Title: From Coulomb to Kerr via the double copy - Donal O'Connell

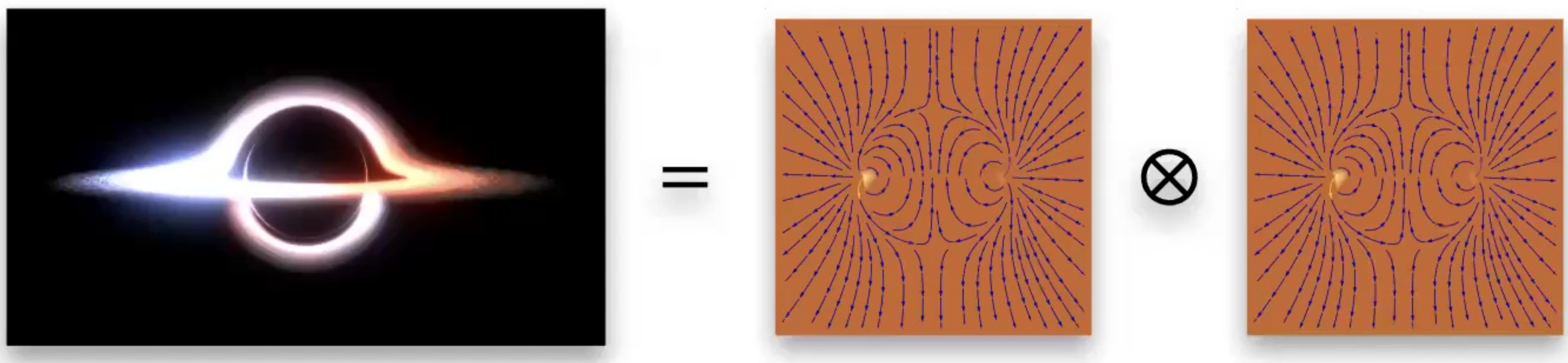
Speakers: Donal O'Connell

Series: Colloquium

Date: October 13, 2021 - 2:00 PM

URL: https://pirsa.org/21100022

Abstract: The double copy relates scattering amplitudes in gauge theory and gravity. But interaction potentials (and spacetime metrics) can be extracted from amplitudes, and so the double copy leads to a relationship between classical solutions of gauge theory and gravity. In this talk I will describe this relationship, provide a perspective on the Schwarzschild metric as a "square" of the Coulomb charge, and take a look at the "square root" of the Kerr metric.



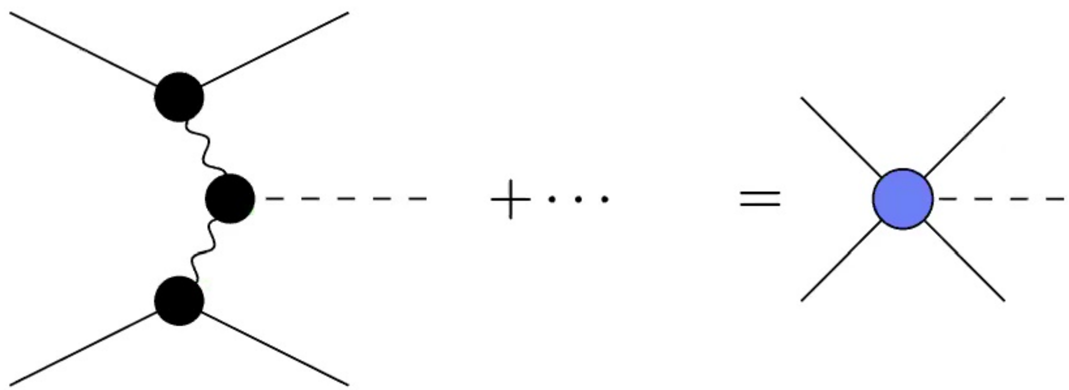
Perimeter Institute, October 2021

From Coulomb to Kerr via the double copy

Donal O'Connell
University of Edinburgh

Invitation

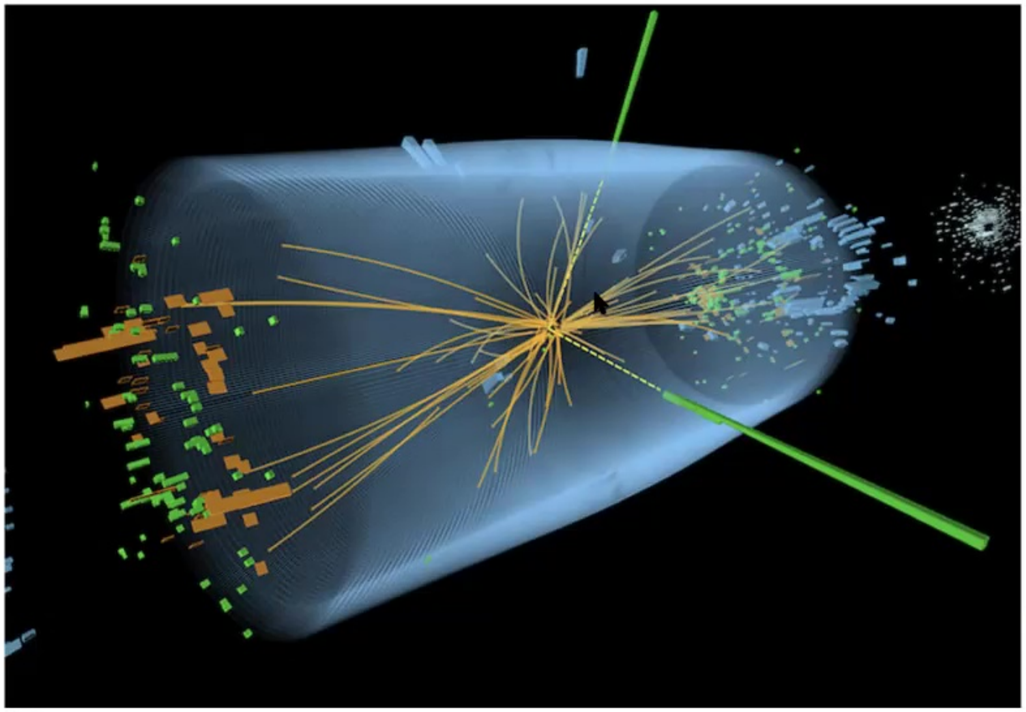
Useful tool: amplitudes



Amplitude: sum of Feynman diagrams

Invitation

Useful tool: amplitudes

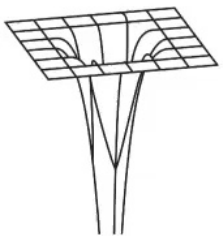


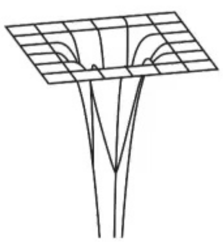
Higgs candidate event

CMS/CERN

Invitation

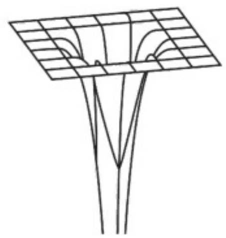
Useful tool: amplitudes… in gravity?

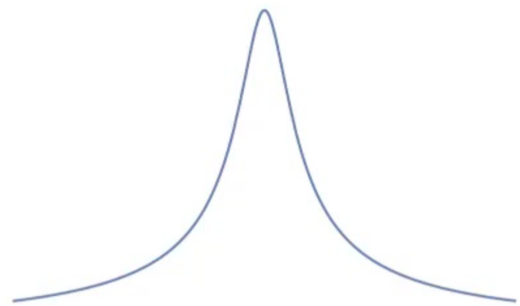


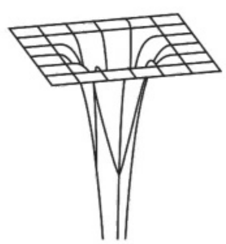


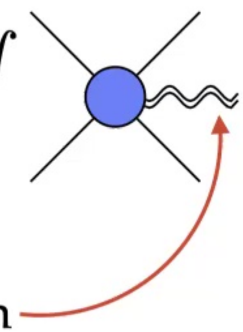
Invitation

Useful tool: amplitudes… in gravity?



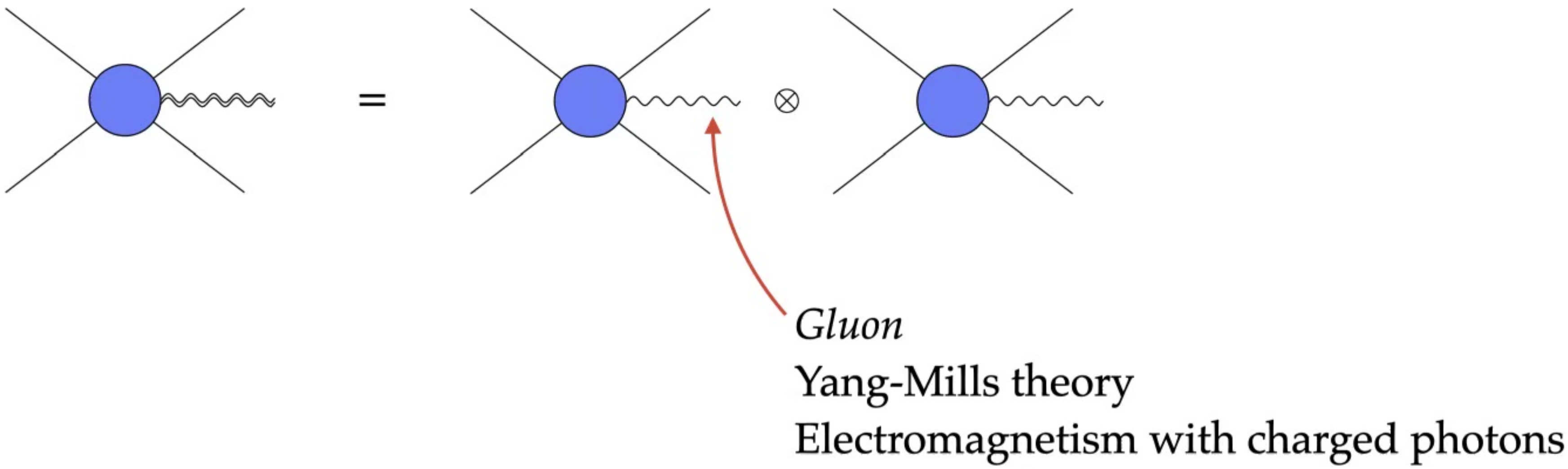
$h(t) =$ 



$= \frac{1}{\text{distance}} \int$ 

Invitation

Good idea: the double copy



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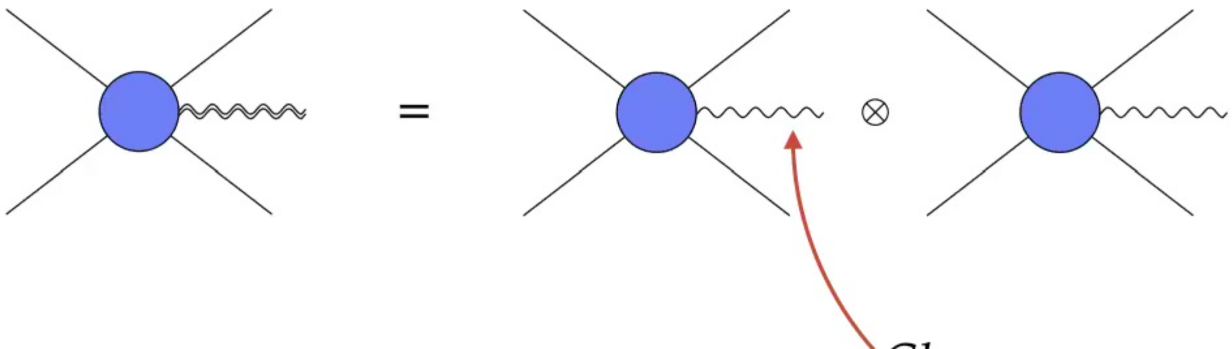


Gluon
Yang-Mills theory
Electromagnetism with charged photons

Gravity = Yang-Mills^2

Invitation

Good idea: the double copy

- ✓ Connects different areas
  - ✓ Use LHC methods for gravitational wave physics
- ✓ Makes calculations in gravity easier
- ✓ Different to geometric viewpoint on gravity
  - ✓ Hidden simplicity in gravity

Invitation

Good idea: the double copy

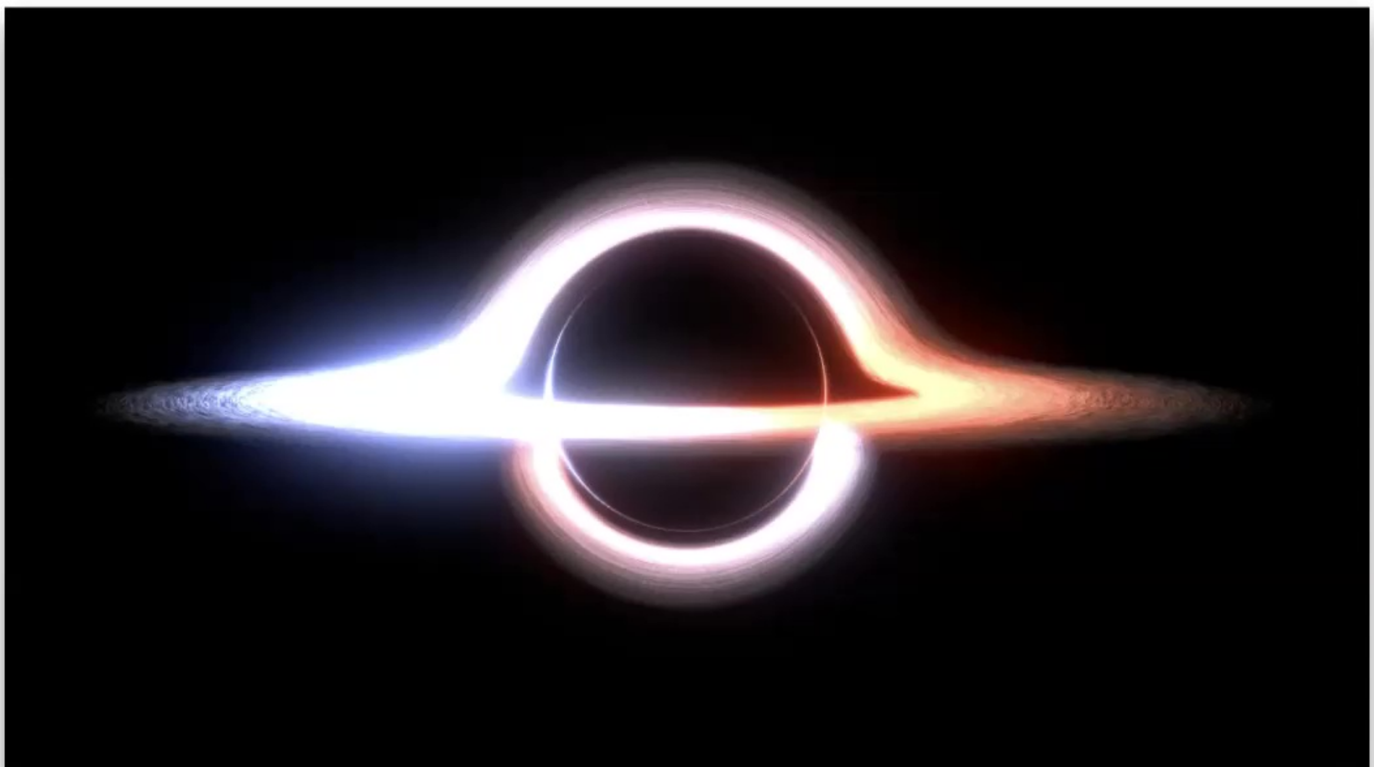


Image credit: Josh Meister

Outline

1. Amplitudes & the double copy
2. From Coulomb to Kerr
3. Beyond perturbation theory
4. Conclusion

Amplitudes & the double copy

Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

Universal time evolution operator in QM

$$|\psi\rangle \xrightarrow{\text{time}\to\infty} U(\infty, -\infty)|\psi\rangle$$

Initial state
Far past

Final state
Far future

Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

Universal time evolution operator in QM

$$|\psi\rangle \xrightarrow{\text{time}\to\infty} S|\psi\rangle \qquad U(-\infty,\infty) = S = 1 + iT$$

Matrix elements of T are the amplitudes

$$\mathcal{A}(p_1 \dots p_n \to q_1 \dots q_m)\delta^4\left(\sum p_i - \sum q_j\right) = \langle q_1 \dots q_m|T|p_1 \dots p_n\rangle$$

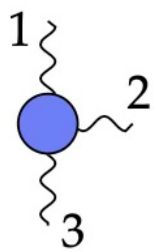
Why amplitudes?

Why are amplitudes relevant to gravitational waves / scattering?

Amplitudes tell us about long-time evolution

The double copy

"Double copy": for amplitudes, gravity = (Yang-Mills)^2 *Kawai, Lewellen, Tye*
Bern, Carrasco, Johansson

YM 

$$\mathcal{A} = g f^{abc}(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3)$$

The double copy

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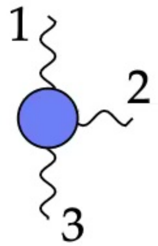
"Colour charge"

YM 

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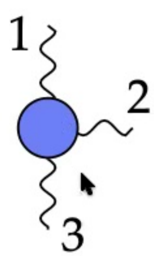

Momenta

$$\mathcal{A} = g f^{abc}(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3)$$

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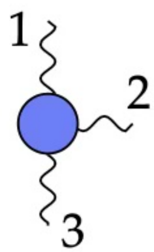

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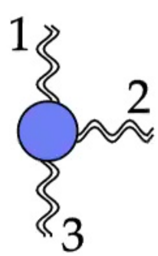
GR



$$\mathcal{M} = \kappa(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3)^2$$

$$e_1^{\mu\nu} = \epsilon_1^\mu \epsilon_1^\nu$$

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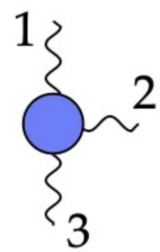

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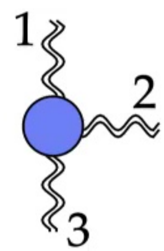
GR



Polarisations

$$\mathcal{M} = \kappa(\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot p_1 + \epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot p_2 + \epsilon_3 \cdot \epsilon_1 \epsilon_2 \cdot p_3)^2$$

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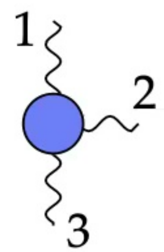
Bern, Carrasco, Johansson

YM 

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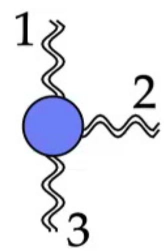
Straight squares

GR 

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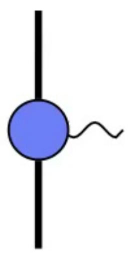
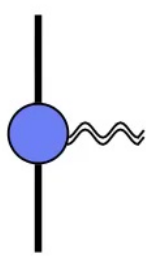
YM 

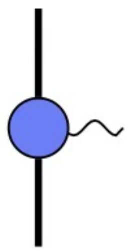
$$\mathcal{A} = gT^a \epsilon \cdot (p_2 - p_1)$$

GR 

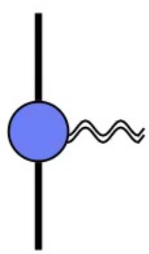
$$\mathcal{M} = \kappa(\epsilon \cdot (p_2 - p_1))^2$$

The double copy

"Double copy": for amplitudes, gravity = (Yang-Mills)^2 *Kawai, Lewellen, Tye*
Bern, Carrasco, Johansson

YM  $\mathcal{A} = gT^a \left(\epsilon \cdot (p_2 - p_1)\right)$

Straight squares

GR  $\mathcal{M} = \kappa \left(\epsilon \cdot (p_2 - p_1)\right)^2$

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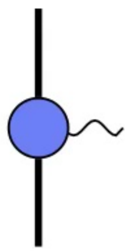
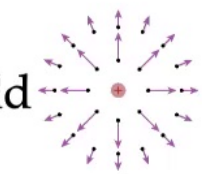
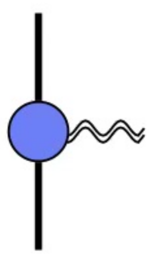
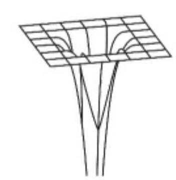
YM  $\mathcal{A} = gT^a \big(\epsilon \cdot (p_2 - p_1)\big)$ Point charge: *Coulomb* field 

Straight squares

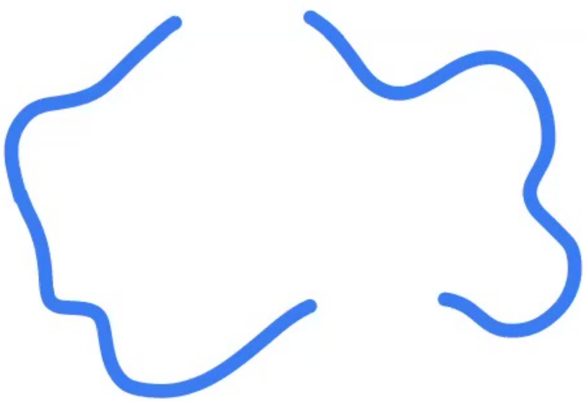
Some kind of square?

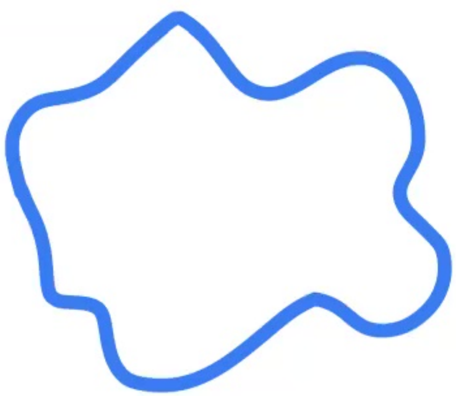
GR  $\mathcal{M} = \kappa\big(\epsilon \cdot (p_2 - p_1)\big)^2$ Point mass: *Schwarzschild* 

The double copy

Why is there a double copy?

1. String theory

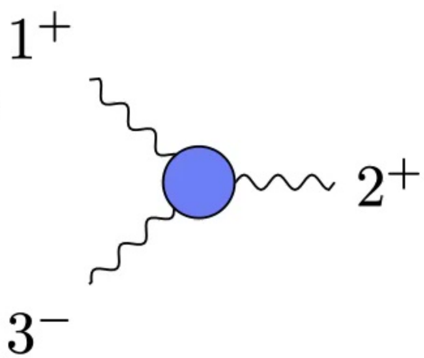




The double copy

Why is there a double copy?

2. Uniqueness $\quad |1] \to e^{i\theta_1/2}|1], \quad |2] \to e^{i\theta_2/2}|2], \quad |3] \to e^{i\theta_3/2}|3]$



$$\Rightarrow \mathcal{A} = \#g \frac{[12]^4}{[12][23][31]} f^{abc}$$

Benincasa & Cachazo

The double copy

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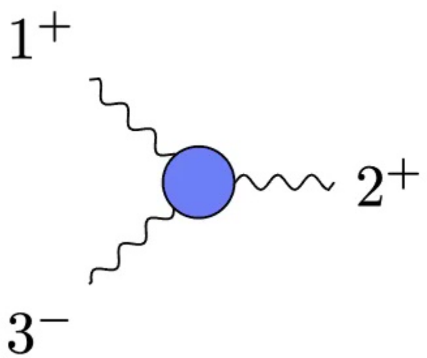

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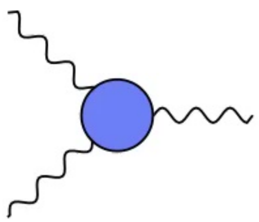
1^+

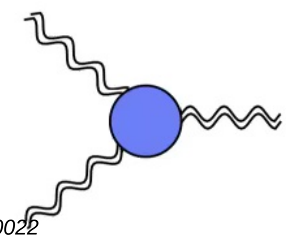


2^+

3^-

$$\Rightarrow \mathcal{A} = \# g \frac{[12]^4}{[12][23][31]} f^{abc}$$

1^{++}



2^{++}

3^{--}

$$\Rightarrow \mathcal{M} = \# \kappa \left(\frac{[12]^4}{[12][23][31]} \right)^2$$

Benincasa & Cachazo

The double copy

When is there a double copy?

- ✓ Tree-level YM & gravity
- ✓ Tree supersymmetric YM & gravity
    - ✓ Many examples of loops

The double copy

When is there a double copy?

- ✓ Tree-level YM & gravity
- ✓ Tree supersymmetric YM & gravity
  - ✓ Many examples of loops
- ✓ Dimensional reductions of these
- ✓ Scalar cases: (non-linear sigma model)^2 = (special Galileon), …

Bern, Carrasco, Johansson
Cachazo, Ellis, Yuan

The double copy

When is there a double copy?

- ✓ Tree-level YM & gravity
- ✓ Tree supersymmetric YM & gravity
  - ✓ Man
- ✓ Dimensional reductions of these
- ✓ Scalar cases: (non-linear sigma model)^2 = (special Galileon), …

Heard of a theory?
Then it's (probably) involved in the double copy

Bern, Carrasco, Johansson
Cachazo, Ellis, Yuan

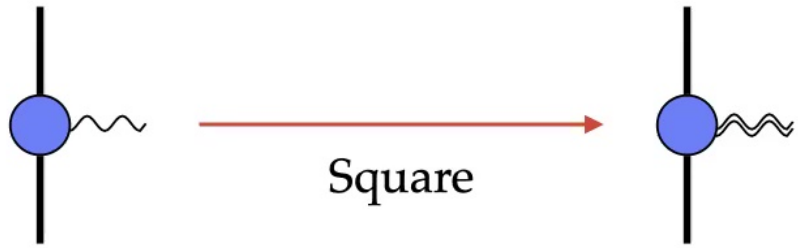
The double copy

Why is there a double copy?

3. We don’t really know

Universality in quantum field theory

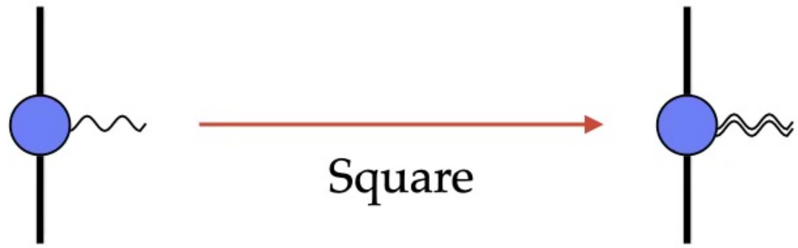
From Coulomb to Kerr



(Gravitational field of point mass) = (Electric field of charge)^2?

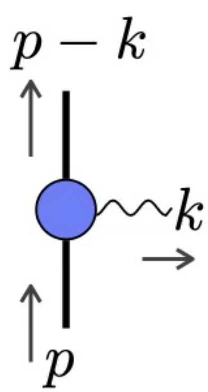
Need interactions to distinguish E&M from YM!

From Coulomb to Kerr



(Gravitational field of point mass) = (Electric field of charge)^2?

There's a bug:



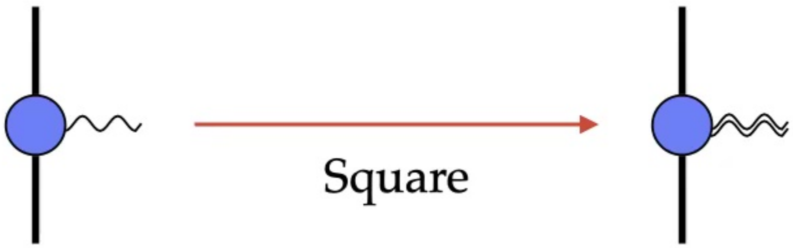
All momenta must be physical

$$(p-k)^2 = m^2 = p^2, \quad k^2 = 0$$

$$\Rightarrow 2p \cdot k = 0$$

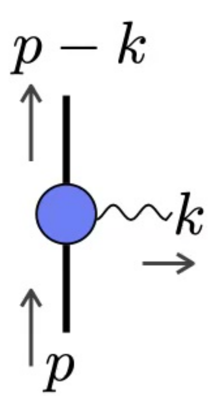
Take $p = \begin{pmatrix} m & 0 & 0 & 0 \end{pmatrix}$

From Coulomb to Kerr



(Gravitational field of point mass) = (Electric field of charge)²?

There's a bug:



All momenta must be physical

$$(p-k)^2 = m^2 = p^2, \quad k^2 = 0$$

$$\Rightarrow 2p \cdot k = 0$$

Take $p = \begin{pmatrix} m & 0 & 0 & 0 \end{pmatrix}$

No non-trivial solution!

From Coulomb to Kerr

Get around this…

1. Analytically continue to complex momenta

Take $p = \begin{pmatrix} m & 0 & 0 & 0 \end{pmatrix}$ $k = \begin{pmatrix} 0 & iE & 0 & E \end{pmatrix}$

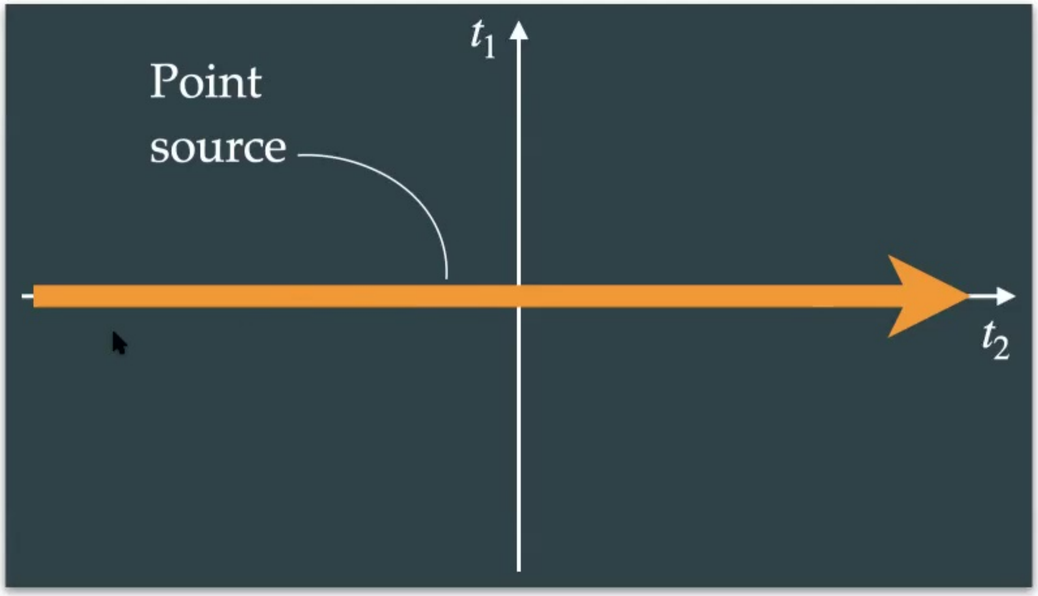
Britto, Cachazo, Feng, Witten

2. Compute classical fields in metric signature $(++--)$

Take $p = \begin{pmatrix} m & 0 & 0 & 0 \end{pmatrix}$ $k = \begin{pmatrix} 0 & E & 0 & E \end{pmatrix}$

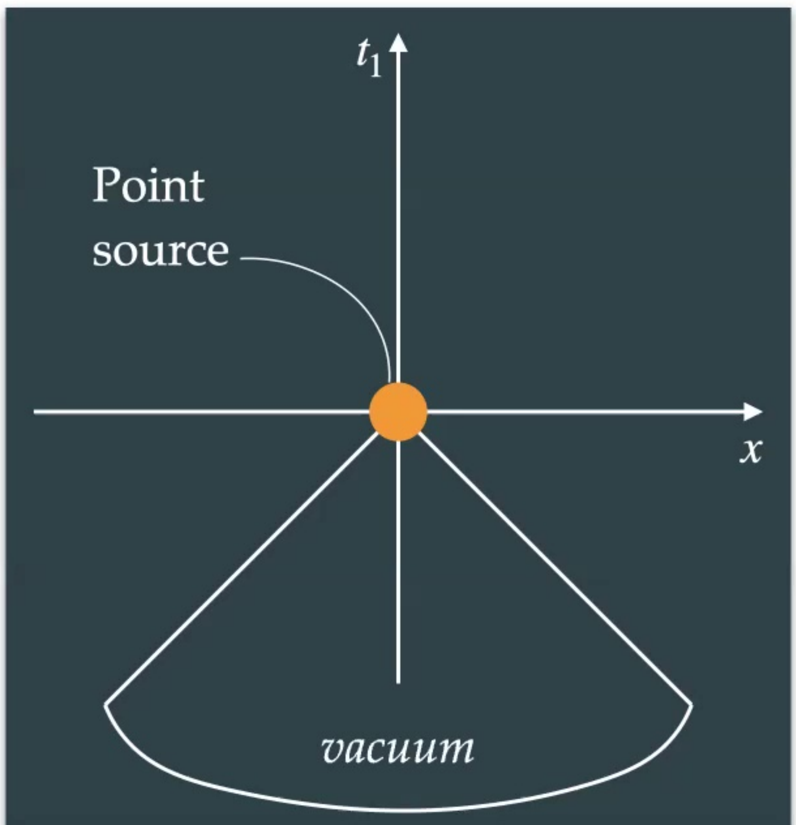
From Coulomb to Kerr

Signature $(+ + - -)$ is a bit unusual…



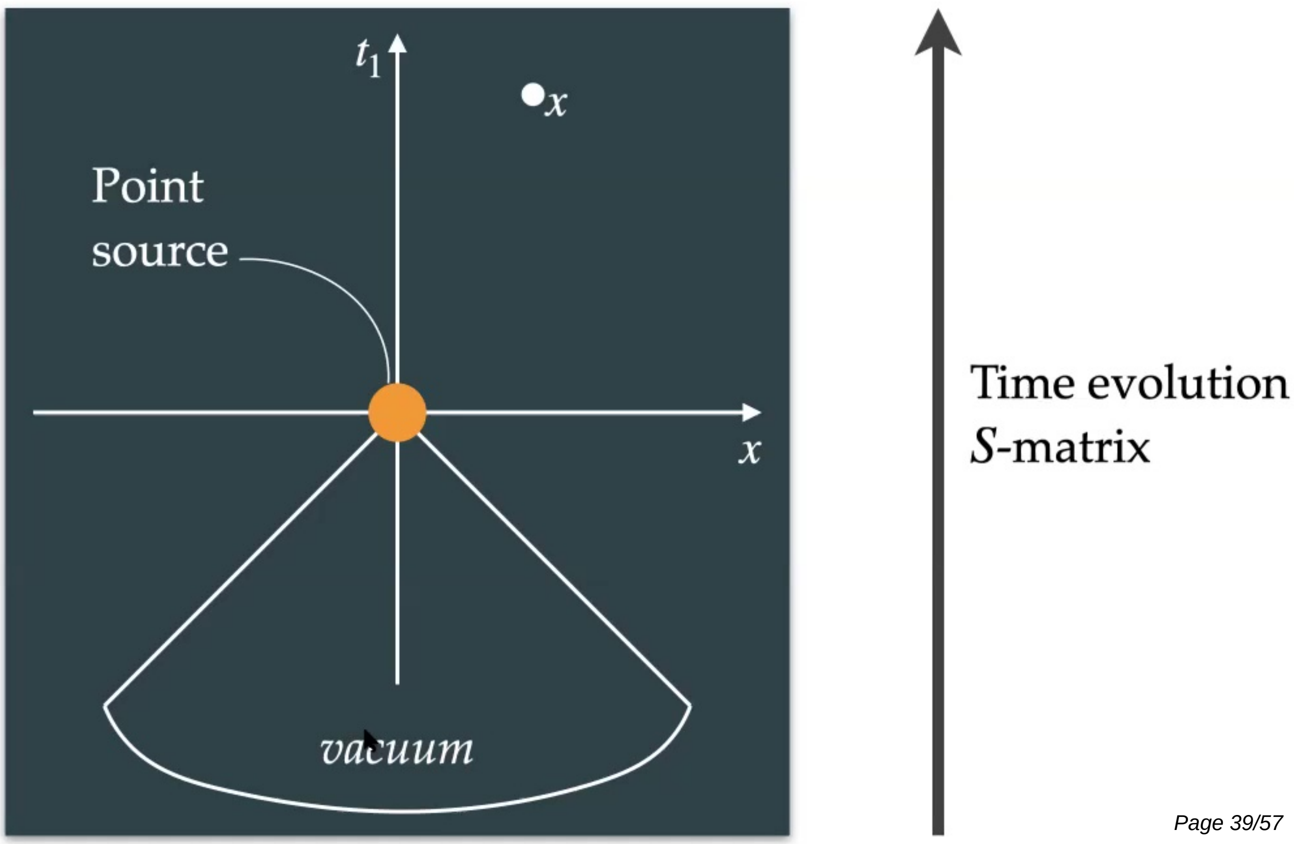
From Coulomb to Kerr

Signature (+ + − −) is a bit unusual…

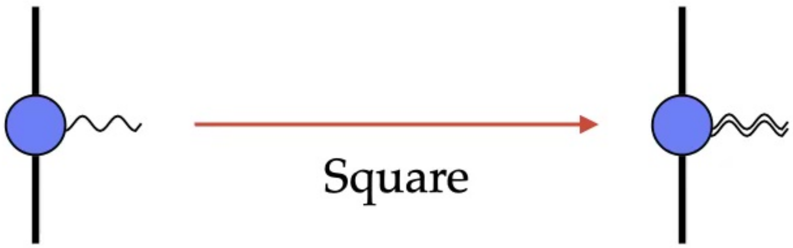


From Coulomb to Kerr

Signature $(++--)$ is a bit unusual…



From Coulomb to Kerr



Measure *expectation* of field strength / curvature

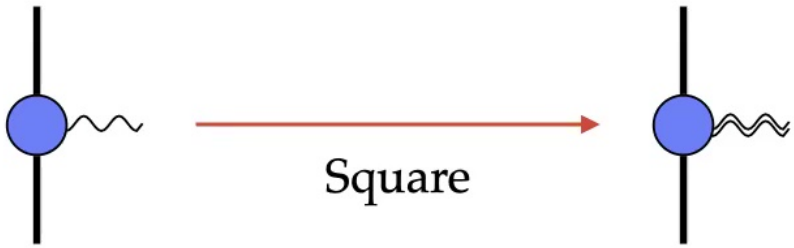
$$\Phi(x) \equiv \langle\psi|S^\dagger F_{..}(x) S|\psi\rangle$$

Field strength operator

$$\Psi(x) \equiv \langle\psi|S^\dagger R_{....}(x) S|\psi\rangle$$

(Weyl) curvature operator
Linearised!

From Coulomb to Kerr



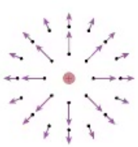
Measure *expectation* of field strength / curvature

$$\Phi(x) \equiv \langle \psi | S^\dagger F_{..}(x) S | \psi \rangle \qquad \Psi(x) \equiv \langle \psi | S^\dagger R_{....}(x) S | \psi \rangle$$

Spinor forms
(closer to amplitudes)

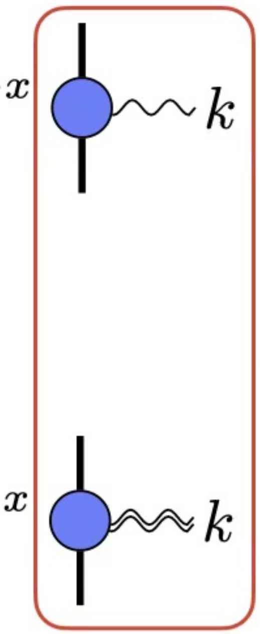
From Coulomb to Kerr

Measure *expectation* of curvature component in classical limit



$$\Phi^{\text{Coul}}(x) = \text{Re} \int \mathrm{d}^4 k \, \delta(k^2)\delta(k \cdot p) \, |k\rangle^2 \, e^{-ik\cdot x}$$

$$\Psi^{\text{Schw}}(x) = \text{Re} \int \mathrm{d}^4 k \, \delta(k^2)\delta(k \cdot p) \, |k\rangle^4 \, e^{-ik\cdot x}$$

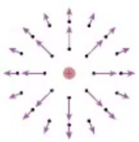


Amplitudes: double copy

Monteiro, Peinador Veiga, Sergola & DOC

From Coulomb to Kerr

Measure *expectation* of curvature component in classical limit



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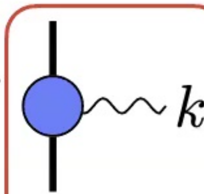
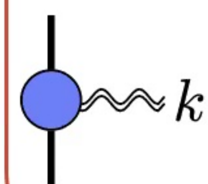


Spinors (derivatives)

Amplitudes: double copy

$$\Psi^{\text{Schw}}(x) = \text{Re} \int \mathrm{d}^4k \, \delta(k^2)\delta(k \cdot p) \, |k\rangle^4 \, e^{-ik\cdot x}$$



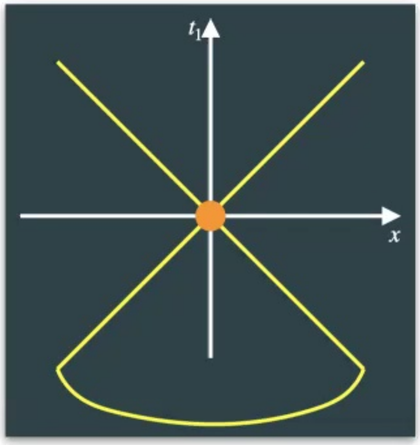
Monteiro, Peinador Veiga, Sergola & DOC

From Coulomb to Kerr

Perform integrals

$$\Phi^{\text{Coul}}(x) = \left[\frac{e}{\sqrt{t_2^2 - x^2 - y^2}^2} (\text{spin structure}) + \delta(t_2^2 - x^2 - y^2)(\cdots) \right] \Theta(t_1)$$

Usual Coulomb: $t_2 \to iz$



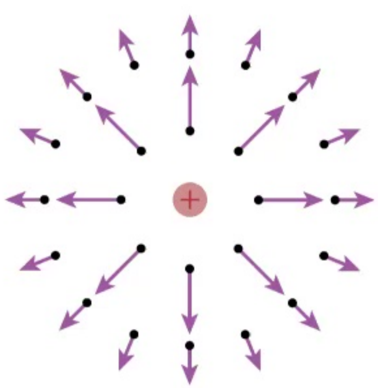
From Coulomb to Kerr

Perform integrals

$$\Psi^{\text{Schw}}(x) = \left[\frac{m}{\sqrt{t_2^2 - x^2 - y^2}^{\,3}} (\text{spin structure})^2 + \right.$$  $$\left. \right] \Theta(t_1)$$

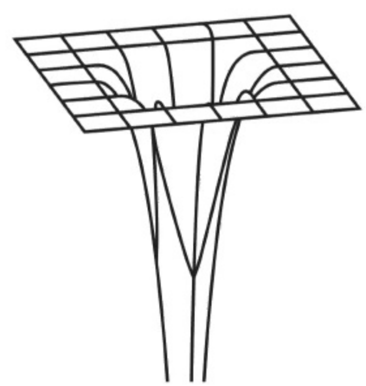
Usual Schwarzschild: $t_2 \to iz$

Coulomb

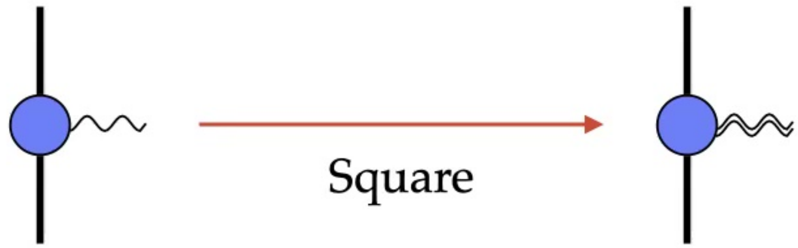


Double copy

Schwarzschild (linearised)



From Coulomb to Kerr



Small step to Kerr (linearised)

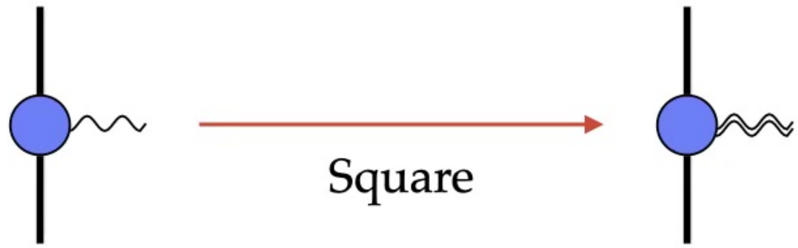
Spin of Kerr

$$\mathcal{M}^{\text{Schw}} = (\mathcal{A}^{\text{Coul}})^2 \qquad \mathcal{M}^{\text{Kerr}} = (\mathcal{A}^{\text{Coul}})^2 e^{k\cdot a}$$

"Minimal coupling"

Arkani-Hamed, Huang & Huang
Arkani-Hamed, Huang & DOC

From Coulomb to Kerr



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Arkani-Hamed, Huang & Huang
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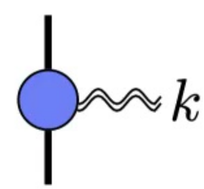
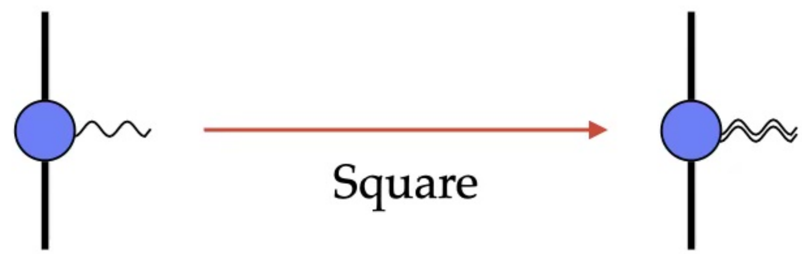
$$\Psi^{\text{Kerr}}(x) = \text{Re} \int \mathrm{d}^4 k \, \delta(k^2) \delta(k \cdot p) \, |k\rangle^4 \, e^{-ik \cdot (x + ia)}$$



From Coulomb to Kerr



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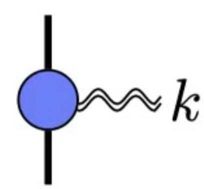
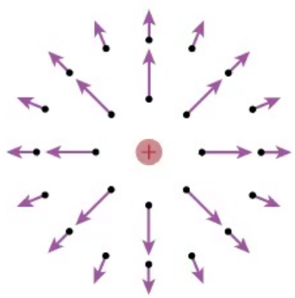


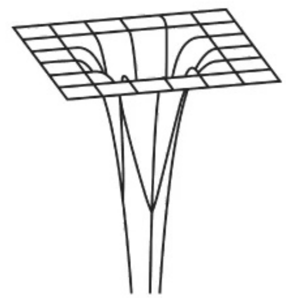
From Coulomb to Kerr

Coulomb



Double copy $\longrightarrow$

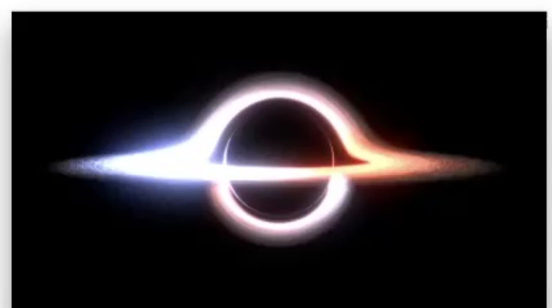
Schwarzschild (linearised)



"Newman-Janis shift"

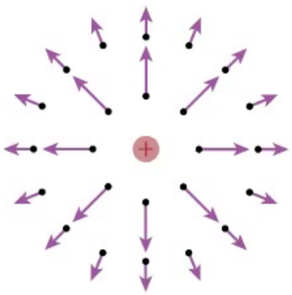
$x \to x + ia$

Kerr (linearised)



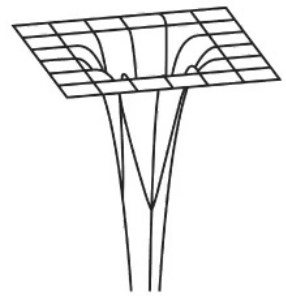
From Coulomb to Kerr

Coulomb



Double copy

Schwarzschild (linearised)

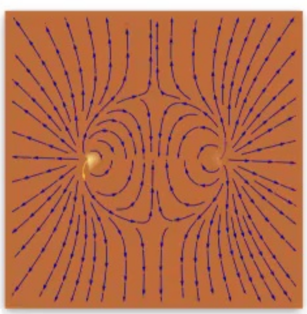


$x \to x + ia$

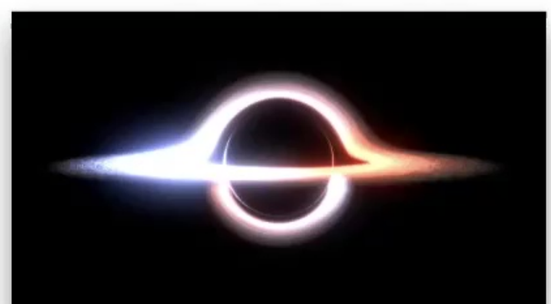
"Newman-Janis shift"

$x \to x + ia$

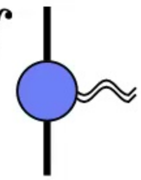
$\sqrt{\text{Kerr}}$

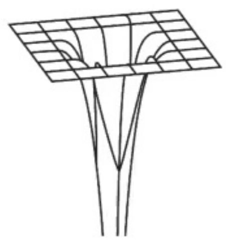


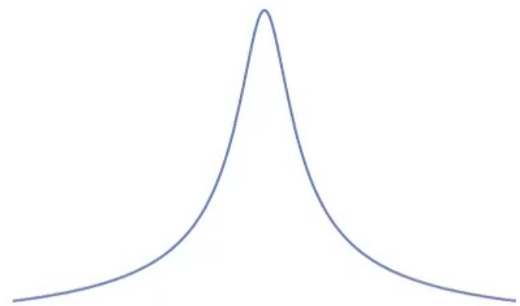
Kerr (linearised)

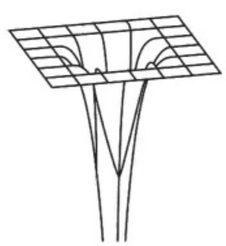


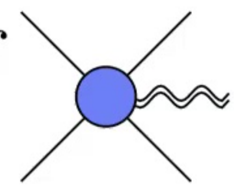
From Coulomb to Kerr

In (2,2) got $\Psi = \int$ . Same idea for *dynamics* in Minkowski:



$\Psi_4 =$ 



$= \dfrac{1}{\text{distance}} \int$ 

Cristofoli, Gonzo, Kosower & DOC

Beyond perturbation theory

Can we make any exact statements?

$$\Psi^{\text{Schw}}_{\alpha\beta\gamma\delta}(x)\xi^\alpha\xi^\beta\xi^\gamma\xi^\delta = -\frac{3m}{2(x^2+y^2+z^2)^{3/2}}\left[\left(\xi^1\right)^2 - \left(\xi^2\right)^2\right]^2$$

Exact classical curvature spinor

Beyond perturbation theory

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Exact classical curvature spinor

Spin structure

Beyond perturbation theory

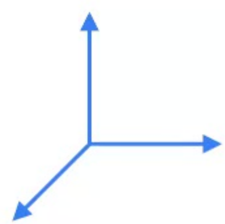
Can we make any exact statements?

Linear approximation is exact!

$$\Psi^{\mathrm{Schw}}_{\alpha\beta\gamma\delta}(x)\,\xi^\alpha\xi^\beta\xi^\gamma\xi^\delta = -\frac{3m}{2(x^2+y^2+z^2)^{3/2}}\left[\left(\xi^1\right)^2-\left(\xi^2\right)^2\right]^2$$

Exact classical curvature spinor

Depends on specific choice of basis vectors



Beyond perturbation theory

Can we make any exact statements?

$$\Phi_{\alpha\beta}^{\sqrt{\mathrm{Kerr}}}(x)\xi^\alpha\xi^\beta\xi^\gamma\xi^\delta = -\frac{Q}{(x^2+y^2+(z+ia)^2)^{3/2}}\left[\left(\xi^1\right)^2 - \left(\xi^2\right)^2\right]$$

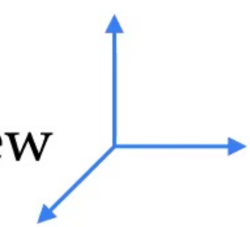
Beyond perturbation theory

Exact relationship $S\,\Psi_{\alpha\beta\gamma\delta} = \Phi_{(\alpha\beta}\Phi_{\gamma\delta)}$

"Weyl double copy"

Luna, Monteiro, Nicholson & DOC

- ✓ Saw that *perturbative* expansion is double copy of amplitudes
- ✓ Many examples!
- ✓ Lack understanding of basis choice in quantum point of view



Conclusions

1. Amplitudes - “atoms of dynamics”
2. Double copy:
   - Aid to calculation
   - New connections between LHC & gravitational wave physics
   - Schwarzschild = Coulomb^2
3. Curiosities:
   - Newman-Janis shift?
   - Physics from $(++--)$ signature?