Title: Towards bootstrapping critical quantum matter

Speakers: Yin-Chen He
Date: October 04, 2021 - 12:00 PM

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Abstract: Critical states of matter are a class of highly entangled quantum matter with various interesting properties and form important bases for emergence of a variety of novel quantum phases. Such states pose serious challenges for the community due to their strongly interacting nature. In this talk, I will discuss our recent progress on tackling critical quantum matter using the method of conformal bootstrap. I will start with introducing several representative examples of critical quantum matter, including the familiar deconfined quantum phase transition, U(1) Dirac spin liquid phase, and the newly proposed Stiefel liquid phase. Next I will focus on the SU(N) deconfined phase transition (i.e. scalar QED), and demonstrate that they can be solved by conformal bootstrap, namely we have obtained their bootstrap kinks and islands.

Zoom Link: https://pitp.zoom.us/meeting/register/tJcqc-ihqzMvHdW-YBm7mYd_XP9Amhypv5vO

Towards bootstrapping critical quantum matter

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Outline

- Critical quantum matter
- A light review of conformal bootstrap
- Bootstrapping critical quantum matter:
  - A toy example: Heisenberg spin-1/2 chain (SU(2) WZW)
  - SU(N) deconfined phase transition (Scalar QED)

Landscape of quantum matter

- Spontaneous symmetry breaking phases and phase transitions.

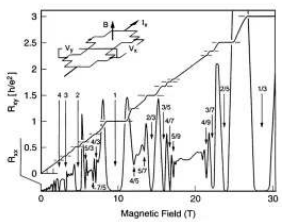
Superconductor

Ferromagnets

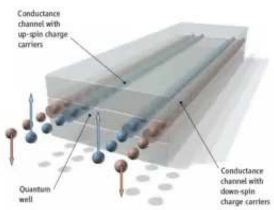
$$\mathcal{L} = (\partial_\mu \phi)^2 + m\phi^2 + \lambda\phi^4$$

- Topological phases (gapped): topological quantum field theory.

Fractional quantum Hall



Topological insulator



$$\mathcal{L} = \frac{k}{4\pi}\epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

- Critical phases:

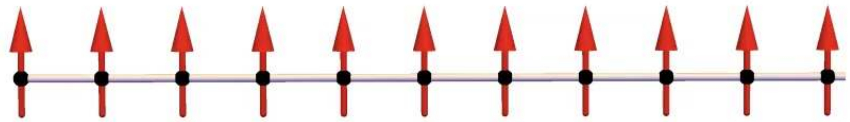
  A. Conformal phases: conformal field theory

  B. Metallic phases: Fermi liquid, non-Fermi liquid, strange metal, etc.

An example of critical phases: Heisenberg quantum spin chain

Spin-1/2

$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



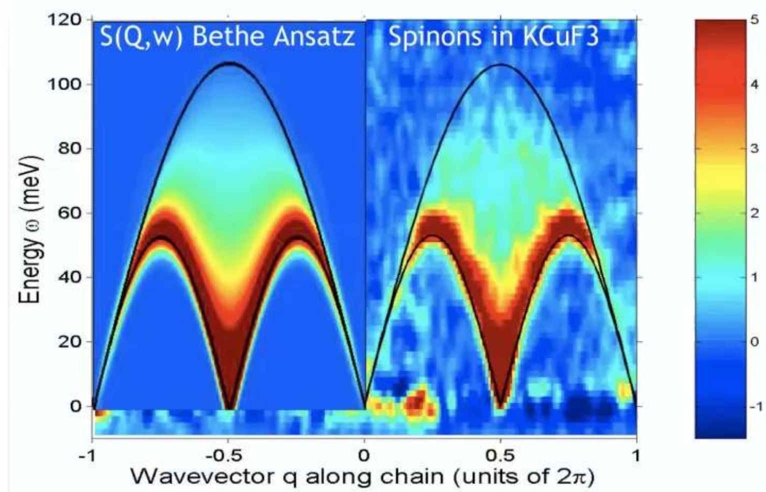
Exactly solvable with Bethe Ansatz (1931)

$$SU(2)_1 \text{ WZW}$$

1+1D critical phase

Material realization: e.g. KCuF3

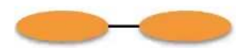
Neutron scattering



Tennant, Cowley, Nagler, and Tsvelik (1995)

Field theory description

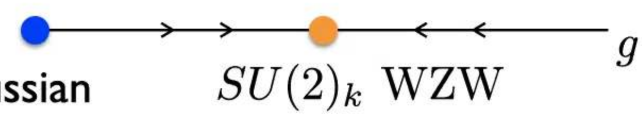
$SU(2)_k$ WZW: $\vec{n} = (n_1, n_2, n_3, n_4)$ $\quad |\vec{n}| = 1$

(n_1, n_2, n_3): 3 spin moments, n_4 valence bond order 

$$\mathcal{S} = \frac{1}{2g}\int d^2x (\partial_\mu \vec{n})^2 + \frac{k}{24\pi}\int d^3x \epsilon_{abcd}\epsilon_{\mu\nu\rho}\tilde{n}_a\partial_\mu\tilde{n}_b\partial_\nu\tilde{n}_c\partial_\rho\tilde{n}_d$$

Kinetic term   Wess-Zumino-Witten (WZW) term

$k > 0$

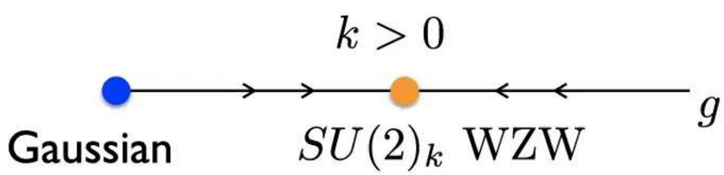


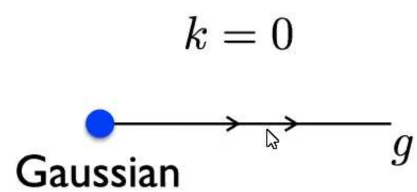
WZW models: continued

(n_1, n_2, n_3): 3 spin moments, n_4 valence bond order $\quad |\vec{n}| = 1$

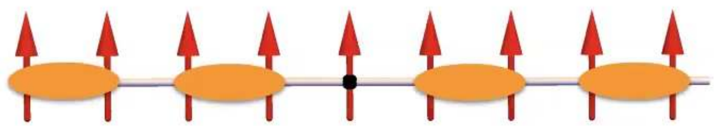
$$\mathcal{S} = \frac{1}{2g}\int d^2x(\partial_\mu \vec{n})^2 + \frac{k}{24\pi}\int d^3x \epsilon_{abcd}\epsilon_{\mu\nu\rho}\tilde{n}_a\partial_\mu\tilde{n}_b\partial_\nu\tilde{n}_c\partial_\rho\tilde{n}_d$$

- WZW term modifies the RG flow.





- WZW term: intertwinement of spin order and VBS order.

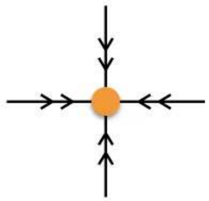


- Topological origin of WZW term.

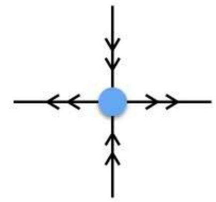
$$\pi_3(S^3) = Z \text{ and } \pi_2(S^3) = 0$$

Critical phase v.s. Critical point

Critical phase
(attractive RG fixed point)



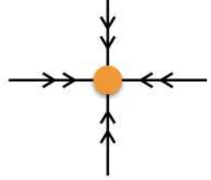
Critical point
(repulsive RG fixed point)



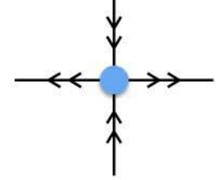
1. $SU(2)_1$ WZW CFT: critical phase in half-integer spin chain.
2. $SU(2)_2$ WZW CFT: critical point in integer spin chain.

Critical phase v.s. Critical point

Critical phase
(attractive RG fixed point)



Critical point
(repulsive RG fixed point)



1. $SU(2)_1$ WZW CFT: critical phase in half-integer spin chain.
2. $SU(2)_2$ WZW CFT: critical point in integer spin chain.

IR symmetry: $SO(4)$

Vector: (n_1, n_2, n_3, n_4)

Symmetric (traceless) rank-2 tensor: T

UV symmetry: $SO(3)_s \times T_x$

$$(j=1,-)\begin{cases} SO(3)_s : (n_1, n_2, n_3) \\ T_x : n_4 \to -n_4 \end{cases}$$

$(j=2,+) + (j=1,-) + (j=0,+)$

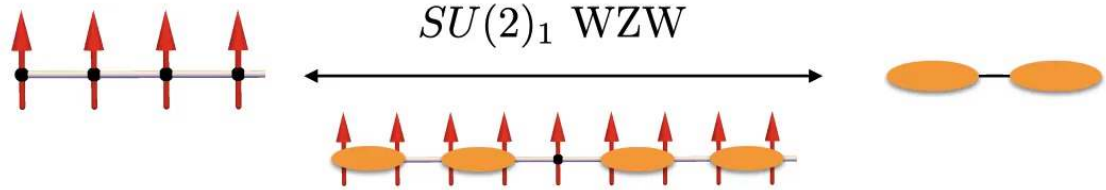
UV singlet

$SU(2)_1$: $\Delta_T = 2$ marginal; $SU(2)_2$: $\Delta_T = 1$ relevant

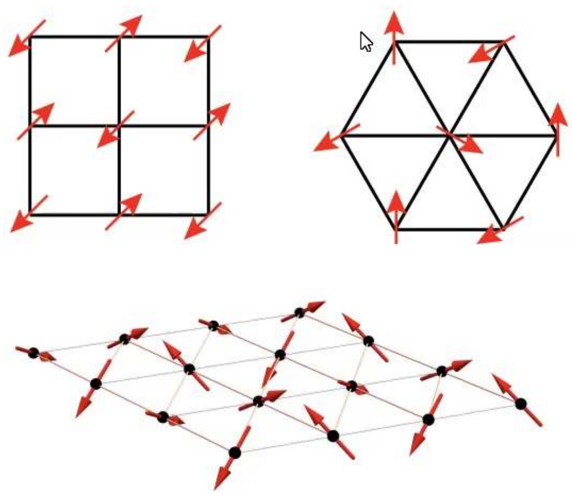
Criticality from intertwining orders

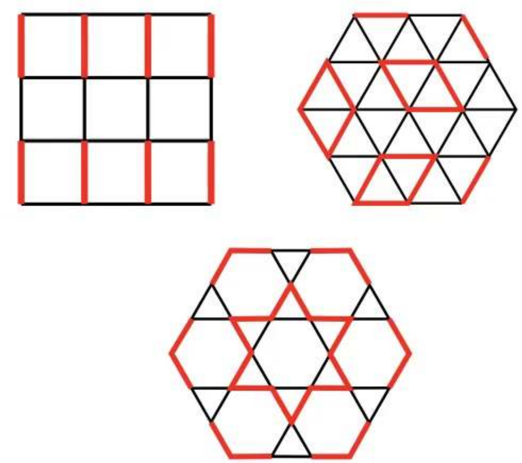
1+1D Critical phase

$SU(2)_1$ WZW



2+1D criticality?





Deconfined phase transition

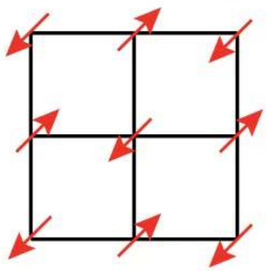
Senthil, Vishwanath, Balents, Sachdev, Fisher

$\vec{n} = (n_1, n_2, n_3, n_4, n_5)$ with $|\vec{n}| = 1$ $\quad \pi_4(S^4) = Z$ and $\pi_3(S^4) = 0$

$$\mathcal{S} = \frac{1}{2g}\int d^3x(\partial_\mu \vec{n})^2 + \mathrm{WZW}[\tilde{n}]$$

Senthil, Fisher; Nahum, et. al.

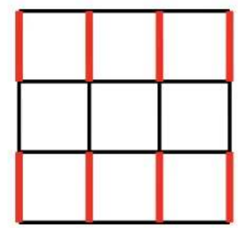
(n_1, n_2, n_3)



Neel state

$$\lambda(n_1^2 + n_2^2 + n_3^2 - n_4^2 - n_5^2)$$

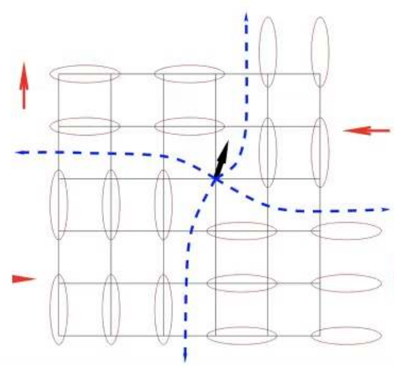
(n_4, n_5)



VBS state

Physics of WZW term: intertwinement of magnetic order and VBS order.

Levin & Senthil



Stiefel manifold SO(N)/SO(N-k)

$N \times k$ real matrices n_{ij} satisfying $n^T n = \mathbf{1}_{k\times k}$.

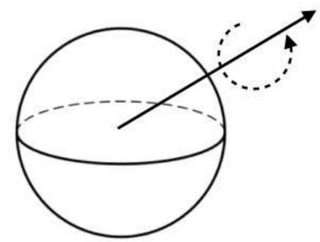
k mutual orthogonal N-component unit vectors.

Examples of Stiefel manifolds:

$S^2 = SO(3)/SO(2)$

- Sphere: $S^N = SO(N+1)/SO(N)$

  $N+1$ component unit vector $|\vec{n}| = 1$



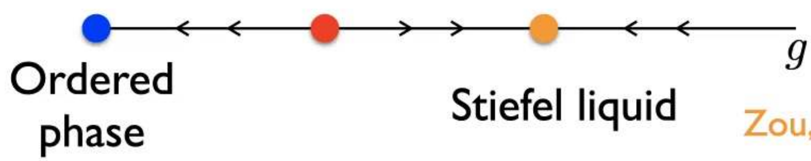
- SO(6)/SO(4):

  Two orthogonal 6-component unit vectors $\vec{n}_1$ and $\vec{n}_2$.

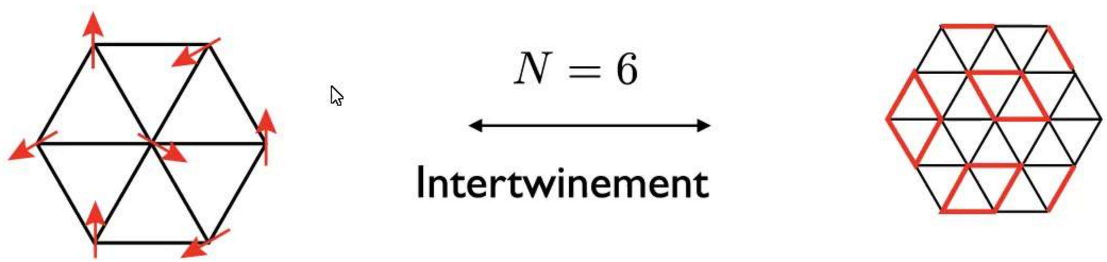
Stiefel liquids: WZW models on Stiefel manifold

- Spin-1/2 Heisenberg chain: 1+1D WZW on SO(4)/SO(3).
- Deconfined phase transition: 2+1D WZW on SO(5)/SO(4).
- 2+1D WZW on SO(N)/SO(4).

$$\pi_4(SO(N)/SO(4)) = Z$$
$$\pi_3(SO(N)/SO(4)) = 0$$



Zou, YCH, Wang (2021)



Dirac spin liquid

Hastings 2000; Zhou & Wen 2002; Ran, Hermele, Lee & Wen 2007

Nf=4 QED3: $\mathcal{L} = \sum_{I=1}^{4} \bar{\psi}_I (i\partial\!\!\!/ - a\!\!\!/)\psi_I$

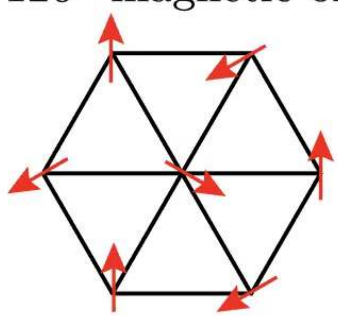
Global symmetry: $\frac{SO(6) \times SO(2)}{Z_2}$

Monopole is the bi-vector of SO(6) and SO(2).

Order states come from the condensation of monopoles.

Song, YCH, Vishwanath, Wang, 1811.11182, 1811.11186

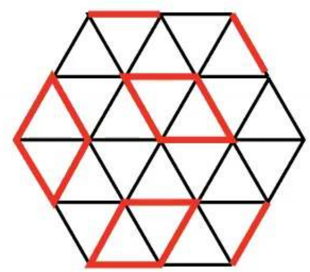
120° magnetic order



6 components

Dirac spin liquid ⟷ N=6 Stiefel liquid

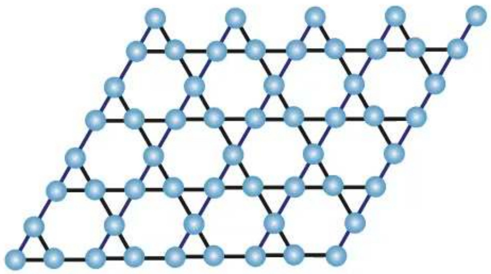
$\sqrt{12} \times \sqrt{12}$ VBS



6 components

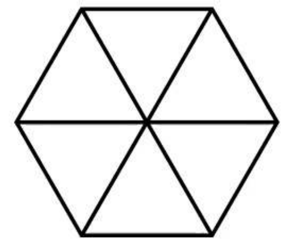
Possible realizations of Dirac spin liquid

Theoretical models



$$H = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

Hastings 2000; Ran, Hermele, Lee & Wen 2007;
YCH, Zaletel, Oshikawa, Pollmann 2016;…



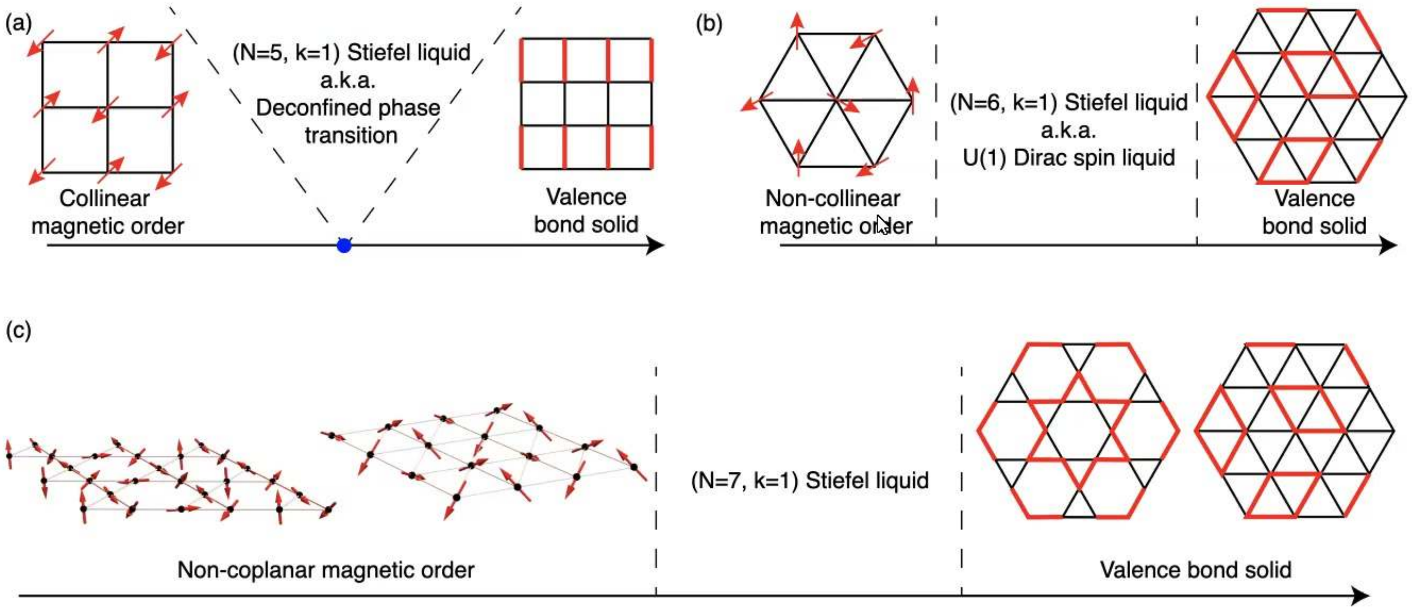
J1-J2 Heisenberg

Y. Iqbal, et.al, Becca 2016;
Hu, Zhu, Eggert, & YCH 2019

Materials: Herbertsmithite (kagome), NaYbO2, etc.

Stiefel liquids in quantum magnetisms

Critical states from melting/intertwining symmetry breaking orders.

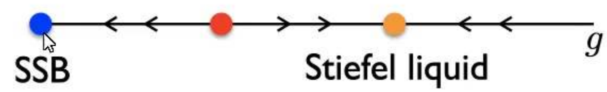


Zou, YCH, Wang (2021)

Fate of Stiefel liquid

2+1D WZW models on the Stiefel manifold SO(N)/SO(4).

$$S_0[n] = \frac{1}{2g}\int d^3x \mathrm{Tr}(\partial_\mu n^T \partial^\mu n) + k \cdot \mathrm{WZW}[\tilde{n}]$$



A. Does it really exist when N>6 (conjectured to be non-Lagrangian)?

Stiefel liquids: WZW models on Stiefel manifold

- Spin-1/2 Heisenberg chain: 1+1D WZW on SO(4)/SO(3).
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- 2+1D WZW on SO(N)/SO(4).

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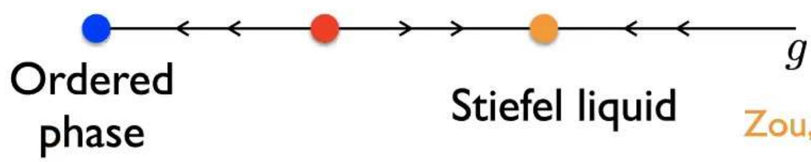
$$\pi_3(SO(N)/SO(4)) = 0$$



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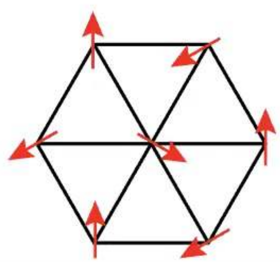
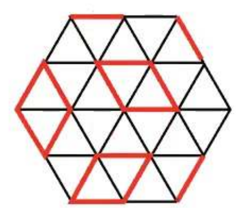


$N = 6$

Intertwinement



WZW models: continued

(n_1, n_2, n_3): 3 spin moments, n_4 valence bond order $\quad |\vec{n}| = 1$

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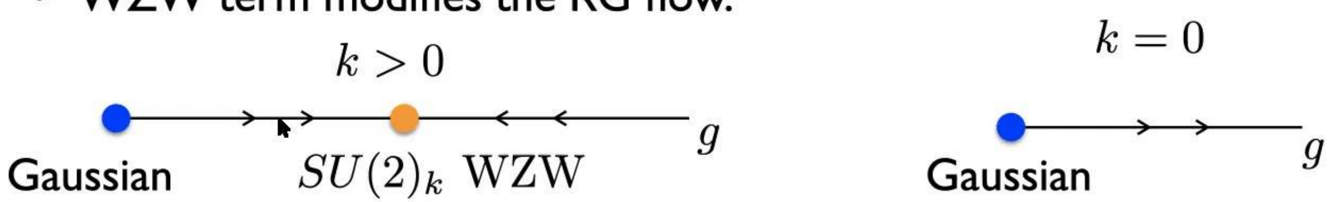
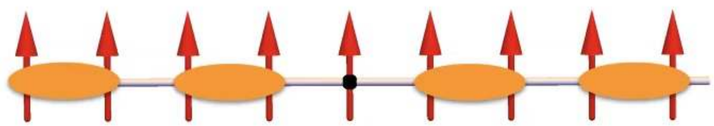
- WZW term modifies the RG flow.



- WZW term: intertwinement of spin order and VBS order.



- Topological origin of WZW term.

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Stiefel liquids: WZW models on Stiefel manifold

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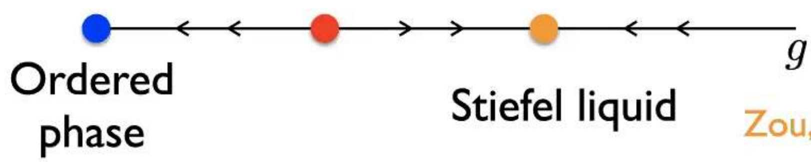

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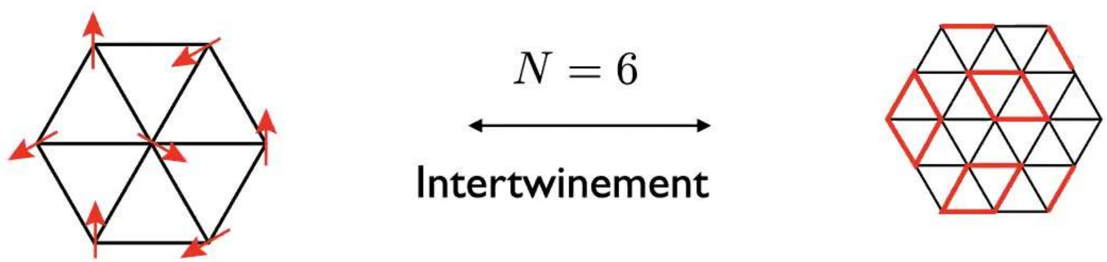

Fate of Stiefel liquid

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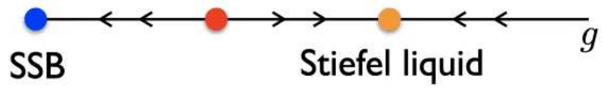


A. Does it really exist when N>6 (conjectured to be non-Lagrangian)?

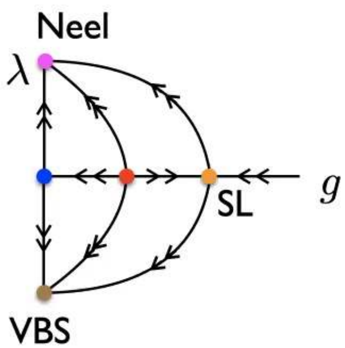
B. Critical phase or critical point?

IR symmetry: $SO(N) \times SO(N-4)$ → Perturbation: λV → UV symmetry: $SO(3)_s \times \text{lattice}$

Critical point

$N = 5$

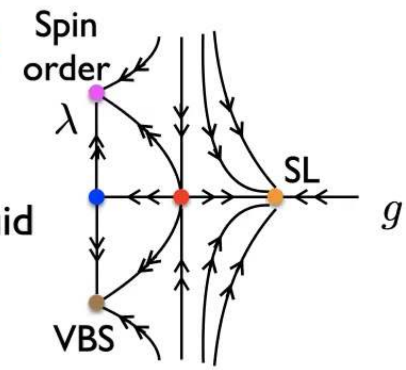
Deconfined critical point



Critical phase

$N \geq 6?$

Dirac spin liquid

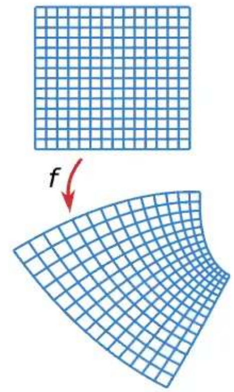


Outline

- Critical quantum matter
- A light review of conformal bootstrap
- Bootstrapping critical quantum matter:
  - A toy example: Heisenberg spin-1/2 chain (SU(2) WZW)
  - SU(N) deconfined phase transition (Scalar QED)

Conformal Symmetry

- Translation symmetry (P)
- Lorentz symmetry (R)
- Scaling symmetry (D)
- Special conformal symmetry (K)



Wiki

$$x^{\mu} \to \frac{x^{\mu} - x^2 b^{\mu}}{1 - 2b \cdot x + b^2 x^2}$$

2pt, 3pt corre. functions are fixed by the conformal symmetry.

$$\langle \mathcal{O}_i(x_1) \mathcal{O}_j(x_2) \rangle = \frac{\delta_{ij}}{|x_{12}|^{2\Delta}}, \quad x_{12} = x_1 - x_2$$

$$\langle \mathcal{O}_i(x_1) \mathcal{O}_j(x_2) \mathcal{O}_k(x_3) \rangle = \frac{\lambda_{ijk}}{|x_{12}|^{\Delta_1 + \Delta_2 - \Delta_3} |x_{13}|^{\Delta_1 + \Delta_3 - \Delta_2} |x_{23}|^{\Delta_2 + \Delta_3 - \Delta_1}}$$

Operator product expansion (OPE):

$$\mathcal{O}_i(x) \mathcal{O}_j(y) = \sum_k \lambda_{ijk} C_k(x - y, \partial_y) \mathcal{O}_k(y)$$

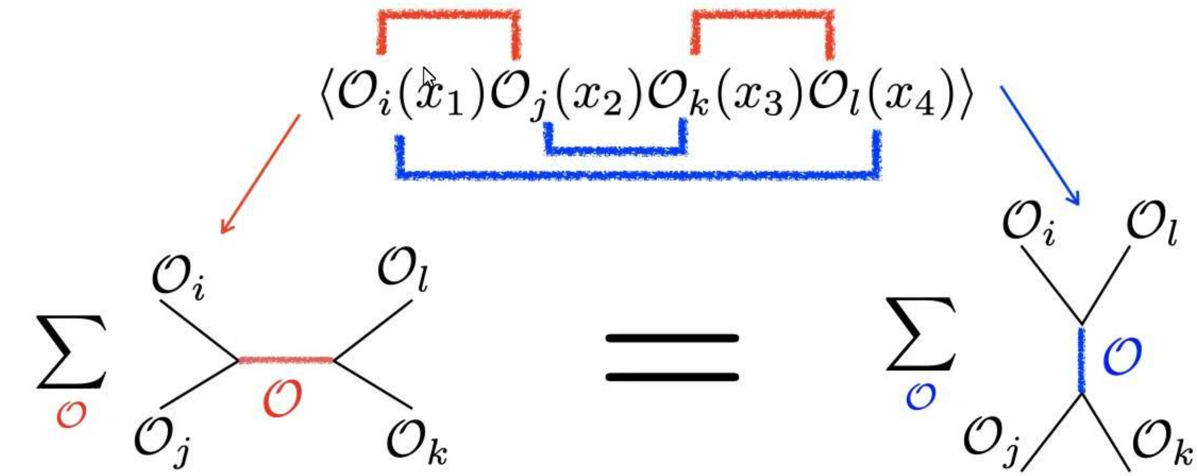
Four point correlation function

NOT completely fixed by conformal symmetry

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\mathcal{O}_l(x_4)\rangle = \frac{1}{|x_{12}|^{2\Delta}|x_{34}|^{2\Delta}} G[u,v]$$

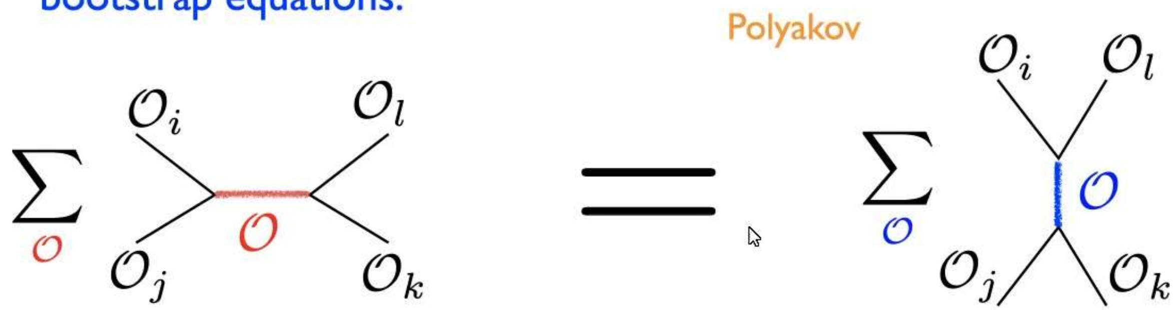
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Crossing symmetry: bootstrap equation



Conformal bootstrap

Conformal bootstrap: find a set of CFT data that solves all the bootstrap equations.



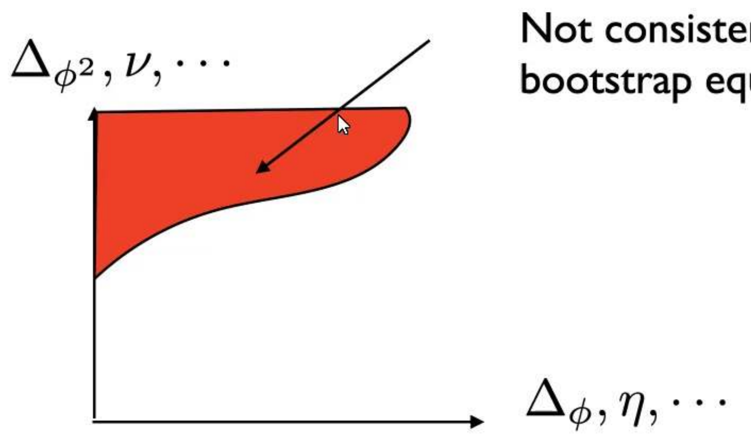
Formidable task due to 3 "infiniteness"!

- Infinite number of equations.
- Each equation involves an infinite sum.
- Each term in the summation is complicated (i.e. infinite series with no closed form for general dimensions).

Numerical bootstrap

Rattazzi, Rychokov, Tonni, Vichi, 2008

- Rather than solving it, let us try to get some constraint from bootstrap equations.



Bootstrap bounds for SO(5) CFTs

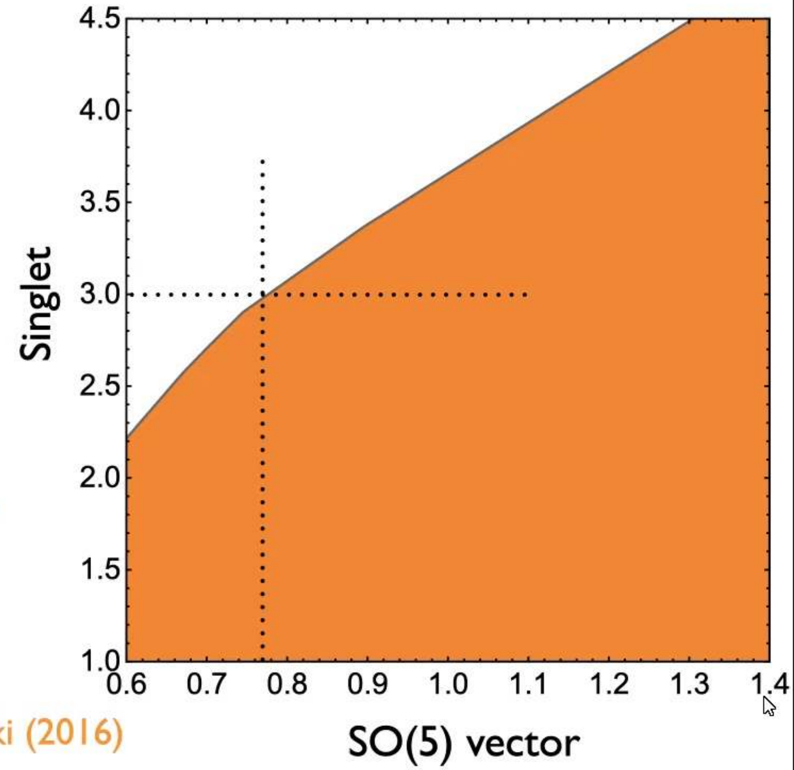
For the SO(5) DQCP, one has

$\Delta_\phi > 0.775 \text{ or } \eta > 0.55$

Simmons-Duffin (unpublished)

But numerically $\eta \approx 0.2 \sim 0.3$

Sandvik, Nahum, Kaul, Melko, Assaad,…



Other bounds: see Nakayama & Ohtsuki (2016)

Bootstrap bounds for Dirac spin liquid

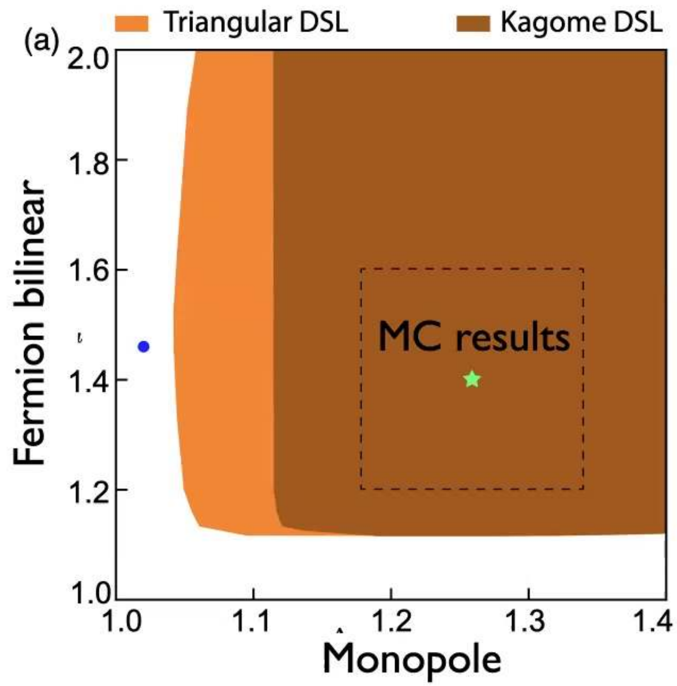
IR symmetry: $SO(6) \times SO(2)$

UV symmetry: $SO(3)_s \times \mathrm{lattice}$

- On the triangular lattice, (T, S) is symmetry allowed.
- On the kagome lattice, (T, S) and (T,T) are symmetry allowed.

Scaling dimension from large-Nf QED:

$$\Delta_{T,S} \approx 2.4, \Delta_{T,T} \approx 2.5$$



YCH, Rong, Su, arXiv:2107.14637

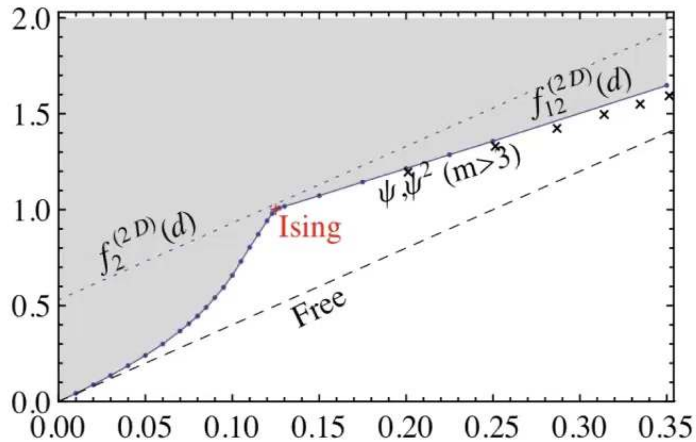
MC results: Karthik & Narayanan

Ising CFT

$\langle \sigma\sigma\sigma\sigma \rangle \qquad \sigma \times \sigma \sim \epsilon + \cdots$

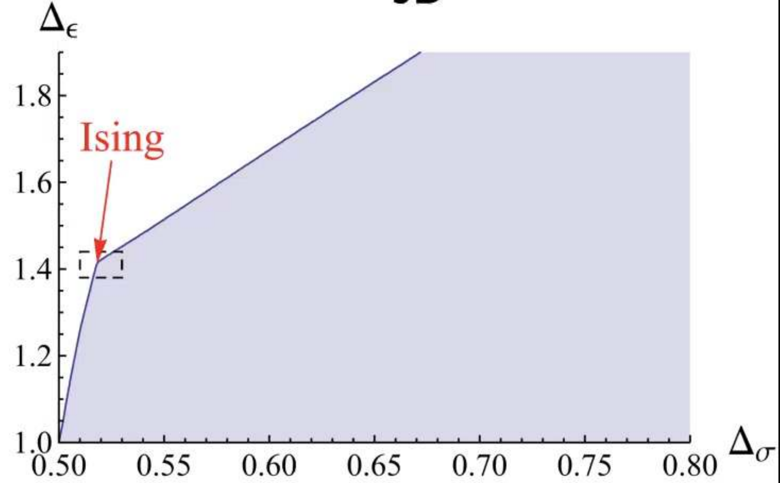
Ising CFT sits at a kink!

2D



Rychkov, Vichi (2009)

3D



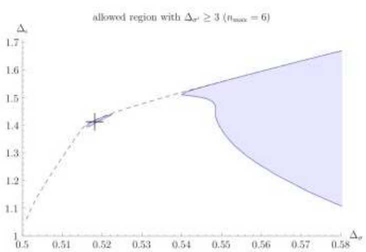
Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi (2012)

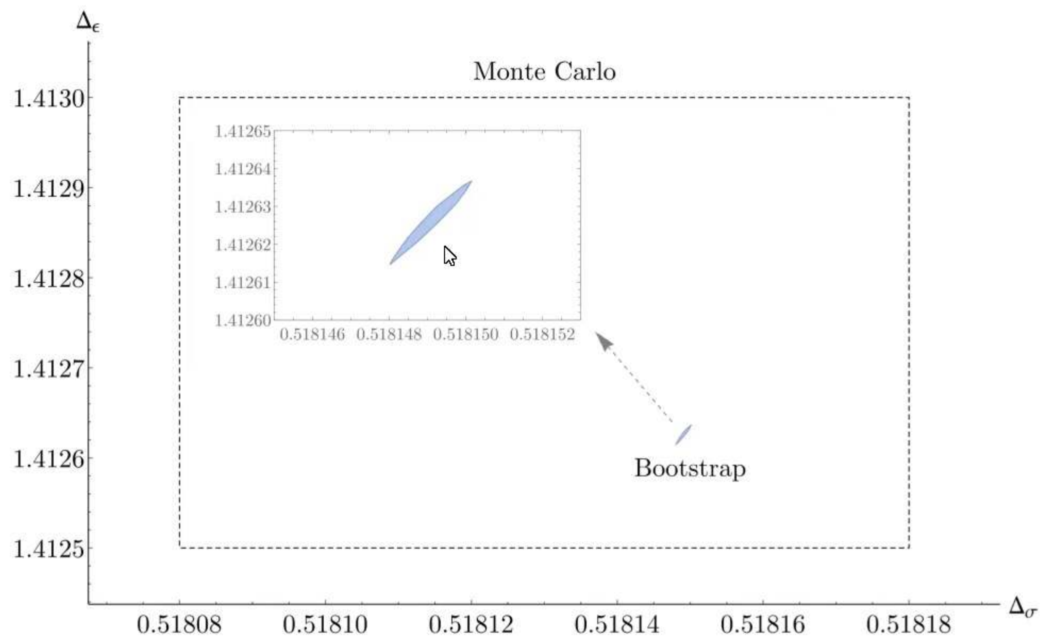
Mixed correlator: CFTs Island

$$\langle\sigma\sigma\sigma\sigma\rangle, \quad \langle\sigma\sigma\epsilon\epsilon\rangle, \quad \langle\epsilon\epsilon\epsilon\epsilon\rangle$$

Most precise critical exponents of 3D Ising!

Kos, Poland, Simmons-Duffin, Vichi, 2016





Outline

- Critical quantum matter
- A light review of conformal bootstrap
- Bootstrapping critical quantum matter:
  - A toy example: Heisenberg spin-1/2 chain (SU(2) WZW)
  - SU(N) deconfined phase transition (Scalar QED)

Bootstrapping critical quantum matter

Challenges: Very often there exists a family of CFTs with identical global symmetries and similar operator spectrum.

A common setup/input of the conformal bootstrap:

	Global symmetry	Operators being bootstrapped
Wilson-Fisher CFT	$O(N)$	$O(N)$ vector, $\Delta \approx 1/2$
Bosonic U(Nc) gauge theory	$\frac{SU(N_f)}{Z_{N_f}} \times U(1)$	$SU(N_f)$ adjoint, $\Delta \approx 1$
$SU(2)_k$ WZW CFT	$SO(4)$	
Stiefel liquid	$SO(N) \times SO(N-4)$	

Gauge fields or WZW levels don't appear in the bootstrap equations!

Bootstrapping SO(4) invariant 1+1D CFT

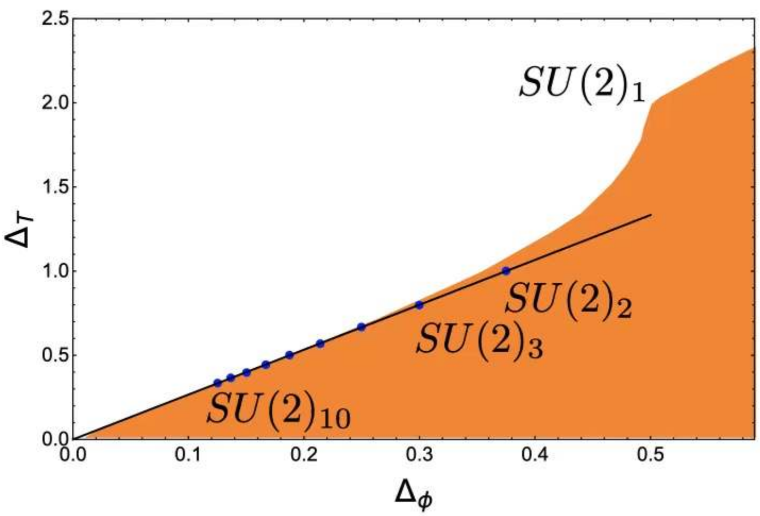
ϕ: $SO(4)$ vector

T: rank-2 symmetric traceless tensor

Single correlator $\langle \phi_i \phi_j \phi_k \phi_l \rangle$

YCH, Rong, Su (2020)

$SU(2)_1$ WZW sits at a kink!



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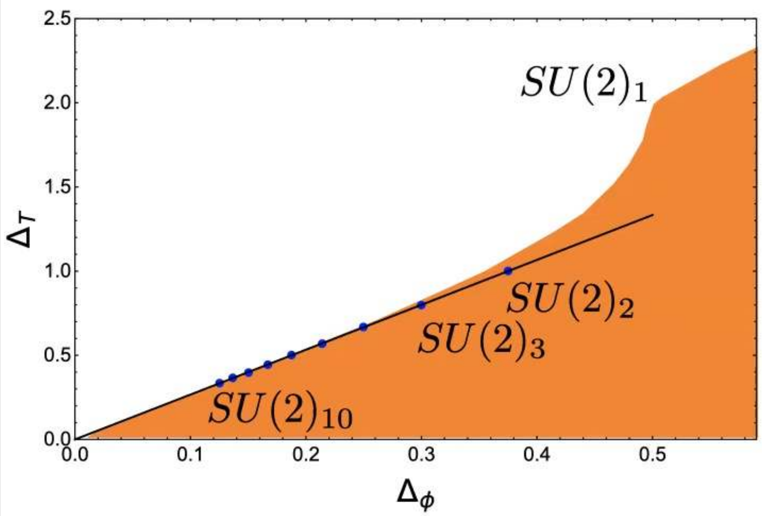
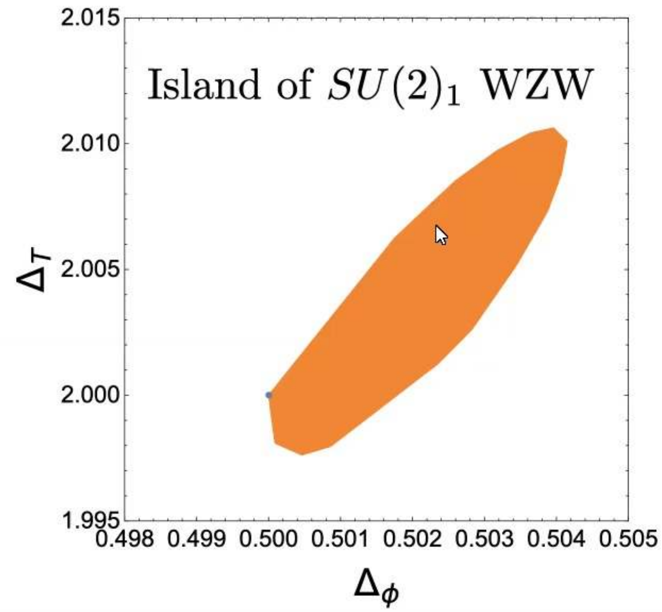


Mixed correlator

$\langle TTTT \rangle$ $\langle \phi\phi\phi\phi \rangle$ $\langle \phi\phi TT \rangle$ $\langle \phi T \phi T \rangle$

YCH, Rong, Su (upblished)



2D CFT kinks and null operators

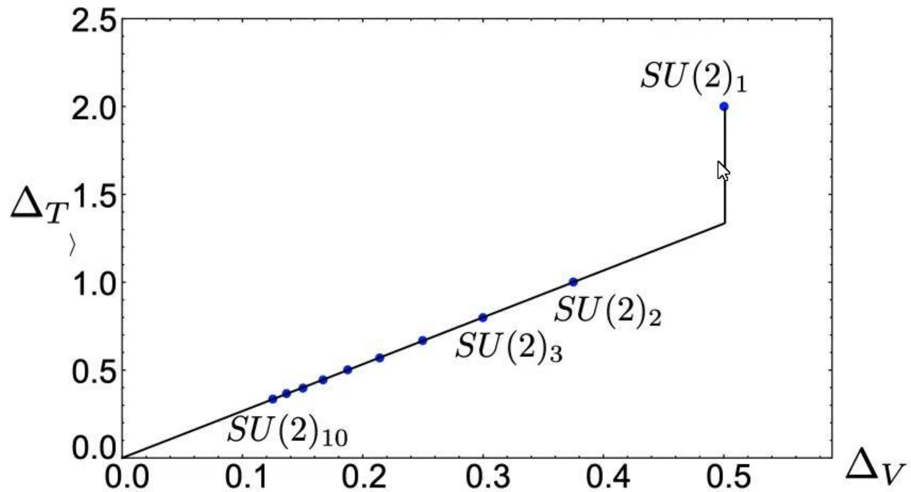
$SU(2)_k$ WZW CFT

Global symmetry: $(SU(2)_L \times SU(2)_R)/Z_2 \cong SO(4)$.

Operator spectrum (lowest weight in each channel):

1. $SO(4)$ V: $\Delta = \frac{3}{2(k+2)}$ (Kac-Moody primary)
2. $SO(4)$ T: $\Delta = \frac{4}{(k+2)}$ for $k > 1$ (Kac-Moody primary)

   $\Delta = 2$ for $k = 1$ $(J^L_{-1} J^R_{-1}|0\rangle)$



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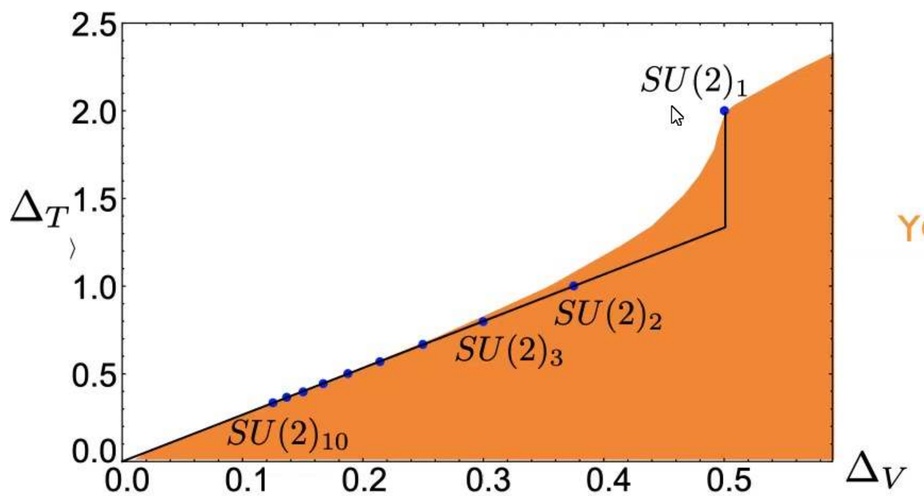

YCH, Rong, Su (2020)

Null/Decoupling operator and gauge theory

$$\text{2D } SU(2)_k \text{ WZW CFT}$$

$\Updownarrow$ Dual — Delmastro, Gomis, Yu (2021)

$$N_f = 2 \text{ 2D Dirac fermions coupled to a } U(k) \text{ gauge field.}$$

A semi-classical interpretation of WZW null/decoupling operators!

YCH, Rong, Su (2021)

$U(k)$ gauge theory	$SU(2)_{k>1}$ WZW	$SU(2)_1$ WZW
$\bar\psi^{[c_1}_{(l_1}\bar\psi^{c_2]}_{l_2)}\psi^{(r_1}_{[c_1}\psi^{r_2)}_{c_2]}$	$\Delta = \frac{4}{k+2}$ (Kac-Moody primary)	Null/Decoupling
$\bar\psi^{c_1}_{l_1}\psi^{l_2}_{c_1}\bar\psi^{c_2}_{r_1}\psi^{r_2}_{c_2}$	$\Delta = 2,\ J_{-1}\bar J_{-1}\lvert 0\rangle$	$\Delta = 2,\ J_{-1}\bar J_{-1}\lvert 0\rangle$

Decoupling operators of gauge theories

N_f flavors of critical bosons coupled to a $U(N_c)$ gauge field in 3d.

YCH, Rong, Su (2021)

$SU(N_f)$ rep		Operator	Scaling dimension
Adjoint		$\bar{\phi}^{f_1}_{c_1}\phi^{c_1}_{f_2}$	$\Delta = 1 + O(1/N_f)$
$A^{[f_1,f_2]}_{[f_3,f_4]}$	$N_c > 1$	$\bar{\phi}^{[f_1}_{[c_1}\bar{\phi}^{f_2]}_{c_2]}\phi^{[c_1}_{[f_3}\phi^{c_2]}_{f_4]}$	$\Delta = 2 + O(1/N_f)$
	$N_c = 1$	$\bar{\phi}^{[f_1}\partial\bar{\phi}^{f_2]}\phi_{[f_3}\partial\phi_{f_4]}$	$\Delta = 4 + O(1/N_f)$

The lowest operator of $U(N_c > 1)$ gauge theories is decoupled at $N_c = 1$.

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	$N_c = 1$	$\bar{\phi}^{[f_1}\partial\bar{\phi}^{f_2]}\phi_{[f_3}\partial\phi_{f_4]}$	$\Delta = 4 + O(1/N_f)$

The lowest operator of $U(N_c > 1)$ gauge theories is decoupled at $N_c = 1$.

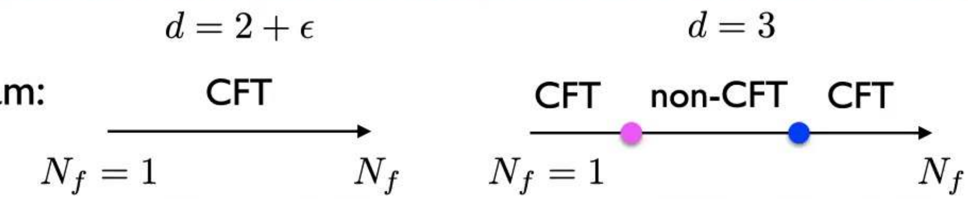
The lowest operator in the anti-symmetric representation $A^{[i_1,\cdots,i_m]}_{[j_1,\cdots,j_m]}$ of $N_c > m-1$ is decoupled at $N_c \leq m-1$.

Also see Reehorst, Rifinetti & Vichi (2020); Manenti & Vichi (2021)

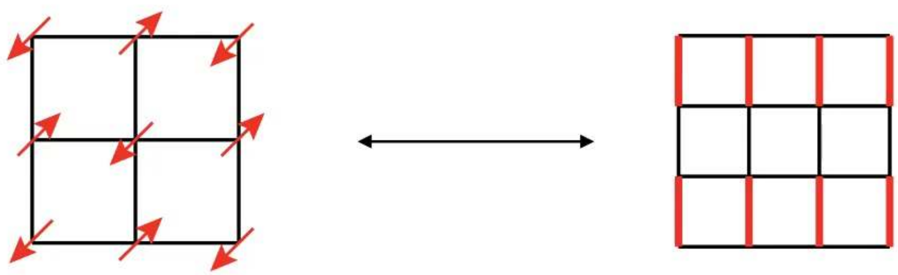
Bootstrap scalar QED

$$\mathcal{L} = \sum_{i=1}^{N_f} |(\partial_\mu - iA_\mu)\phi_i|^2 + \frac{g}{4}|\phi|^4 + \frac{1}{4e^2}F_{\mu\nu}^2.$$

Schematic phase diagram:



- Nf=1: particle-vortex duality. Peskin 1978; Dasgupta & Halperin 1981
- Nf=2: deconfined phase transition. Senthil, et al. 2004
- General Nf: SU(N) deconfined phase transition. Kaul & Sandvik 2012



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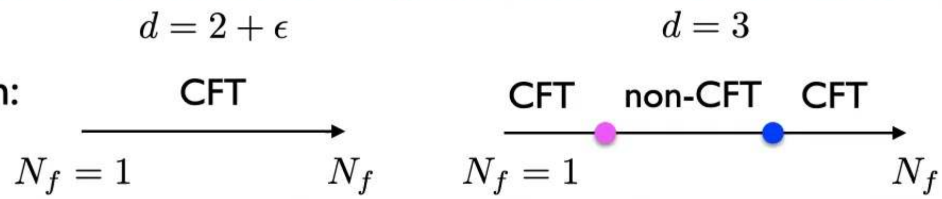


$SU(N_f)$ adjoint

$a = \bar{\phi}^i \phi_j - \delta^i_j / N_f |\phi|^2$

Bootstrap correlator: $\langle a(x_1)a(x_2)a(x_3)a(x_4)\rangle$

OPE: $a \times a = S^+ + Adj^\pm + A\bar{A}^+ + S\bar{S}^+ + S\bar{A}^- + A\bar{S}^-.$

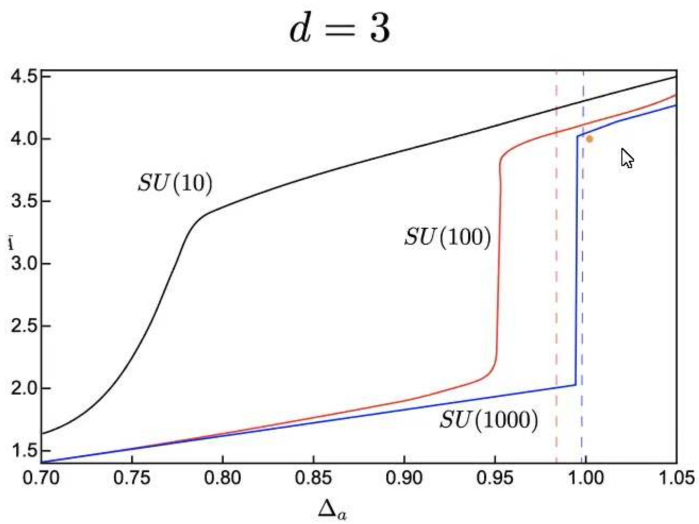
$A\bar{A}$: $T^{[f_3,f_4]}_{[f_1,f_2]}$ $S\bar{S}$: $T^{(f_3,f_4)}_{(f_1,f_2)}$ $S\bar{A}$: $T^{(f_3,f_4)}_{[f_1,f_2]}$ $A\bar{S}$: $T^{[f_3,f_4]}_{(f_1,f_2)}$

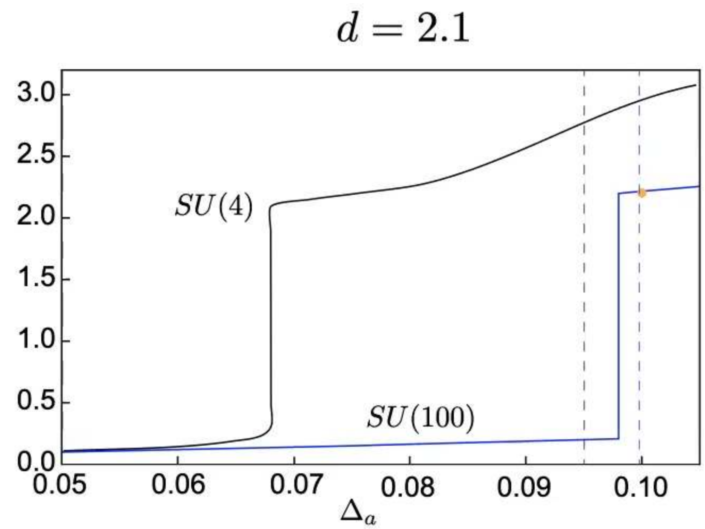
Decoupling operator

Bounding the decoupling operator

YCH, Rong, Su (2021)

The bootstrap bound can capture the essential physics of the operator decoupling, but the scalar QED does not precisely sit at the kink.





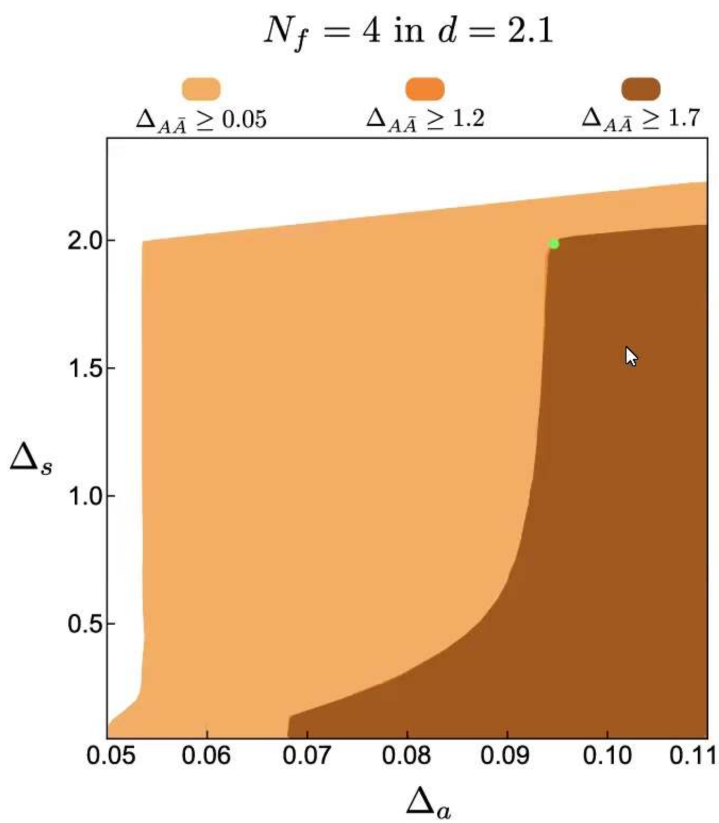
Large-N_f: $\Delta_a = 1 - \frac{48}{3\pi^2 N_f} + O(1/N_f^2)$

$2+\epsilon$: $\Delta_a = \epsilon - \frac{2}{N_f}\epsilon^2 + O(\epsilon^3)$

Kaul & Sachdev (2008);

Hikami, 1979, 1981

Scalar QED kink

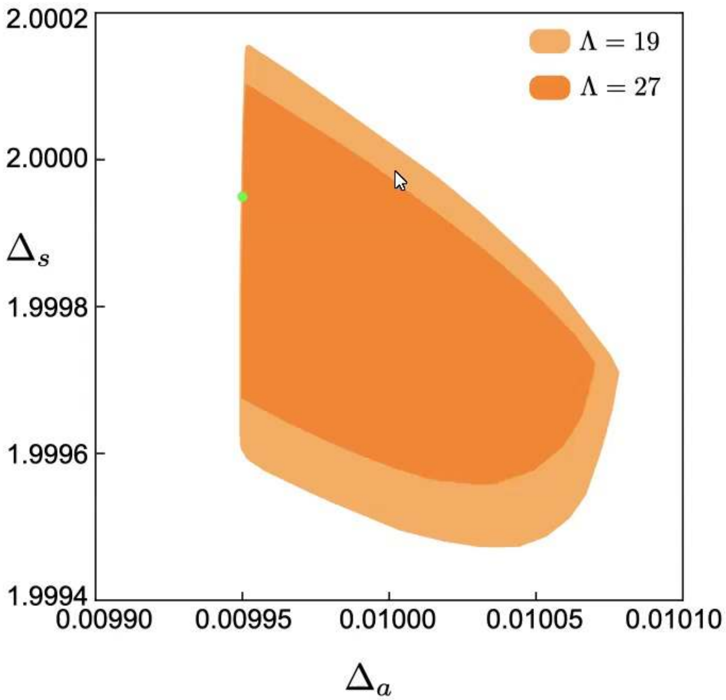


YCH, Rong, Su (2021)

The scalar QED appears as a kink once a mild gap is added for the decoupling operator, and it is stable against the change of the gap.

Scalar QED island

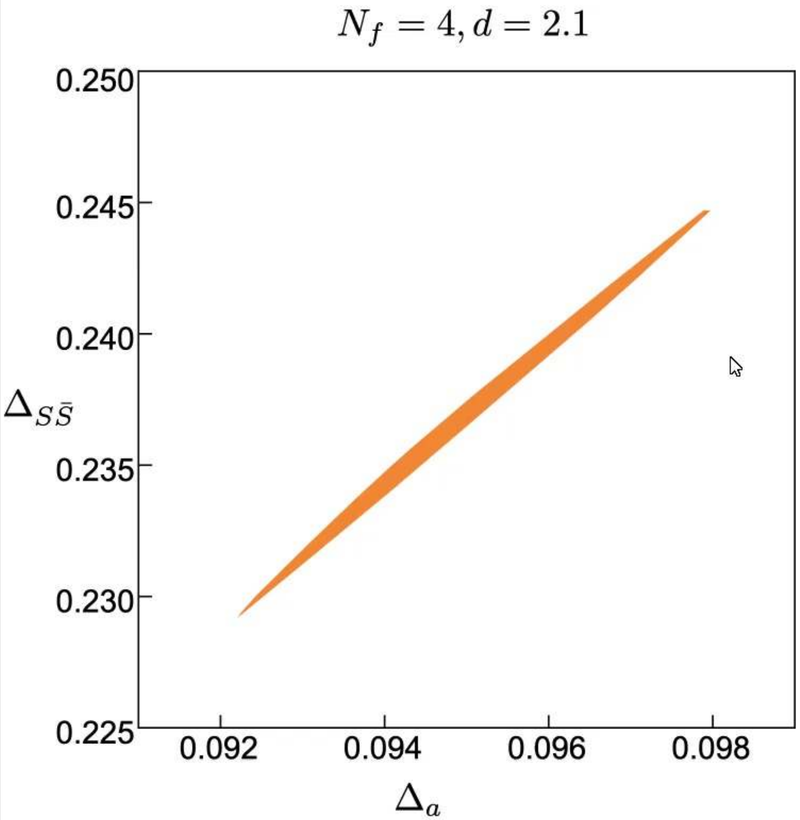
$N_f = 4, d = 2.01$



Single correlator

	gap imposed	physical gap
$\Delta_{A\bar{A}}$	$2d-3$	$2d-2$
$\Delta_{s'}$	3	4
$\Delta_{J'_\mu}$	$d-0.8$	$d+1$
$\Delta_{S\bar{S}}$	Δ_a	$2\Delta_a$

Scalar QED island



Single correlator

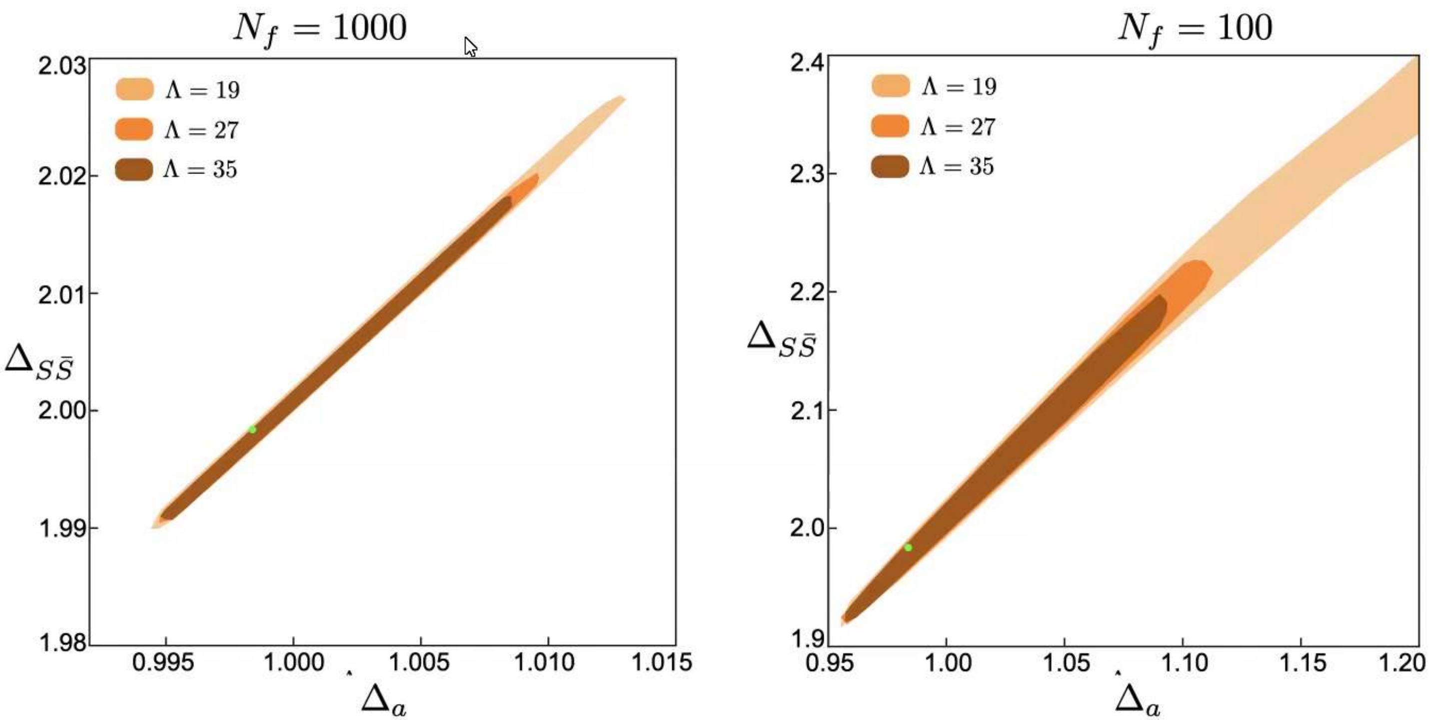
	gap imposed	physical gap
$\Delta_{A\bar{A}}$	$2d-3$	$2d-2$
Δ_s	1	2
$\Delta_{J'_\mu}$	d	$d+1$
$\Delta_{S\bar{S}'}$	$2d-3$	$2d-2$

Scalar QED islands in 3d

YCH, Rong, Su (2021)

	gap imposed	physical gap
$\Delta_{A\bar{A}}$	3	4
$\Delta_{J'_\mu}$	3.1	4
$\Delta_{S\bar{S}'}$	3	4

Single correlator



Summary and outlook

- I talk about several representative examples of critical quantum matter, including deconfined phase transition, Dirac spin liquid and Stiefel liquid.
- We proposed a recipe to bootstrap critical quantum matter described by gauge theories and WZW theories, and benchmarked it with scalar QED by obtaining its bootstrap kink and islands.
- Future: We plan to use the mixed correlator to solve scalar QED, QED and Stiefel liquid.
- Open question: Can some properties of 2d Kac-Moody algebra be generalized to higher dimensions?

Thanks!

arXiv:2005.04250
arXiv: 2101.07262
arXiv:2107.14637