Title: Quantum Many-body theory in the Quantum Information era

Speakers: Matthew Fisher

Series: Quantum Matter

Date: October 18, 2021 - 12:00 PM

URL: https://pirsa.org/21100027

Abstract: Traditionally, quantum many-body theory has focussed on ground states and equilibrium properties of spatially extended systems, such as electrons and spins in crystalline solids. In recent years "noisy intermediate scale quantum computers" (NISQ) have emerged, providing new opportunities for controllable non-equilibrium many-body systems. In such dynamical quantum systems the inexorable growth of non-local quantum entanglement is expected, but monitoring (by making projective measurements) can compete against entanglement growth. In this talk I will overview theoretical work exploring the behavior of "monitored" quantum circuits, which can exhibit a novel quantum dynamical phase transition between a weak measurement phase and a quantum Zeno phase, the former which we characterize in detail. Accessing such physics in the lab is challenged by the need for post-selection, which might be circumnavigated by decoding using active error correction, conditioned on the measurement outcomes, as will be described in systems with Z2 symmetry.

Zoom Link: https://pitp.zoom.us/meeting/register/tJcqc-ihqzMvHdW-YBm7mYd_XP9Amhypv5vO

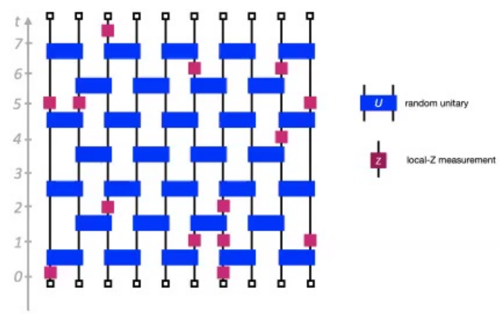
# Quantum many-body theory in Quantum Information era

Quantum Matter Frontier Seminar
10/18/21

MPA Fisher

Traditionally: Equilibrium phases of Quantum Matter in solids

This talk: Open, non-equilibrium, dynamical, quantum phases/transitions

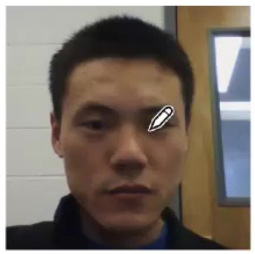



- Measurement driven entanglement transition
- Active error correction in dynamical memory


# Thanks to my collaborators!

Shengqi Sang, Zhi-Cheng Yang, Tianci Zhou, Tim Hsieh, Andreas Ludwig

Yaodong Li



Xiao Chen

Sagar Vijay

- Y. Li, X. Chen and M.P.A. Fisher, Phys. Rev. B, 98, 205136 (2018)
- Y. Li, X. Chen and M.P.A. Fisher, Phys Rev B 100, 134306 (2019)
- Y. Li, S. Vijay and MPA Fisher, arXiv:2105.13352
- Y. Li and M.P.A. Fisher, arXiv:2108.04274

HEISING-SIMONS FOUNDATION

# Outline

1) Measurement induced entanglement transition in random hybrid circuits

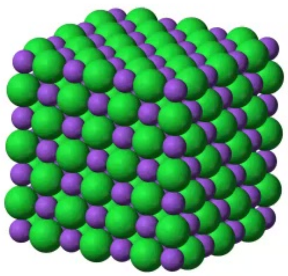
2) Nature of the “volume-law” phase (in random circuits)

3) Active Quantum error correction in dynamical memories

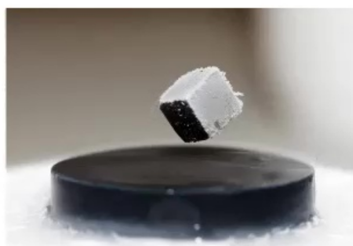
# Quantum Matter: Emergence


Traditional Quantum condensed matter physics: Studies very large collections of electrons/atoms/spins, usually in solids

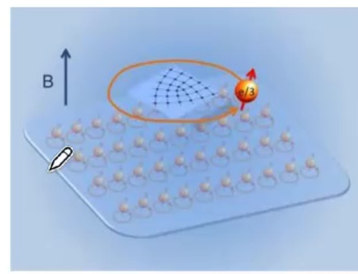
Emergence: The whole is greater than the sum of the parts, collective phases of matter



Crystalline solids



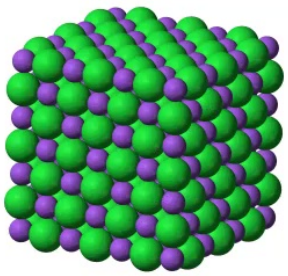
Superconductors



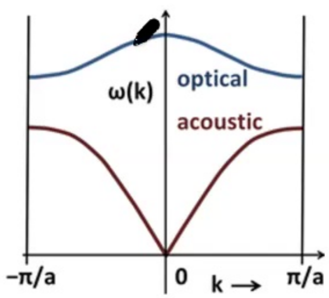

Fractional Quantum Hall effect

# Quantum Matter: Universality

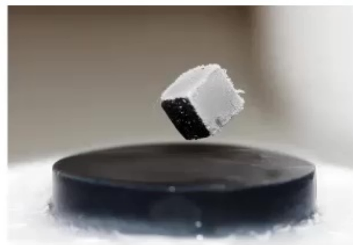
Emergent phases have properties that are independent of the microscopic details



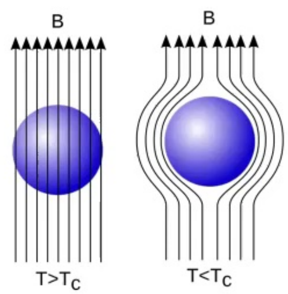
Crystalline solids



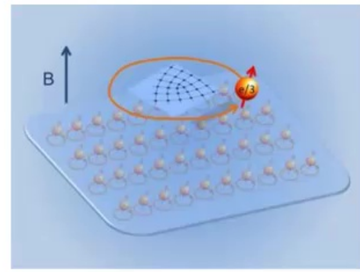

support (quantized) phonon excitations



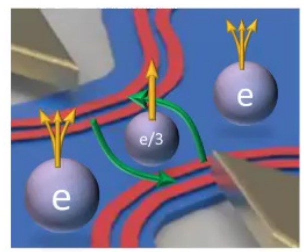
Superconductors,




always have a Meissner effect and zero resistance




Fractional Quantum Hall effect,

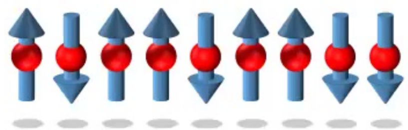



support excitations w/ fractional charge


# Quantum matter theory

Analyze simple Hamiltonians; eg Heisenberg spin model

$$\hat{\mathcal{H}} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

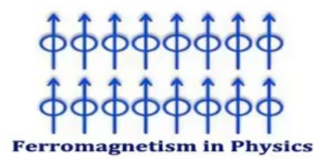


Focus on Ground states and thermal equilibrium states

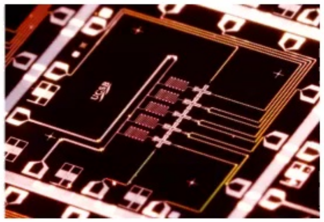
$$\hat{\rho}_{eq} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$$

Characterize quantum phases/transitions by order parameters and topology
e.g. magnetization

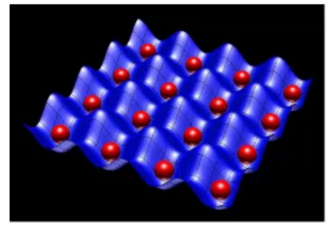
$$\vec{M} = Tr(\hat{\rho}_{eq} \vec{S}_i)$$



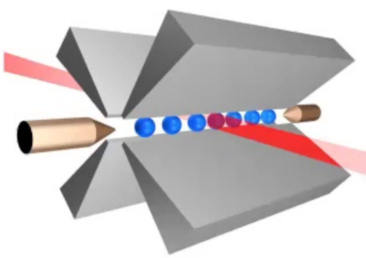
# New Experimental Platforms for many-body physics: (NISQ computers)

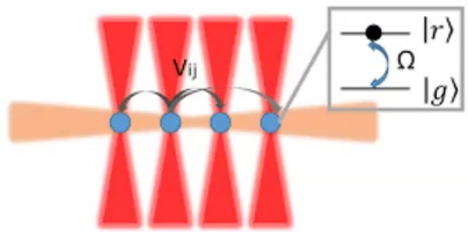
Superconducting QuBit arrays



Ultra-cold atoms



Trapped ions



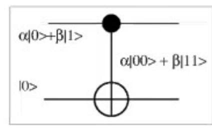

Rydberg atoms

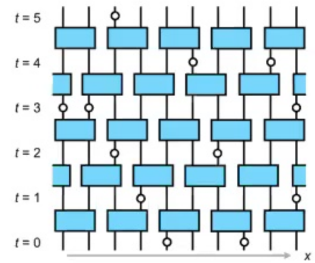

# New opportunities for quantum many-body theory

Quantum Hamiltonians;

$$\hat{\mathcal{H}} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Quantum circuits




Ground states, equilibrium,

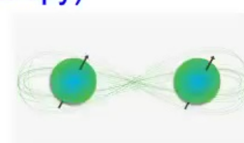
$$\hat{\rho}_{eq} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$$

Non-equilibrium dynamics,
open quantum systems,
role of measurement

Order parameters

$$\vec{M} = Tr(\hat{\rho}_{eq} \vec{S}_i)$$

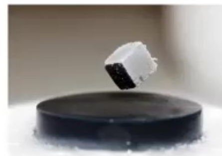
Quantum entanglement;
(entanglement entropy)

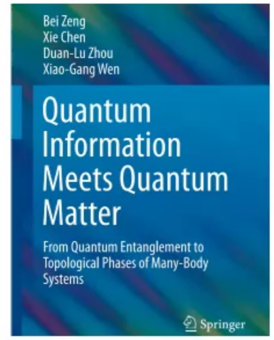


# “Quantum matter meets quantum information”

**Quantum matter theory (thermodynamic limit)**

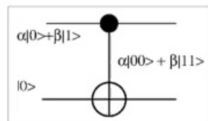
Ground states, exotic order, quantum criticality






**Quantum Information theory (often few qubit)**

Open, non-equilibrium systems, w/ measurements





This talk:
**Quantum phases/transitions driven by measurements**
**(in open, non-equilibrium systems, in thermodynamic limit)**

## Common thread: Entanglement Entropy


# Entanglement entropy

Single eigenstate $|\psi\rangle$

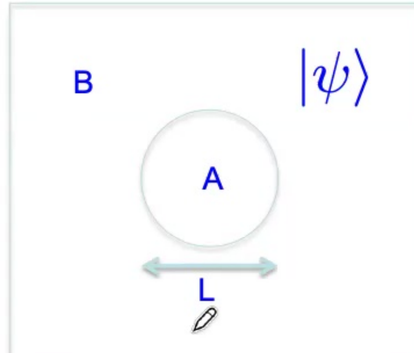
Pure-state density matrix: $\hat{\rho} = |\psi\rangle\langle\psi|$

**Spatial Bi-partition: R**egions A and B

Reduced density matrix in A

$$\hat{\rho}_A = Tr_B(\hat{\rho})$$

Entanglement entropy: $S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$




Scaling of entanglement entropy in equilibrium:

Ground states: Area law $S_A(L) \sim L^{d-1} \sim |\partial A|$ Ground states manifest spatial locality

Highly excited states: volume law $S_A(L) = sL^d \sim |A|$

Finite energy-density eigenstates are non-local

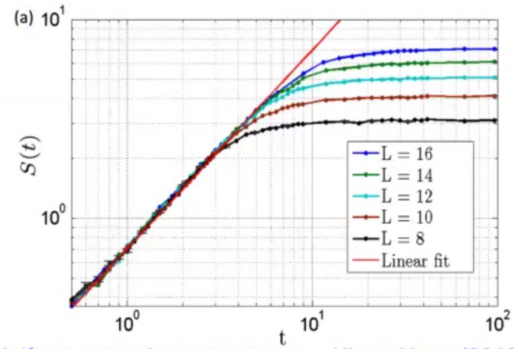
# Entanglement **dynamics** out of equilibrium in **closed system**


## 1) Quantum Quench

Evolve unentangled initial state w/ Hamiltonian

$$H = \sum_{i=1}^{L} \left( g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

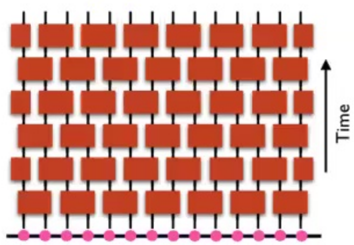
Entanglement spreads ballistically,
even though energy diffuses

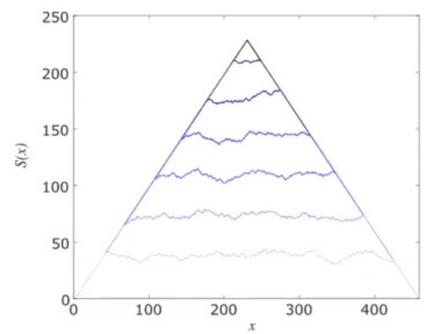



Half-cut entanglement entropy Kim + Huse (2013)

## 2) Unitary Dynamics with no energy conservation

Quantum circuit: evolve Qubits w/ (random) unitary gates
Initial state: unentangled product state
Entanglement spreads ballistically, into maximal entropy state




Nahum, Ruhman, Vijay, Haah (2017)

# Control Entanglement growth via measurements



Alice and Bob share singlet, $S_A = S_B = \ln(2)$

Alice measures spin

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB}] \Rightarrow \begin{cases} |\psi'\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \\ |\psi'\rangle = |\downarrow\rangle_A \otimes |\uparrow\rangle_B \end{cases}$$

After measurment, direct product state

$$S_A = S_B = \ln(2) \Rightarrow S'_A = S'_B = 0$$

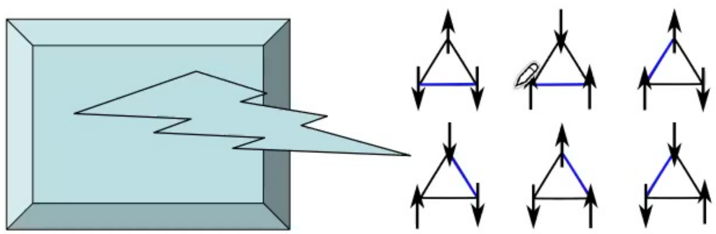
(Local) Measurement induces disentanglement

w/ measurements have an **Open system**


# Open Quantum Systems

## Two classes:

System coupled to a bath (environment)



- Initial pure density matrix becomes mixed
- Environment "measures" system, but results lost
- Decoherence
- Dynamics of density matrix evolves w/ (e.g.) Lindblad equation

**Active Decoding with decoherence**

System is monitored by an "observer"

- Initial pure state is measured and stays pure
- "Observer" keeps track of measurements
- Wavefunction evolves as a pure state
- Dynamics described in terms of (wavefunction) quantum trajectories

**Measurement-driven entanglement transition**

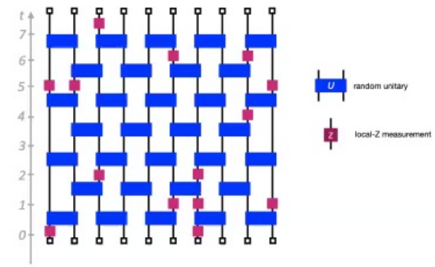
# “Hybrid” Quantum Circuit (“monitored”)

## w/ both unitary and measurement gates

- Unitary evolution induces entanglement growth
- QuBit Measurements induce disentanglement

***Explore competition between unitary evolution and measurements***

(by following wavefcn quantum trajectories)





- Li, Chen, MPAF (2018/2019)
- Skinner, Ruhman, Nahum (2018)
- Chan, Nandkishore, Pretko, Smith (2018)
- Choi, Bao, Qi, Altman (2019)
- Gullans, Huse (2019)
- Many more...

“Canonical” Non-unitary circuit:
- (Random) 2 Qubit unitary gates
- Single qubit measurements (w/ probability p)

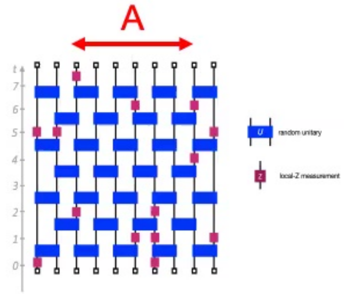
***Single parameter:*** $p \in [0, 1]$


# Phase diagram for hybrid circuit?

Initial state (say unentangled) $\hat{\rho}_0 = |\psi_0\rangle\langle\psi_0|$

- Run circuit dynamics to long times $\hat{\rho}_t = |\psi_t\rangle\langle\psi_t|$
- Density matrix stays **pure**
- Compute bipartite entanglement entropy $S_A(t) = -Tr_A(\hat{\rho}_A \log \hat{\rho}_A)$

## Phase Diagram?

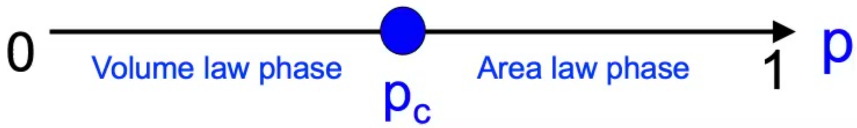


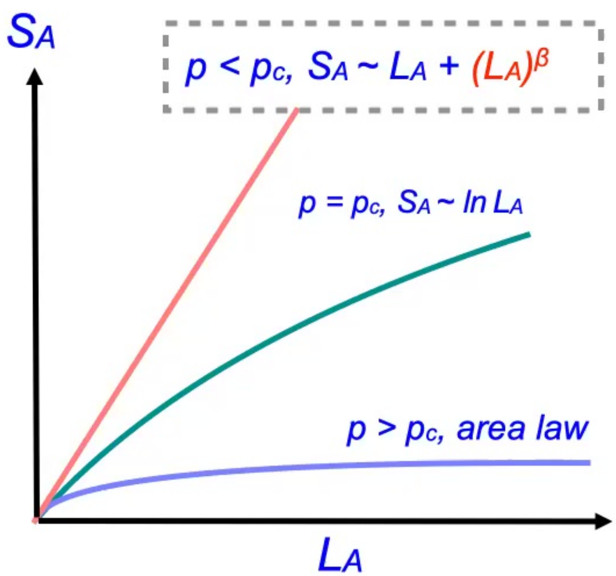

- p=0; No measurement, Volume law entanglement $S_A \sim |A|$
- p=1; Measure every Qubit, no entanglement (area law) $S_A \sim |\partial A|$
- Transition at p=$p_c$?

0 — Volume law phase — $p_c$ — Area law phase — 1 → p

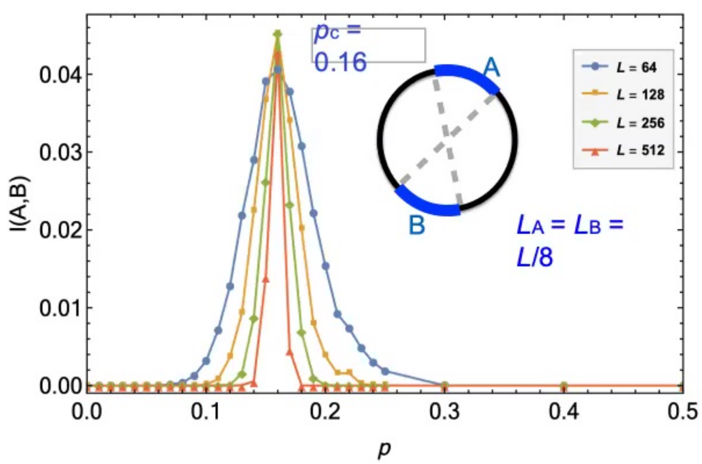
**Entanglement transition**

# Measurement induced entanglement transition




Clifford circuit numerics, ~500 qubits




Li, Chen, MPAF '19


# Conformal Symmetry at criticality ($p=p_c$)

Li, Chen, Ludwig, MPAF (2020)

$$I_{AB} = S_A + S_B - S_{AB}$$

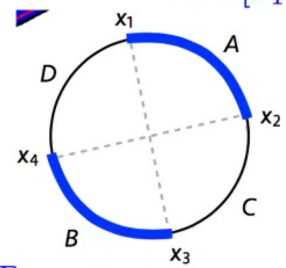
*If* have underlying conformal field theory, then mutual information depends only on the cross ratio

$$I_{AB} = f(\eta)$$

$$\eta \equiv \frac{x_{12}x_{34}}{x_{13}x_{24}}$$

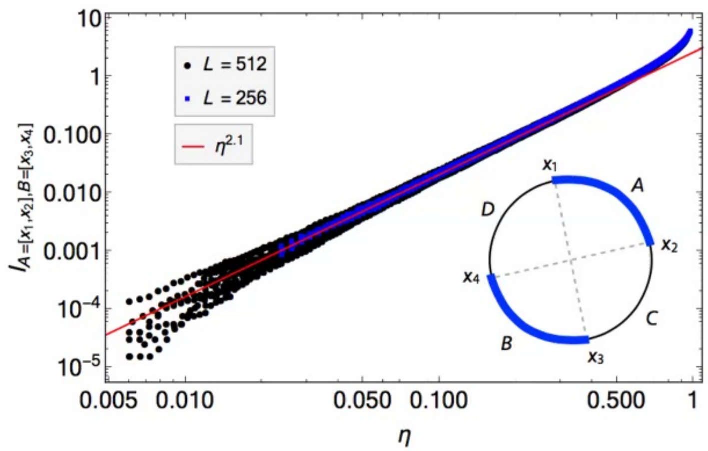
$$x_{ij} = \frac{L}{\pi}\sin\left(\frac{\pi}{L}|x_i - x_j|\right)$$

$A = [x_1, x_2]$




$B = [x_3, x_4]$

$I_{AB}$




$\eta$

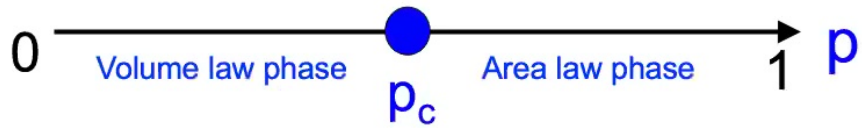
# Outline

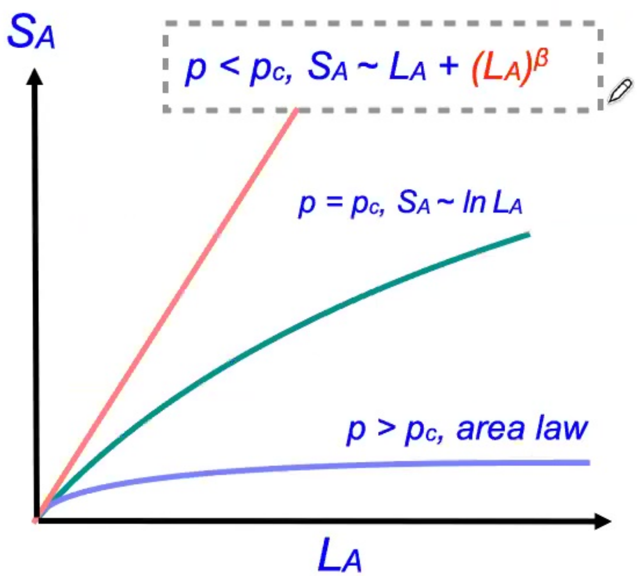
1) Measurement induced entanglement transition in random hybrid circuits

**2) Nature of the "volume-law" phase (in random circuits)**

3) Active Quantum error correction in dynamical memories

# Measurement induced entanglement transition

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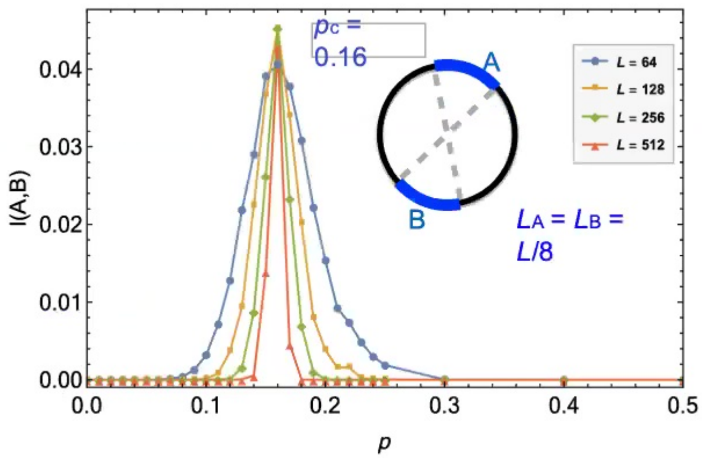
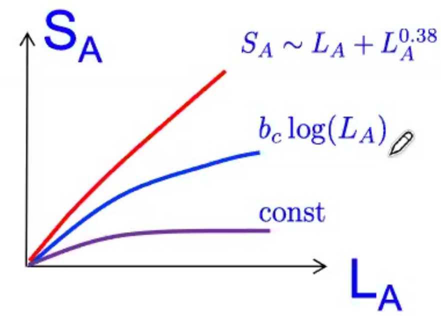



Li, Chen, MPAF '19

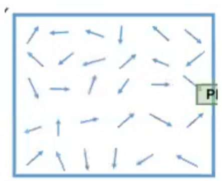
# Nature of the volume law phase?

"Background" in volume law phase
(Clifford numerics)

$$S_A \sim L_A + L_A^{0.38}$$



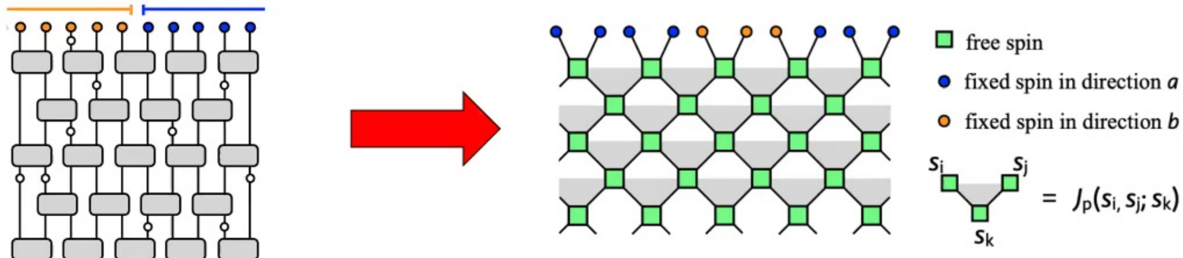

"Understand" via mapping to stat mech model



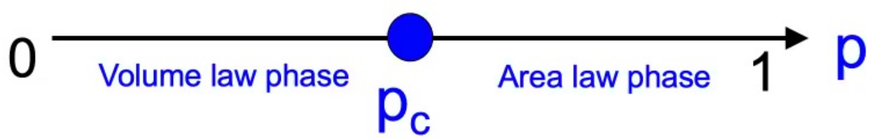

# Mapping to Stat Mech (spin) model

Random Haar circuit w/ measurements mapped to 2d “Generalized Potts” model (in space-time)

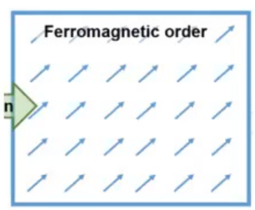
T. Zhou, A. Nahum (2018)
Jian, You, Vasseur, Ludwig (2019)
Bao, Choi, Altman (2019)



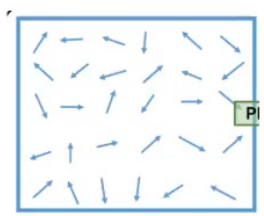

## Phases in Stat mech model




“Ordered” phase




“Paramagnetic” phase

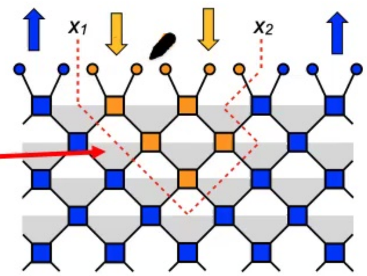


# Entanglement entropy from Stat Mech model

$S_A = F_A =$ free energy cost for changing boundary conditions in region A

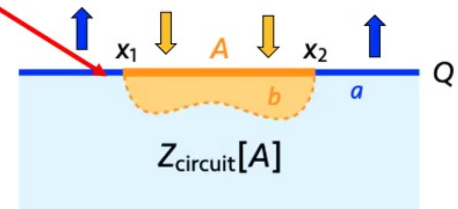
Volume law (ordered) phase:

Expect an “entanglement domain wall”




Surface tension gives

$$S_A = F_A \approx \sigma L_A$$

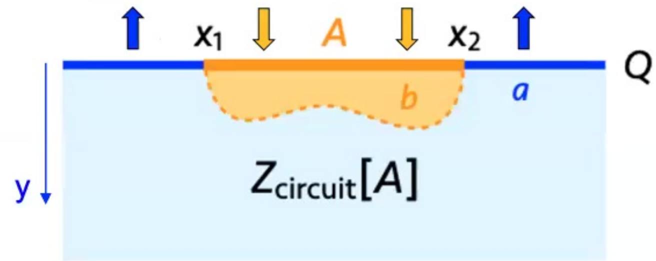



Area law (paramagnetic) phase: $S_A = F_A = O(1)$

Domain walls have proliferated (zero surface tension)

# Fluctuations of Entanglement domain wall

Assume **Ising order (MFT)** w/ simple domain wall, use capillary wave theory




$$F_{CW} = -\ln(Z_{CW})$$

$$Z_{CW}(L_A) = e^{-\sigma L_A} \int Dy(x) e^{-\sigma \int_0^{L_A} dx (\partial_x y)^2}$$

$y(x)$ parameterizes domain wall

$$S_A \approx F_{CW}(A) = \sigma L_A + \frac{3}{2} \ln L_A$$

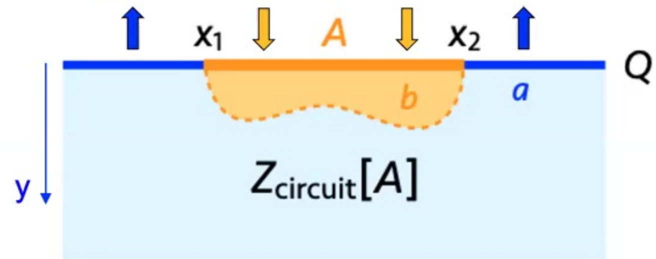
Fan, Vijay, Vishwanath, You (2020)
Li, MPAF (2020)

Volume law term (surface energy)

Log from Transverse fluctuations of entanglement domain wall

# Fluctuations of Entanglement domain wall

Assume **Ising order (MFT)** w/ simple domain wall, use capillary wave theory





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Fan, Vijay, Vishwanath, You (2020)
Li, MPAF (2020)

Volume law term (surface energy)

Log from Transverse fluctuations of entanglement domain wall

Clifford Numerics $S_A \sim L_A + L_A^{\beta}$ $\beta \approx 0.38$

- **Capillary wave theory does not capture quantitatively nature of volume law phase**
- **Entanglement domain walls are (somehow) more complicated**


# Role of disorder in volume law phase

Stat mech model for entanglement entropy requires a “replica limit”: index $m \rightarrow 0$

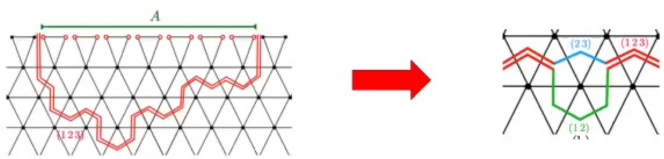
$$\langle S_A \rangle = \overline{S_{AE}} - \overline{S_E} \qquad \overline{S_{AE}} = \lim_{m \to 0} \frac{1}{m}[\overline{Tr\rho_{AE}^{m+1}} - 1]$$

Average over quantum trajectories and random unitaries

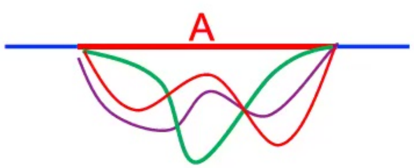
Average over random unitaries
E = environment of ancillas

m=1 spin model is “clean” Ising model, capillary wave theory
m=2,3,... entanglement domain wall splits into m domain walls with an attractive interaction

m=2




m=3




m domain walls w/ attractive interaction, in the replica limit $m \rightarrow 0$

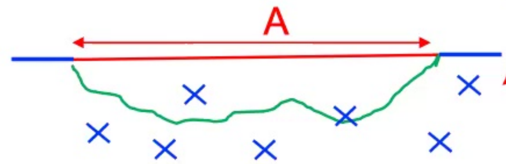

# Directed Polymer in a Random Environment (DPRE)

M. Kardar, Nucl. Phys. B (1987)

$$Z_{DP}(L_A) = e^{-\sigma L_A} \int Dy(x) e^{-\sigma \int_0^{L_A} [(\partial_x y)^2 + V(x,y)]}$$

Random potential

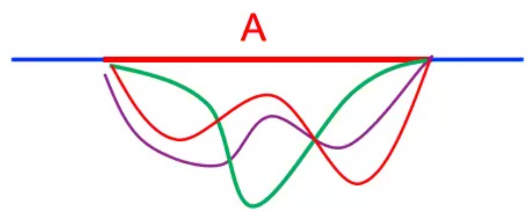
$$\langle V(x,y)V(x',y')\rangle = \delta(x-x')\delta(y-y')$$




Average free energy over disorder, using replica trick

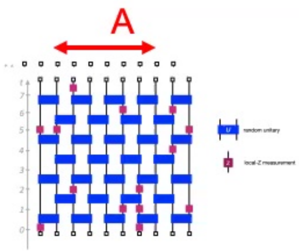
$$\langle F_A \rangle = -\langle \ln Z_{DP} \rangle = \lim_{m \to 0} \frac{1}{m} [\langle Z_{DP}^m \rangle - 1]$$

Average free energy is m-directed polymers w/ an attractive interaction, in replica limit, $m \to 0$

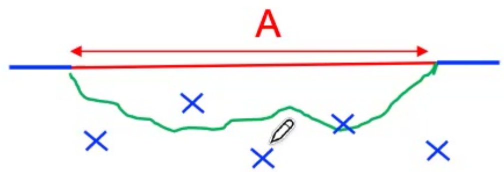



# Entanglement entropy in volume law phase of hybrid circuit given by free energy of DPRE

Li, Vijay, MPAF (2021)




**Entanglement in hybrid circuit**

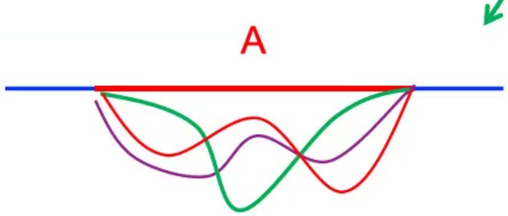



**DPRE Free energy**

Average over (Haar) unitary gates

$$\langle S_A \rangle \approx \langle F_A \rangle$$

Replica trick




m attracting, directed paths in the replica limit $m \to 0$

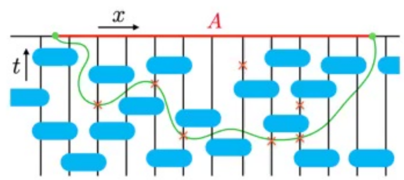
# Universal critical exponents for DPRE

M. Kardar, Nucl. Phys. B (1987)

Subdominant free energy corrections;

$\langle F_A \rangle = s_0 L_A + b L_A^{\beta}$ $\quad F_A^{sub} = 2\langle F_A \rangle - \langle F_{2A} \rangle = b L_A^{\beta}$ $\quad \beta = 1/3$

Sample-to-sample free energy fluctuations;




$\delta F_A = \sqrt{\langle F_A^2 \rangle - \langle F_A \rangle^2} = c L_A^{\beta}$

Wandering Exponent of DPRE

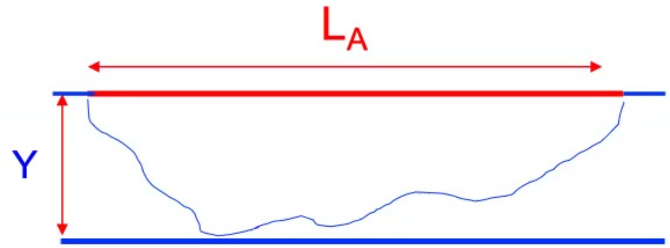

$L_A^{\zeta} \qquad \zeta = 2/3$

The universal exponents $\quad \beta = 1/3 \qquad \zeta = 2/3$

# DPRE in confined geometry: Finite-size scaling


Scaling form for sub-dominant free energy

$$F_A^{sub}(Y) = L_A^{\beta}\Phi(Y L_A^{-\zeta})$$

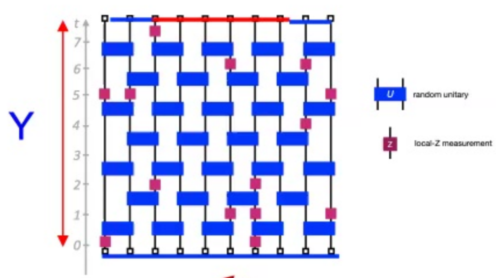



Scaling form for free energy fluctuations

$$\delta F_A(Y) = L_A^{\beta}\Psi(Y L_A^{\zeta})$$

---

## Clifford hybrid circuit in confined geometry

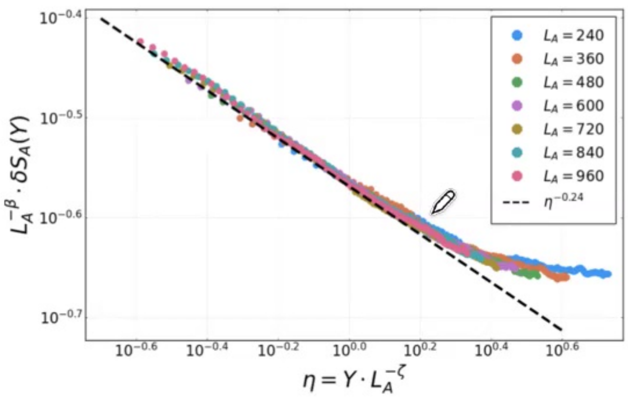



$$\hat{\rho}_0 = \frac{1}{2^L}\hat{1}$$

Maximally mixed initial state

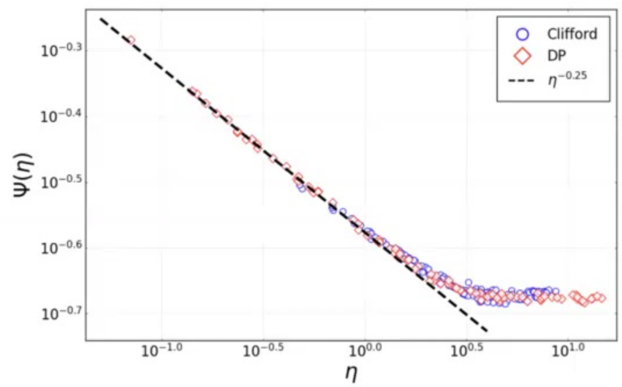
# Clifford hybrid circuit (volume law) versus DPRE


### Entanglement entropy fluctuations




$$\delta S_A(Y) = L_A^{\beta}\Psi(Y L_A^{\zeta})$$

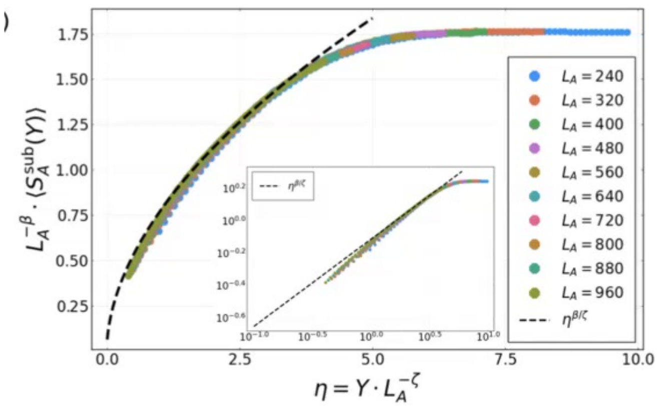
### DPRE free-energy fluctuations




$$\delta F_A(Y) = L_A^{\beta}\Psi(Y L_A^{\zeta})$$

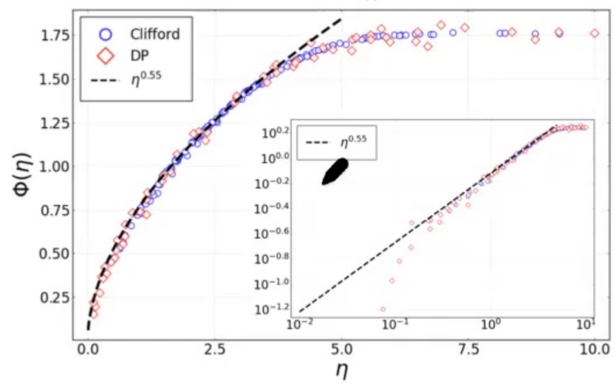
# Size scaling for sub-dominant entanglement-entropy/free-energy


### Sub-dominant Entanglement-entropy in Clifford circuit




$$S_A^{sub}(Y) = L_A^{\beta}\Phi(Y L_A^{-\zeta})$$

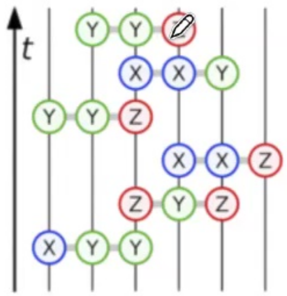
### Sub-dominant free-energy for DPRE




$$F_A^{sub}(Y) = L_A^{\beta}\Phi(Y L_A^{-\zeta})$$

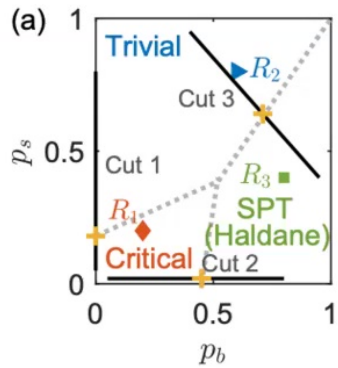
# Generalizations "Enriched" phases in measurement circuits

## Measurement-only models




Ippoliti, Gullans, Gopalakrishnan *et. al.* '20
Lavasani, Alavirad, Barkeshli '20

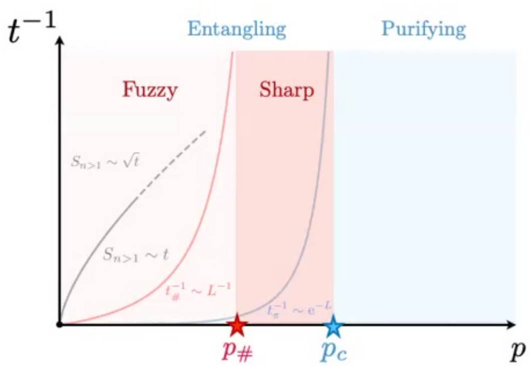
## Symmetry enriched phases




Bao, Choi, Altman '21

## U(1) Symmetric dynamics

Agrawal *et. al.* '21




# Open system measurement-protected $Z_2$ quantum phase

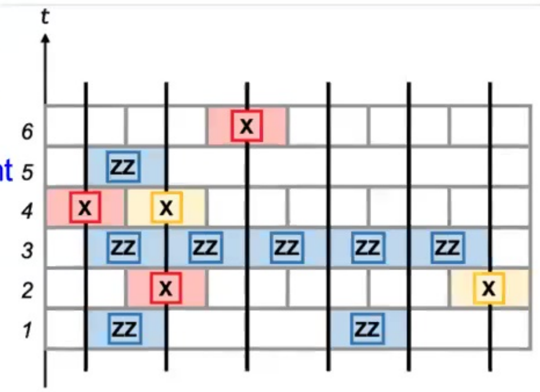
Lang and Buchler 2020
Sang and Hsieh, 2021
Bao, Choi, Altman 2021
Li and MPAF 2021


$$\mathbf{X} = \prod_i X_i$$

X (yellow box) $\rho \rightarrow \rho_\pm = P^x_\pm \rho P^x_\pm$ X-measurement

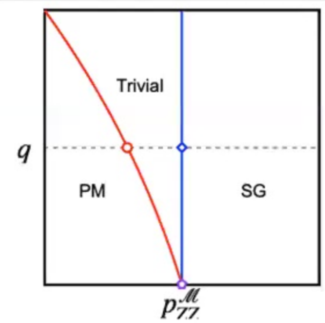
X (red box) $\rho \rightarrow (\rho + X\rho X)/2$ X-depolarization

ZZ (blue box) $\rho \rightarrow \rho_\pm = (P^{zz}_\pm \rho P^{zz}_\pm)$ ZZ-measurement




Spin glass order? $\chi_{SG} = \frac{1}{L} \sum_{i,j} |\langle Z_i Z_j \rangle_c|^2$

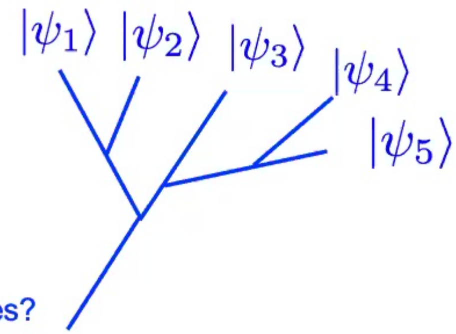
**Spin-glass order survives in 1d open system (w/ decoherence), protected by measurement (not possible in 1d equilibrium system)**



# Experimental Access?

Quantum trajectories reveal phases/transitions

Averaging over quantum trajectories washes out all effects $\rho_{mixed} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$




- Require multiple copies of same pure state $|\psi\rangle$
- Post selection on O(Lt) measurement outcomes to get copies?
  No! Must choose among $2^{Lt}$ possible (random) outcomes, to get each copy

Proposals addressing this challenge:

- Clifford circuits: Accessed via local probe

  Gullans, Huse '19
  **Noel, Niroula, Zhu *et. al.* '20**

- Space-time duals of unitary dynamics, which looks like unitary plus measurements

  Ippoliti, Khemani '20
  Ippoliti, Rakovszky, Khemani '21
  Lu, Grover '21

- Use measurement outcomes to "decode" (essentially active "error correction")

  Dennis, Kitaev, Landahl, Preskill. **Topological quantum memory**. 2000
  Li, MPAF 2021

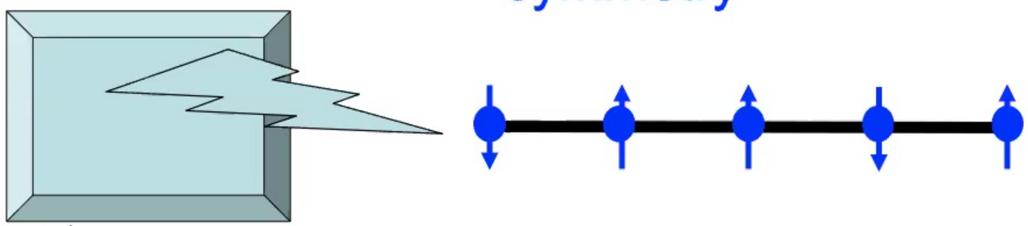
# Outline

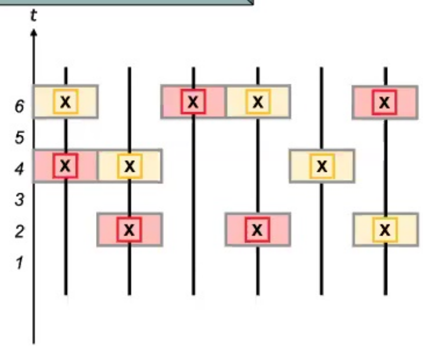
1) Measurement induced entanglement transition in random hybrid circuits

2) Nature of the "volume-law" phase (in random circuits)

**3) Active Quantum error correction in dynamical memories**

# Protecting an open (noisy) 1d quantum system with $Z_2$ symmetry






$$\mathbf{X} = \prod_i X_i$$

X $\rho \rightarrow X \rho X$ (unitary) bit-flip error

X $\rho \rightarrow (\rho + X \rho X)/2$ X-depolarization error (decoherence)

Employ active error correction

Dennis, Kitaev, Landahl, Preskill.
**Topological quantum memory**. 2000

# Toy example of error correction: Repetition "code"



"Code space" defined by conditions $Z_jZ_{j+1} = +1$ (check operators)
2d code space (1 logical qubit) has basis states $\{|\mathbf{0}\rangle = |000000\rangle, |\mathbf{1}\rangle = |111111\rangle\}$

"Code state" $\alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle$

Encode: $\alpha|0\rangle + \beta|1\rangle \Rightarrow |\Psi_0\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle$

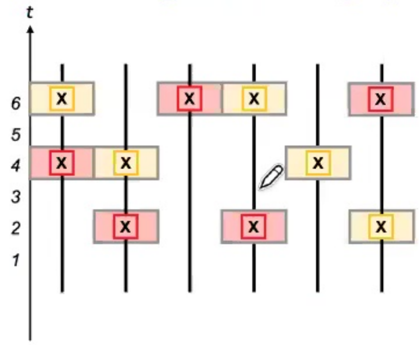
Encoded information are 2 complex numbers (unknown to us)

Bit-flip error ($3^{rd}$ Qubit $X_3$) $X_3|\Psi_0\rangle \Rightarrow |\Psi_1\rangle = \alpha|001000\rangle + \beta|110111\rangle$

1-step error-detection: Measure check operators and find "syndrome"
$Z_1Z_2 = Z_4Z_5 = Z_5Z_6 = 1$, but $Z_2Z_3 = Z_3Z_4 = -1$ detects error (but does not destroy quantum info)

Correct error: $X_3|\Psi_1\rangle = |\Psi_0\rangle$

# Protecting a Noisy system with $Z_2$ Symmetry
# Symmetry protected quantum memory



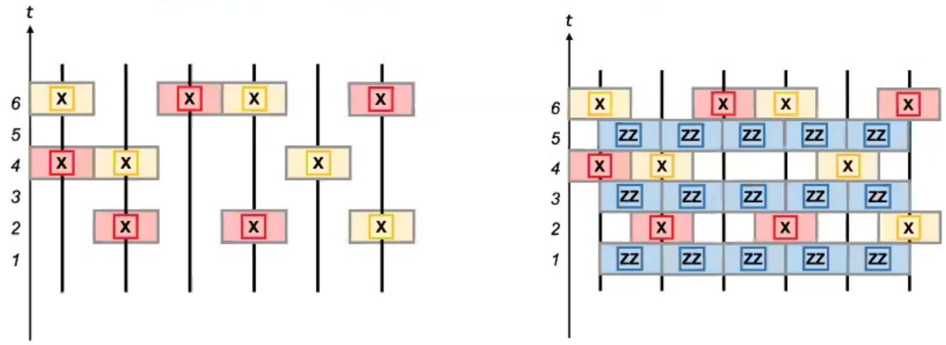

Dennis, Kitaev, Landahl, Preskill.
**Topological quantum memory**.
2000

Y. Li, MPAF arXiv: 2108.04274


X $\rho \rightarrow X\rho X$ bit-flip

X $\rho \rightarrow (\rho + X\rho X)/2$ X-depolarization (decoherence)

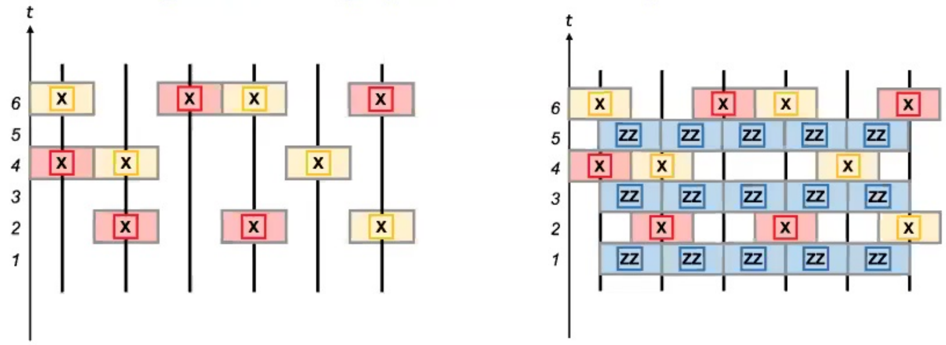
# Protecting a Noisy system with $Z_2$ Symmetry
# Symmetry protected quantum memory





| | | |
|---|---|---|
| X (yellow) | $\rho \to X\rho X$ | bit-flip |
| X (red) | $\rho \to (\rho + X\rho X)/2$ | X-depolarization (decoherence) |
| ZZ | $\rho \to \rho_\pm = (P_\pm^{zz}\rho P_\pm^{zz})$ | syndrome ZZ-measurement |

# Protecting a Noisy system with $Z_2$ Symmetry
# Symmetry protected quantum memory




X $\rho \to X\rho X$ bit-flip

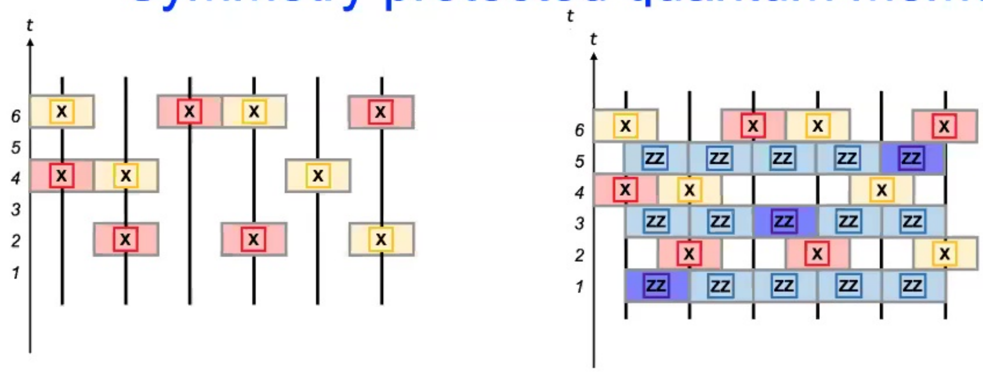
X $\rho \to (\rho + X\rho X)/2$ X-depolarization (decoherence)

ZZ $\rho \to \rho_{\pm} = (P_{\pm}^{zz}\rho P_{\pm}^{zz})$ syndrome ZZ-measurement

Assume some ZZ measurement are “faulty”,
w/ readout of ZZ being opposite to ZZ on the state

Fault tolerant error model

# Protecting a Noisy system with $Z_2$ Symmetry
# Symmetry protected quantum memory





X $\rho \to X\rho X$ bit-flip

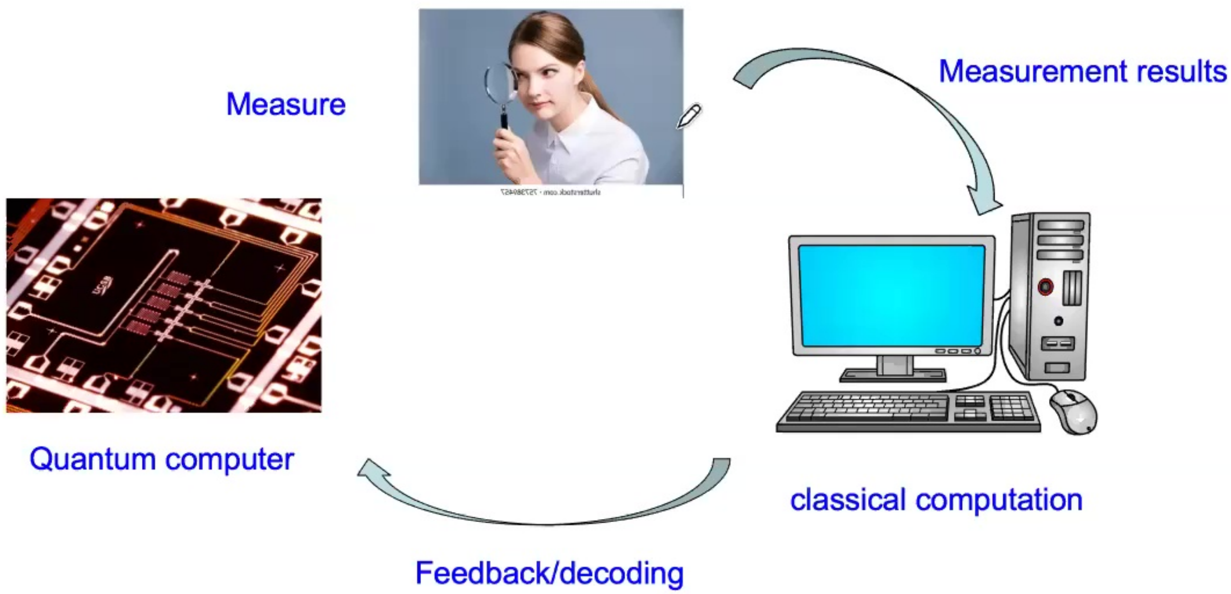
X $\rho \to (\rho + X\rho X)/2$ X-depolarization (decoherence)

ZZ $\rho \to \rho_{\pm} = (P_{\pm}^{zz}\rho P_{\pm}^{zz})$ syndrome ZZ-measurement

ZZ Faulty syndrome measurement "errors"

Assume X and ZZ errors occur with the same probability, $p_{err}$

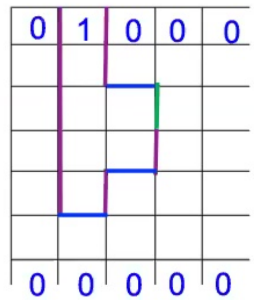
# Active Quantum dynamics




# Mapping Syndrome to (classical) Random Bond Ising model

Dennis, Kitaev, Landahl, Preskill.
**Topological quantum memory**. 2000

System dynamics
(in bit-flip model)



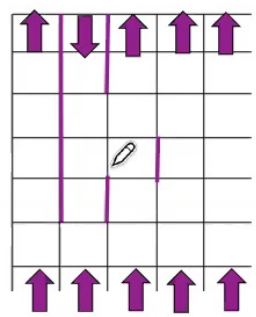

| ZZ=-1 non-trivial syndrome

— X bit-flip $p_{err}$

ZZ=+1 faulty syndrome $p_{err}$

## Map to Ising model

$$Z[\eta] = \sum_{S_i=\pm 1} e^{J \sum_{ij} \eta_{ij} s_i s_j}$$



| negative Ising bond $\eta_{ij} = -1$

| — positive Ising bond $\eta_{ij} = +1$

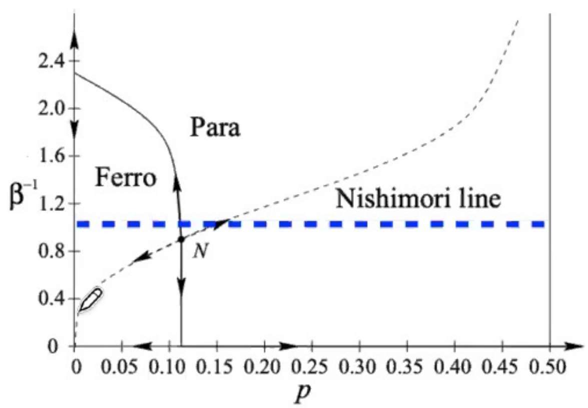
# Gauge transform to +/-J RBIM

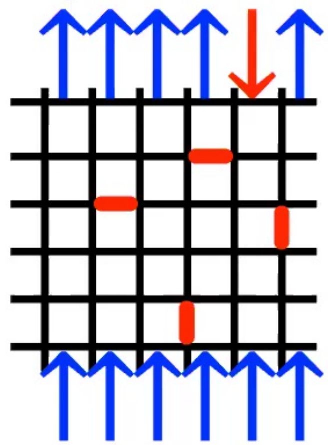
Up to a gauge transformation (same as Dennis et.al. 2000)

$$Z[\eta] = Z[\tilde{\eta}]$$

$\tilde{\eta} = +1$ probability $1 - p_{err}$

$\tilde{\eta} = -1$ probability $p_{err}$



# Encode Information in initial state

$|\mathbf{0}\rangle = |000...0\rangle \qquad |\mathbf{1}\rangle = |111...1\rangle$

Encode

$\alpha|0\rangle + \beta|1\rangle \rightarrow |\Psi_{in}\rangle = \alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle$

$\rho_{in} = |\Psi_{in}\rangle\langle\Psi_{in}|$

Quantum information

$\alpha, \beta$


## Run circuit dynamics:

$\rho_{in} \rightarrow \rho_{out} = \mathcal{C}[\rho_{in}]$

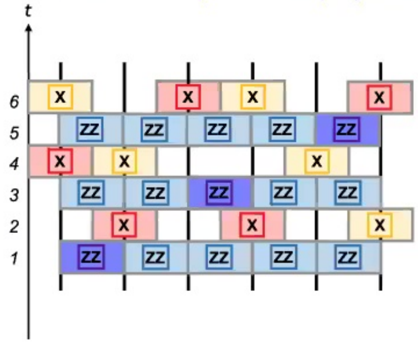
$\rho_{out} = |\Psi_{out}\rangle\langle\Psi_{out}|$

$|\Psi_{out}\rangle = \alpha|\mathbf{m}\rangle + \beta|\overline{\mathbf{m}}\rangle$

$|\mathbf{m}\rangle = |m_1, m_2, ..., m_L\rangle$

$m_i = -\overline{m}_i = 0, 1$

$\rho_{out} = |\Psi_{out}\rangle\langle\Psi_{out}|$




$\rho_{in} = |\Psi_{in}\rangle\langle\Psi_{in}|$

# Decoding protocol

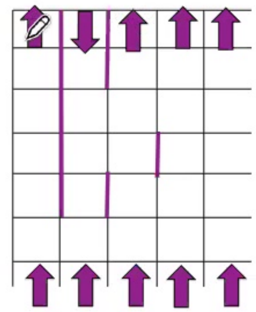
Monte Carlo on Ising model, with b.c. all up at t=0

Compute $\langle s_i \rangle_{t=T}$ $\quad \langle s_j \rangle = \frac{1}{Z[\eta]} \sum_{s_i = \pm 1} s_j e^{J \sum_{ij} \eta_{ij} s_i s_j}$

$(-1)^{\tilde{m}_i} = sgn \langle s_i \rangle_{t=T}$

Success if $\tilde{m}_i = m_i; \quad \forall i$

(assuming final syndrome measurement is free of errors)



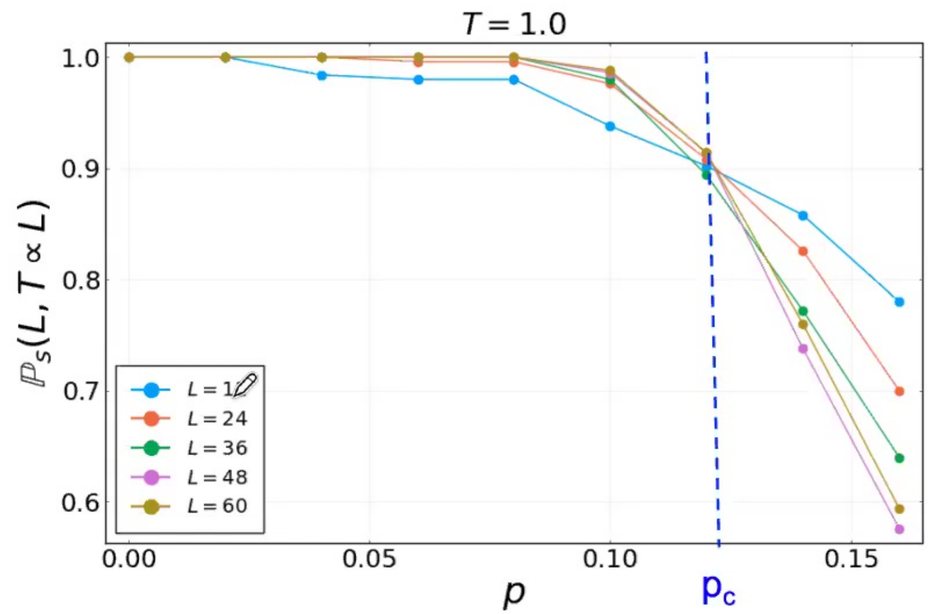
Decode: $U = \prod_i X_i^{\tilde{m}_i} \qquad \rho_{out} \rightarrow U \rho_{out} U^\dagger$

Success implies; $\quad U \mathcal{C}[\rho_{in}] U^\dagger = \rho_{in}$

# Success probability in noisy Clifford $Z_2$ simulation

p=probability of X-depolarizing errors
p=probability of faulty ZZ measurements

Y. Li, MPAF arXiv: 2108.04274



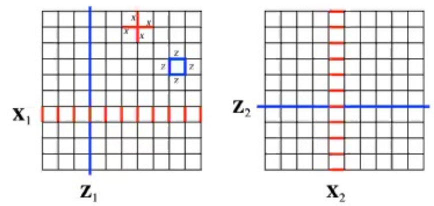

For $p<p_c$ as L becomes large, probability of recovery approaches $P_s = 1$


# Some generalizations

- Repetition code in higher dimensions, d.
  Map to random d+1 classical Ising model
  Decoding via Monte Carlo:

- Toric code in 2+1

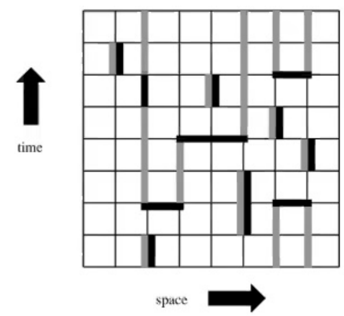
  Map to 3d Random $Z_2$ Gauge theory (Dennis et al)
  Decode: (in progress)
  compute Wilson loop in $Z_2$ gauge theory via Monte Carlo




Comparing results with previous work;

Dennis, Kitaev, Landahl, Preskill. 2000
Wang, Harrington, Preskill. 2002

Decoding RBIM and Random $Z_2$ Gauge theory by using minimal weight perfect matching (MWPM), to find the most likely error history



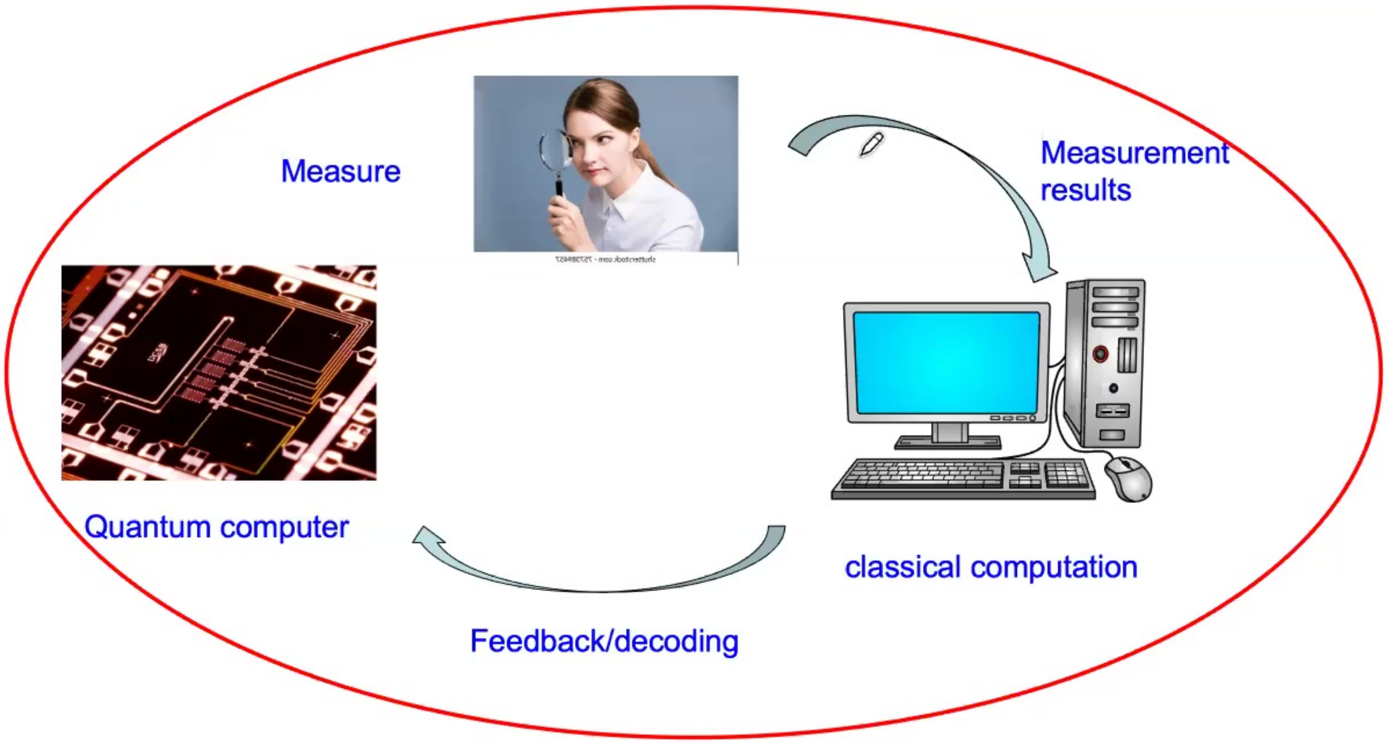

# Comparison with MWPM (Dennis et.al.)

Size scaling of (classical) decoding times for quantum circuits running for T=O(L)

| | (1+1)-d rep. code | (2+1)-d rep. code | (2+1) Toric code |
|---|---|---|---|
| $t_{decode}$ (MWPM) Dennis et.al. 2000 | $O(L^6 \ln L)$ | NP-Hard (exponential scaling) | $O(L^9 \ln L)$ |
| $t_{decode}$ (Monte Carlo) (Li, MPAF 2021) | $O(L^4)$ | expect $O(L^5)$ (Path summation - rather than Monte Carlo) $O(L^3)$ | ?? perhaps $O(L^5)$ ?? |

# Novel Quantum Dynamical Phases?

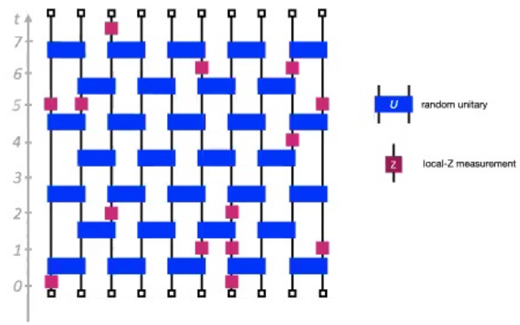
## (beyond active error correction…)




# Summary: Entanglement Transitions

**Quantum Entanglement Transition in "monitored" systems**:

Competition between unitary induced entanglement and measurement induced disentanglement





Open questions

- Universality class of transition in d=1 (CFT)?
- Experimental observation of entanglement transition (ion trap quantum computer)?
- (Fault tolerant) Active Quantum error correction in non-Abelian topological phases?
- Active Quantum error correction in fracton codes (Ising model generalizations)?
- Novel Quantum dynamical phases in monitored open systems w/ decoherence?

# Novel Quantum Dynamical Phases?

## (beyond active error correction…)