

Title: Holomorphic surfaces from determinants

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Series: Mathematical Physics

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Abstract: I will describe a concrete, computable example of holographic geometry emerging purely from the combinatorics of large matrices. Correlation functions of determinant operators in a matrix-valued free chiral algebra will be matched to holomorphic curves in  $SL(2, \mathbb{C})$ .

Zoom Link: <https://pitp.zoom.us/j/99341570607?pwd=TVVmSkF3d1haa0hBOWIPeDhGNi9aQT09>

# GIANT GRAVITONS IN TWISTED HODOGRAPHY



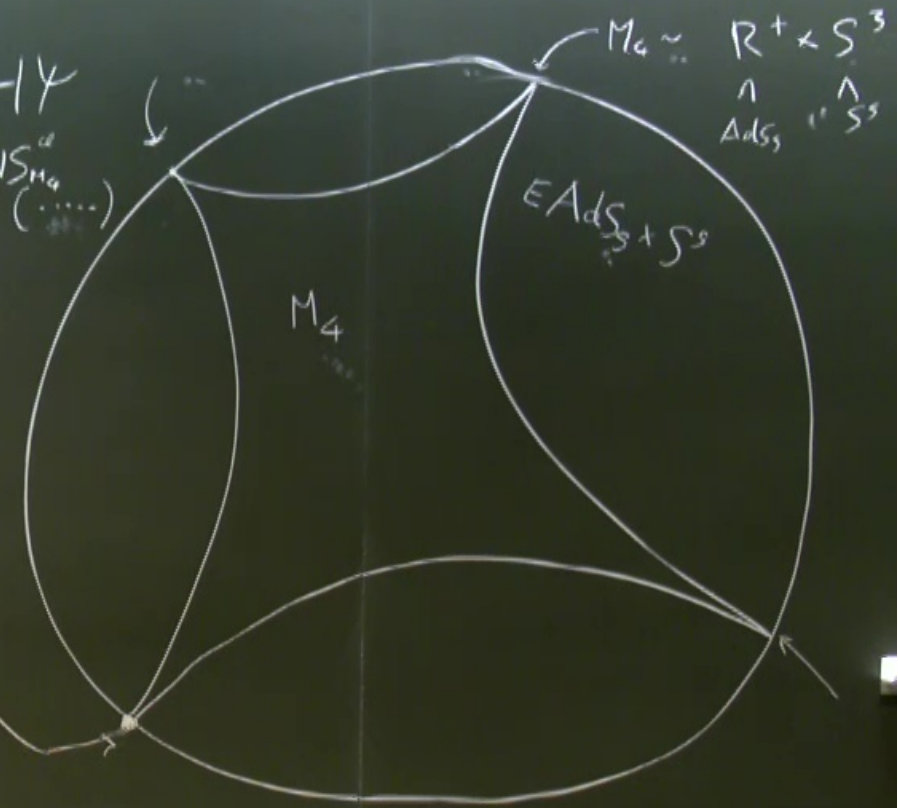
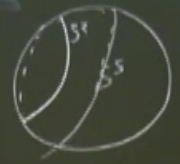
# TWISTED HOLOGRAPHY

$$\langle \det [m_\mu + \vec{m}_\mu \cdot \vec{\Phi}(x_\mu)] \dots \det [m_{\mu'} + \vec{m}_{\mu'} \cdot \vec{\Phi}(x_{\mu'})] \rangle \sim e^{NS_{M_4}}$$

$\vec{m}^2 = 0$

$$\det_{N \times N} [m_\mu + \vec{m}_\mu \cdot \vec{\Phi}(x)]$$

$N=4$  SYM  $\phi_1 \dots \phi_6$

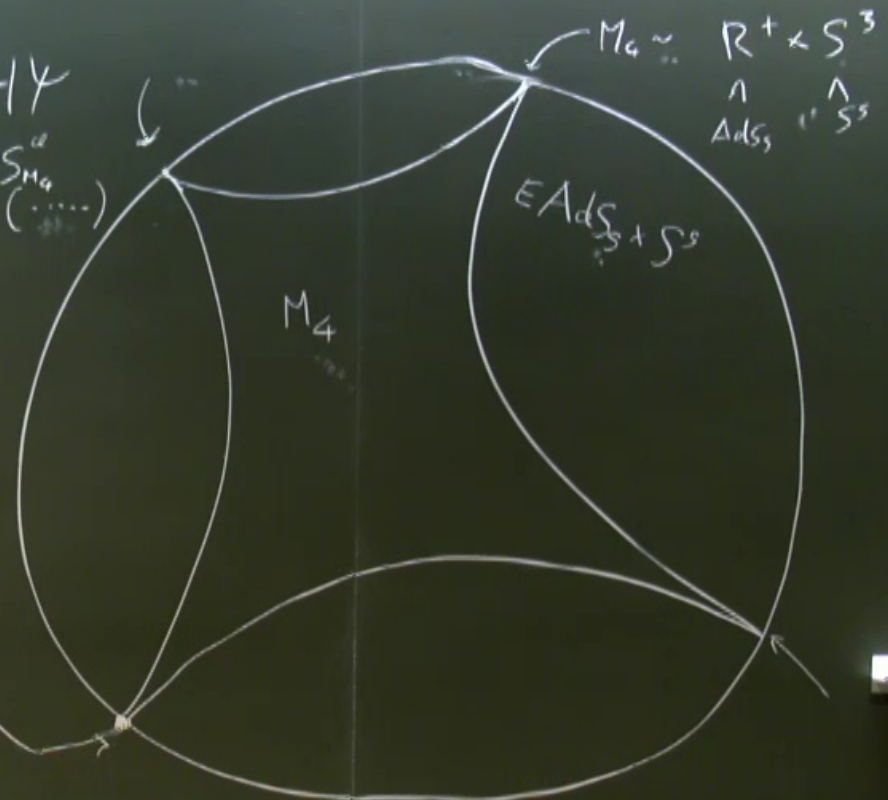
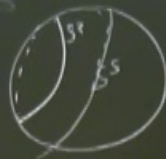


# TWISTED HOLOGRAPHY

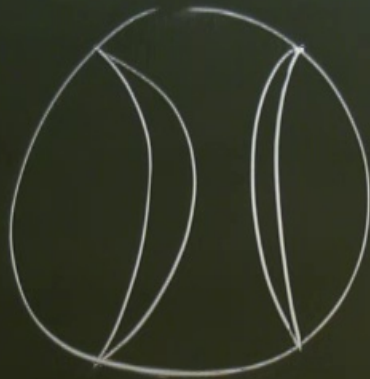
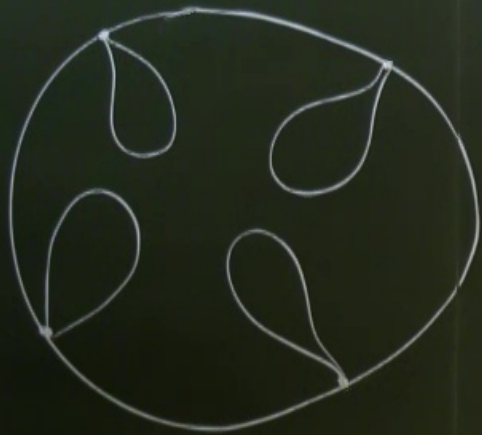
$$\langle \det [m_i + \vec{m}_i \cdot \vec{\Phi}(z_i)] \dots \det [m_u + \vec{m}_u \cdot \vec{\Phi}(z_u)] \rangle \sim e^{NS_{M_4}}$$

$$\begin{matrix} \vec{m}^t = 0 \\ \downarrow \\ \det_{N \times N} [m_i + \vec{m}_i \cdot \vec{\Phi}(z_i)] \end{matrix}$$

$N=4$  SYM  $\phi_1 \dots \phi_6$



# GIANT GRAVITONS IN



# TWISTED HO

$$\langle \det[m_i, \bar{m}_i, \phi_i] \dots \det[m_i, \bar{m}_i, \phi_i] \dots \rangle$$

$$\det_{N \times N} [m_i, \bar{m}_i, \phi_i]$$

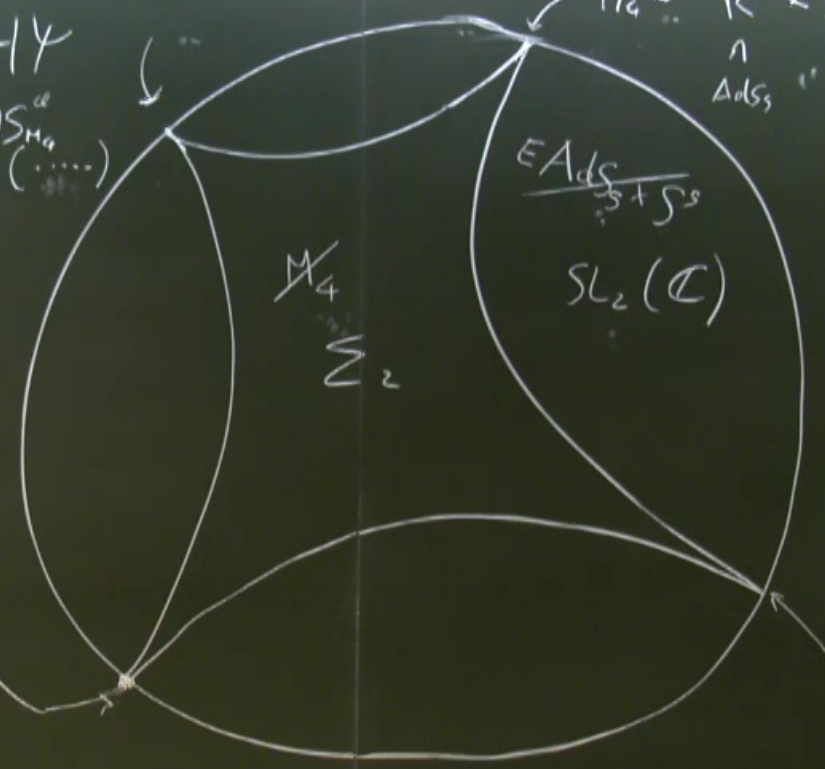
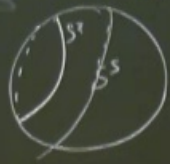
$N=4$  SYM  $\phi_1 \dots$

# LISTED HOLOGRAPHY

$$\det_{N \times N} [m_i + \vec{m}_i \cdot \vec{\phi}_i(x_i)] \dots \det_{N \times N} [m_N + \vec{m}_N \cdot \vec{\phi}(x_N)] \sim \sum_{M_0} e^{NS_{M_0}(\dots)}$$

$$\begin{matrix} \vec{m}^2 = 0 \\ \downarrow \\ \det_{N \times N} [m + \vec{m} \cdot \vec{\phi}(x)] \end{matrix}$$

$N=4$  SYM  $\phi_1 \dots \phi_6$



$$\begin{matrix} \Sigma_2 \sim \mathbb{C}^* \\ M_4 \sim \mathbb{R}^+ \times S^3 \\ \wedge \\ \Delta AdS_5 \quad \wedge \\ \quad \quad \quad S^5 \end{matrix}$$

# GIANT GRAVITONS IN TWISTED

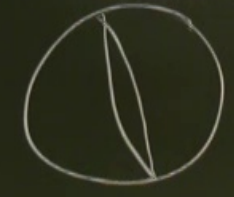
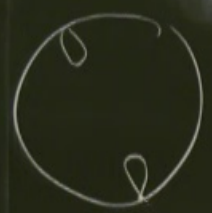


$\Downarrow$   
 $\sum \mathbb{R}^2$

$$\langle T_2 \gamma^n \det(m+X) \rangle = m^{N-a} c_n$$

$$\stackrel{N \cdot T_2 \downarrow \ln m+X}{=} e^{N \ln m}$$

$$\langle T_2 T_2 T_2 T_2 \dots \det(m+X) \rangle \sim e^{N \cdot S_{\text{eff}}} (\dots)$$



$$\langle \det X \det \bar{X} \rangle$$

$X = \phi + i\psi$

$$\langle \det_{N \times N} [m_i + \bar{m}_i \cdot \phi_i \cdot \bar{\phi}_i] \dots \rangle$$

# GRAVITONS IN TWISTED HOLOGRA

$$\det(m + \vec{m} \cdot \phi) = \int d\vec{r} dt e^{\vec{\psi} [m + \vec{m} \cdot \phi] \psi}$$

$$\langle \vec{m}_a \phi(x_a) \vec{m}_b \phi(x_b) \rangle = \frac{\vec{m}_a \cdot \vec{m}_b}{|x_a - x_b|^2} = G_{ab}$$

$$\langle \det \dots \det \rangle = \int \underbrace{DA D\lambda D\phi}_{\text{FREE}} \prod_{a=1}^k [d\bar{\psi}_a d\psi_a] e^{N S_{FS} + \sum_{a=1}^k \bar{\psi}_a [m_a + \vec{m}_a \cdot \vec{\phi}(x_a)] \psi_a}$$

$$= \int \pi d\bar{\psi}_a d\psi_a e^{\sum_a \bar{m}_a \bar{\psi}_a \psi_a + \frac{1}{N} \sum_{a \neq b} G_{ab} (\bar{\psi}_a \psi_b) (\bar{\psi}_b \psi_a)}$$



# $\in \mathcal{D}$ Holography

$$\langle \vec{n}_a \phi(x_a) \vec{n}_b \phi(x_b) \rangle = \sum_{M_a} e^{NS_{M_a}(\dots)}$$

$$= \frac{\vec{n}_a \cdot \vec{n}_b}{|x_a - x_b|^2} = G_{ab}$$

$$NS_{FS}^{FREE} + \sum_{\alpha=1}^k \bar{\psi}_\alpha [m_\alpha + \vec{n}_\alpha \cdot \vec{\phi}(x_\alpha)] \psi_\alpha$$

$$+ \frac{1}{N} \sum_{\alpha \neq \beta} G_{\alpha\beta} (\bar{\psi}_\alpha \psi_\beta) (\bar{\psi}_\beta \psi_\alpha)$$

$M_4 \sim \mathbb{R}^+ \times S^3$   
 $\uparrow \quad \uparrow$   
 $AdS_5 \quad S^5$

$$= \int \prod_a d\vec{n}_a d\vec{\phi}_a e^{-\sum_a m_a \bar{\psi}_a \psi_a + \sum_{a,b} e_{ab} \bar{\psi}_a \psi_b + N(G_{ab})^{-1} e_{ab} e_{ba}}$$

$$= \int \prod_a d\vec{e}_{ab} e^{-\sum_{a,b} N(G_{ab})^{-1} e_{ab} e_{ba} - \text{det}(m_a \delta_{ab} + e_{ab})} N$$

# GIANT GRAVITONS IN TWISTED Holography



$\downarrow$   
 $\sum_{\mathbb{R}^3}$

$$e_{ab} = G_{ab} (e^{-1})_{ab}$$

$$e_{aa} = m_a$$

$$e = \begin{pmatrix} // & 0 & 0 \\ 0 & // & 0 \\ 0 & 0 & // \end{pmatrix}$$

$$\langle \vec{n}_a \phi(x_a) \vec{n}_b \phi(x_b) \rangle = \frac{\vec{n}_a \cdot \vec{n}_b}{|x_a - x_b|^2} = G_{ab}$$

$$= \int \prod d\bar{\psi}_a d\psi_a e^{\sum_a \bar{\psi}_a \psi_a + \frac{1}{N} \sum_{a \neq b} G_{ab} (\bar{\psi}_a \psi_b) (\bar{\psi}_b \psi_a)}$$

# $\in \mathcal{D}$ Holography

$M_4 \sim \mathbb{R}^+ \times S^3$   
 $\uparrow \quad \uparrow$   
 $AdS_5 \quad S^5$

$$\langle \vec{n}_a \phi(z_a) \vec{n}_b \phi(z_b) \rangle = \frac{\vec{n}_a \cdot \vec{n}_b}{|z_a - z_b|^2} = G_{ab}$$

$$x = (\text{Re } z, \text{Im } z, 0, 0)$$

$$n = (\dots, 0, 0)$$

$\rightarrow m \cdot \vec{n}_a + \sum p_{ab} \vec{n}_a \vec{n}_b$

$$\vec{n} \cdot \vec{\phi} = \frac{(v_a - v_b)(\bar{z}_a - \bar{z}_b)}{|z_a - z_b|^2} = \frac{v_a - v_b}{z_a - z_b}$$

$$X(z) + v Y(z)$$

$n \in \mathbb{C}^2 \otimes \mathbb{C}^2$

$$n = \begin{pmatrix} 1 \\ v \end{pmatrix} \otimes \begin{pmatrix} 1 \\ \bar{z} \end{pmatrix}$$

defines

$$+ \frac{1}{N} \sum_{a \neq b} G_{ab} (\bar{\psi}_a \psi_b) (\bar{\psi}_b \psi_a)$$

GIANT



$\downarrow$   
 $\mathbb{Z} \amalg \mathbb{B}$

$$[a, e] = [z, e^{-1}]$$

$$e_{aa} = m_a$$

$\mu = \text{diag}(v_a)$   
 $\mathbb{Z} = \text{diag}(z_a)$

$$e = \begin{pmatrix} // & 0 & 0 \\ 0 & // & 0 \\ 0 & 0 & // \end{pmatrix}$$

$$= \int \pi d\bar{\psi} d\psi e^{\sum_a m_a \bar{\psi}_a \psi_a + \frac{1}{i\epsilon} \sum}$$

$\langle \det [m_a + X] \rangle$

D-BRANES ON  
 HOLONOMY CURVES  
 IN  $SL(2, \mathbb{C})$

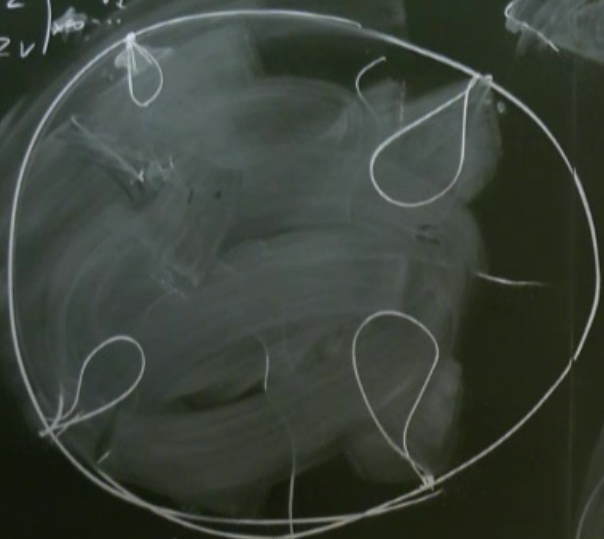
$\vec{m} \cdot \vec{\phi} = X$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad ad - bc = 1$$

$$g \in SL(2, \mathbb{C})$$

$m_1, z_1, v_1$

$$g \sim \begin{pmatrix} 1 & z \\ v & zv \end{pmatrix}$$



$\Downarrow$   
 $\Sigma \cup \mathbb{R}, \mathbb{C}$

$$[\mu, e] = [\zeta, e^{-1}]$$

$-e_{aa} = m_a$

vs

$$\mu = \text{diag}_{\text{diag}}(v_a)$$

$$\zeta = \text{diag}(z_a)$$

$m_a$

$\theta$ -CRANES  
 ON  
 HOL CURV  
 IN  $SL(2)$

$$[B(a), C(a)] = 0$$

$$\boxed{[a, e] = [\zeta, e^{-1}]}$$

$$-e_{aa} = m_a$$

vs

B-PLANES  
ON  
HOL CURVES  
IN  $SL(2, \mathbb{C})$

$$\langle \det [m_i + X(z) v_i, Y(z)] \dots \rangle$$

$\mu = \text{diag} (v_i)$   
 $\zeta = \text{diag} (z_i)$

$(a, b, c, d)$   
 $a \in \mathbb{C}$   
 $ad - bc = 1$

$$B(a) = a\zeta + e$$

$$C(a) = a\mu + e^{-1}$$

$$aD(a) - B(a)C(a) = 1$$

$$D(a) \neq a\zeta\mu + \mu e + e^{-1}\zeta$$

$$\boxed{\begin{aligned} [b - B(a)]\zeta &= 0 \\ [c - C(a)]\zeta &= 0 \\ [d - D(a)]\zeta &= 0 \end{aligned}}$$

$$N_{a \neq b} \quad G_{ab} (4, 1) (4, 4)$$

$$\langle \det[m_+ X(z) + v_+ Y(z)] \dots \det(\dots) \tau_2 [X(z) + vY(z)]^n \rangle$$

0-DRAWES  
ON  
HOL CURVES  
IN  $SL(2, \mathbb{C})$

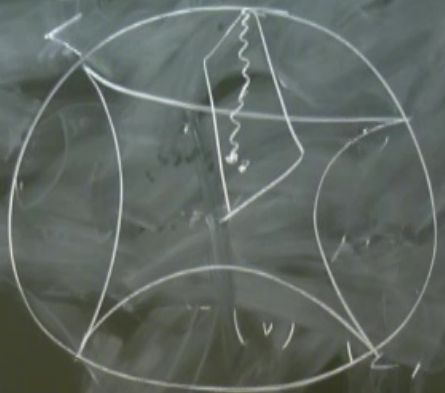
$$\begin{aligned} [b - B(a)] \xi &= 0 \\ [c - C(a)] \xi &= 0 \\ [d - D(a)] \xi &= 0 \end{aligned}$$

$(b, c, d)$   
 $ad - bc = 1$   
 $+ e$   
 $= e^{-1}$

$$C(a) = 1$$

$$\mu + \mu \rho = e^{i\theta}$$

$$\sum_{a \neq b} \dots$$



$$\Sigma_2 \sim \mathbb{C}^*$$

$$M_0 \sim \mathbb{R}^+ \times S^3$$

$$m \dots \dots \dots$$

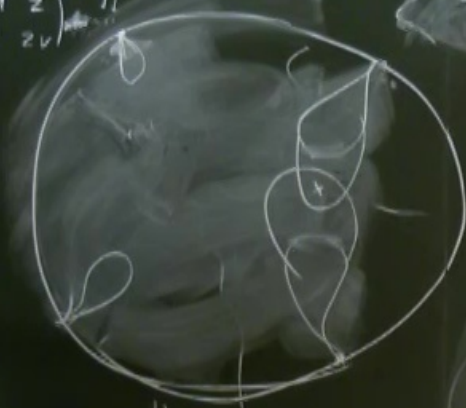
$d \dots$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad ad - bc = 1$$

$$g \in SL(2, \mathbb{C})$$

$$m, z, v_1$$

$$g \sim \begin{pmatrix} 1 & z \\ v & zv \end{pmatrix}$$



$$\downarrow$$

$$\mathbb{Z} \amalg \mathbb{R}$$

$$[B(a), C(a)] = 0$$

$$\boxed{[a, e] = [z, e^{-1}]}$$

$$-e_{aa} = m_a$$

IN VS

0-CRANES LISTED ON HOL CURVES IN  $SL(2, \mathbb{C})$

Major diag:  $(v_i)$   
 $\mathbb{Z} = \text{diag}(z_n)$

$$\begin{pmatrix} a & b & c & d \\ & & & \end{pmatrix} \quad ad - bc = 1$$

$$B(a) = a\beta + e$$

$$C(a) = a\mu + e^{-1}$$

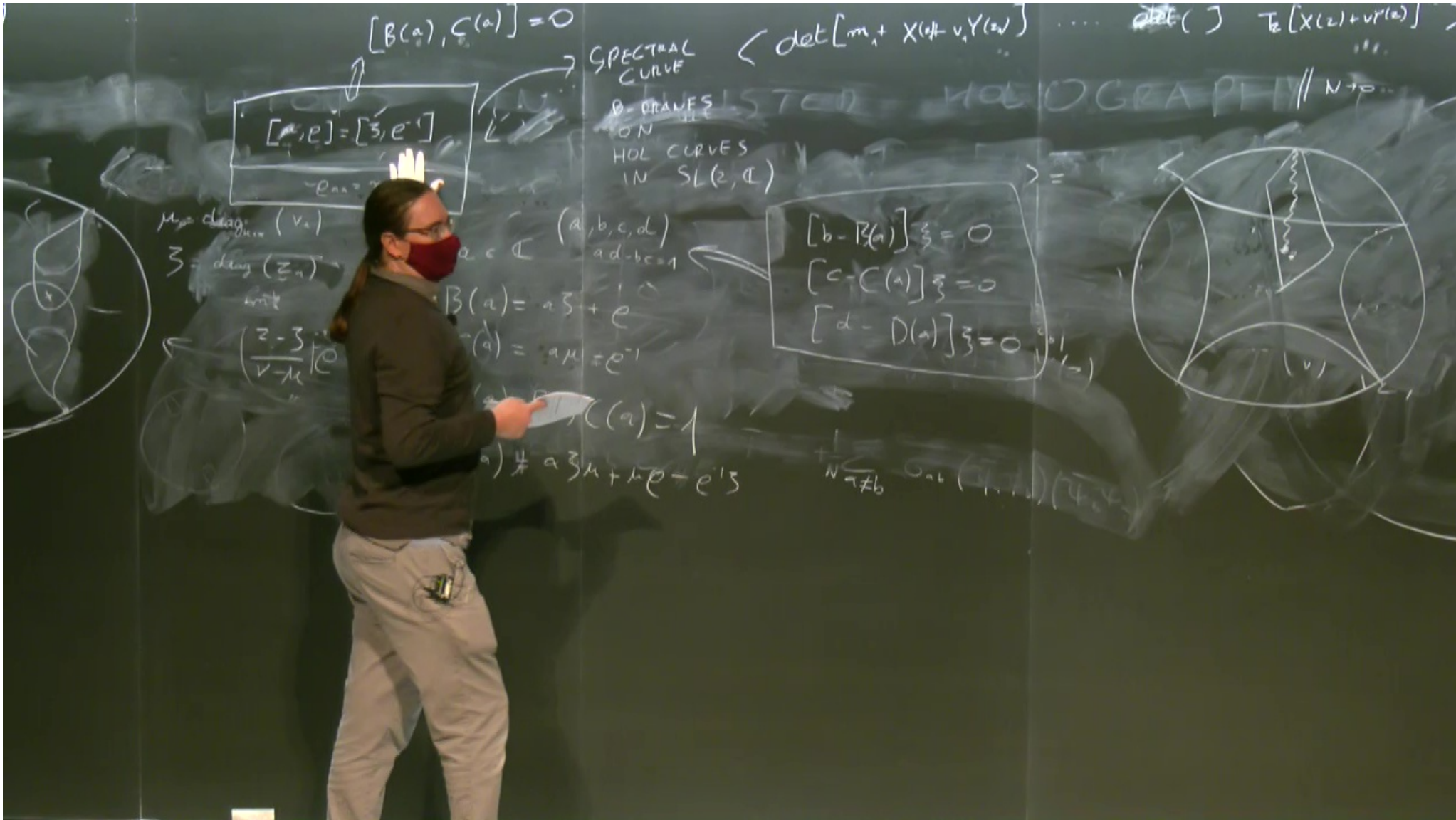
$$B(a)C(a) = 1$$

$$3\mu + \mu e + e^{-1}\beta$$

$$\boxed{\begin{matrix} [b - B(a)] \\ [c - C(a)] \\ [d - D] \end{matrix}}$$

$$N \neq b$$





$$[B(a), C(a)] = 0$$

$$[z, e] = [z, e]$$

SPECTRAL CURVE  
 B-PLACES ON  
 HOL CURVES  
 IN  $SL(2, \mathbb{C})$

$$\langle \det[m_1 + X(z) + Y(w)] \dots \det(\cdot) T_2[X(z) + Y(w)] \dots \rangle$$

$\mu = \text{diag}(\dots, v, \dots)$   
 $\zeta = \text{diag}(\zeta_1, \dots)$   
 with  
 $\begin{pmatrix} z - \zeta \\ v - \mu \end{pmatrix} \Big|_e$

$(a, b, c, d)$   
 $ad - bc = 1$   
 $B(a) = a\zeta + e$   
 $C(a) = a\mu = e^{-1}$

$$\begin{aligned} [b - B(a)] \zeta &= 0 \\ [c - C(a)] \zeta &= 0 \\ [d - D(a)] \zeta &= 0 \end{aligned}$$

