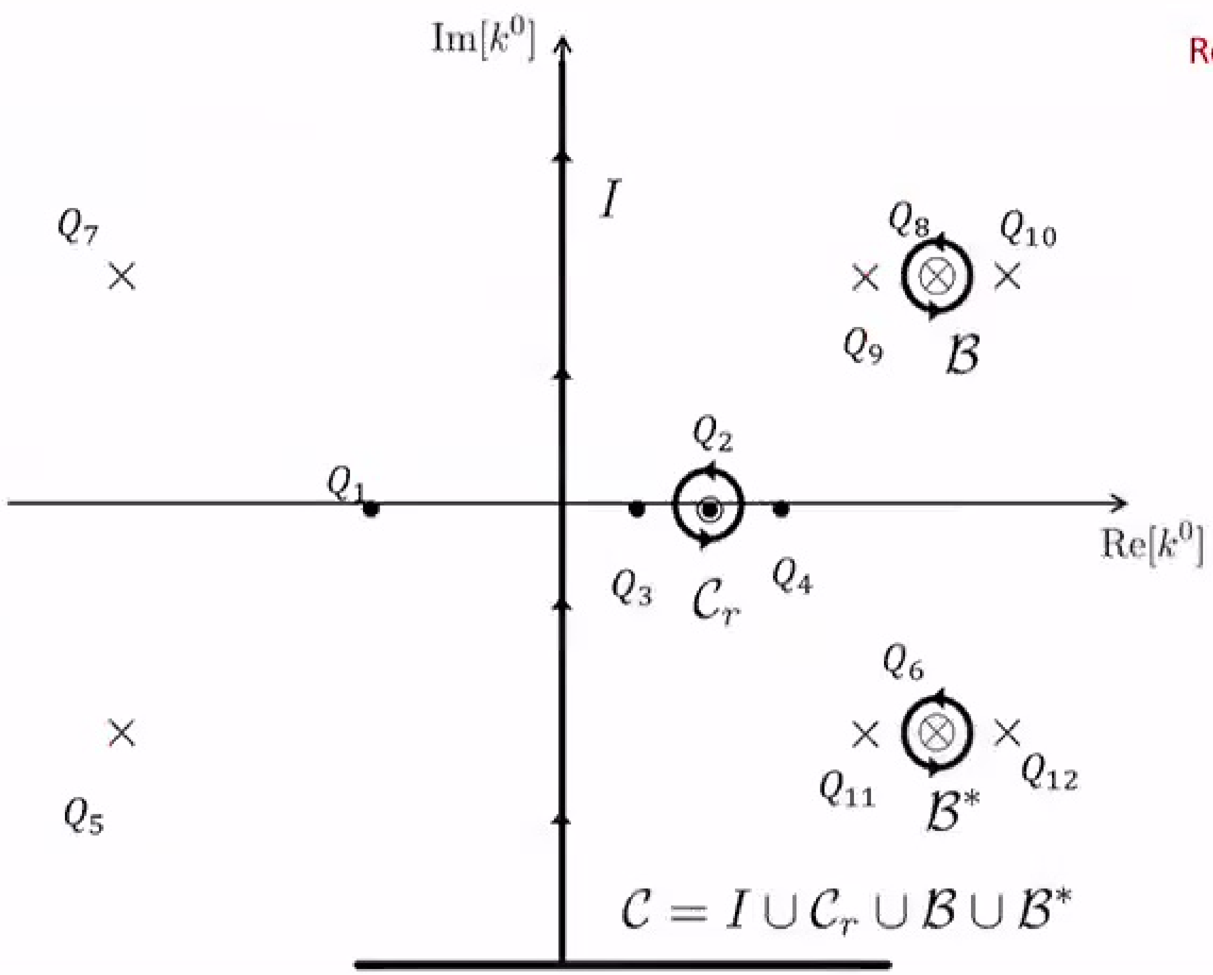
Gives an imaginary part!

$$LHS = 2Im\{\mathcal{M}(p)\} = 2\pi\lambda^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \theta(p^0-\omega_{k-p})\delta(p^0-\omega_k-\omega_{k-p})$$

Same Cutkosky rules of local two-derivative case!

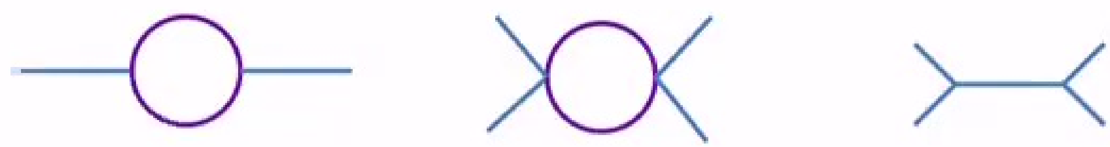
Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$



$$\mathcal{C} = I \cup \mathcal{C}_r \cup \mathcal{B} \cup \mathcal{B}^*$$

Some remarks



- Higher loops investigated in the case of real masses [Sen & Pius 2015]
- Higher loops complex conjugate masses not yet [Work in progress...]
- More complicated vertexes will not affect the result as long as they do not change the pole structure
- In the gravitational case, proving the Cutkosky rules is not sufficient. We also need to project away unphysical states due to gauge invariance
- What about infinite pairs of complex conjugate poles? The same prescription should apply.

Some open problems

- How to define an Hamiltonian for nonlocal theories?
- How to define a 'good' classical limit?
  (…correspondence principle?)
- Quantification of the causality violation?
  (Only at short distances…?)
- Singularity resolution at non-linear level?
- Huge arbitrarity in the choice of the entire function !?!
  (constraints from phenomenology, e.g. inflationary cosmology:
  see Koshelev, Kumar, Starobinsky JHEP 2017,2020)
- Nonlocal Lagrangians from first principles…?

23:35 Thu Oct 28

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Some open problems

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23:36 Thu Oct 28 32%

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REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Reciprocity Theory of Elementary Particles

MAX BORN
University of Edinburgh, Edinburgh, Scotland

I. INTRODUCTION

THE theory of elementary particles which I propose in the following pages is based on the current conceptions of quantum mechanics and differs widely from the ideas which Einstein himself has developed in regard to this problem. I hope that it may nevertheless be acceptable as a contribution to this volume in honor of his 70th birthday, as it is based on his famous relation between energy E and mass m of a physical system, $E=mc^2$, and as it can be interpreted as a rational generalization of his ("special") theory of relativity.

Relativity postulates that all laws of nature are invariant with respect to such linear transformations of space time $x^k=(\mathbf{x},t)$ for which the quadratic form $R=x^kx_k=t^2-\mathbf{x}^2$ is invariant (the velocity of light is taken to be unity). The underlying physical assumption is that the 4-dimensional distance $r=R^{\frac{1}{2}}$ has an absolute significance and can be measured. This is a natural and plausible assumption as long as one has to do with macroscopic dimensions where measuring rods and clocks can be applied. But is it still plausible in the domain of atomic phenomena?

Doubts have been expressed a long time ago, e.g., by Lindemann (Lord Cherwell) (14) in his instructive little book. I think that the assumption of the observability of the 4-dimensional distance of two events inside atomic dimensions is an extrapolation which can only be justified by its consequences; and I am inclined to interpret the difficulties which quantum mechanics encounters in describing elementary particles and their interactions as indicating the failure of that assumption.

The well-known limits of observability set by Heisen-

sponding to the particles with which one has possibly to do. This is the problem which is now in the center of interest: by estimating $\mathbf{p}$ and E for a particle observed in the Wilson chamber or in a photographic emulsion, one obtains a rough value of the rest mass which may permit one to recognize the kind of particle with which one has to do. If the value of P thus obtained is however incompatible with the known particles a new one is discovered. During the last year this has happened several times, and one gets the impression that there may be no end of it. New types of mesons are found almost every week, and it seems to be not an extravagant extrapolation to suppose that there is an infinite number.

It looks, therefore, as if the distance P in momentum space is capable of an infinite number of discrete values which can be roughly determined while the distance R in coordinate space is not an observable quantity at all.

This lack of symmetry seems to me very strange and rather improbable. There is strong formal evidence for the hypothesis, which I have called *the principle of reciprocity*, that the laws of nature are symmetrical with regard to space-time and momentum-energy, or more precisely, that they are invariant under the transformation

$$x_k \rightarrow p_k, \quad p_k \rightarrow -x_k. \qquad \text{(I.1)}$$

The most obvious indications are these: The canonical equations of classical mechanics

$$\dot{x}^k = \partial H/\partial p_k, \quad \dot{p}_k = -\partial H/\partial x^k \qquad \text{(I.2)}$$

are indeed invariant under the transformation (1), if where $P = p^k p_k$, and the auxiliary condition $\frac{\partial}{\partial x_\mu} = 0$ is imposed. It can be shown for the pure radiation field that the factor $e^{-(P+P')/2b^2}$ does not lead to any consequences different from those of the orthodox theory; but if the electron is considered in the usual ... singularity in the radiation field, ... a moving electron is

$$e^{-a^2\Box}\Box A_k = \frac{e v_k}{c}\,\delta\{x - x_e(t)\}\,,\quad v = \left(\frac{dx_e}{dt}, c\right), \qquad (2)$$

... reducing to the usual solution at large distance, and to the difference between the retarded and advanced potentials introduced by Dirac[6] in the first approximation at small distance from the singularity. The energy and momentum of the field are also finite, and are identified with the energy and momentum of the electron. In this way the idea of Abraham[7] has been realized without the relativistic difficulties associated with a rigid electronic structure.

Before quantization, a Hamiltonian formulation is required, for which, because of the appearance of high derivatives in the Lagrangian, either the well-known method of Lagrangian multipliers or a method of successive approximation may be used. The interaction energy then has the form:

$$H_i = e\,\alpha\,.\,\Sigma e^{-(k^2-(k\,.\,p)^2/E^2)/2b^2}\,B(k)e^{ik\,.\,x/\hbar} + B^*(k)e^{-ik\,.\,x/\hbar}\,e^{-(k^2-(k\,.\,p)^2/E^2)/2b^2}, \qquad (3)$$

where $E = (p^2 + mc^2)^{1/2}c$ is the energy of the

discrete character (or characters) is r! multiplied by the expected number for r of the specified character (or characters).

The binomial and the hypergeometric distributions are the simplest cases where this theorem can be applied direct. For these distributions, the rth factorial moments, $\mu'_{[r]}$, are

$$r!\binom{n}{r}p^r = n^{(r)}p^r \text{ and}$$

$$\frac{r!\binom{n}{r}(Np)^{(r)}}{N^{(r)}} = \frac{n^{(r)}(Np)^{(r)}}{N^{(r)}},$$

respectively. They are evidently r! times the expectation for r of the events.

Some of the other distributions which can be applied in this theorem are: (a) the theory of the distribution of black–black, black–white and other joins arising from points possessing any one of k colours arranged on a line or in the form of a rectangular lattice; (b) the distribution for the number of runs in ascending or descending order discussed by Kermack and McKendrick[2], which ultimately is the same thing as the distribution of peaks and troughs discussed by Kendall[3]; and (c) the distributions arising in the matching theory dealt with by Batin[4], Anderson[5] and Wilks[6]. It is also felt that the theorem will be useful in evaluating the factorial moments of many other distributions. The application and the proof of this theorem will be discussed in detail elsewhere shortly.

23:38 Thu Oct 28 32%

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Thank you
for
your attention!

23:39 Thu Oct 28 32%

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Bubble diagram: local 2-derivative case

$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2-i\epsilon} \frac{1}{(k-p)^2+m^2-i\epsilon}$$

$Q_1 = -\omega_k + i\epsilon$

$Q_2 = p^0 + \omega_{k-p} + i\epsilon$

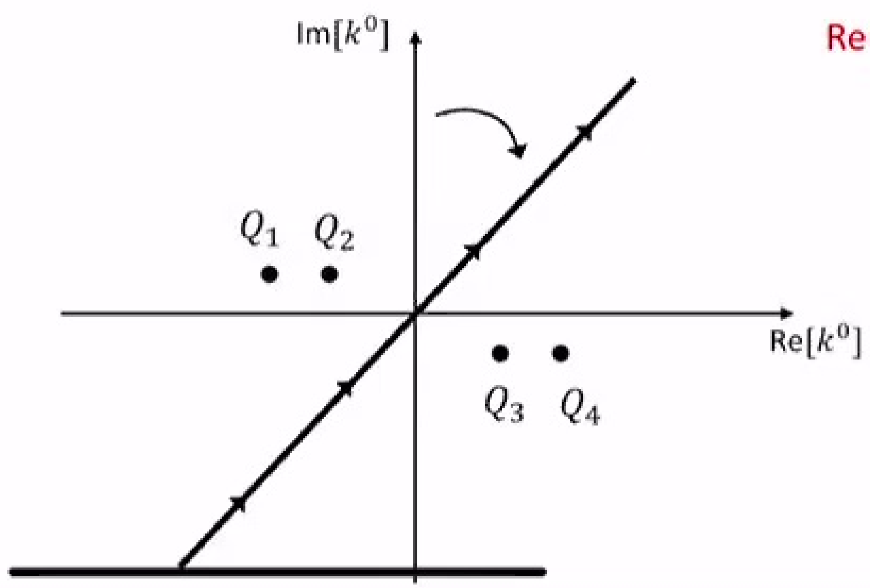
$Q_3 = \omega_k - i\epsilon$

$Q_4 = p^0 + \omega_{k-p} - i\epsilon$

$\omega_k = \sqrt{\vec{k}^2 + m^2}$

$\omega_{k-p} = \sqrt{(\vec{k}-\vec{p})^2 + m^2}$

$\mathrm{Re}(p^0) > 0$



Some open problems

- How to define an Hamiltonian for nonlocal theories?
- How to define a 'good' classical limit?
  (…correspondence principle?)
- Quantification of the causality violation?
  (Only at short distances…?)
- Singularity resolution at non-linear level?
- Huge arbitrarity in the choice of the entire function !?!
  (constraints from phenomenology, e.g. inflationary cosmology:
  see Koshelev, Kumar, Starobinsky JHEP 2017,2020)
- Nonlocal Lagrangians from first principles…?

23:56 Thu Oct 28

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$$R + R F(\Box) R$$

$$e^{-f(\nabla^2)} \nabla^2 = \rho$$

$$R_{\mu\nu} F(\Box) R^{\mu\nu}$$

$$f(\nabla^2)$$

$$R = \sum_n f_n \omega^n$$

$$R_{\mu\nu} = 0$$

$$R_{\mu\nu}$$