Title: Dynamical friction in scalar dark matter

Speakers: Kathy Clough

Series: Strong Gravity

Date: October 14, 2021 - 1:00 PM

URL: https://pirsa.org/21100018

Abstract: I will discuss the dynamical friction experienced by black holes moving through scalar or "wave-like" dark matter. This has been studied analytically and numerically in the non relativistic case, which is applicable to its effect in "fuzzy dark matter" halos where the scalar wavelength is on galactic scales. On smaller scales, dynamical friction from dark matter clouds formed from accretion or superradiance may cause a dephasing in the gravitational wave signal in LISA EMRIs. I will discuss our recent numerical work (https://arxiv.org/abs/2106.08280) to extend the description of dynamical friction to relativistic scalars, in order to treat this case.

Zoom Link: https://pitp.zoom.us/j/98815587081?pwd=a3FvcUtTaWFUejBpR1Q3QUhRRzF4dz09

Dynamical friction from light dark matter in the relativistic regime

Katy Clough

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UNIVERSITY OF OXFORD

Science & Technology Facilities Council

Queen Mary University of London

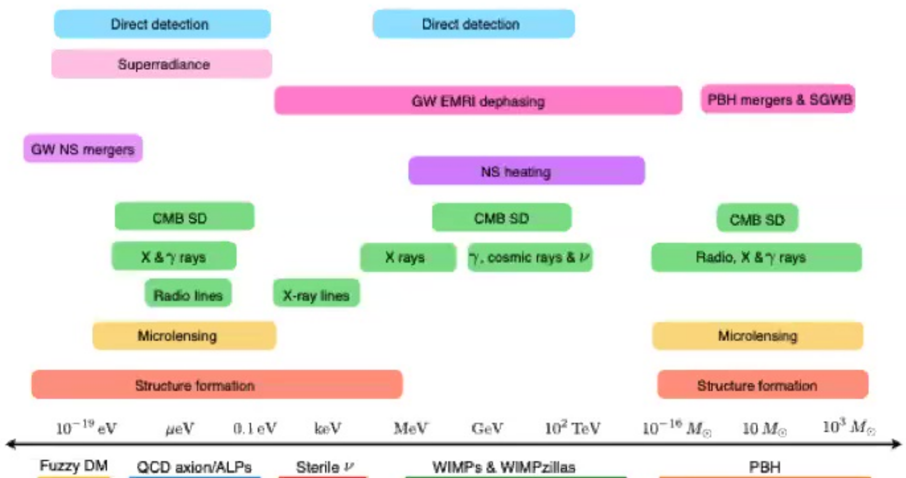
Plan for the talk

- A brief background on light dark matter candidates
- Dynamical friction in the non relativistic regime
- Dynamical friction in the relativistic regime
- Next steps

A brief background on light DM

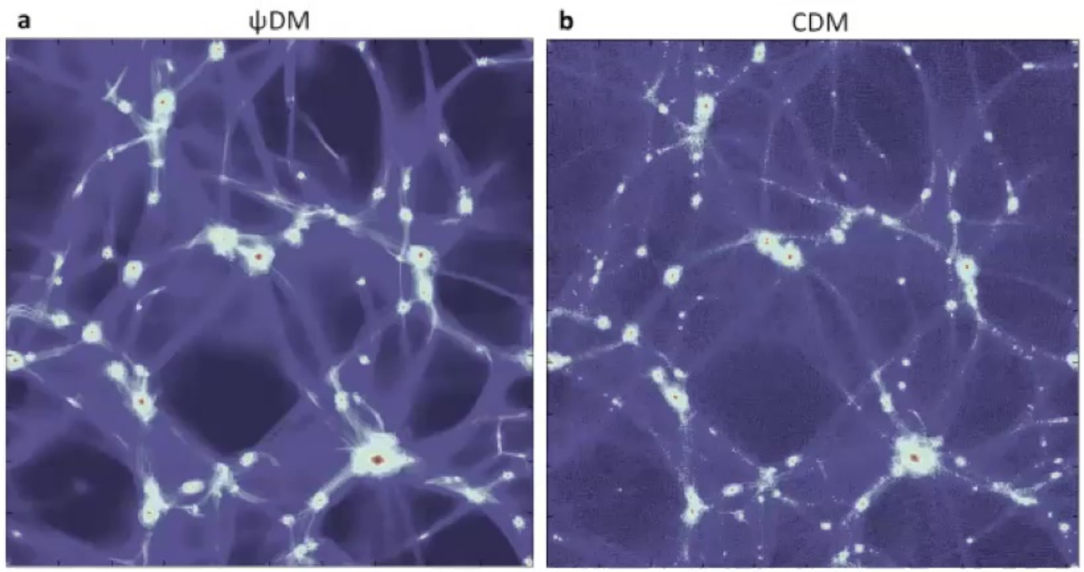
See Wave Dark Matter review by Lam Hui
(Annual Review of Astronomy and Astrophysics, arXiv 2101.11735)

Why light dark matter?



EuCAPT white paper, in prep

Why light dark matter?



Nature Phys. 10, 496-499
Schive et al. 2014

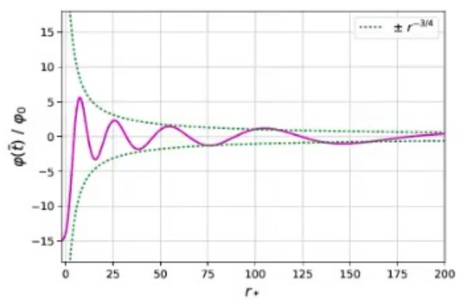
Light dark matter is wave-like

- In the limit of small mass m (sub eV), the number density is high, and dark matter is well described by a classical field, obeying (for a spin 0 particle) the Klein Gordon equation:

$$\nabla^\mu \nabla_\mu \phi = \mu^2 \phi$$

- It has a **wave like behaviour**, and **pressure support** on length scales of order 1/mu
- The underlying properties of the particles (number density, velocity) at each point can be inferred by considering properties of the field, for example:

$$k_i \sim \dot{\phi}\ \partial_i \phi \qquad \rho \sim \dot{\phi}^2 + \mu^2 \phi^2 + (\partial_i \phi)^2$$



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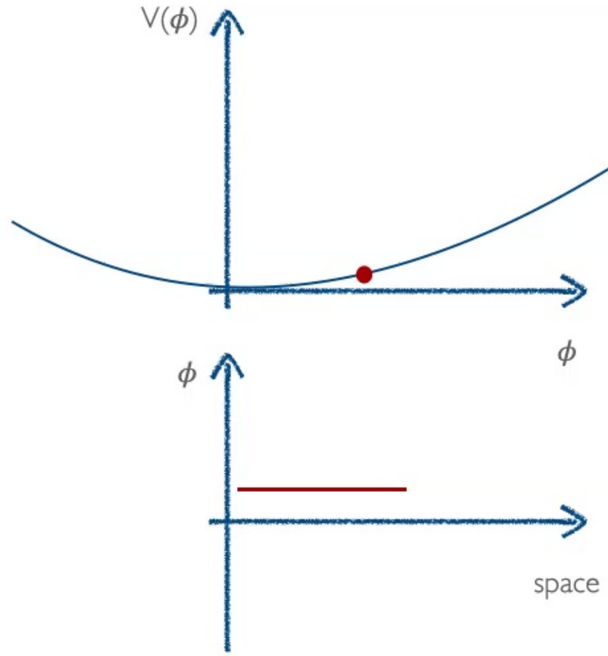

Cold!

$$k_i \sim \dot{\phi}\, \partial_i \phi \sim 0$$

Energy density set by local amplitude

$$\rho \sim \dot{\phi}^2 + \mu^2 \phi^2 \implies \langle \rho \rangle \sim \mu^2 \phi^2$$

Pressureless (on average)

$$p \sim \dot{\phi}^2 - \mu^2 \phi^2 \implies \langle p \rangle \sim 0$$

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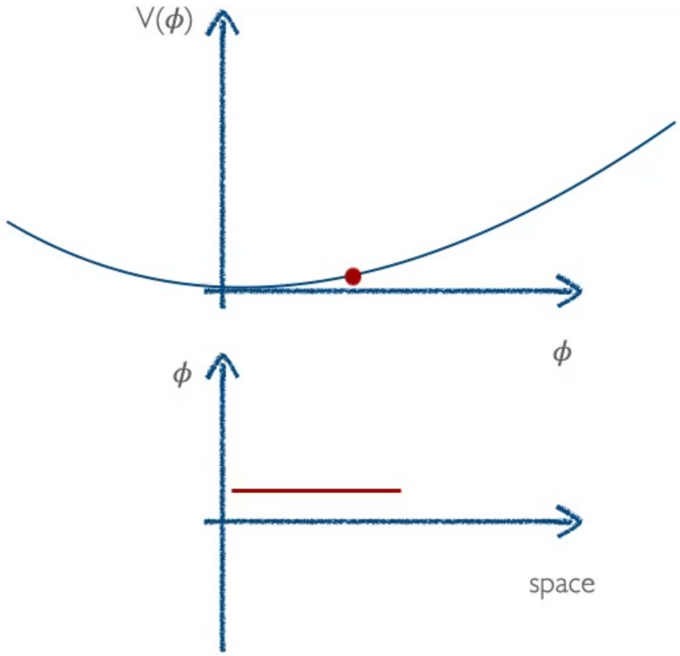

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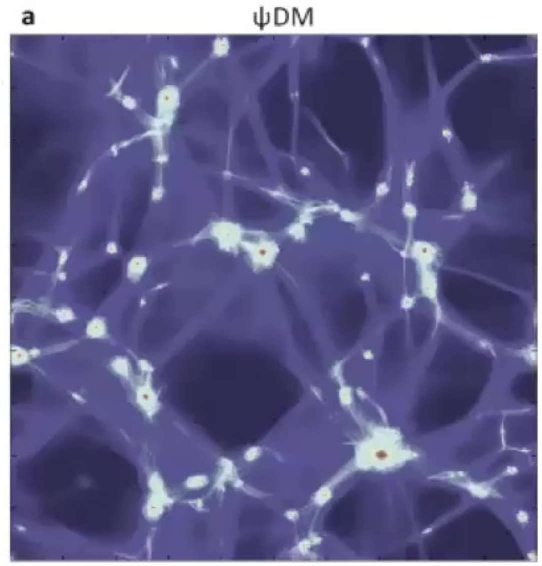
Non relativistic description

- Modulo out the oscillations:

$$\phi(x,t) = \psi(x,t)\ e^{i\mu t} + c\,.\,c\,.$$

- Assuming that $\dot{\psi} \ll \mu\psi$ one can show that ψ obeys the Schroedinger-Poisson equations

- Density is related to the amplitude of ψ as $\rho \sim \psi\psi^*$



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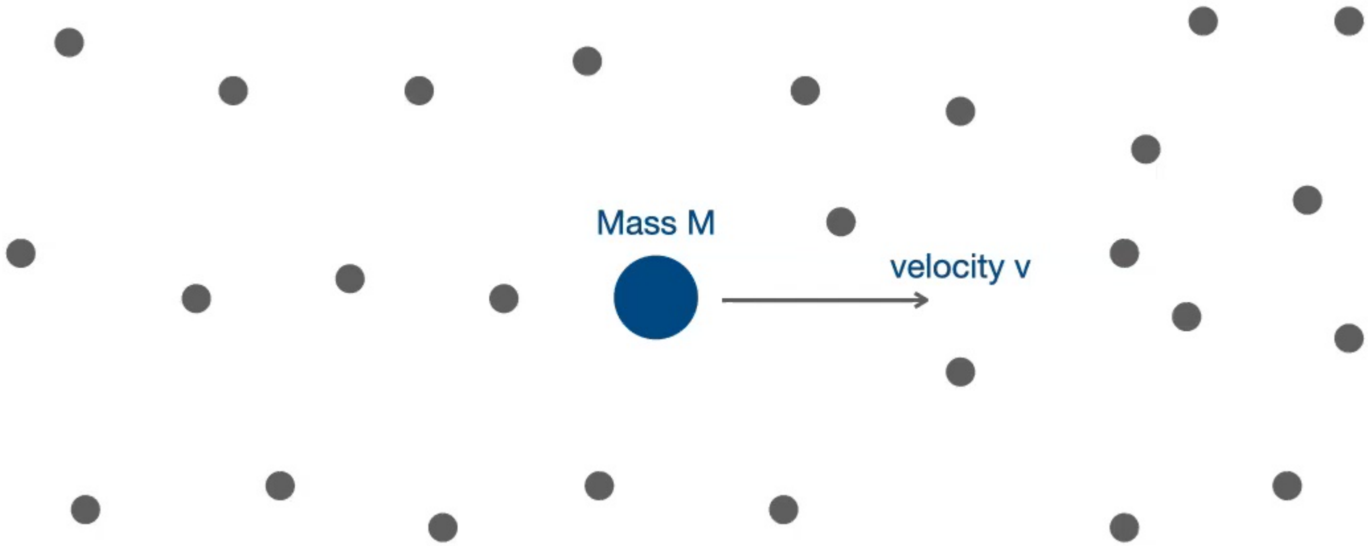
Dynamical friction



Dynamical Friction. I. General Considerations: the Coefficient of Dynamical Friction.
S. Chandrasekhar, Astrophysical Journal, vol. 97, p.255

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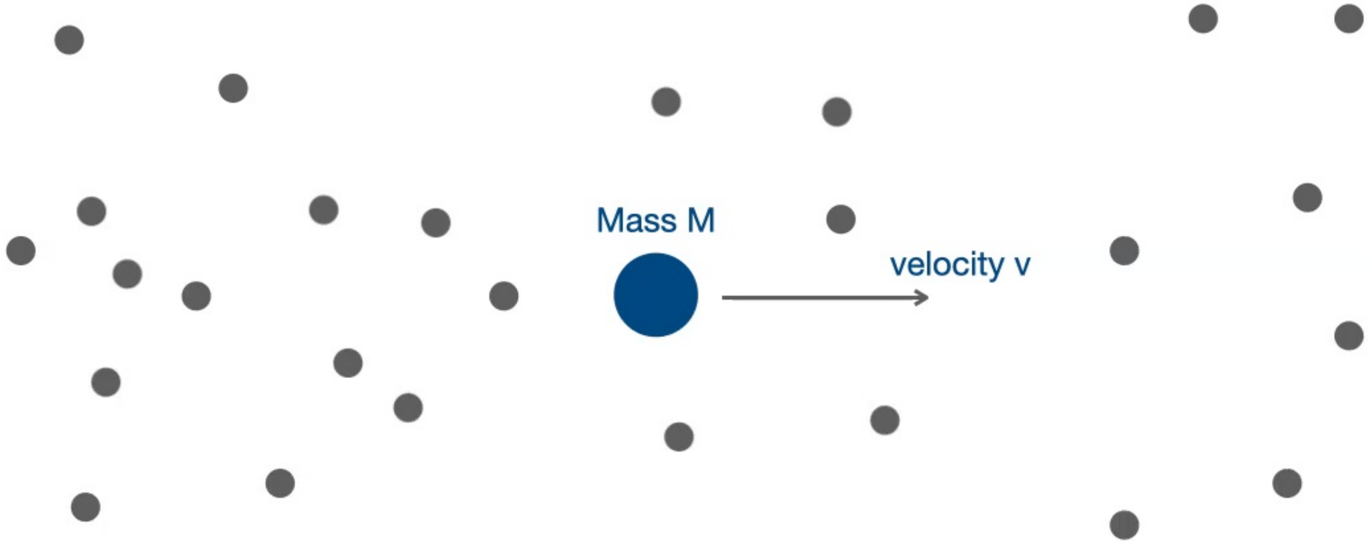

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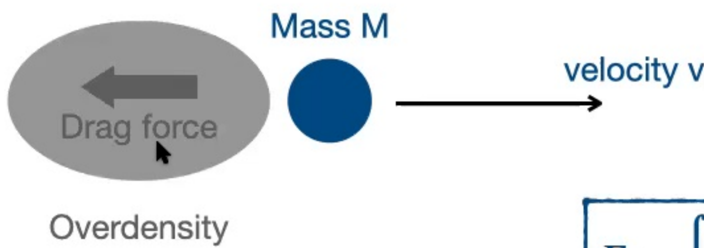
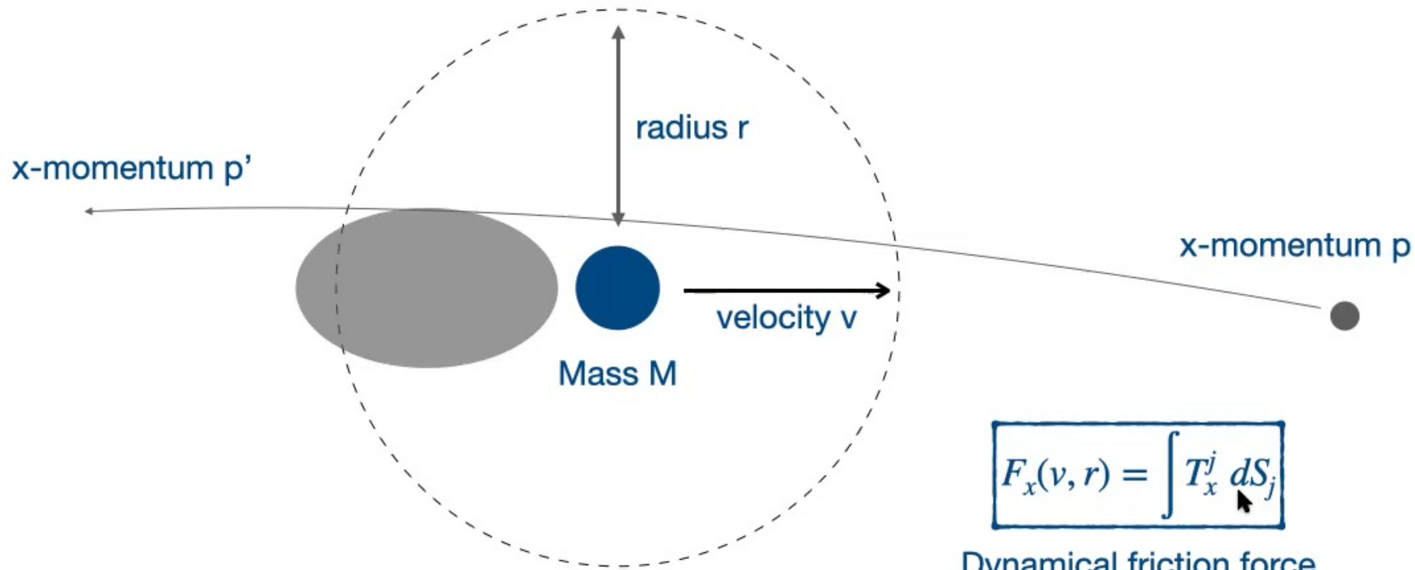
Dynamical friction - Newtonian calculation



$$F_i = \int \frac{GM\rho}{r^2} . \frac{x^i}{r} \, dV$$

Dynamical friction force

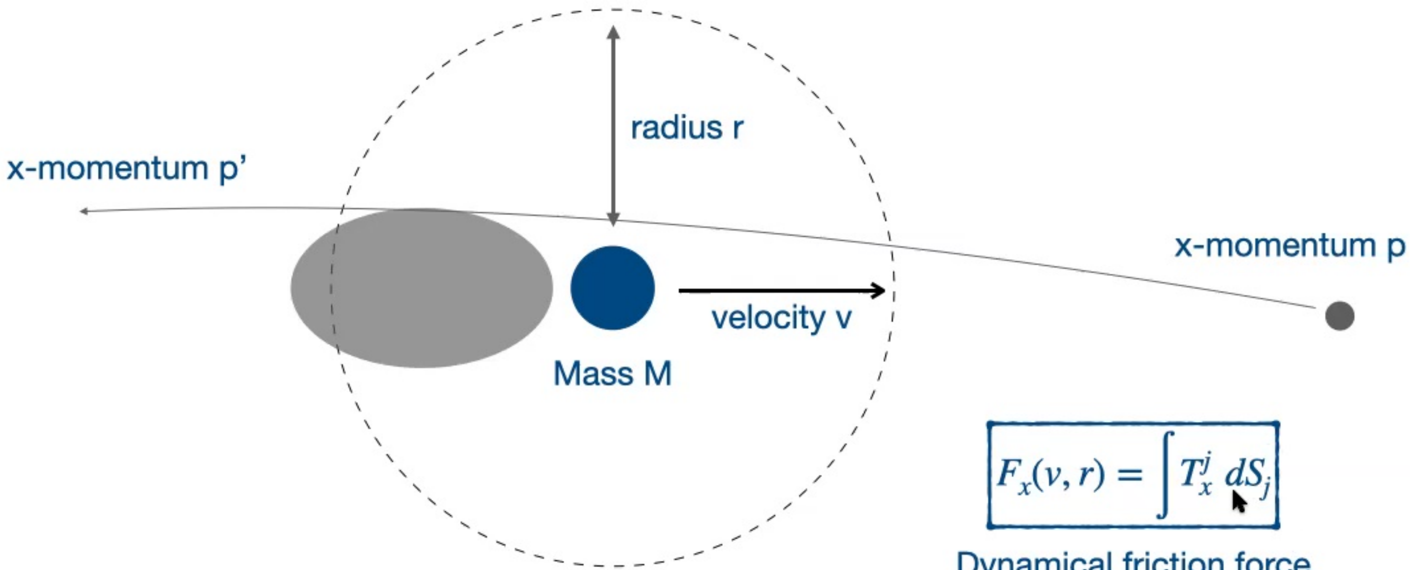
Dynamical friction - Newtonian calculation



$$F_x(v, r) = \int T_x^j \, dS_j$$

Dynamical friction force
(in the stationary limit)

Dynamical friction - Newtonian calculation



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Dynamical friction - Newtonian calculation for scalar DM

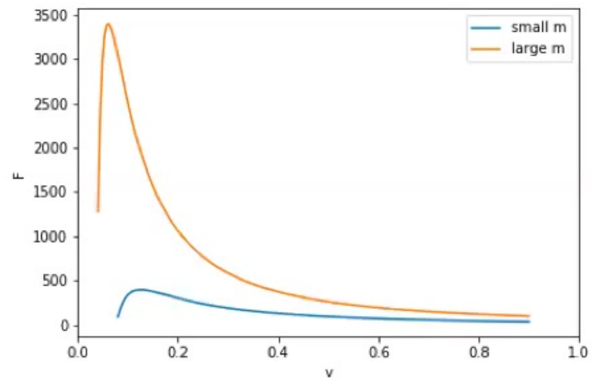
For scalar mass m the behaviour is parametrised by the ratio

$$\beta = \frac{GM}{v^2} \Big/ \frac{1}{mv}$$

And the force is

$$F = 4\pi\rho \left(\frac{GM}{v}\right)^2 \left(\ln(2mvr) - 1.0 - Re(\Psi(1 + i\beta))\right)$$

Ultralight scalars as cosmological dark matter
L. Hui, J. Ostriker, S. Tremaine, E. Witten, Phys. Rev. D 95, 043541 (2017)



Force versus velocity

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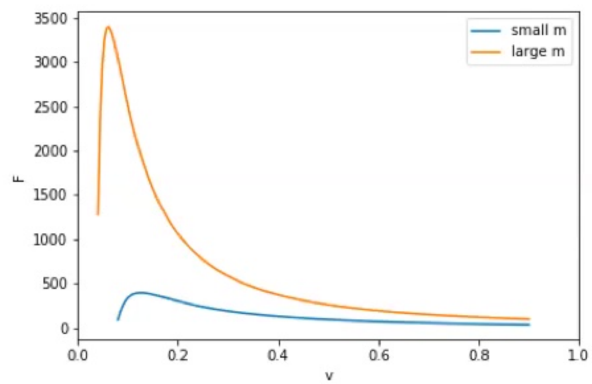

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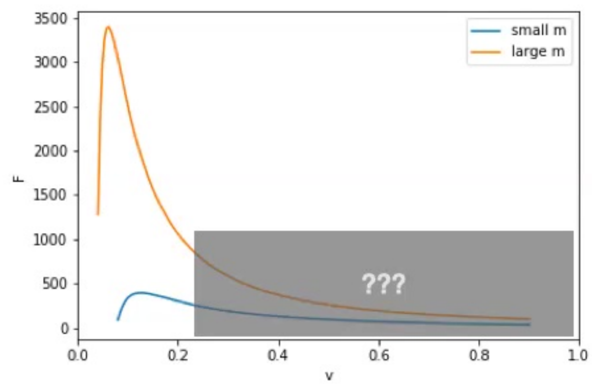

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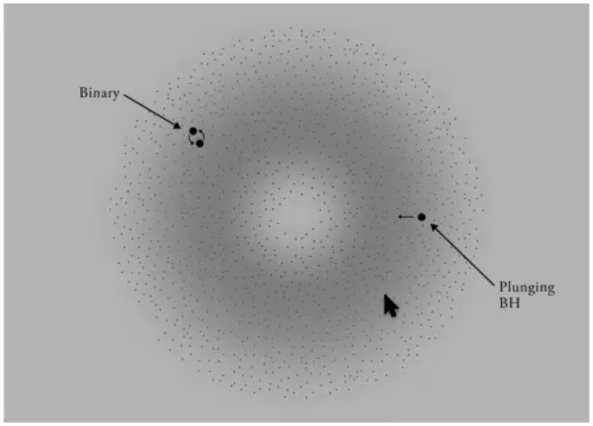

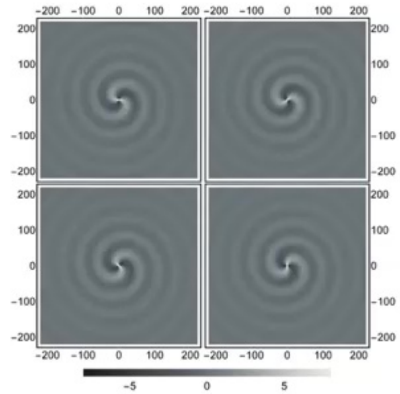

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Stirred and shaken: Dynamical behavior of boson stars and dark matter cores
Phys.Lett.B 811 (2020) 135944
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Dynamical Friction From Ultralight Dark Matter
Y. Wang, R. Easther
arXiv:2110.03428

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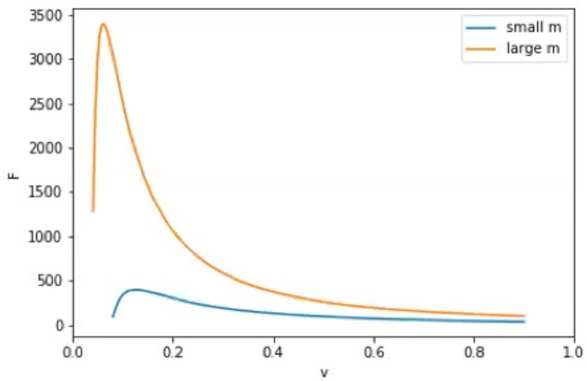

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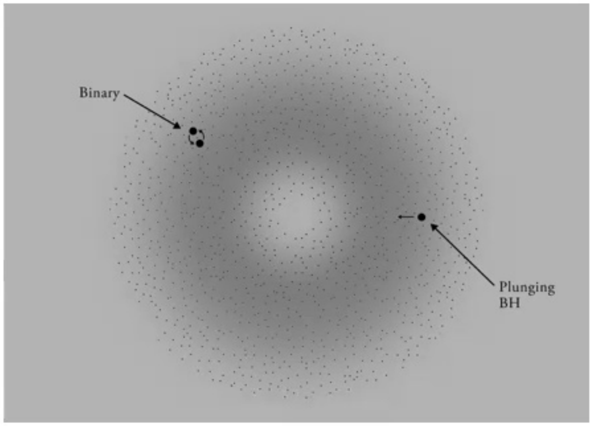

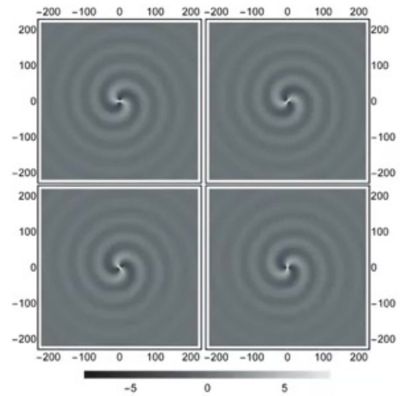

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arXiv:2110.03428

Relativistic dynamical friction

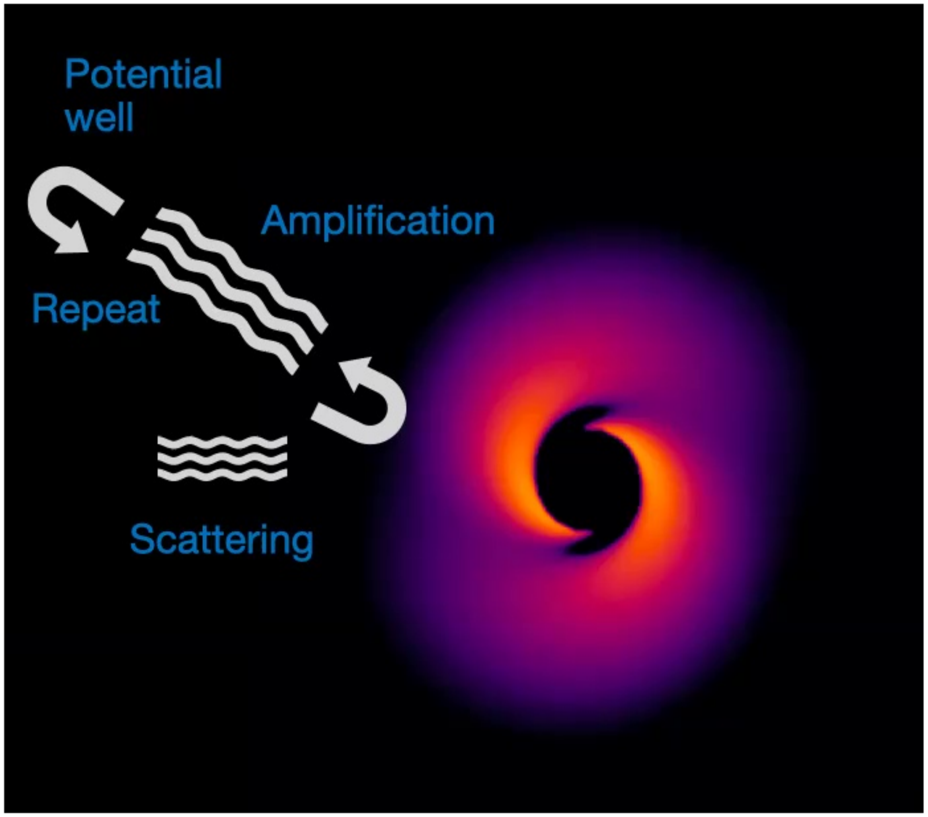
Superradiance

Superradiant Instability and Backreaction of Massive Vector Fields around Kerr Black Holes
W.E. East and F. Pretorius
Phys.Rev.Lett. 119 (2017) 4, 041101

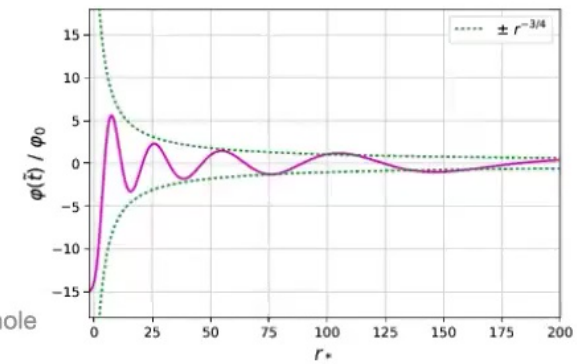
String Axiverse
A. Arvanitaki et. al.
Phys.Rev.D 81 (2010) 123530

see R Brito's review Superradiance : New Frontiers in Black Hole Physics

$$\frac{\rho}{[R_s^{-2}]} \sim \frac{0.1 M_{BH}}{\frac{4\pi}{3}(10R_s)^3} \sim 10^{-5}$$



Accretion

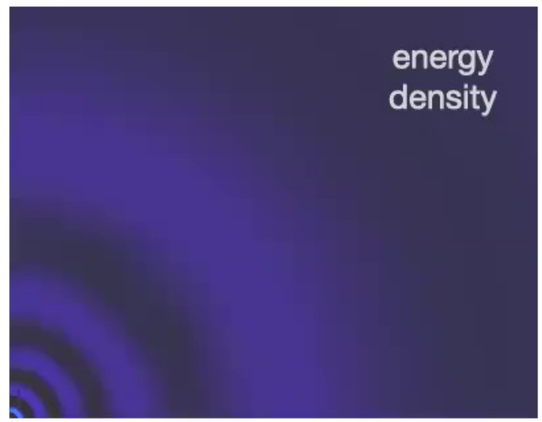


radial
field profile

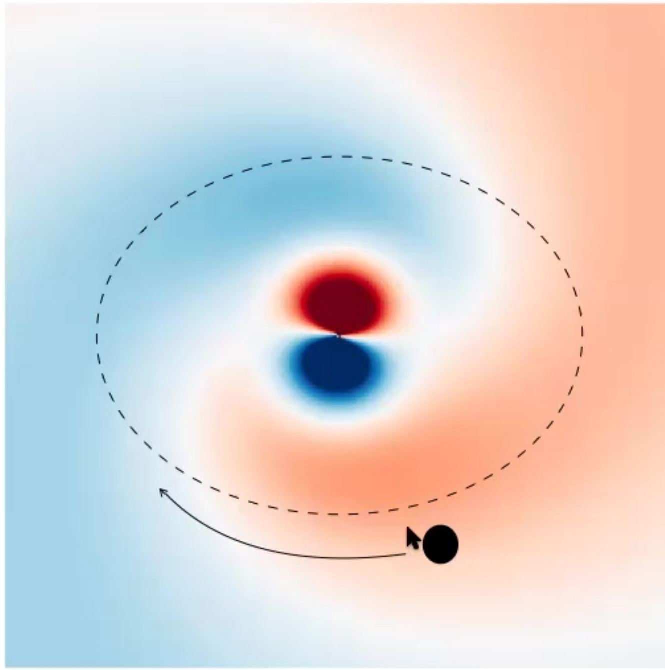
Black Hole Hair from Scalar Dark Matter
L Hui, D Kabat, X Li, L Santoni, S.S.C. Wong
JCAP 06 (2019) 038

Growth of massive scalar hair around a Schwarzschild black hole
KC, P.G. Ferreira, M Lagos
Phys.Rev.D 100 (2019) 6, 063014

$$\frac{\rho}{[R_s^{-2}]} \sim 10^{-12} \left(\frac{M_{BH}}{10^9 M_\odot}\right)^2 \left(\frac{\rho_0}{\mathrm{GeV\ cm^{-3}}}\right) \left(\frac{v_0}{10^{-3}c}\right)^{-3}$$



Can we detect these environments in EMRI orbits?

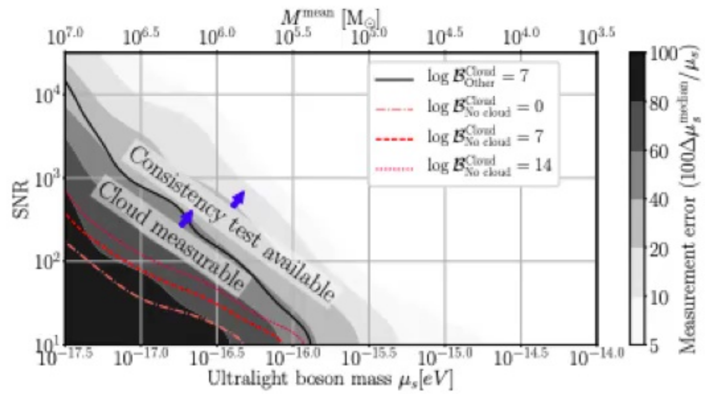


Dynamical friction - impact on EMRI orbits

- Can work out presence of a superradiant cloud by studying the dephasing of the inspiral of a smaller object into a supermassive BH

- Impact of dynamical friction not included, only backreaction of cloud on the metric

- For inner orbits, relativistic corrections potentially matter



Probing the existence of ultralight bosons with a single gravitational-wave measurement
OA Hannuksela , K Wong , R Brito , E Berti , TGF Li
Nature Astronomy volume 3, pages 447–451(2019)

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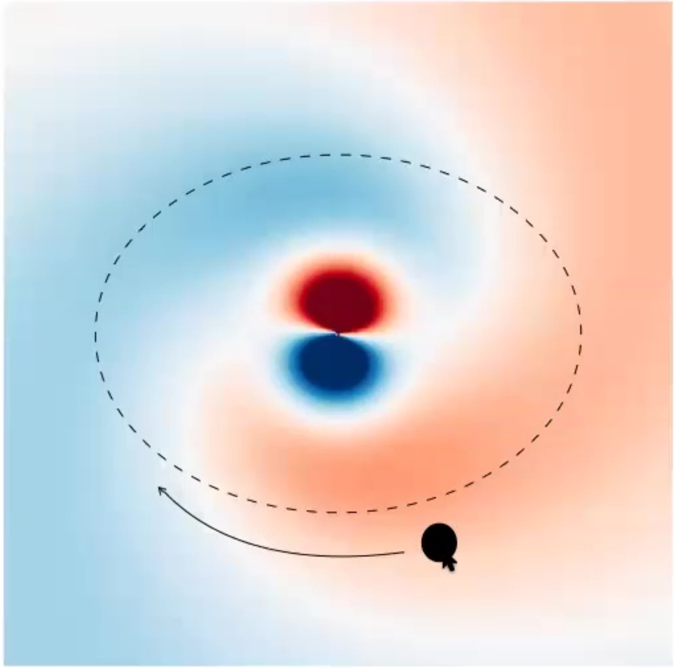
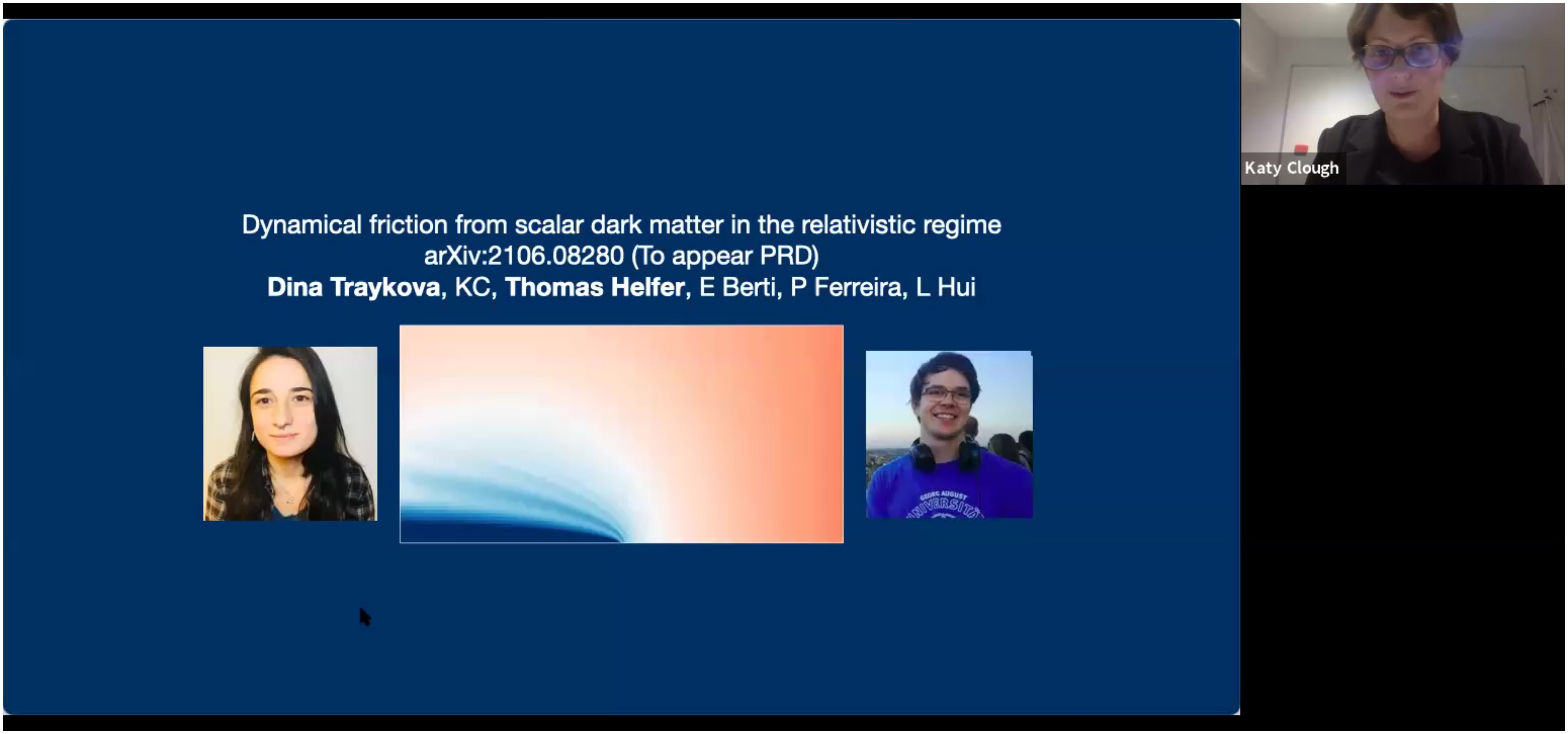



Dynamical friction from scalar dark matter in the relativistic regime
arXiv:2106.08280 (To appear PRD)
Dina Traykova, KC, **Thomas Helfer**, E Berti, P Ferreira, L Hui
Katy Clough

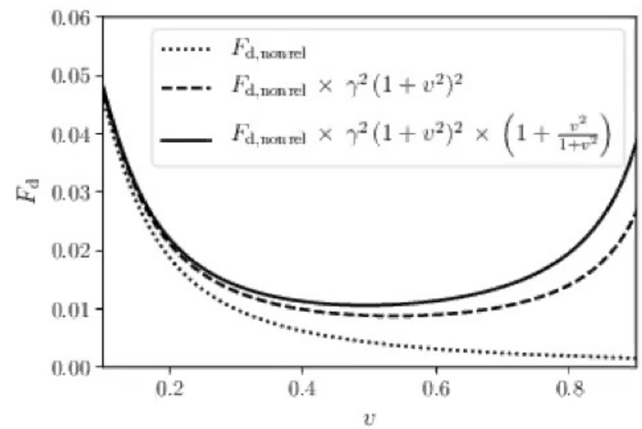
Dynamical friction - expected relativistic corrections

- Overall relativistic correction

$$F_{nonrel} \times \gamma^2 (1 + v^2)^2$$

- Also replace density $\rho \rightarrow \rho + p$

$$F_{nonrel} \times \frac{\rho + p}{\rho}$$



Force versus velocity

Accretion onto a Moving Black Hole: A Fully Relativistic Treatment
L Petrich, SL Shapiro, FL Stark, SA Teukolsky ApJ v.336, p.313

Relativistic dynamical friction in a collisional fluid
E. Barausse, MNRAS, Volume 382, Issue 2, December 2007

Dynamical friction - expected relativistic corrections

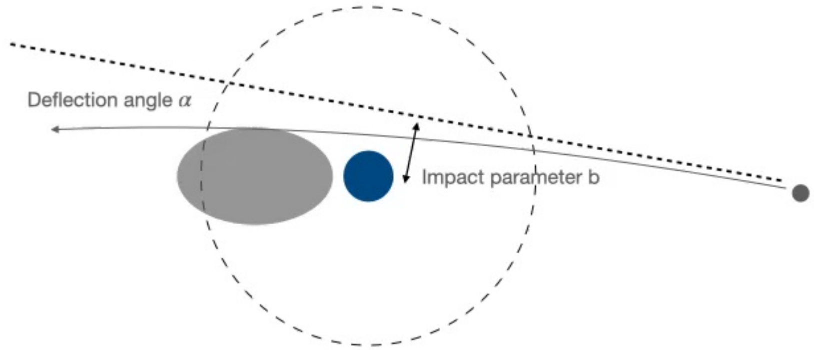
- First relativistic correction

$$F_{nonrel} \times \gamma^2 \times (1 + v^2)^2$$

- γ^2 relates to the increase in the momentum of the fluid as seen in the rest frame of the perturber

- The second part comes from the increase in the deflection angle at relativistic velocities

$$\alpha = \frac{2M}{bv^2} \times (1 + v^2)$$



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- Pressure is not isotropic, and is zero in the rest frame of the dark matter, so we use the pressure in the direction of motion as in the BH rest frame:

$$F_{nonrel} \times \frac{\rho + p_x}{\rho}$$

- Scalar field does not have a well defined equation of state, consider:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i \phi)^2 - \frac{1}{2}\mu^2\phi^2$$

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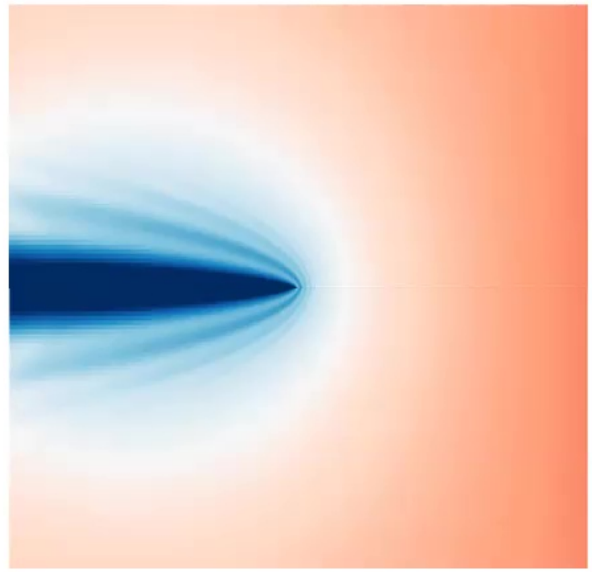
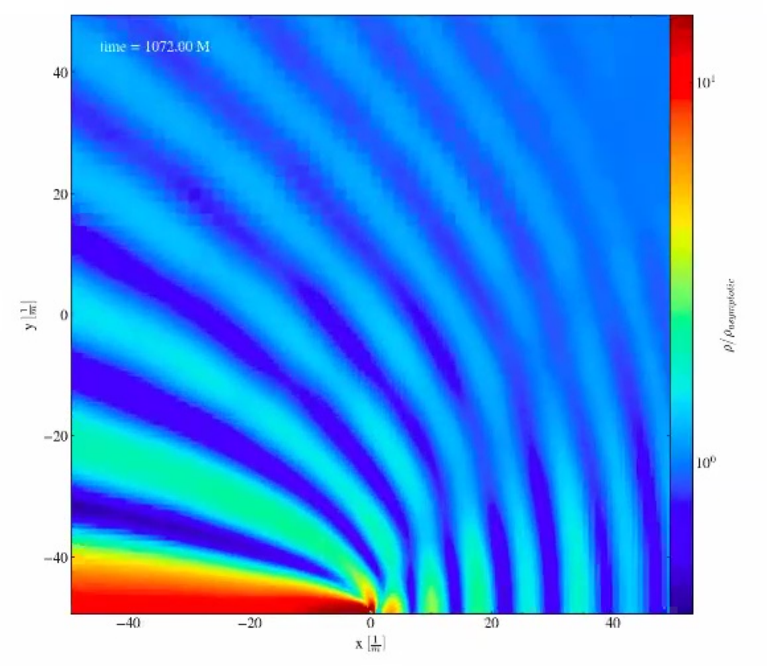
- Assuming a plane wave in the x direction, we get a **velocity dependent** correction factor of:

$$F_{nonrel} \times 1 + \frac{v^2}{1 + v^2}$$



Dynamical friction simulations



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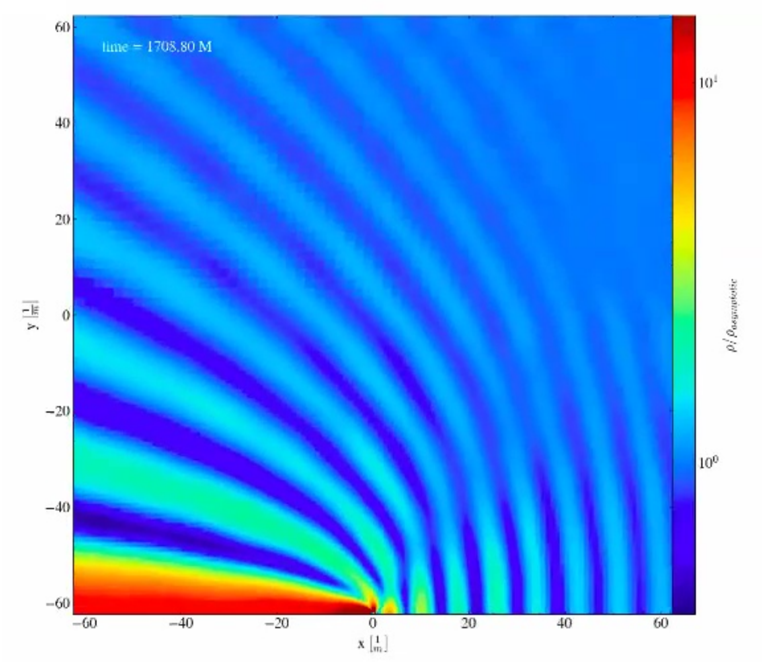

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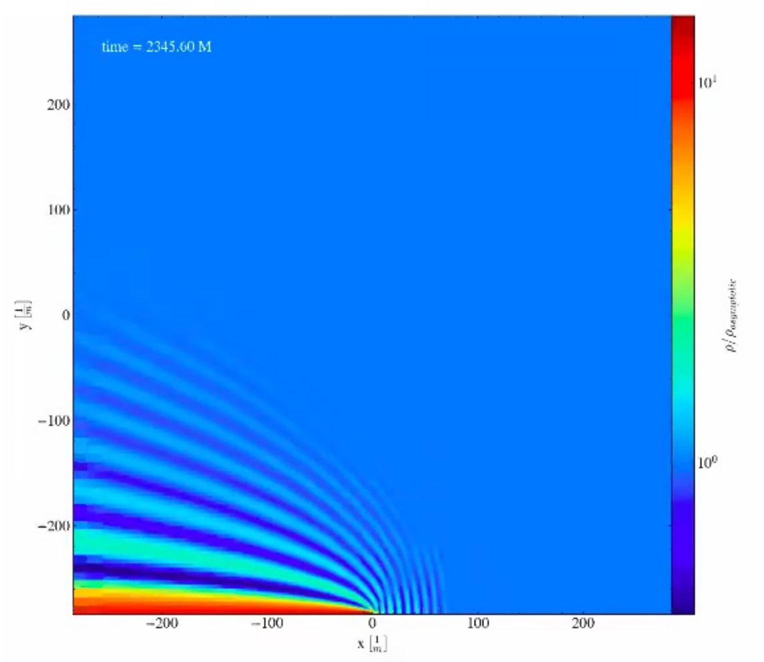

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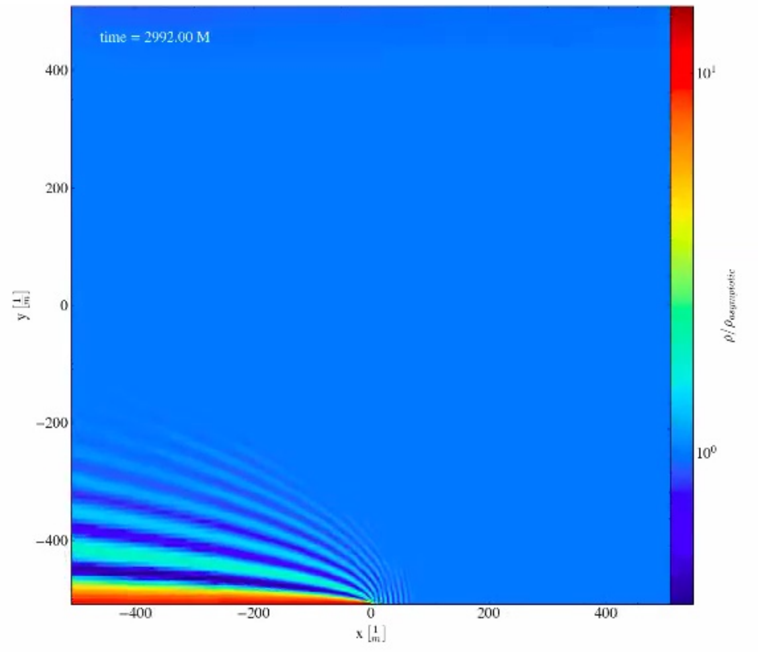

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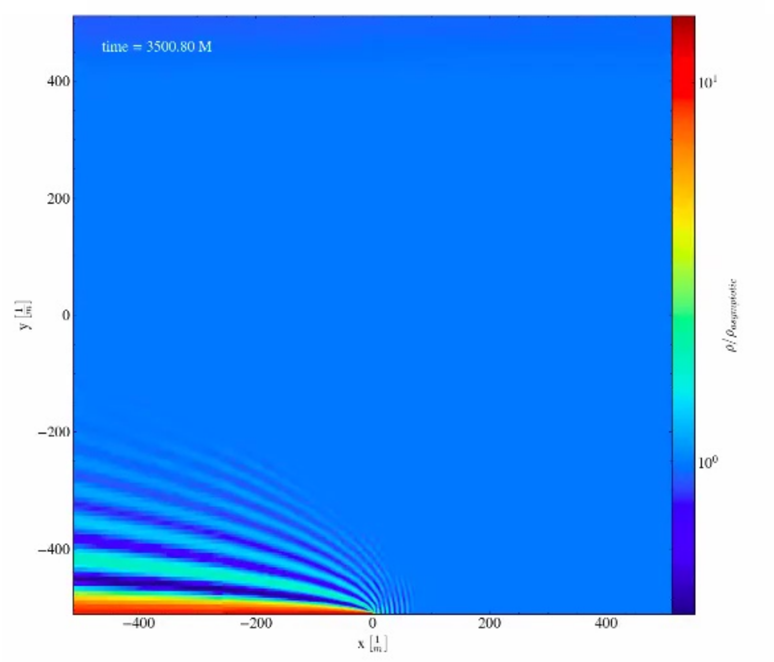

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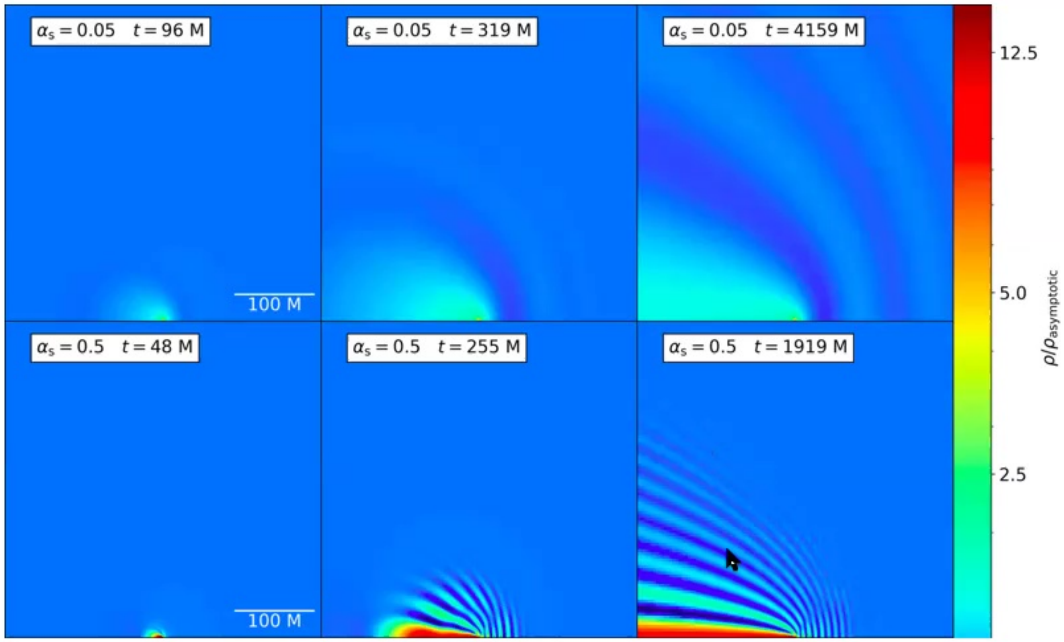
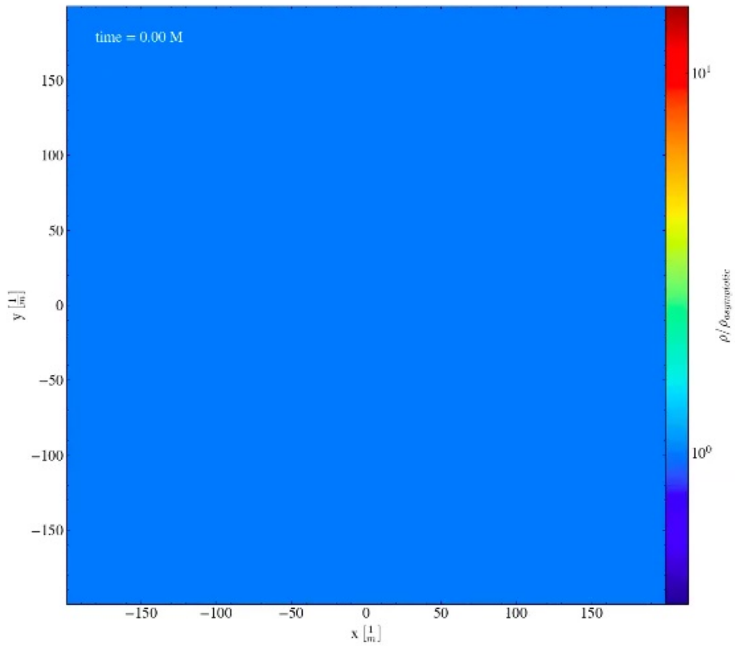


Smaller scalar mass

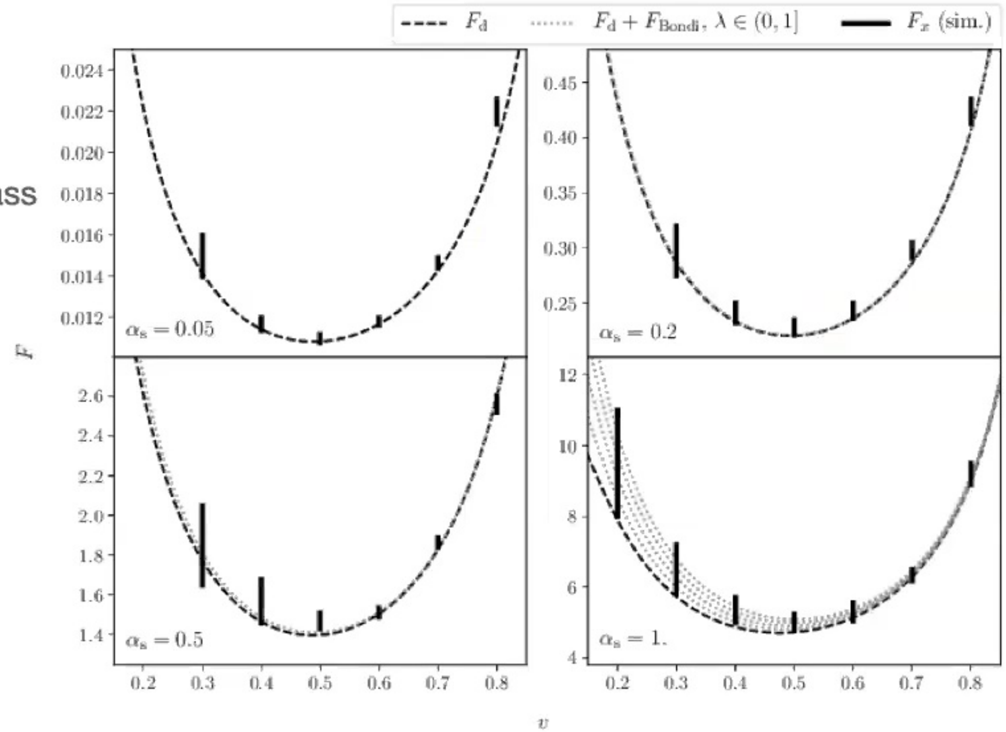
Larger scalar mass

Dynamical friction simulations



Dynamical friction - results

Smaller scalar mass



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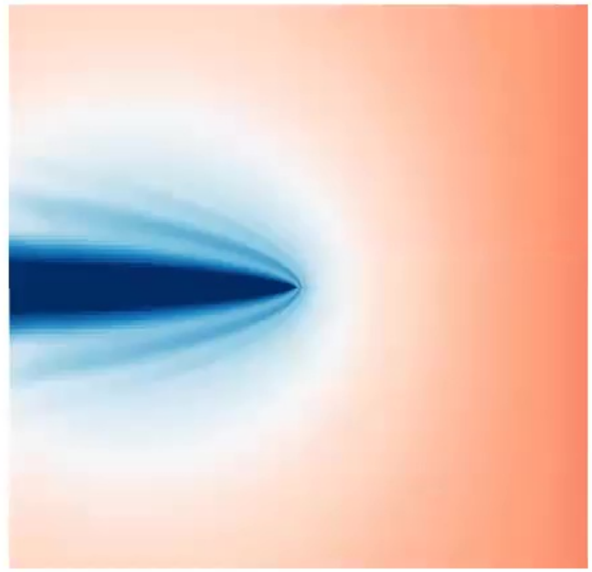

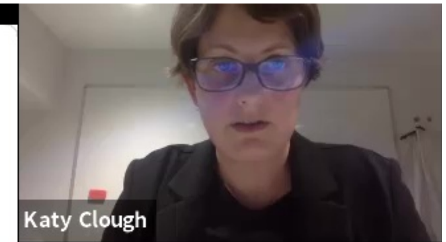

Reminder - Newtonian calculation for scalar DM

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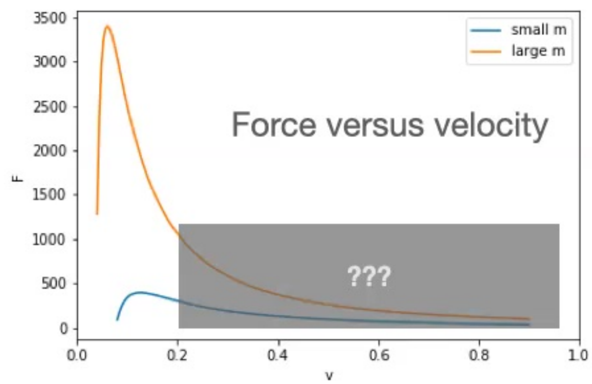

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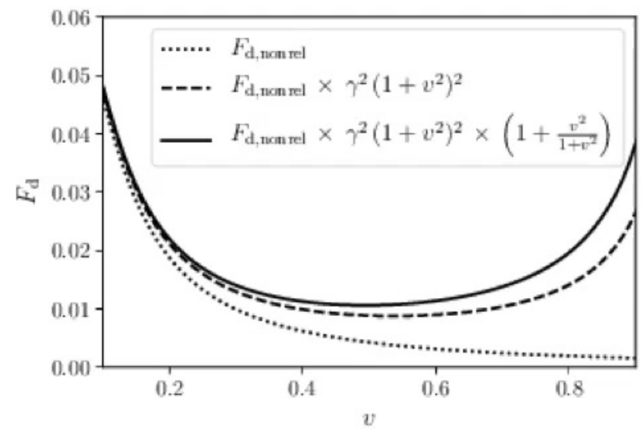

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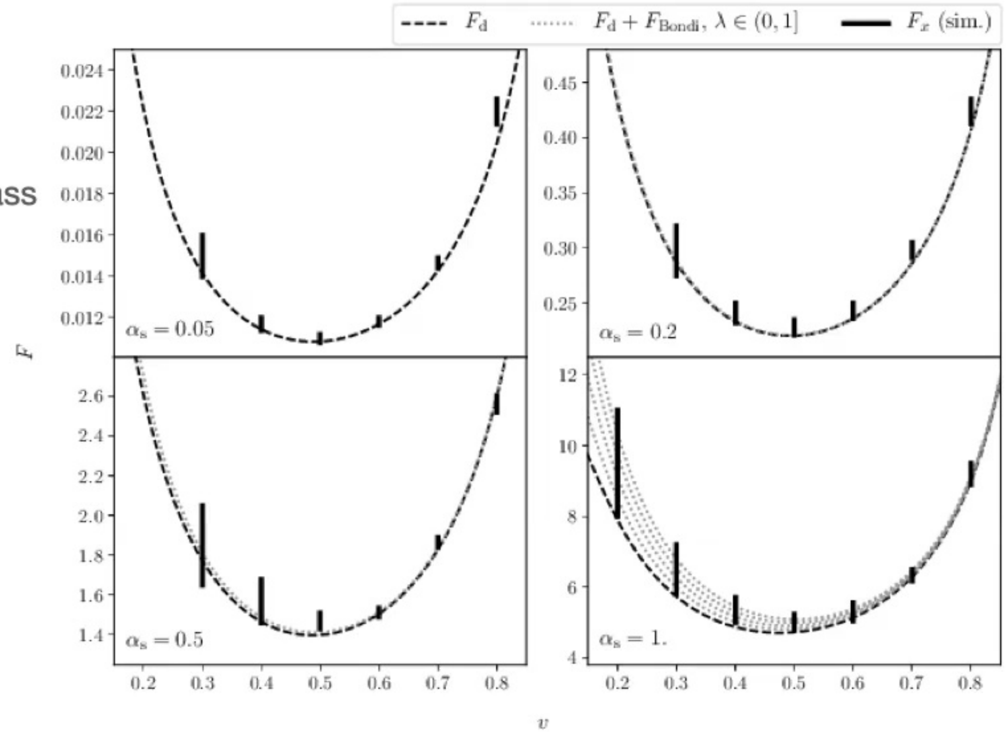
Accretion onto a Moving Black Hole: A Fully Relativistic Treatment
L Petrich, SL Shapiro, FL Stark, SA Teukolsky ApJ v.336, p.313

Relativistic dynamical friction in a collisional fluid
E. Barausse, MNRAS, Volume 382, Issue 2, December 2007

Dynamical friction - results

Smaller scalar mass



Larger scalar mass

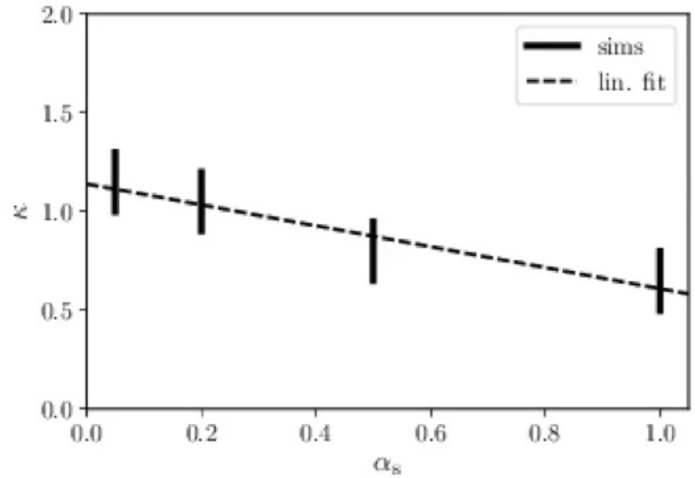
Dynamical friction - results

- Conclusion: Expected relativistic expression fits pretty well

$$F_{rel} = F_{nonrel} \times \underbrace{\left(1 + \kappa \frac{v^2}{1+v^2}\right)}_{pressure} \times \underbrace{\gamma^2 \left(1+v^2\right)^2}_{deflection/boost}$$

- "Fudge factor" kappa encodes dependence of pressure correction on scalar mass - in masses $\mu M \gg 1$ we should recover the pressureless limit, for small masses it is of order 1.

kappa



Scalar mass

Relativistic gravitational "forces"

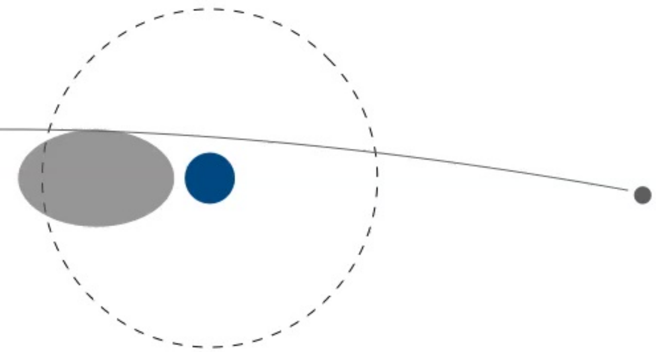
- We can define a "force" as the change in the ADM momentum of the spacetime, sourced by a net flux of momentum*:

$$F_i = \partial_t P_i^{ADM} = -\int_{\partial\Sigma} \alpha N_j T^j_i \; dS$$

- However this is only really the dynamical friction in the stationary limit, otherwise one is just "growing the cloud"

- It takes quite a while for our simulations to become stationary

**Accurate to first order assuming a small asymptotic energy density - we neglect backreaction on metric*



Relativistic gravitational "forces"

- In a dynamical situation, the growth in the ADM momentum will include the momentum of the growing DM cloud as well as the BH

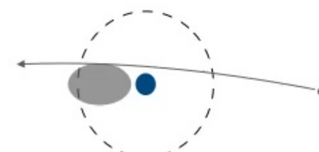
$$P_i^{ADM} = \int_\Sigma S_i \, dV + P_i^{curvature}$$

- In an asymptotically flat spacetime (in appropriate coordinates), one can show that at any instant in time

$$\partial_t P_i^{ADM} = \underbrace{\partial_t \int_\Sigma S_i \, dV}_{cloud\ growth} - \underbrace{\int_\Sigma \alpha T^\mu_\nu \nabla_\mu (\delta^\nu_i) \, dV}_{dynamical\ friction}$$

- Then we can use instead the volume integral to find directly the quantity of interest

$$F_i \equiv \partial_t P_i^{curvature} = - \underbrace{\int_\Sigma \alpha T^\mu_\nu \, \Gamma^\nu_{\mu i} \, dV}_{dynamical\ friction}$$

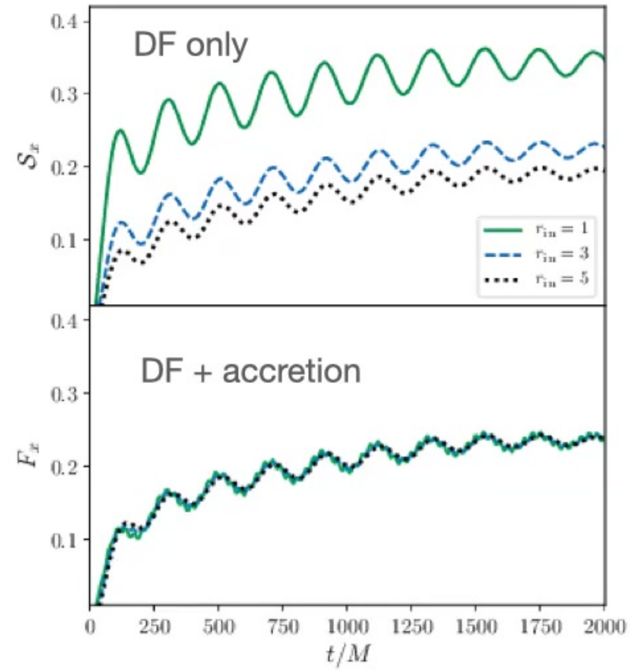


Relativistic gravitational "forces"

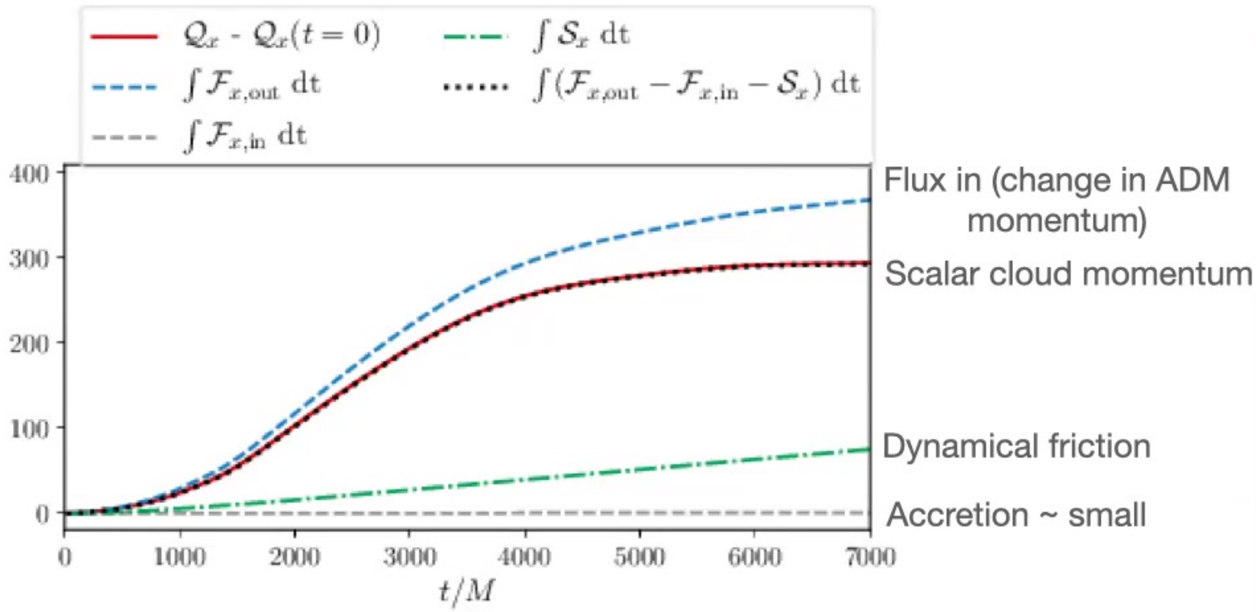
- Also need to take account of singularity

$$\partial_t P_i^{curvature} = -\underbrace{\int_{\Sigma-\Sigma_{BH}} \alpha T^\mu_\nu \nabla_\mu(\delta^\nu_i)\, dV}_{dynamical\ friction} - \underbrace{\int_{\partial\Sigma_{BH}} \alpha N_j T^j_i\, dS}_{accretion}$$

- In the relativistic case the split between accretion and dynamical friction is slicing dependent
- It is less coordinate dependent to measure the sum of the two



Relativistic gravitational "forces"



Checking the (non) conservation of momentum over time

What next?

- Feed result into EMRI calculations
- Treat more realistic scenarios
  - Impact of cloud having a finite size
  - Impact of circular orbit
- Can be easily extended to other cases - e.g. self interactions/vector field - that are analytically more challenging