Title: Beyond Chance and Credence

Speakers: Wayne Myrvold

Series: Quantum Foundations

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Abstract: This talk is about how to think about probabilistic reasoning and its use in physics. It has become commonplace, in the literature on the foundations of probability, to note that the word "probability" has been used in at least two distinct senses: an objective, physical sense (often called "objective chance"), thought to be characteristic of physical situations, independent of considerations of knowledge and ignorance, and an epistemic sense, having to do with gradations of belief of agents with limited information about the world. I will argue that in order to do justice to the use of probabilistic concepts in physics, we should go beyond this familiar dichotomy, and make use of a third concept, which I call "epistemic chance," which combines epistemic and physical considerations.

Beyond Chance and Credence

Perimeter Institute Quantum Foundations Seminar
October 22, 2021

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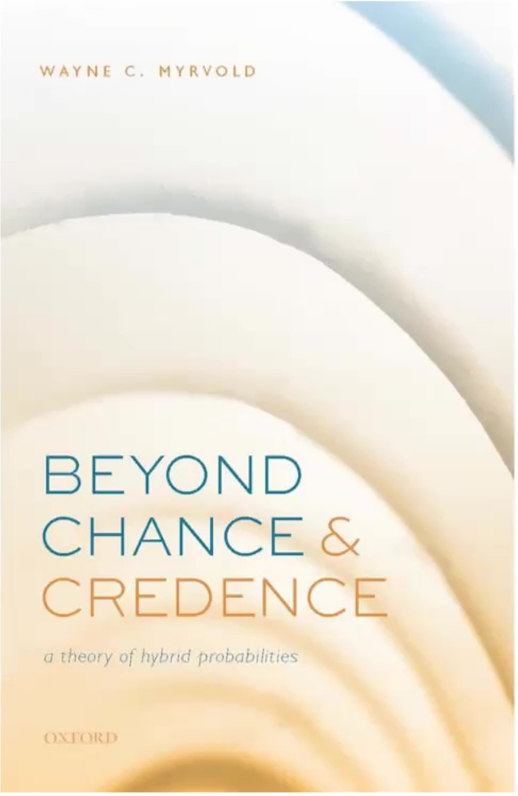
Based on…

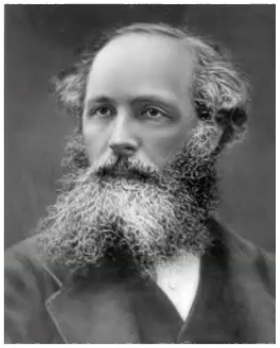


WAYNE C. MYRVOLD
BEYOND CHANCE & CREDENCE
a theory of hybrid probabilities
OXFORD

Probabilities… who needs them?

- *Statistical* v.*dynamical* regularities
  - The former are explained via the (weak) law of large numbers, and hold only with high probability.
- J.C. Maxwell (1873):

those uniformities which we observe in our experiments… are uniformities of the same kind as those explained by Laplace and wondered at by Buckle, arising from the slumping together of multitudes of cases, each of which is by no means uniform with the others.



Two senses of the word "probability"

Epistemic

- Degrees of belief of an (ideally rational) agent.
- I will call these: *Credences*

Aleatory

- Characteristic of the physical circumstances leading up to an event.
- I will call these: *Chances*

Not two rival theories of probability.

Is Chance compatible with determinism?

- In the 18th and 19th centuries, many writers on probability (*e.g.* Jacob Bernoulli 1713, Laplace 1814, de Morgan 1838, Jevons 1874) avowed a conception of probability that is wholly epistemic.
- Here's Bernoulli:
  - In themselves and objectively, all things under the sun, which are, were, or will be, always have the highest certainty. …Seen in relation to us, the certainty of things is not the same for all things, but varies in many ways, increasing and decreasing. . . . *Probability*, indeed, is degree of certainty

A tension

- Here's Bernoulli, again, on games of chance:
  - The originators of these games took pains to make them equitable by arranging that the numbers of cases that result in profit and loss be definite and known and that all the cases happen equally easily [*pari facilitate obtingere possent*].
- Is a die being fair (all faces equiprobable) a feature of the physical set-up?
- There seems to be a need for a concept of chance on which
  - there is a matter of fact about the chance of getting a six on a die, and
  - claims about these chance can be tested empirically.

This tension remains

- In statistical mechanics textbooks, probabilities are usually said to be introduced because of our ignorance of the microstate of the system.
- These textbooks also stipulate particular probability distributions for various equilibrium states, and say that the choice is justified by agreement with experiment.

The tension

- Does determinism make nonsense of the idea that there is a matter of fact about such things as the bias of a coin, claims about which are subject to empirical test?

Two things that don't work

- Attempts to define objective probabilities by mere counting of possibilities.
  - Requires a judgment about which possibilities are "equally possible."
- Attempts to define objective probabilities in terms of frequencies.

On counting possibilities: a dialogue

Laplace

the definition of probability is ... the ratio of the number of favorable cases to that of all the cases possible.

But that supposes the various cases equally possible. If they are not so, we will determine first their respective probabilities, whose exact appreciation is one of the most delicate points of the theory of chance.

Also Laplace

Probabilities in statistical mechanics

- Is the probability of a macrostate in stat. mech. just the size of the state?
- This means, for an isolated system with definite energy:
  - In classical mechanics, microcanonical measure.
  - In quantum mechanics, dimension of a subspace.
- With Laplace, we ask: why *this* way of counting?
- I claim: rationale for this has to do with the process of equilibration, and doesn’t transfer to non-equilibrium situations.

Chance and frequency

- There are links between chances and relative frequencies.
  - Suppose there are n balls in an urn, m of which are black, and that a ball is drawn in such a way that each is equally likely. Then the chance of a black ball being drawn is m/n.
  - (Strong Law of Large Numbers) In an infinite series of probabilistically independent events, each of which has probability p, with probability one, the relative frequency will converge to p.

Chance and frequency

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  - Suppose there are n balls in an urn, m of which are black, and that a ball is drawn in such a way that each is equally likely. Then the chance of a black ball being drawn is m/n.
  - (Strong Law of Large Numbers) In an infinite series of probabilistically independent events, each of which has probability p, with probability one, the relative frequency will converge to p.
- But these don't suffice for defining objective probability in terms of relative frequencies.
- Just saying them requires a notion of probability distinct from frequency!

Where does this leave us?

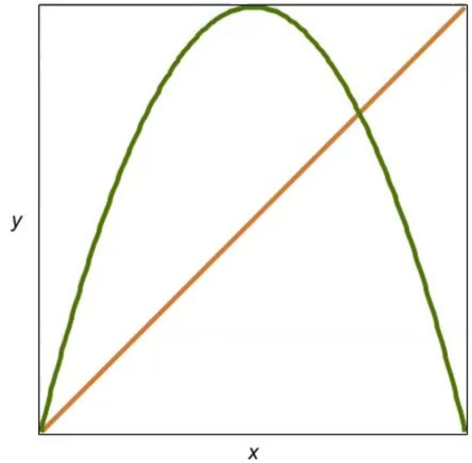
- There is an important and legitimate role for a notion of "chance" such that
  - The value of a chance does not depend on what anyone thinks of it
  - The value of a chance *does* depend on physical features of the chance set-up
  - We can formulate and test hypothese about values of chances
  - We can explain why we should expect there to be statistical regularities of the sort that we observe
  - Use of this notion does not require commitment to indeterminism in fundamental physics

A clue (on the right track)

- Bernoulli, Laplace, Cournot, and others, talk of some things occurring more easily than others….
- There's something to that! For example, it is harder to balance a pencil on its point than it is to lay it down flat.

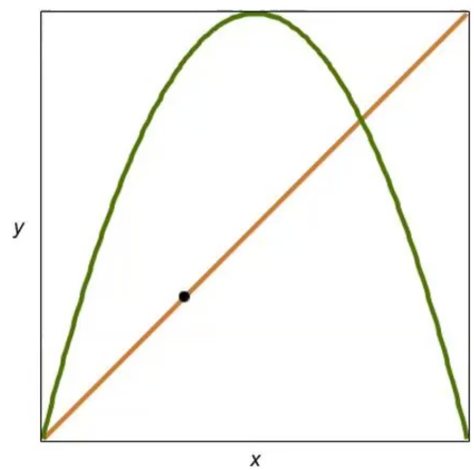
An example: the parabola gadget

- Parabola gadget:
  - A board, one meter square, on which is inscribed a parabola and a diagonal.



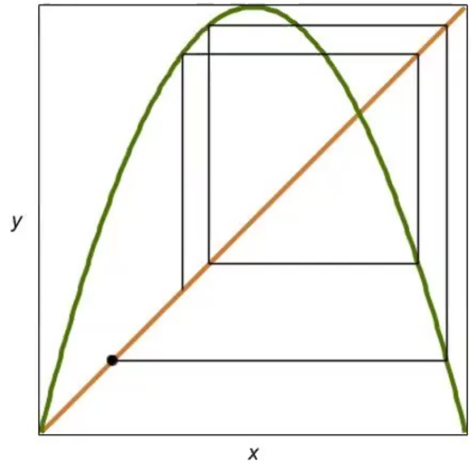
Dynamics of the gadget

- A ball starts out somewhere on the diagonal.



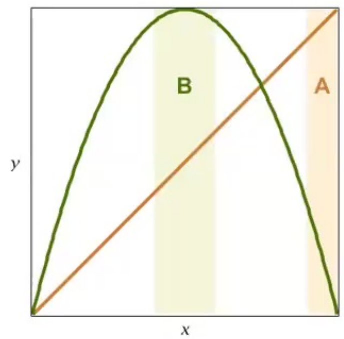
Dynamics of the gadget

- A ball starts out somewhere on the diagonal.
- It moves vertically towards the parabola until it hits it.
- It then moves horizontally toward the diagonal until it hits it.
- Repeat.



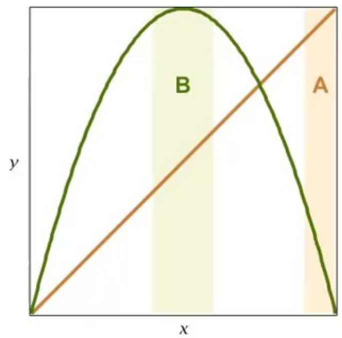
Let's play a game

- A parabola gadget has been running for a while (at least ten iterations). Which do you regard as more probable?



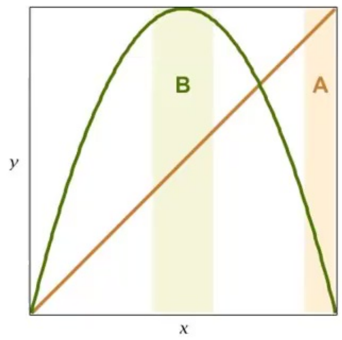
A. The ball is within 10 cm. of the right side.
B. The ball is within 10 cm. either side of the center.

Bob's reasoning



$$\frac{\text{Prob}(B)}{\text{Prob}(A)} = \frac{\textit{measure of B states}}{\textit{measure of A states}} = 2$$

Bob's reasoning

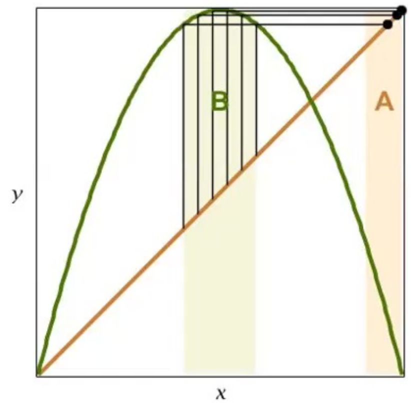


$$\frac{\mathrm{Prob}(B)}{\mathrm{Prob}(A)} = \frac{measure\ of\ B\ states}{measure\ of\ A\ states} = 2$$

- Bob bets on *B*.

Alice's reasoning

- There's something unstable about Bob's credences, that ascribe equal probabilities to equal lengths on the diagonal.



Alice's reasoning

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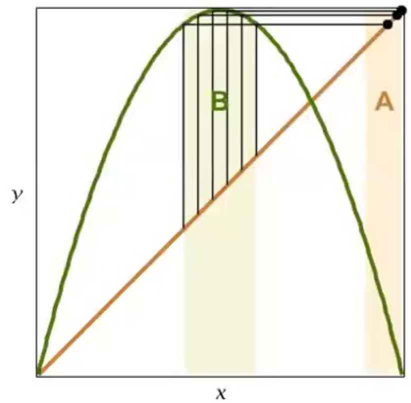


- In one iteration, all points in *B* end up in *A*!

Alice's reasoning

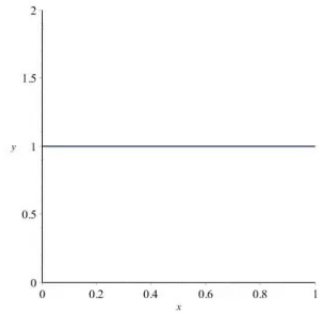
- The range of *x*-values that map into [.99,1] has width much greater than 0.01.
- Initial probability distributions will tend to go into ones that give more weight to the extremes.

Evolving probability distributions

- Suppose we have a physical system, and a dynamical map T that takes state at time t_0 to state at time t_1.

Evolution of initial distributions

- After just 5 iterations,



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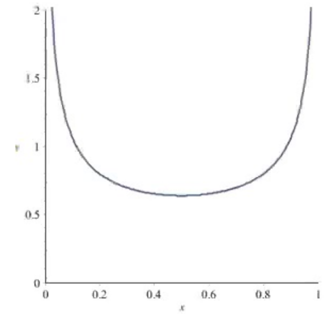
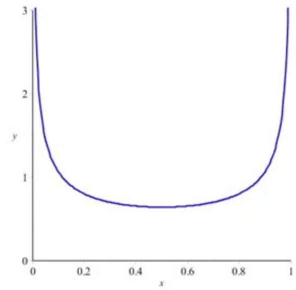
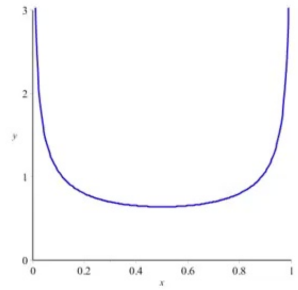


A “attractor”measure



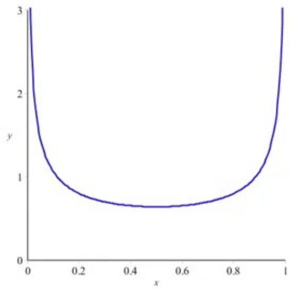
- A wide range of measures over initial conditions yield virtually the same probabilities for conditions after say, 10 iterations.

A "attractor"measure



- A wide range of measures over initial conditions yield virtually the same probabilities for conditions after say, 10 iterations.
- This "attractor" measure is invariant under evolution.

A "attractor" measure



- A wide range of measures over initial conditions yield virtually the same probabilities for conditions after say, 10 iterations.
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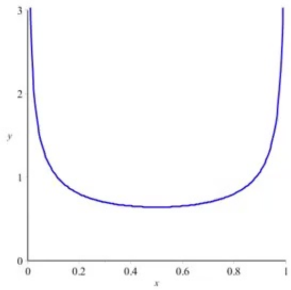
- It is picked out by the dynamics, not just the structure of the state space.

A "attractor" measure

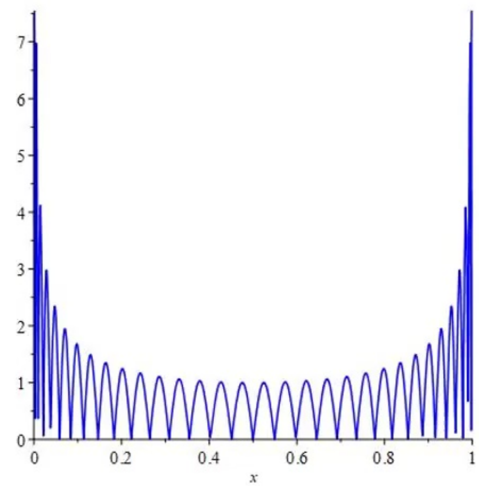


- A wide range of measures over initial conditions yield virtually the same probabilities for conditions after say, 10 iterations.
- This "attractor" measure is invariant under evolution.
- It is picked out by the dynamics, not just the structure of the state space.
- On this measure,

$$\frac{measure(A)}{measure(B)} \approx \frac{8}{5}$$

Example

- Suppose Bob thinks that, after 5 iterations, equal intervals of the diagonal are equally probable.
- Are there a probability distributions over initial conditions that will do this?
- Yes. Here's one density function that does it.

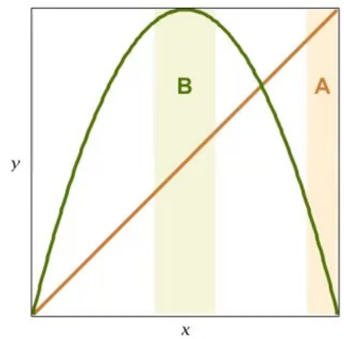


Can *B* be more probable than *A*?

- There *are* probability distributions over initial conditions that make *B* more probable than *A* after *n* iterations.
- For large *n*, these vary *very rapidly* over the space of initial conditions.
- Moreover, provided they can be represented by a density function, after sufficiently many iterations *A* becomes more probable than *B*.

Another game

- 1,000 parabola gadgets have been running for a while (at least ten iterations). Which do you regard as more probable?



A. More of them have balls in region *A* than in region *B*.
B. More of them have balls in region *B* than in region *A*.

- No particular care is taken in setting initial conditions.
- Believing more strongly in *B* requires *absurdly* precise knowledge of initial conditions.

Ingredients

- Suppose we have for some physical system:
  - Limitation of control over state at some time t_0 and of knowledge about it.
  - Judgment (perhaps vague) of which credences about initial state are reasonable, given this knowledge (not too fine-grained).
  - Dynamical laws taking initial state into later state.
  - Restriction to probabilities of results of feasible measurements.

Epistemic Chances

- Suppose we have:
  - A physical system;
  - A dynamical map T on its state space, taking state at time t_0 to state at time t_1;
  - A class C of credences, which are those that could represent to credence of a reasonable agent about the state of the system at time t_0;
  - A threshold ε, below which differences in credences are to be regarded as negligible;
- Any credence function P about the state at time t_0, can be evolved, via T, to yield probabilities P_T about the state at time t_1.

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  - A physical system;
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  - A threshold ε, below which differences in credences are to be regarded as negligible;
- Any credence function P about the state at time t_0, can be evolved, via T, to yield probabilities P_T about the state at time t_1.
- If, for some number p, every credence function P in C, $P_T(A)$ is within ε of p, then we say that p is an *epistemic chance* of A at t_1.

Neither chance nor credence

- If an objective chance is meant to be something an event has “by its nature,” without consideration of limitations of knowledge, these epistemic chances are not objective chances.

Epistemic chances

- Set-ups of games of chance, and the sorts of systems we successfully apply statistical mechanics to, exhibit the sort of sensitivity to initial conditions that permits the sort of washing out of differences between credences about initial conditions required for the existence of epistemic chances.

Neither chance nor credence

- If an objective chance is meant to be something an event has “by its nature,” without consideration of limitations of knowledge, these epistemic chances are not objective chances.
- They are not to be identified with credences: an agent might be uncertain about the result of evolving, *via the actual dynamics*, her credences about states at t_0 to t_1.

Epistemic chances

- Set-ups of games of chance, and the sorts of systems we successfully apply statistical mechanics to, exhibit the sort of sensitivity to initial conditions that permits the sort of washing out of differences between credences about initial conditions required for the existence of epistemic chances.
- I claim: examination of the use of probability in statistical mechanics shows that they are well-suited to play that role.

Conclusions

- Dynamics can pick out a special class of measures, for systems that have been evolving for a while.
- Foundational work in statistical mechanics should pay more attention to equilibration results:
  - Theorems to the effect that, under mild conditions on initial state and the dynamics, probability distributions over restricted set of variables tend towards thermal distributions.
  - These exist for both classical and quantum physics.
- These are of no avail in forming judgments about Initial State of the Universe.

Thank you

- For more on this topic…

Overview

Description

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A Theory of Hybrid Probabilities

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