Title: Resolution of the black hole information paradox: The Fuzzball Paradigm

Speakers: Samir Mathur

Series: Colloquium

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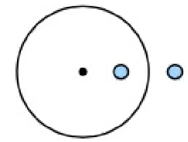
Abstract: We will begin by explaining the black hole information paradox, starting from first principles. We will then explain how computations in string theory yield a resolution of this paradox. When we make a bound state of strings and branes, then this bound state is found to swell up into a horizon sized `fuzzball'; this fuzzball radiates like a normal body without any information loss. The existence of these fuzzballs implies a new picture for the quantum gravitational vacuum, where the virtual fluctuations resemble the scale free fluctuations at a second order phase transition, rather than being confined to within the planck scale. We will see how this `vecro' picture of the vacuum might give a resolution to several puzzles we face in cosmology, like the origin of energy needed for inflation and the existence of a cosmological constant.

Resolution of the black hole information paradox: The fuzzball paradigm

Samir D. Mathur

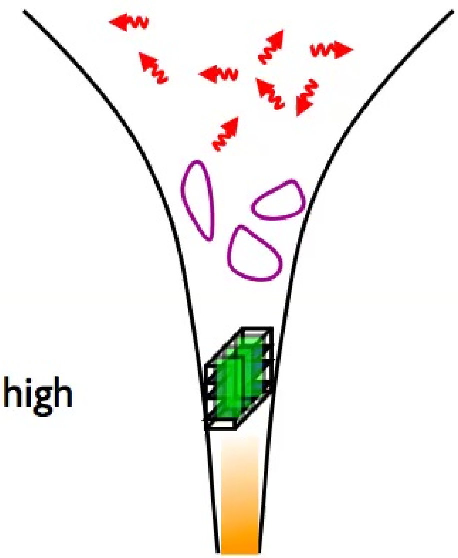
The Ohio State University

Black holes present us with a sharp question:



What is the resolution of the black hole information paradox ?

Cosmology presents us with many puzzles



(i) What is the cosmological constant so small?

(ii) What gives the energy needed to drive inflation?

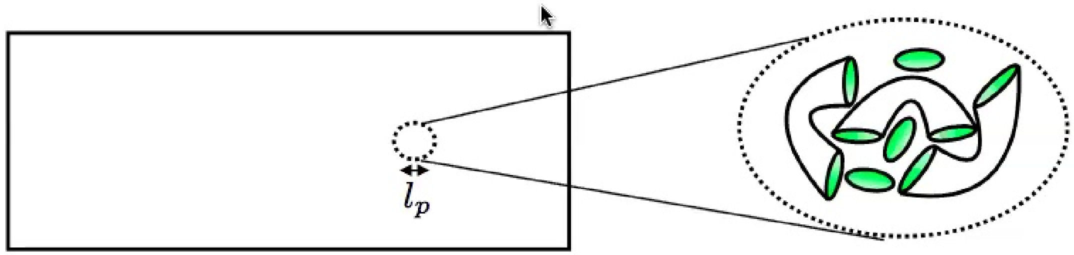
(iii) Why are the Hubble constant values between low and high redshift observations not agreeing?

In this talk we will see how string theory gives a resolution of the paradox.

The picture that emerges is called the fuzzball paradigm

The results also indicate a path to understanding the above questions in cosmology

Our conventional picture is that quantum gravity effects are relevant only at distances smaller than the planck length



Quantum gravity

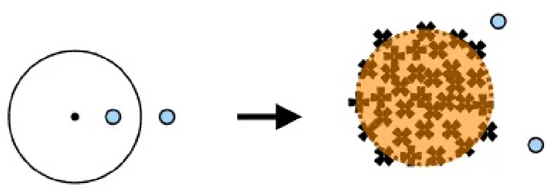
Quantum fields, Classical gravity

$$l_p \sim \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-33}\, cm$$

But if a large number N quanta are involved, then is the scale of quantum gravity

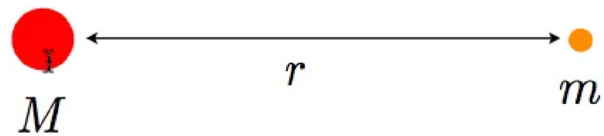
l_p or $N^{\alpha}\, l_p$??

In string theory we can explicitly construct microstates of black holes, and we find that the length scale is $N^{\alpha}\, l_p$, with α such that the length scale is the radius of the horizon



The fuzzball paradigm

Gravity is an attractive force



$$PE = -\frac{GMm}{r}$$

By itself, a mass has an intrinsic energy $E = mc^2$

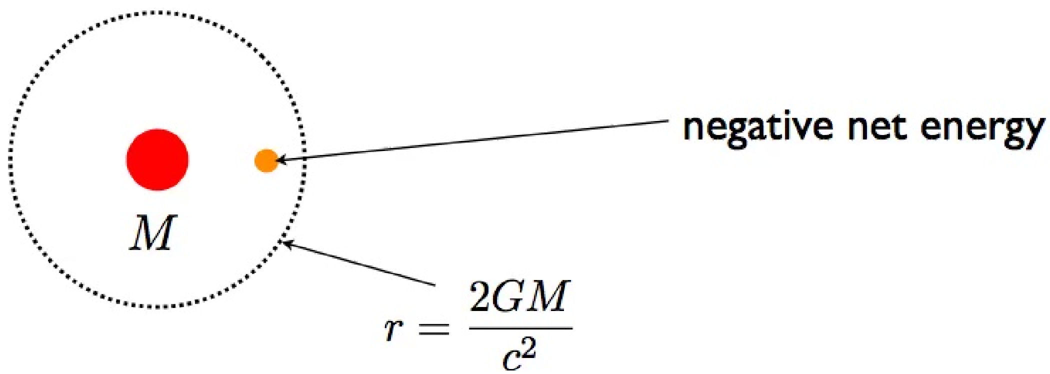
Thus we can schematically write the total energy of the mass m as

$$E = mc^2 - \frac{GMm}{r}$$

We see that $E < 0$ for $r < \frac{GM}{c^2}$

Doing this properly with general relativity changes this only by a numerical factor

We find $E < 0$ for $r < \frac{2GM}{c^2}$



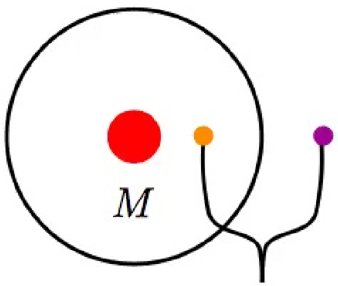
The radius $r = \frac{2GM}{c^2}$ is called the horizon radius

If we now consider quantum mechanics, then we find that the vacuum around the horizon becomes unstable

In quantum mechanics, the vacuum can have fluctuations which produce a particle-antiparticle pair

$$\Delta E \, \Delta t \sim \hbar$$

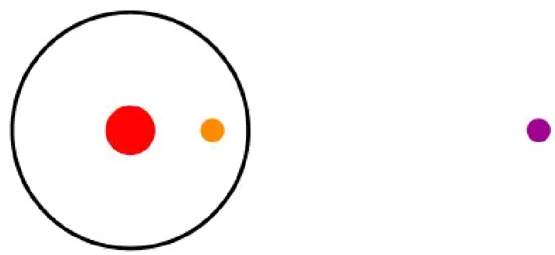
If a fluctuation happens near the horizon, the particles do not have to re-annihilate

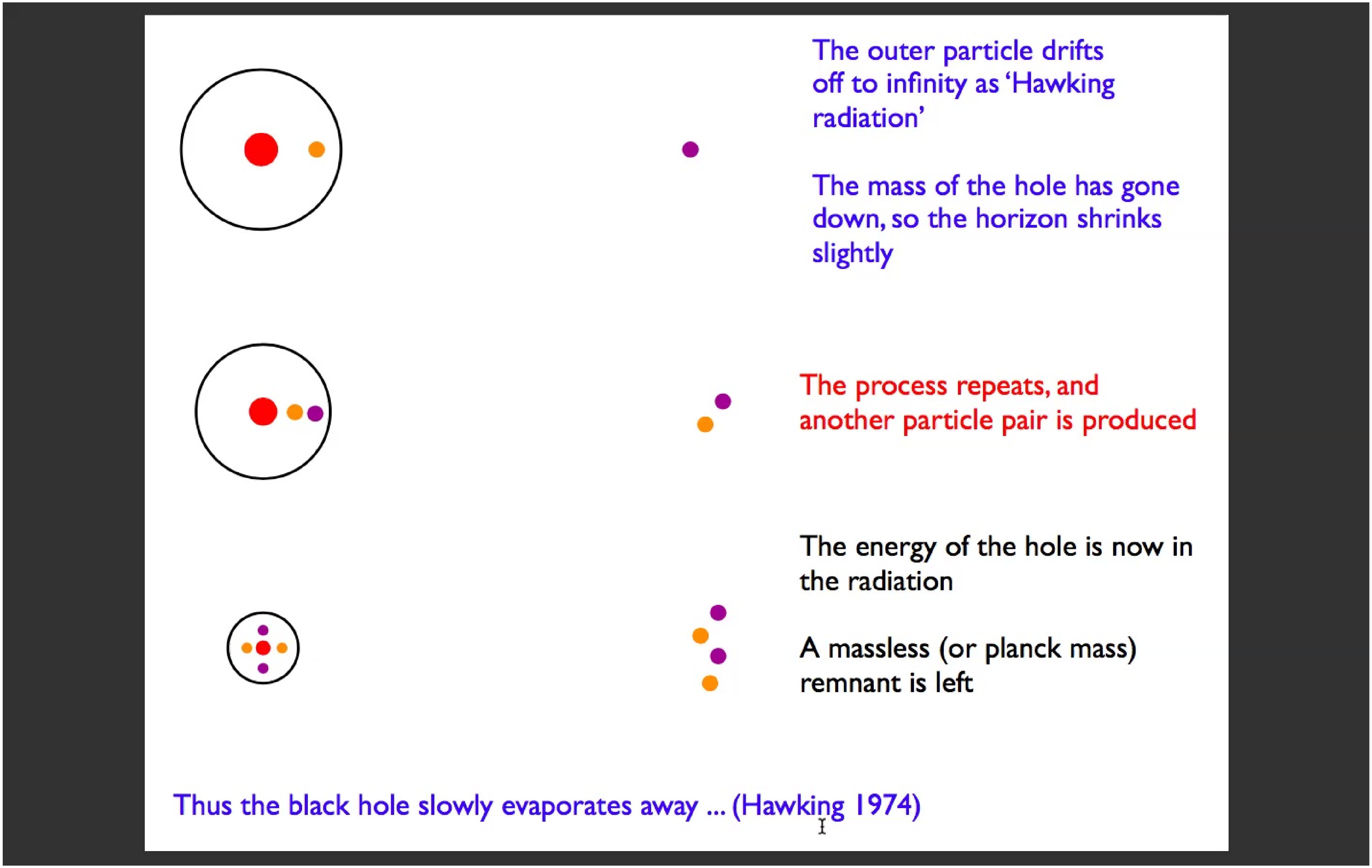


$$\Delta E = 0 \quad \rightarrow \quad \Delta t = \infty$$

The outer member drifts off to infinity as Hawking radiation

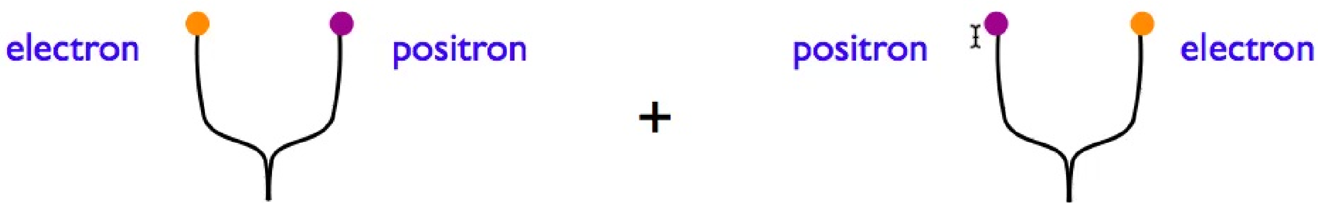
The inner member falls into the hole and reduces its mass



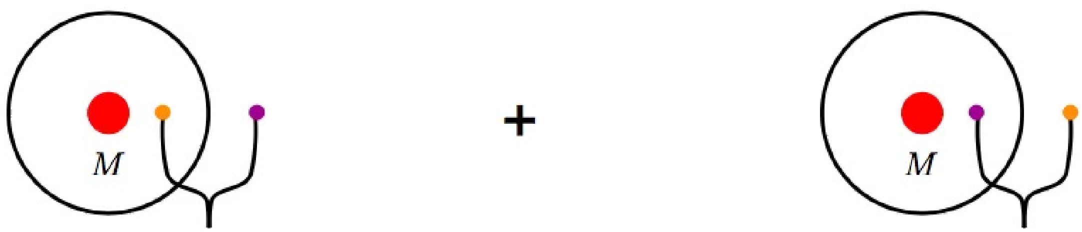

The outer particle drifts off to infinity as 'Hawking radiation'
The mass of the hole has gone down, so the horizon shrinks slightly
The process repeats, and another particle pair is produced
The energy of the hole is now in the radiation
A massless (or planck mass) remnant is left
Thus the black hole slowly evaporates away ... (Hawking 1974)

But in 1975 Hawking noticed that this radiation process led to a severe problem

Vacuum fluctuations produce entangled states ...

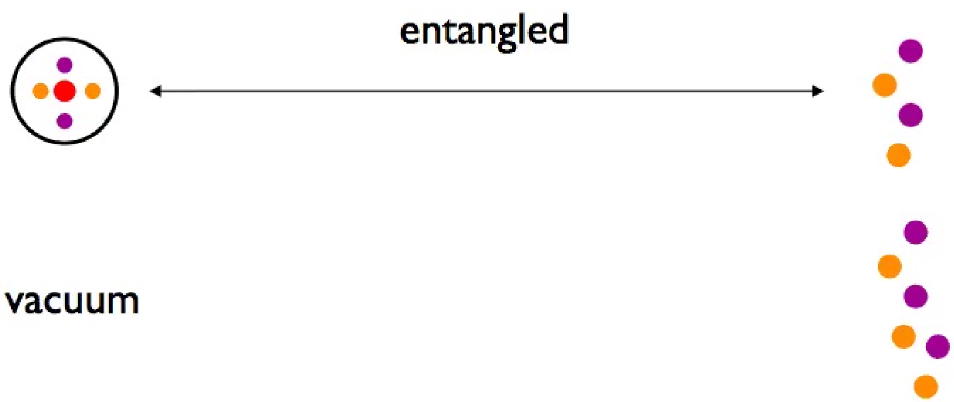


So the state of the radiation is entangled with the state of the remaining hole



There is of course nothing wrong with having entanglement between two well separated systems

The problem comes if one of these spins vanishes from the universe; then there is no well defined state for the other electron



Entanglement keeps growing with each emission

There is no quantum state for these radiation quanta (violation of quantum theory)

This is Hawking's black hole information paradox (1975)

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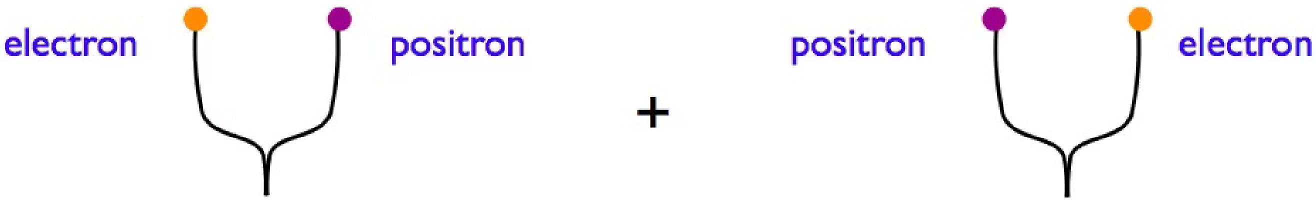

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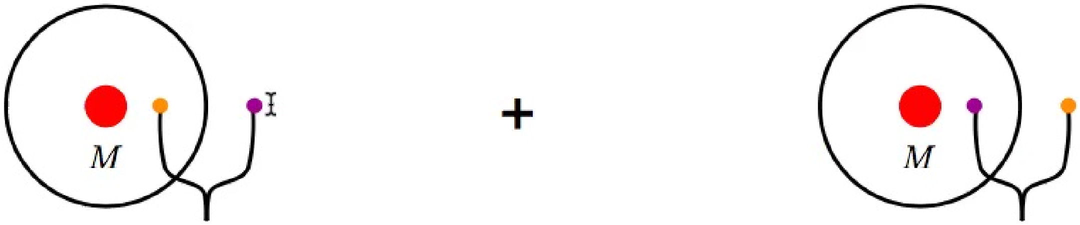

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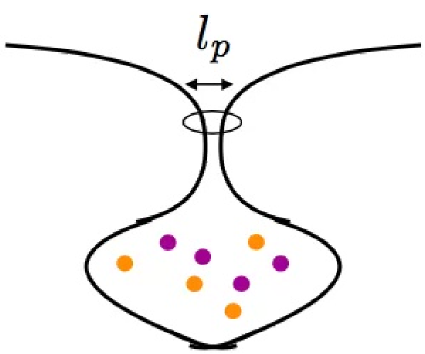
People were not happy with Hawking's claim that black holes violate quantum theory

Perhaps some unknown quantum gravity effect stops further evaporation when the horizon radius reaches planck length ?

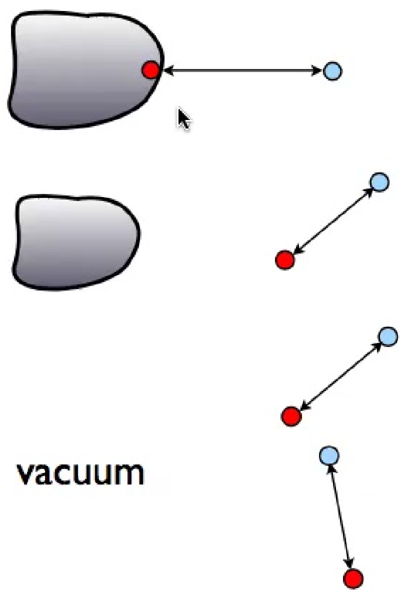
But the planck sized remnant must hold an arbitrary number of internal states, since we could have started with a black hole of arbitrary size



One model is that the matter in the hole disappears into a baby universe

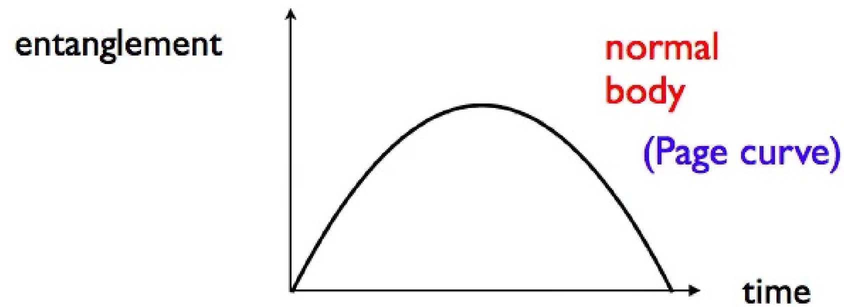
But classical general relativity does not allow such geometries if the matter energy-momentum tensor is assumed to have the usual positivity properties

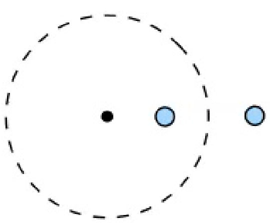
How do black holes differ from other radiating bodies ?



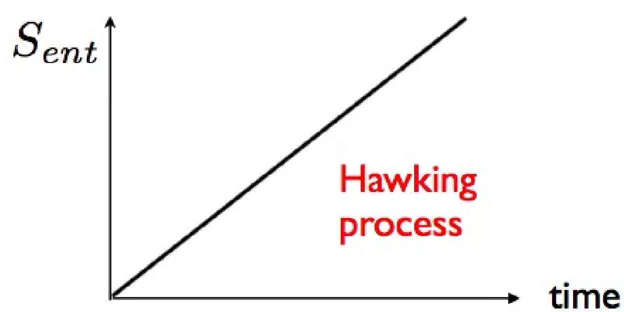
Emitted photon will be entangled with the atom which emitted it

But after some time the atom floats out like ash, so only things at infinity are entangled with each other





A black hole creates entangled pairs from the vacuum



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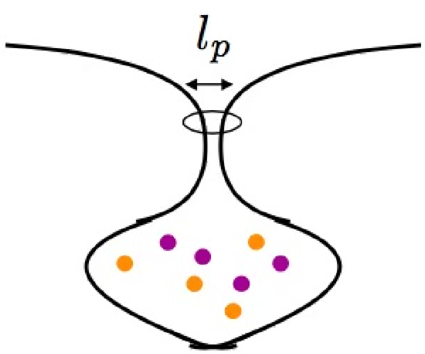

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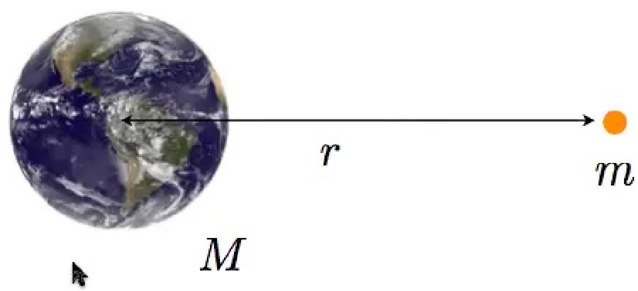
But classical general relativity does not allow such geometries if the matter energy-momentum tensor is assumed to have the usual positivity properties

Why do we think a black hole has this structure? A planet is not like this ...

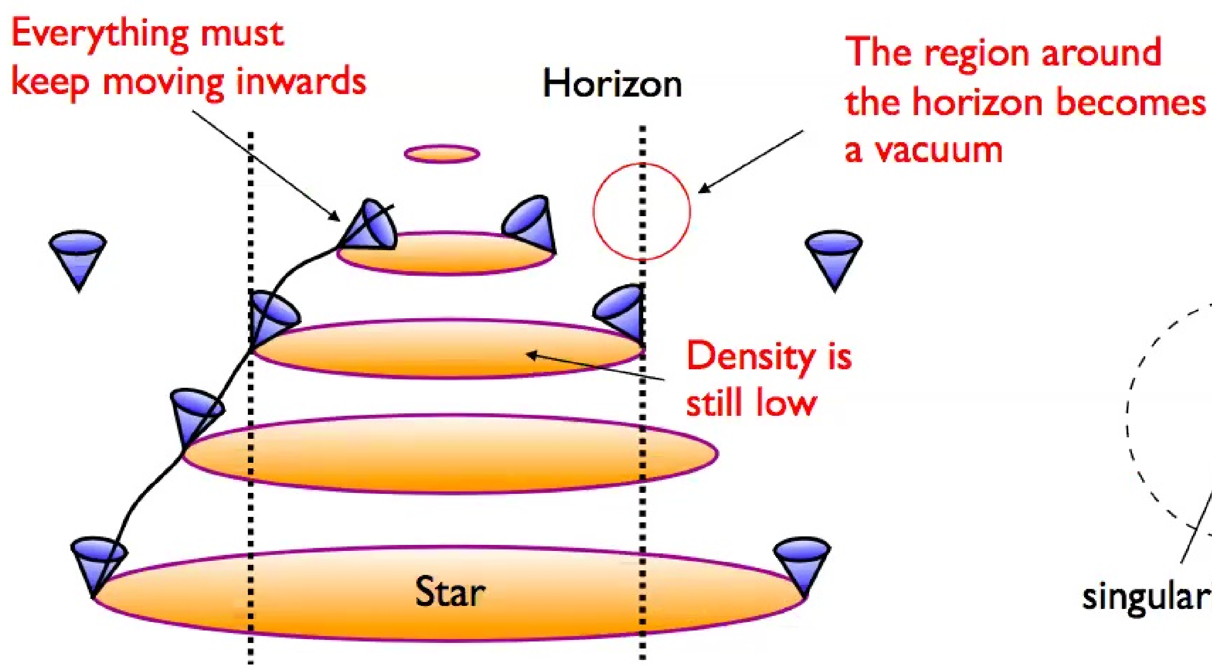


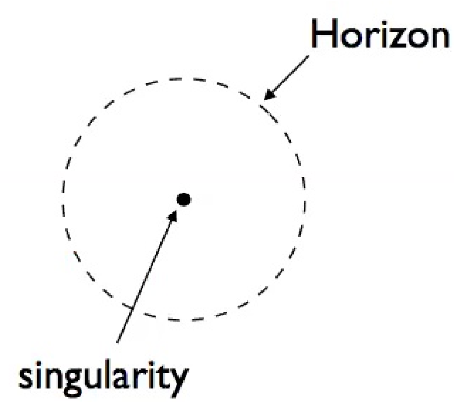
The expression

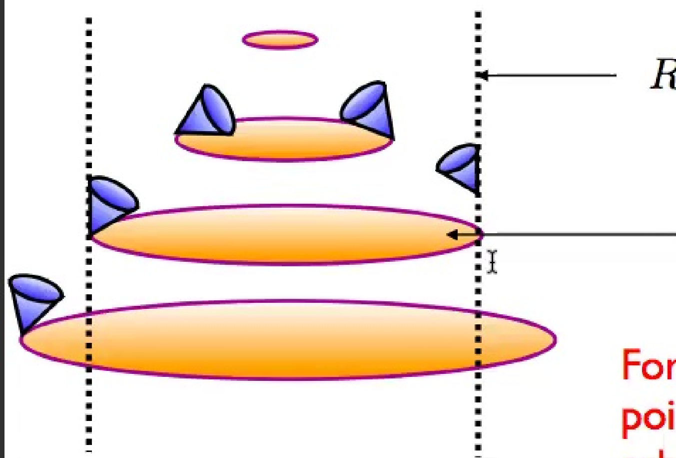
$$PE = -\frac{GMm}{r}$$

works only upto the surface of the earth

But in a black hole, matter suffers a runaway collapse to a singularity at $r = 0$





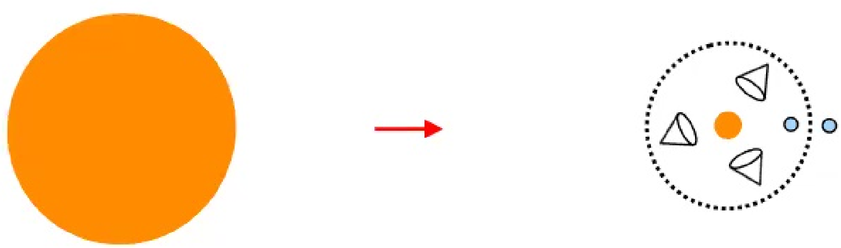


$$R = \frac{2GM}{c^2}\,, \qquad M \sim \rho R^3$$

$$\rho \sim \frac{c^6}{G^2}\frac{1}{R^2}$$

For sufficiently large black holes, the density at the point where the star passes into its horizon is arbitrarily low !!

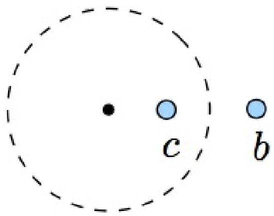
Locally, the spacetime is gently curved, so by the equivalence principle, the atoms in the star just fall smoothly through the horizon



The light cone structure then maintains the horizon, the Hawking process creates entangled pairs, and we get a sharp puzzle at the endpoint of evaporation

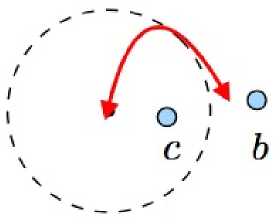
Given the importance of this puzzle, why was more time not spent by string theorists and others in trying to resolve it?

There was quite a pervasive belief that the problem could be resolved in one way or another by small corrections



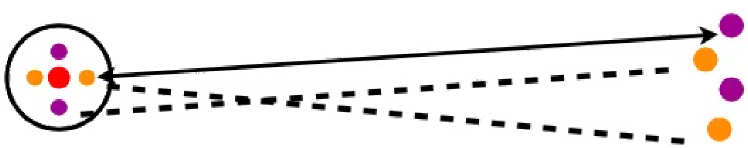
$$\frac{1}{\sqrt{2}}(\uparrow_b\downarrow_c - \downarrow_b\uparrow_c)$$

Hawking's original calculation

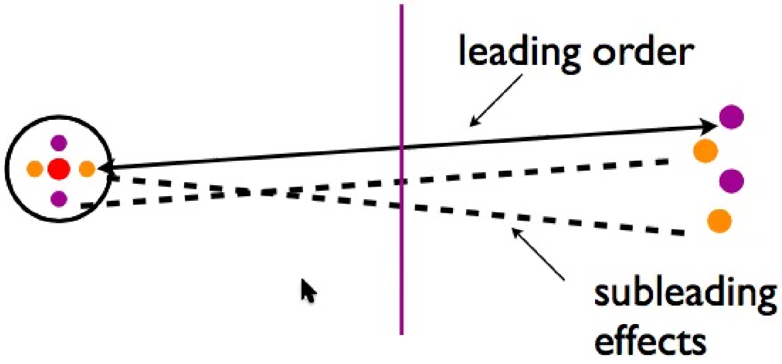


$$\frac{1}{\sqrt{2}}(\uparrow_b\downarrow_c - \downarrow_b\uparrow_c) \; + \; O(\epsilon)$$

Includes any small quantum gravity effects that may leak outside the light cone



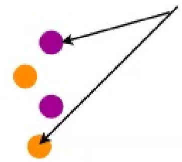
These corrections can induce slightly different entanglements than Hawking's leading order calculation



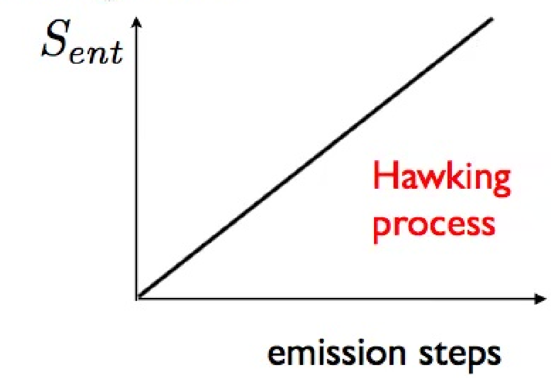
Number of emitted quanta is very large $\sim (M/m_p)^2$

Can small corrections remove the entanglement between the radiation and the planck sized hole?

vacuum

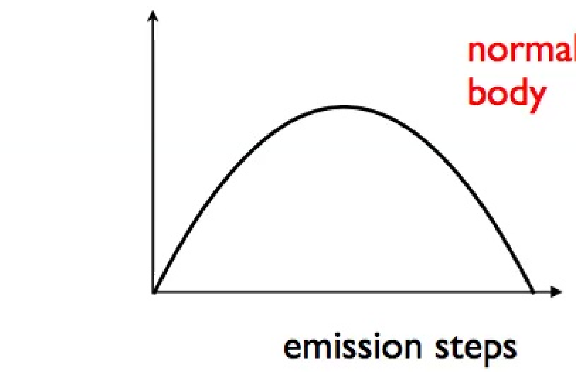


⁇

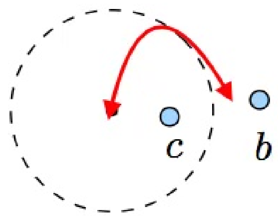


small corrections

⁇



But the small corrections theorem showed that this cannot happen

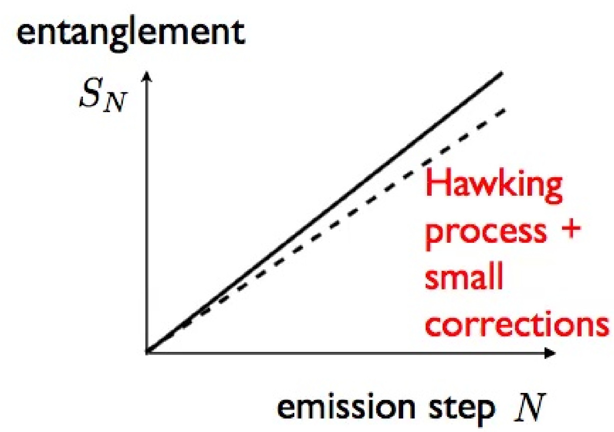


$$\frac{1}{\sqrt{2}}(\uparrow_b\downarrow_c - \downarrow_b\uparrow_c) \; + \; O(\epsilon)$$

Small corrections theorem: The entanglement must keep growing as

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

(SDM arxiv: 0909.1038)



The only assumption needed is that once the quanta recede sufficiently far from the hole, they cannot be modified significantly by further interactions with the hole (this happens also for photons emitted from burning coal)

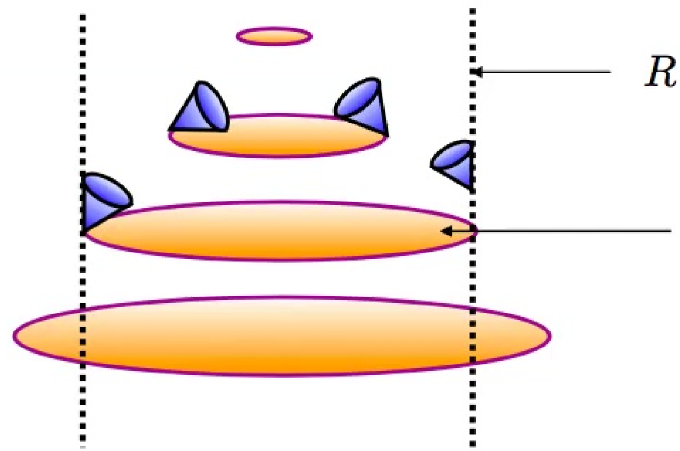
No small correction to the semiclassical picture of the hole (i.e., using quantum fields on curved space) can change Hawking's conclusion

Summary of the information paradox

(A)

Curvatures are very low at the horizon

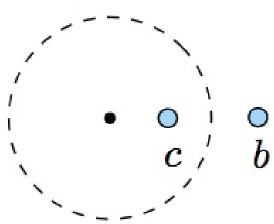
Thus we expect the equivalence principle to hold here, star falls smoothly through horizon



$$R = \frac{2GM}{c^2},$$

$$\rho \sim \frac{c^6}{G^2}\frac{1}{R^2}$$

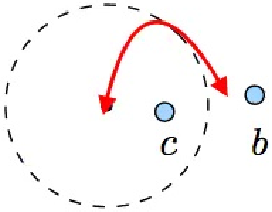
(B) Hawking pair creation process generates entangled pairs from the vacuum



$$\frac{1}{\sqrt{2}}(\uparrow_b\downarrow_c - \downarrow_b\uparrow_c)$$

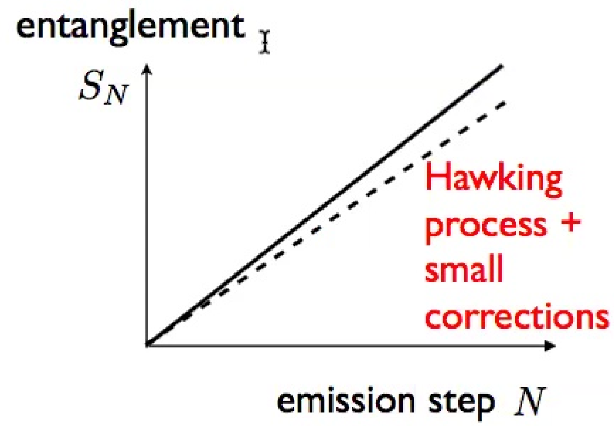
Radiation is highly entangled with remaining hole

(C) Small corrections cannot stop this entanglement from growing monotonically



$$\frac{1}{\sqrt{2}}(\uparrow_b\downarrow_c - \downarrow_b\uparrow_c) \; + \; O(\epsilon)$$

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

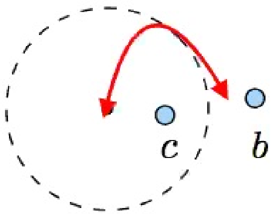


(D) There is a sharp puzzle at the endpoint of evaporation

There is no quantum state for these radiation quanta (violation of quantum theory)

How do we fit an arbitrarily large number of states in the tiny remnant?

(C) Small corrections cannot stop this entanglement from growing monotonically



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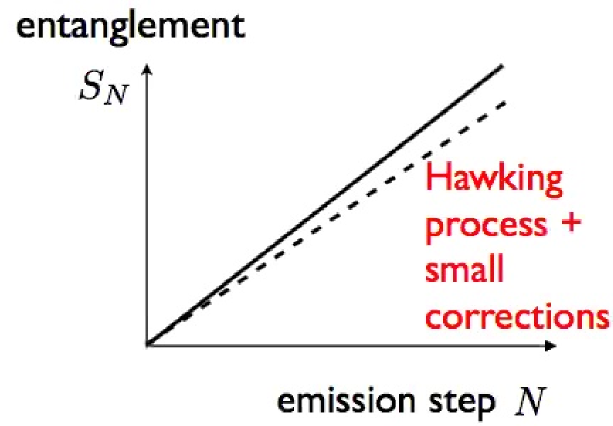

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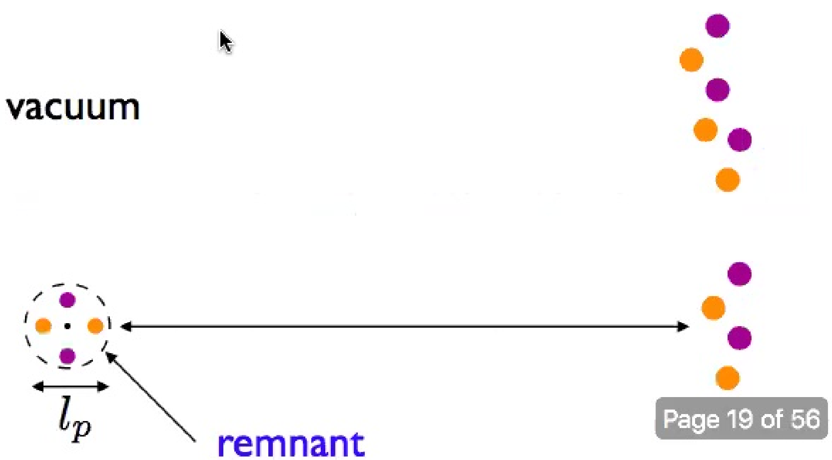

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How do we fit an arbitrarily large number of states in the tiny remnant?

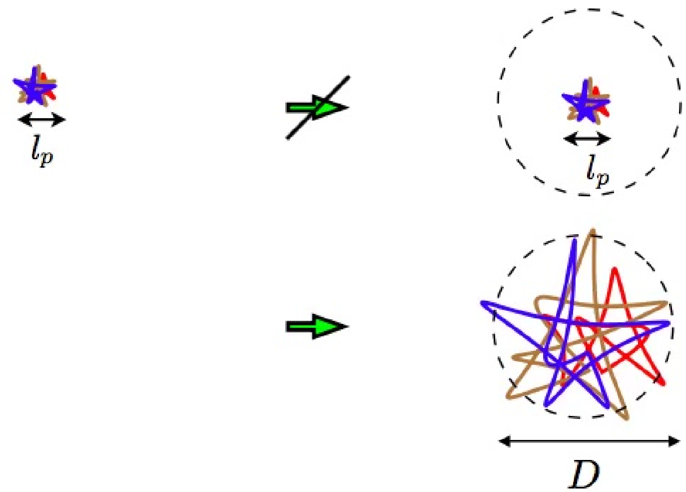
Resolution of the paradox: the fuzzball paradigm

Afshordi, Avery, Bah, Balasubramanian, Bena, Bianchi, Bobev, de Boer, Bossard, Carson, Ceplak, Chowdhury, Craps, Gimon, Giusto, Guo, Hampton, Heidmann, Houppe, Jejjala, Katmadas, Kanitscheider, Keski-Vakkuri, Kraus, Levi, Li, Lunin, Madden, Maldacena, Maoz, Martinec, Massai, Mayerson, Morales, Niehoff, Pani, Park, Peet, Potvin, Puhm, Ross, Ruef, Russo, Rychkov, Saxena, Shigemori, Simon, Skenderis, Srivastava, Taylor, Titchner, Turton, Tyukov, Vasilakis, Walker, Wang, Warner ... and many others

In string theory we have to make black holes by taking bound states of strings and branes

We find that the size of the bound state is never smaller that the horizon radius

weak coupling strong coupling



The size of the bound state grows with the number of branes, and a horizon never forms ⟶ *fuzzball*

Bound states of D1, D5, P charges:

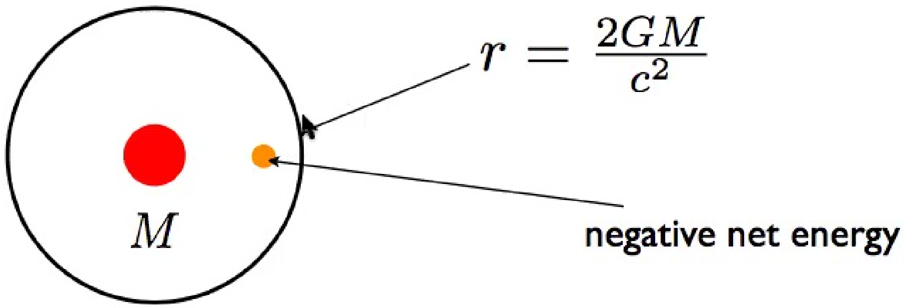
$$D \sim \left[\frac{\sqrt{n_1 n_5 n_p}\, g^2\, \alpha'^4}{V\, L}\right]^{\frac{1}{3}} \sim R_H$$

(SDM hep-th/9706151)

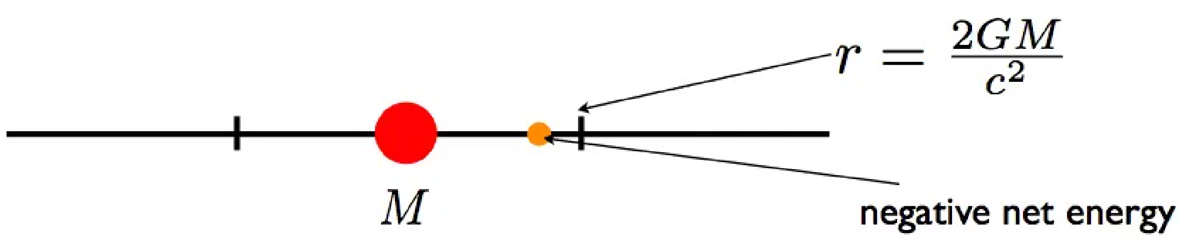
Many examples of string solutions have been constructed, and in each case we have found that the solution has no horizon (All 2 charge extremal states, many families of 3 and 4 charge extremal, some families of nonextremal, radiation from near extremal ...)

What is the structure of a fuzzball ?

We live in 3 space and 1 time dimension. Recall the black hole ...

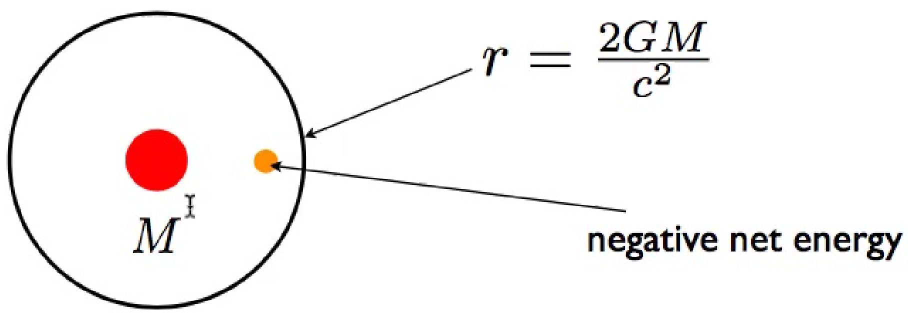


Let us draw just one space direction for simplicity



What is the structure of a fuzzball ?

We live in 3 space and 1 time dimension. Recall the black hole ...



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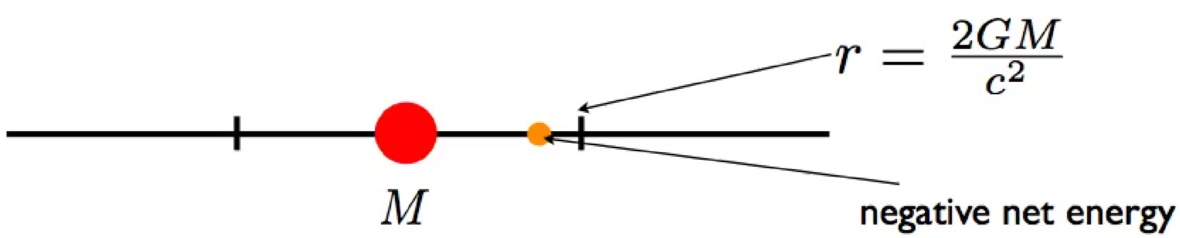
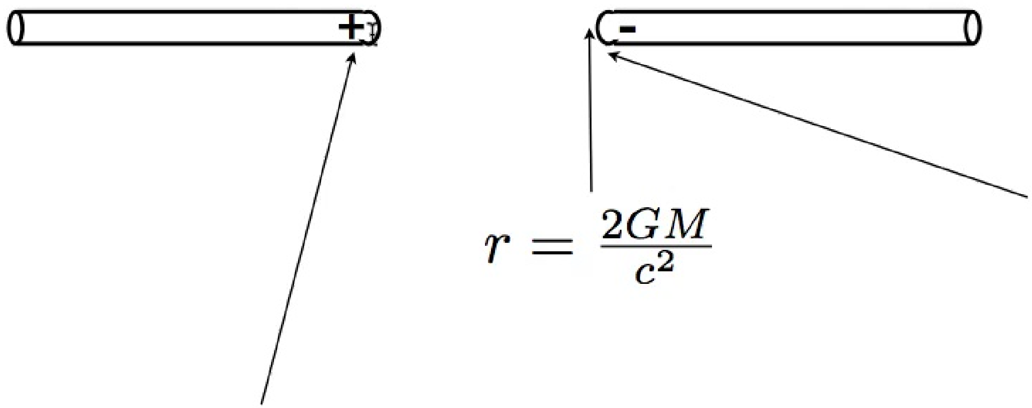


But there is a completely different structure possible with compact dimensions ...

No place to put particles with net negative energy



$$r = \frac{2GM}{c^2}$$

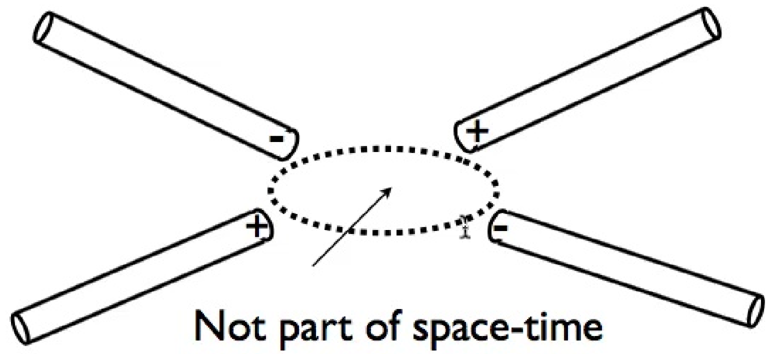
The mass M is captured by the energy in the curved manifold

There is an extra 'twist' in the space-time which makes it consistent to have both boson and fermion wave functions

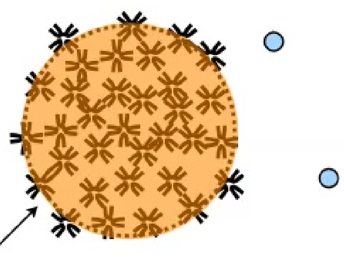
(Kaluza Klein monopoles and anti-monopoles)

1-dimension

In more dimensions :



We will draw only the structure near the horizon :



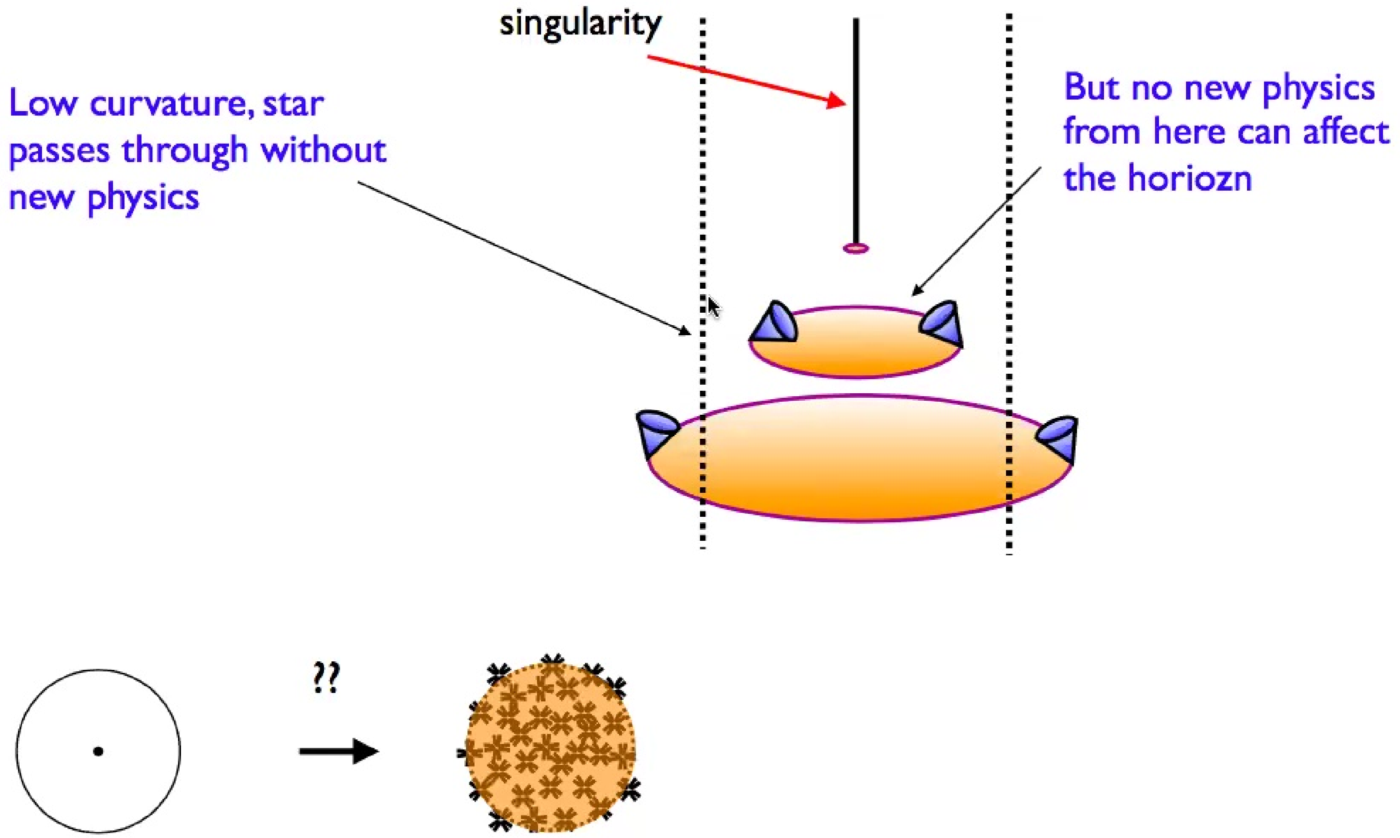
no horizon

"Fuzzball"

Nothing can fall 'into the hole' because it is like a normal body with no horizon

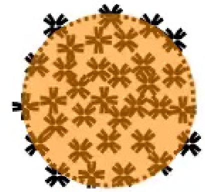
This resolves the information paradox

How did the semiclassical approximation break down?

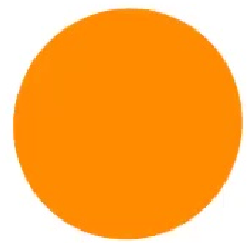


In principle, once we know the eigenstates of a system, we can find the evolution of any initial state

We can think of fuzzballs as the eigenstates of the full string theory



The initial collapsing star can be written as a linear superposition of eigenstates



$$|\Psi\rangle = \sum_i |E_i\rangle$$

The state at any later time is give by

$$|\Psi(t)\rangle = \sum_i e^{-iE_i t}|E_i\rangle$$

This is hard to do in practice, so let us see if we can arrive at the dynamics of the evolution using what we know about fuzzballs ...

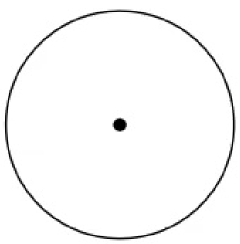
In 1972, Bekenstein used thermodynamic arguments to deduce that black holes have an entropy

$$S_{bek} = \frac{c^3}{\hbar} \frac{A}{4G}$$

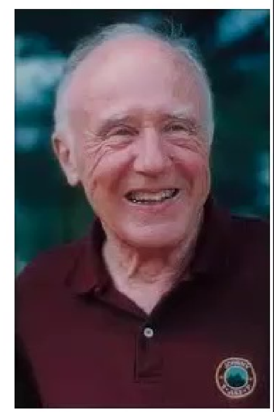
This would imply that a solar mass black hole has $e^{S_{bek}} \sim 10^{10^{144}}$ states

This is far larger than the number of states of normal matter with the same energy

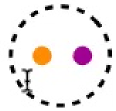
In the past we did not know what to do with this large value of the entropy, because we could not see the different states that were counted by the entropy

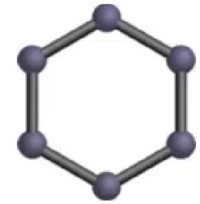


Wheeler: Black holes have no hair



Can there be virtual fluctuations of more complicated objects ?

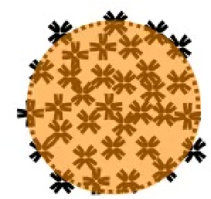
positronium 

Benzene ring 

All such fluctuation are present in the usual vacuum, but the probability of fluctuations describing heavy objects is small ...

The argument:

(A) If fuzzballs exist as on-shell configurations describing the microstates of black holes, then the gravitational vacuum must contain virtual fluctuations corresponding to fuzzball type configurations

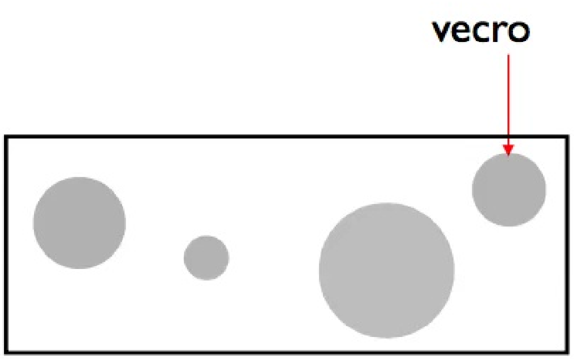


(B) Fluctuations corresponding to large mass configurations are expected to be suppressed. But the degeneracy of such configurations rises quickly with the energy, similar to the large degeneracy of on-shell states

$$\mathcal{N} \sim e^{S_{bek}}$$

The Bekenstein entropy is so large, that the suppression of a fuzzball-type fluctuation with large energy is offset by the large degeneracy of different possible fuzzball fluctuations of this energy

(C) Thus the gravitational vacuum has these extended-size fluctuations everywhere



VECRO:
Virtual Extended Compression-Resistant Objects

Why are large vecro fluctuations $R_v \gg l_p$ not suppressed?

The fluctuation to any large fuzzball type configuration is indeed highly suppressed:

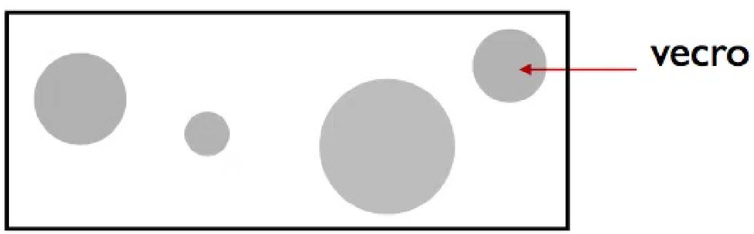
$$P \sim e^{-S} \sim e^{-ET}$$

$$E \sim M \sim \frac{R^{d-2}}{l_p^{d-1}}, \quad T \sim R \quad \longrightarrow \quad S \sim \left(\frac{R}{l_p}\right)^{d-1}$$

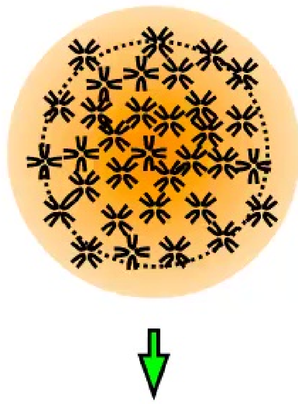
But there are a very large number of such fuzzball type configurations:

$$\mathcal{N} \sim e^{S_{bek}} \quad , \quad S_{bek} \sim \frac{A}{G} \sim \left(\frac{R}{l_p}\right)^{d-1}$$

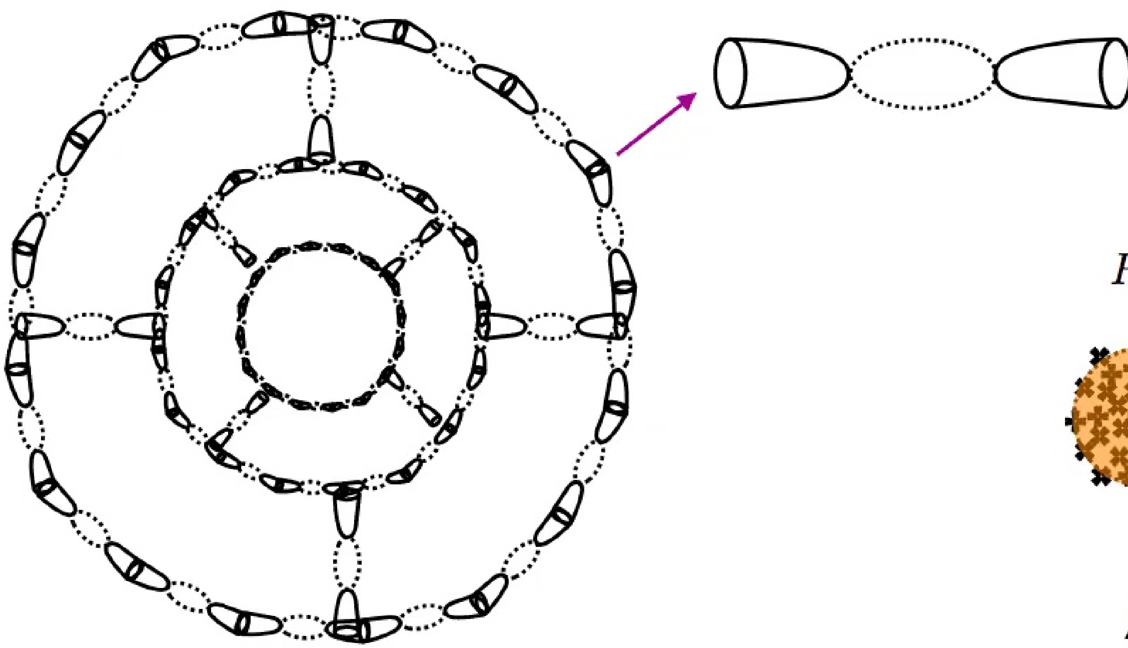
Thus we can have $P\mathcal{N} \sim 1$; i.e., the suppression is offset by the large degeneracy



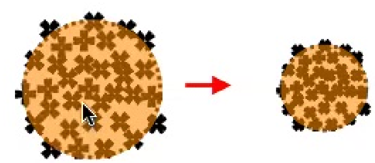
Compression-Resistance of fuzzball type configurations



Fuzzball configurations are extended structures, that resist being deformed

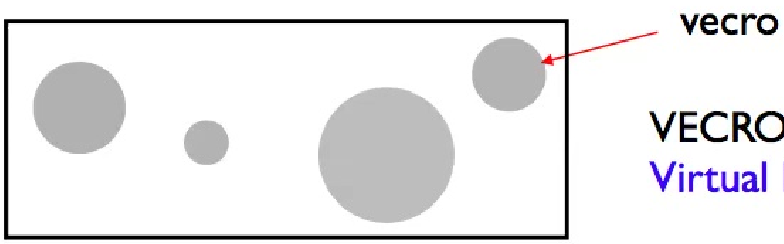


$$R \to (1 \pm \delta) R$$



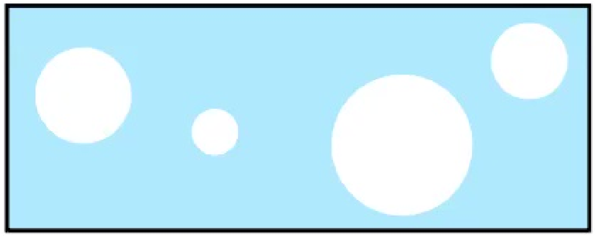
$$\Delta E \sim M(R)$$

We get the following picture of the vacuum of quantum gravity

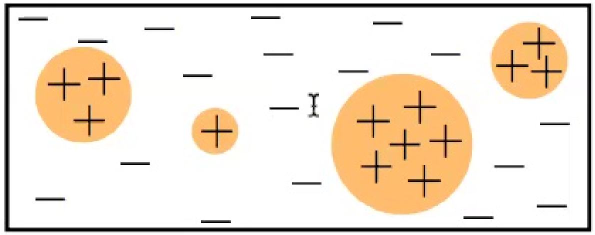


VECRO:
Virtual Extended Compression-Resistant Objects

So in the fuzzball paradigm the vacuum has a structure similar to that in phase transitions ...



Steam bubbles form in water near boiling point

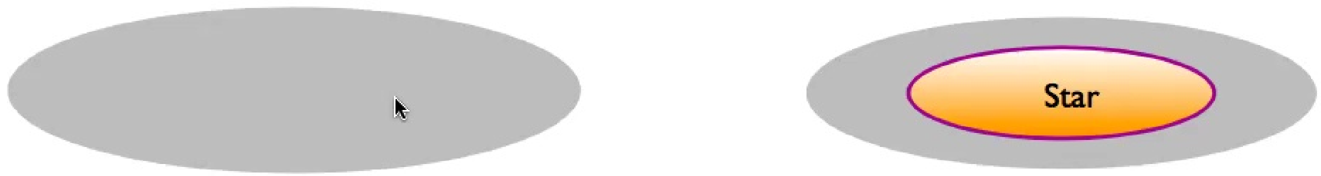


Ising model near criticality

(We cannot get this in a theory with no fuzzballs; e.g. 3+1 canonically quantized gravity)

What is the effect of this vecro structure of the vacuum ?

First consider a low curvature object like a star

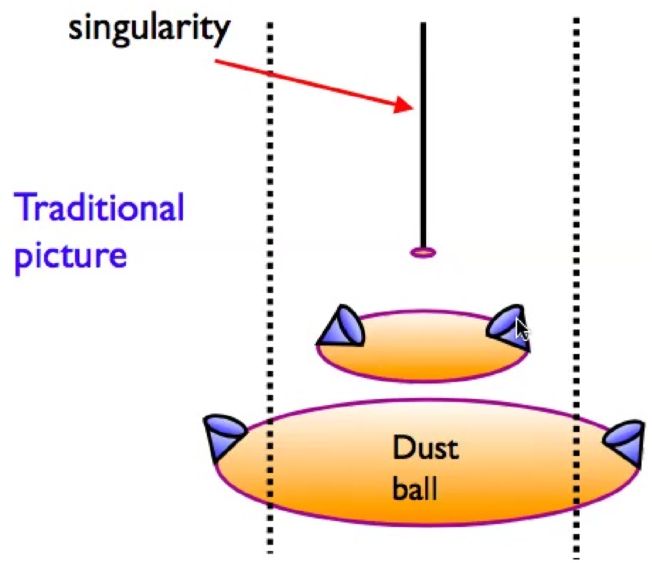


A star has a weak gravitational pull

So the vecro compresses slightly and stabilizes. This distortion of the wavefunctional is included in the Einstein action dynamics

Let us now ask what happens in a black hole ....

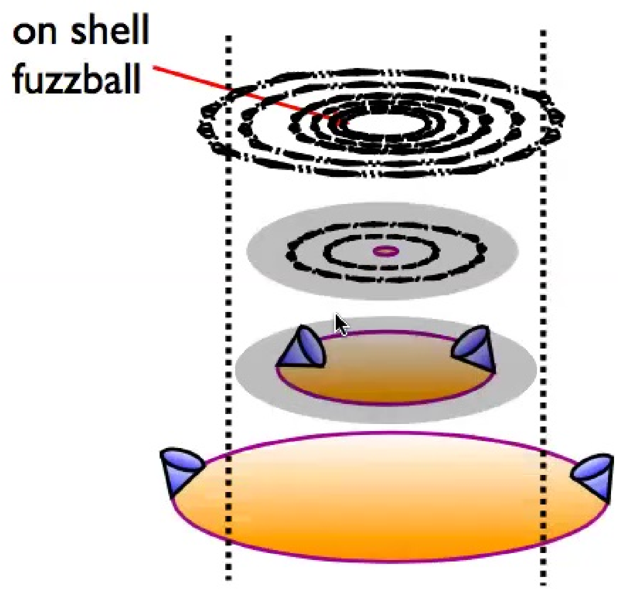
This is the semiclassical picture of black hole formation



But now we have to consider what the vecro fluctuations are doing in this spacetime ....

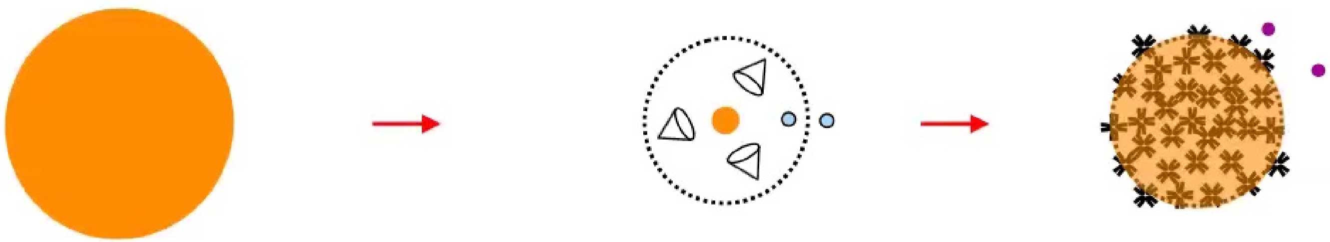
Inside the horizon, the light cones point inwards, so a vecro must keep on compressing (i.e., it cannot stabilize in size) ....

Inside a closed trapped surface, the vecros are forced to keep compressing



The resulting distortion of the vacuum wavefunctional turns the vecros to on-shell fuzzballs

This is analogous to how a large distortion of a vacuum mode of scalar field gives pair creation



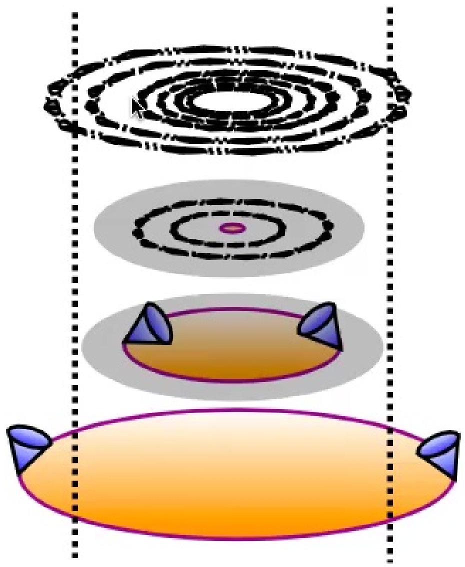
The extended nature of the vecro fluctuation allows it to detect the formation of a closed trapped surface, violating the equivalence principle

The only input we need to reach this picture of the vacuum is that causality hold to leading order in gently curved spacetime ...

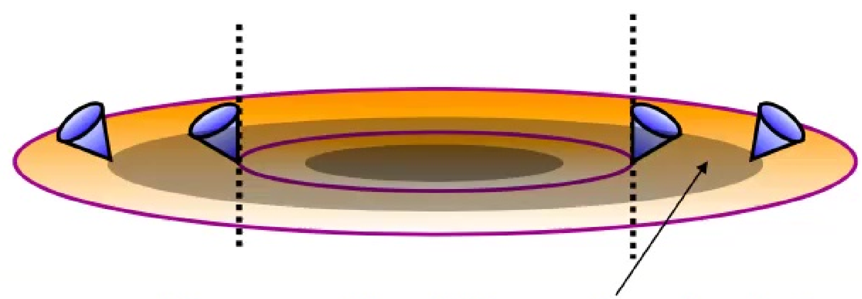
(This is the power of Hawking's information paradox supplemented with the small corrections theorem)

Can we observe the difference between a traditional black hole and a fuzzball?

A long wavelength coherent beam of photons will scatter off a fuzzball but not of a traditional black hole (Chua+Afshordi 21)



With a black hole, vecros inside a closed trapped surface were forced to compress

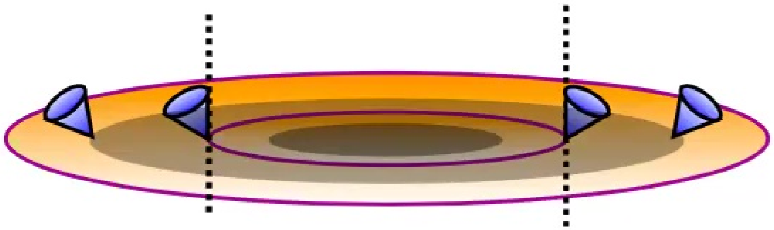


In a cosmology, vecros larger than the cosmological horizon will be forced to stretch

Vecros with radii lager than the horizon

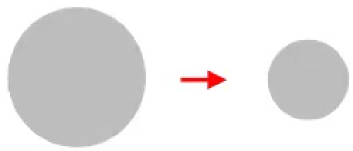
$$R_v > H^{-1}$$

forced to stretch



Vecros with radii lager than the horizon

$R_v > H^{-1}$ stretch



R_v $(1 \pm \delta) R_v$

(Order unity compression or stretching)

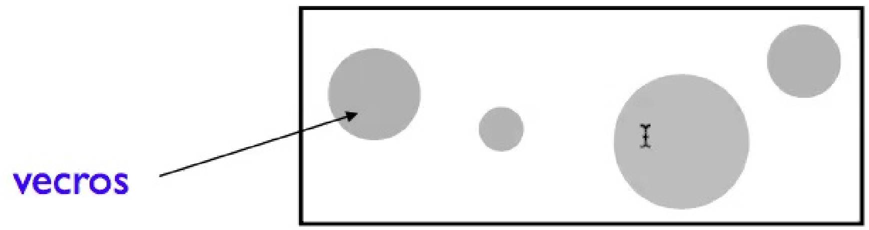
$\Delta E \sim M(R_v)$

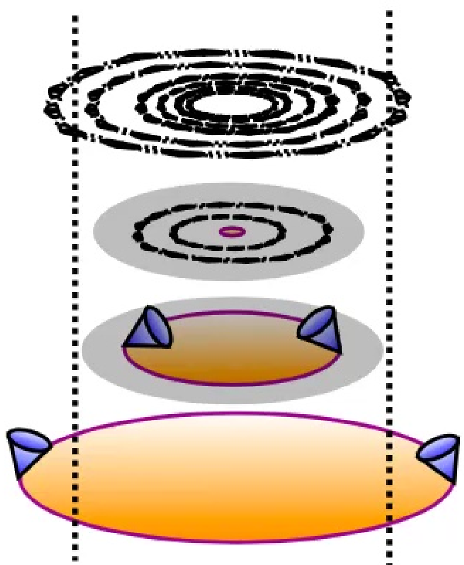
The energy from this stretching will be the source of energy for the effects we seek

(Energy for inflation, dark energy ...)

At this point we note an important difference between Minkowski spacetime and the cosmological spacetime ...

In Minkowski space, we have vecros with all radii $0 < R_v < \infty$





This is necessary in order that we can resolve the Hawking puzzle for black holes of all sizes

But in an expanding cosmology, extended structures cannot have a radius larger than the distance that light has been able to travel since the big bang

We take a flat cosmology

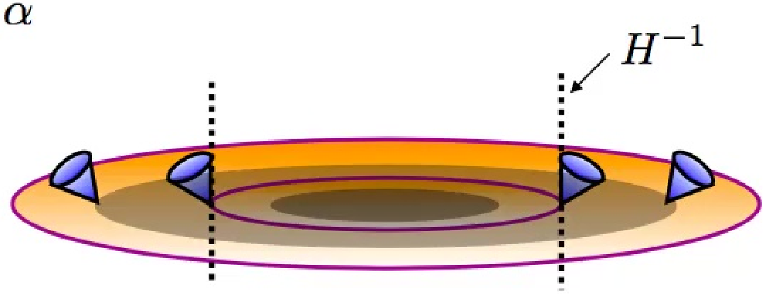
$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$

$$a(t) = a_0 t^\alpha, \quad H = \frac{\dot{a}}{a} = \frac{\alpha}{t}, \quad H^{-1} = \frac{t}{\alpha}$$

The distance that light has travelled since the big bang is

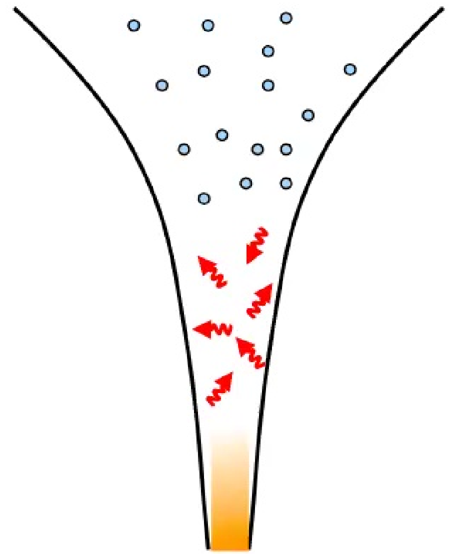
$$R_{max}(t) = a(t) \int_{t'=0}^{t} \frac{dt'}{a(t')} = \frac{t}{1-\alpha}$$

$$R_v \leq R_{max} \longrightarrow \frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$

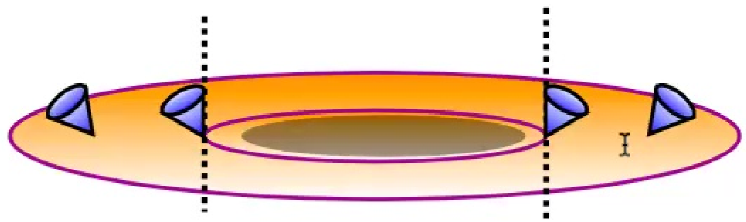


So we can form vecros with all radii upto this value

(A) $$\frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$



In the radiation phase, $\alpha = \frac{1}{2}$ so $\frac{R_v}{H^{-1}} \leq 1$



So there cannot be any energy from the forcible stretching of vecros in this phase ...

This is important, since observations do not allow much freedom in the amount of energy in this phase of the universe

(Extra energy would lead to faster expansion, which would alter the predictions of Big Bang Nucleosynthesis (BBN))

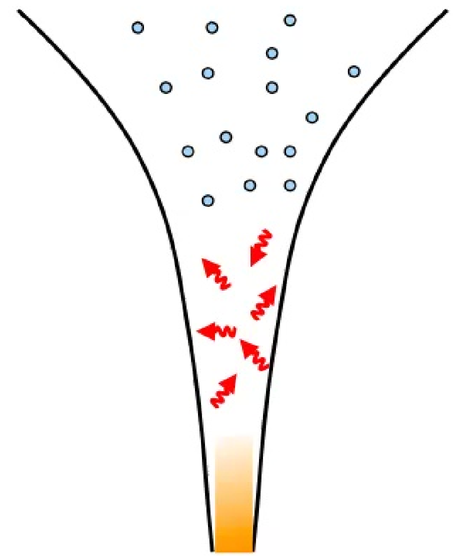
(B) $$\frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$

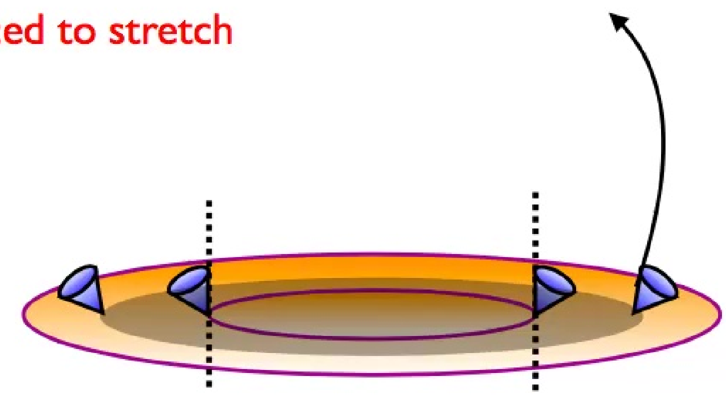
For a dust cosmology, $\alpha = \frac{2}{3}$

So vecros in the range

$$1 \leq \frac{R_v}{H^{-1}} \leq 2$$

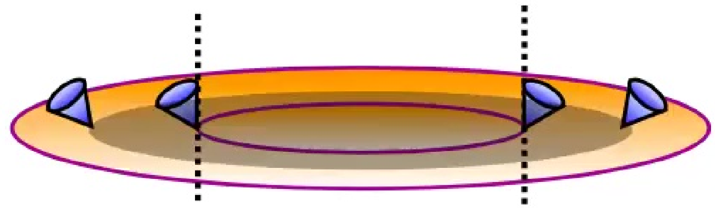
will be forced to stretch





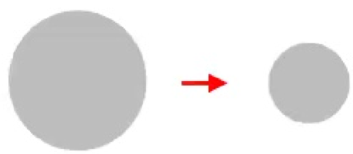
When the radiation phase turns to the dust phase, there will be a stretching of vecros

This will lead to an extra energy in the universe which is not part of our usual semiclassical stress tensor



How much is this energy ?

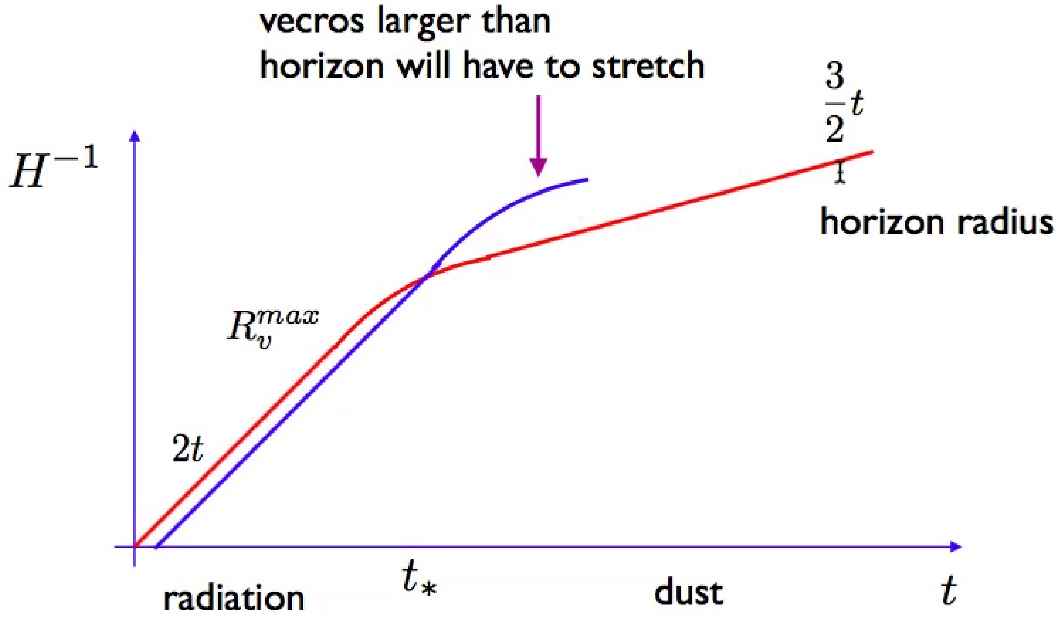
We had noted the scale of energies associated with an order unity distortion of the vecro distribution function



$$\Delta E \sim M(R_v)$$

Using the Hubble relation giving the size of the cosmological horizon we find that the energy density from the stretching of vecros is of order the closure density

$$\rho_v \sim \rho_{closure}$$



If the changeover were adiabatic, then the vecro distribution will adjust to a minimum energy one, and there will be no extra energy.

If the changeover were sudden, we will get a stretching of horizon scale vecros by a factor of order unity

The actual changeover in the power law is somewhere in between, happening over a few Hubble timescales

Thus we expect an energy density $\Delta\rho = \mu\,\rho_{closure}$ with μ much smaller than 1

where $\rho_{closure}$ is the closure energy density at the epoch of matter radiation equality $z \approx 2500$

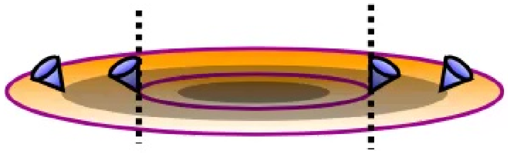
Interestingly, just this scale of energy is required at around just this epoch to resolve a tension in two different observations of the Hubble constant

Measurements on local objects suggest $H_0 \approx 74\,Km/s/Mpc$

The ΛCDM model applied to cosmic microwave background measurements suggests $H_0 \approx 67\,Km/s/Mpc$

An extra energy density of order $\sim 10\%$ of the closure density at t_* can explain this tension ... such energy is called Early Dark Energy (EDE)

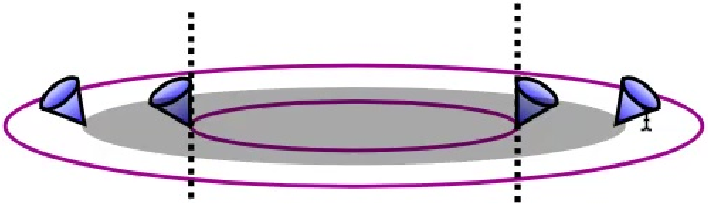
We see that the energy required may arise naturally from the dynamics of vecros ...



(C) Dark energy ?

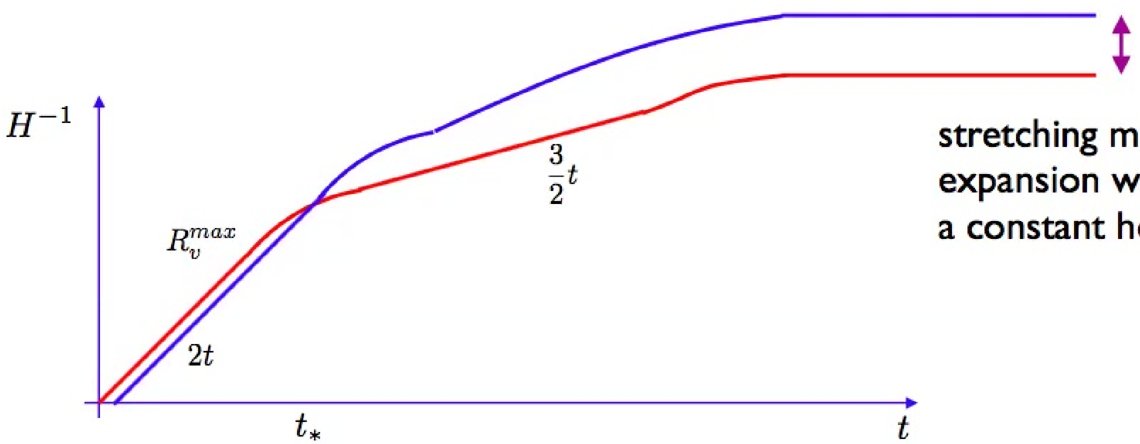
We have noted that $\Delta\rho = \mu\, \rho_{closure}$

from the stretching of vecros



Suppose the stretching of vecros is such that $\Delta\rho = \rho_{closure}$

Then we do not need any matter to support the expansion: the energy of stretching vecros maintains the expansion



stretching maintains the expansion with a constant horizon radius

Then we get a universe with expansion dominated by dark energy ....

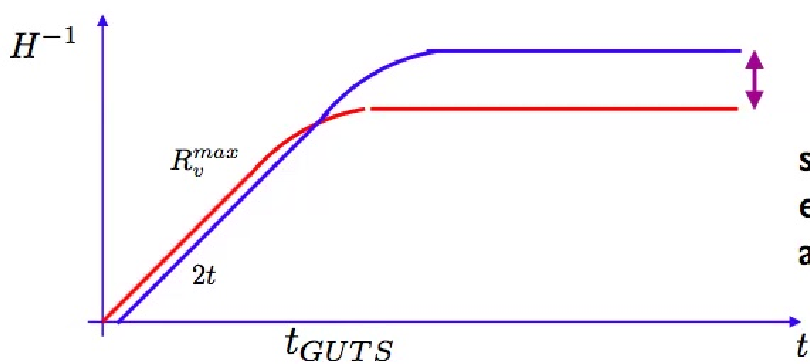
(D) Inflation ?

We have seen that we get a stretching of vecros when the slope of the Hubble expansion decreases suddenly

This happens when the pressure drops suddenly

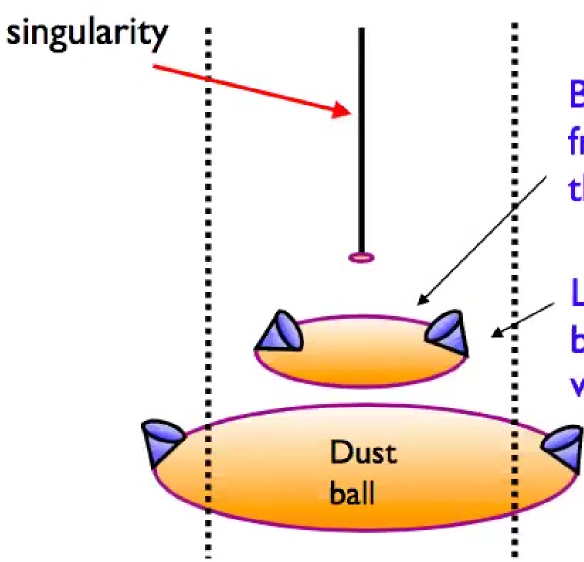
At the end of the GUTS epoch, the heavy GUTS particles become nonrelativistic,, leaving only the light standard model particles to provide pressure

We again have the possibility of getting stuck in a phase where the energy of vecro stretching maintains the expansion, with a fixed Hubble radius



$$\Delta\rho = \rho_{closure}$$

stretching maintains the
expansion with
a constant horizon radius

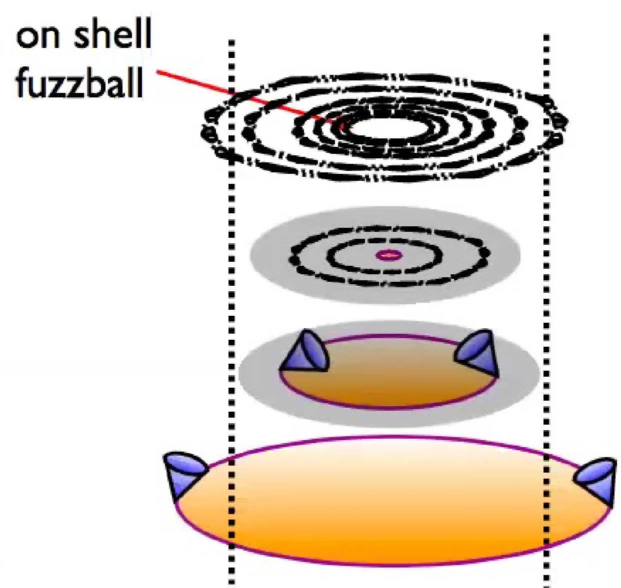


But no new physics from here can affect the horiozn

Low curvature, dust ball passes through without new physics

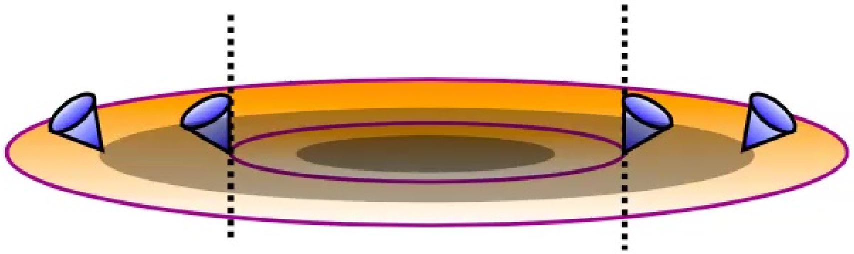
Small corrections cannot help

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$



The extended nature of the vecro fluctuation allows it to detect the formation of a closed trapped surface, violating the equivalence principle

In a cosmology, the vecro fluctuations cannot have arbitrary size since they cannot be larger than the particle horizon (the distance that light can travel since the big bang)



The stretching of vecros generates an extra energy at the scale of the cosmological horizon ...

But this is the scale at which we need new physics to explain many features of the universe : inflation, dark energy, Hubble constant tensions ...

Thus there is a beautiful interplay between the black hole puzzles and the puzzles of cosmology ..

THANK YOU !!