Title: Perturbative lessons for nonperturbative quantum gravity

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Abstract: In the asymptotic safety approach to quantum gravity it is conjectured that a nonperturbative ultraviolet completion exists for Einstein's metric gravity and its perturbative effective description in four dimensions. Such completion would circumvent the notorious problems of renormalizability and predictivity at high energies of the quantum theory. I want do discuss the conjecture armed with pragmatism and avoiding preconceived ideologies. I also want to present some perturbative results in two dimensions, that can be used to argue the validity of the conjecture above two dimensions, and to discuss the implications for the current status of the approach.

Perturbative lessons for nonperturbative quantum gravity

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talk at Perimeter Institute, October 2021
based on work with R. Martini, A. Ugolotti, F. Del Porro

Disclaimers and trivia

Thank you for the invitation!

My first ever (relevant) talk was at PI in 2009 at the conference:
Asymptotic safety: 30 Years Later
It was a great experience, but just trust my word and *do not* watch the talk

I am going to give a *very partial* overview of some topics and neglect the rest

My current time is 20:30 after a long day of work, please be understanding

A non-comprehensive history of quantum gravity

General relativity

Einstein's

$$S[g_{\mu\nu}, \phi] = S_{\rm EH}[g] + S_{\rm mat}[g, \phi] = -\frac{1}{G}\int d^4x \sqrt{g}\, R[g] + S_{\rm mat}[\phi]$$

Interaction with matter mediated by Newton's G

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = G\, T^{\rm mat}_{\mu\nu}$$

But also, schematically, see the expansion in $E/M_{\rm Pl} \sim \sqrt{G}E$

$$S_{\rm EH}[\eta + \sqrt{G}h] = \int d^4x \Big\{ (\partial h)^2 + \sqrt{G} h (\partial h)^2 + G h^2 (\partial h)^2 + \ldots \Big\}$$

Early inclusions of quantum effects

Quantum mechanics in curved space with *fixed* $g_{\mu\nu}$, example:

$$T_H = \frac{\kappa}{2\pi} \to \frac{1}{8\pi M}$$

Implies that a black hole radiates with T_H, incompatibility/incompleteness of GR

Backreactions effects through QFT in curved space

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = G \langle T^{\rm mat}_{\mu\nu} \rangle$$

Potential applications in the correct regimes (e.g. inflation) but ask the experts

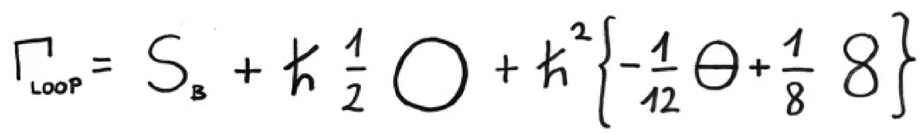
Naive quantization

GR's symmetries are "broken" (compare with gauge theories), therefore $g_{\mu\nu} = 0$ is not a natural expansion point

Using a background field and Euclidean PI

$$e^{-\frac{1}{\hbar}\Gamma_{\text{eff}}[\bar{g}]} = \int Dh\, e^{-\frac{1}{\hbar}S_{\text{EH}}[\bar{g}+h]}$$

Can be computed perturbatively (ex: two loops)



Loop results: successes

At one loop and in dimensional regularization $d = 4 - \epsilon$

$$\Gamma_{\rm div} = \frac{2}{\epsilon} \int d^4x \sqrt{g} \left\{ \frac{7}{20} R_{\mu\nu} R^{\mu\nu} + \frac{1}{120} R^2 + E_4 \right\}$$

Counterterms outside S_{EH} would be required. However for pure gravity

$$R_{\mu\nu} = 0$$

Therefore $\Gamma_{\rm div} = 0$ on-shell, generally off-shell one redefines

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + \frac{1}{\epsilon} \Big(c_1 R_{\mu\nu} + c_2 g_{\mu\nu} R \Big)$$

Loop results: failures

At one loop including a cosmological constant Λ

$$\Gamma_{\rm div} = \frac{2}{\epsilon}\int d^4x\sqrt{g}\Big\{\frac{53}{90}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \frac{1}{120}\Lambda^2\Big\}$$

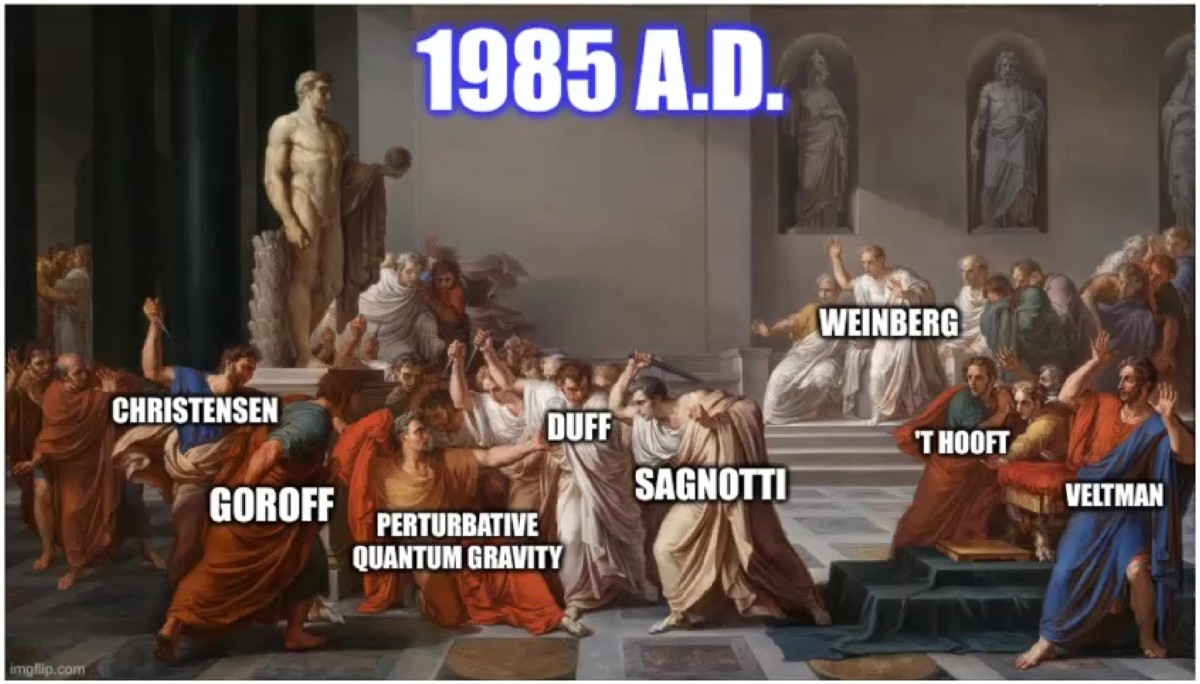
Or more generally including matter fields (e.g. a scalar)

$$\Gamma_{\rm div} = \frac{G}{\epsilon}\int d^4x\sqrt{g}\Big\{\frac{5}{4}(\partial_\mu\phi\partial^\mu\phi)^2 - \frac{1}{3}R(\partial_\mu\phi\partial^\mu\phi)\Big\}$$

The final nail in the coffin for pure gravity comes at two loops

$$\Gamma_{\rm div} = \frac{1}{\epsilon}\int d^4x\sqrt{g}\Big\{\frac{209}{2880}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\theta}R^{\rho\theta}{}_{\mu\nu} + \cdots\Big\}$$

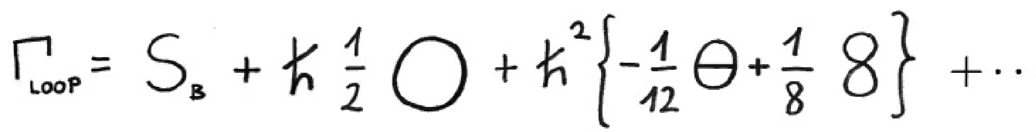
Gravity is not perturbatively renormalizable in $d = 4$



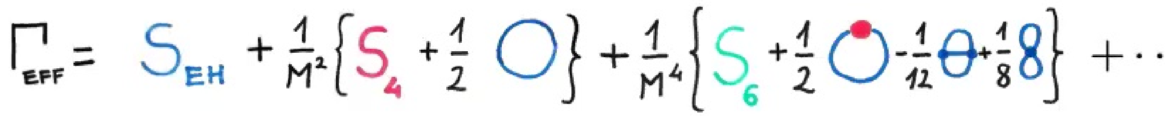
"The Death of Julius Caesar" by Vincenzo Camuccini (1806)

Gravity as an effective theory

Failure of renormalizability from dimful coupling $[G] = -2 \implies$ expansion $\sim \frac{E}{M_{\rm Pl}}$
Replace the perturbative expansion...



... with an effective one at a scale $M \approx M_{\rm Pl}$ working for $E \ll M_{Pl}$ (see **Donoghue**)



$$\Gamma[g] \approx M^2 \int d^4x\sqrt{g}\left\{\mathcal{R} + \frac{1}{M^2}\mathcal{R}^2 + \frac{1}{M^4}\mathcal{R}^3 + \text{nonlocal terms} + \cdots\right\}$$

Asymptotic safety

ANOTHER POINT OF VIEW: $d = 2 + \epsilon$

In general dimension $[G] = 2 - d \implies [G] = 0$ in $d = 2$. Turns out that S_{EH} is perturbatively renormalizable and asymptotically free in $d = 2$

$$\beta_G = -\frac{19}{24\pi} G^2$$

In $d = 2 + \epsilon$ we expect $G \to G\mu^{-\epsilon}$ (μ: some RG scale)

$$\beta_G = \epsilon G - \frac{19}{24\pi} G^2$$

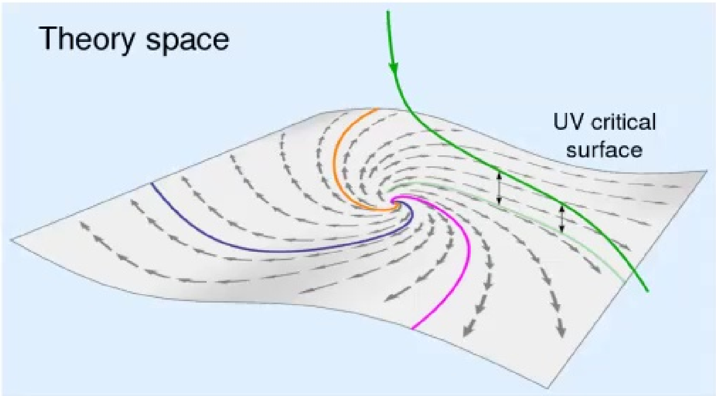
$G^* = 0$ IR attractive FP versus $G^* \sim \epsilon$ UV "attractive" FP

Idea: for $E \gtrsim M_{\mathrm{Pl}}$ maybe $\Gamma \to S^*$

Weinberg's asymptotic safety

Weinberg's idea: meaningful and UV complete theory if

- There is a scale invariant RG fixed point
- The FP has finitely many UV-relevant directions



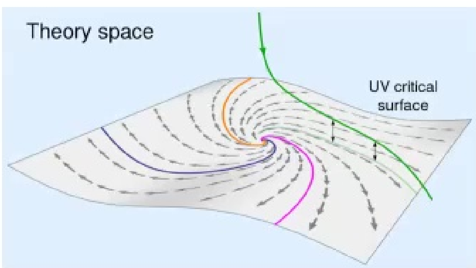
Wilsonian framework: RG lines as trajectories of equal IR physics

Predictivity

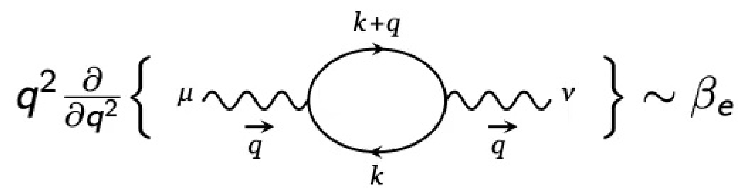
Naively: locate oneself on critical surface with finitely many "experiments"

More precisely

- Trajectories flow to some fixed IR physics (e.g. a phase in stat-mech language)
- Integration from FP leaves Γ with dependence on finite parameters/couplings



versus



Note however:

- Parameter/coupling interpretation is not easy nor straightforward...
- ... especially when lacking a mesoscopic interpretation
- IR should also interpolate with EFT

Reuter's 1996 approach

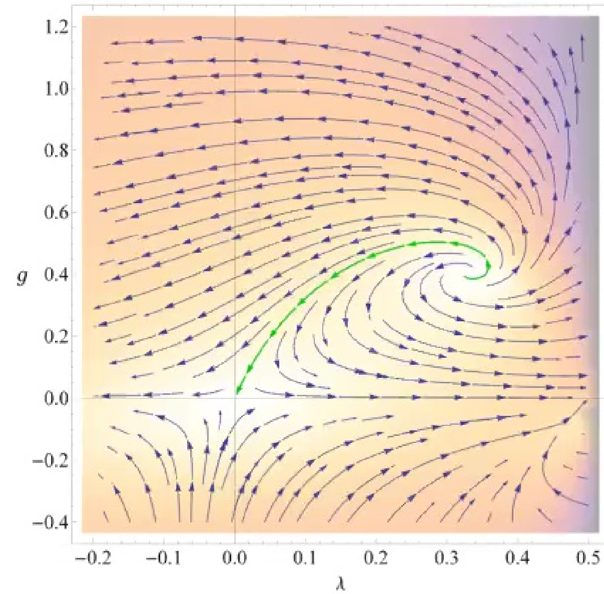
1990-1995: Key results in $d = 2 + \epsilon$ with transfer from String Theory

However convergence to $\epsilon = 2$ unclear $\Longrightarrow$ desire to work in $d = 4$

Necessary a non-perturbative method **Reuter** used **Wetterich**'s equation:

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k$$

The good:



The bad: some open problems

How it was done:

- Projection to a truncation ansatz

$$\Gamma_k = \int d^4x\sqrt{g}\Big\{g_0(k) - g_1(k)R[g]\Big\} + \text{20yrs worth of additional operators}$$

- Gauge dependence

$$S_{\text{g.f.}} = \frac{1}{2\alpha}\int d^4x\sqrt{g}F_\mu F^\mu\,, \qquad F_\mu = \nabla^\nu h_{\mu\nu} + \beta\,\nabla_\mu h^\nu{}_\nu$$

- Parametric dependence

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{\lambda}{2}h_{\mu\rho}h^\rho{}_\nu + \cdots$$

Qualitatively ok, but the quantitative results depend heavily on all parameters!

The ugly: nonperturbative misinformation (way too many words, sorry about this)

Probably I will attract some hate, but we have to be honest as scientists:

- To cutoff, or not to cutoff, that is the question...
  Necessary to communicate the meaning of powerlaw divergences
- Truncation misleadingly referred to as "nonperturbative approximation" intended as "better than perturbation theory", but is it?
- Nonperturbative Wetterich's RG suggested as the only way to formulate but interpolation to IR is not naive. Alternative in continuum? Compensations?
- Effort spent towards other issues (e.g. unitarity)?
  Especially in comparison to the practice "truncate and compute"

Do I have full solutions to these problems? Honestly, no

Universal results in $d = 2 + \epsilon$

18/28

Back to $d = 2 + \epsilon$

Take the *bare* action

$$S = \int d^4x\sqrt{g}\Big\{g_0 - g_1 R[g]\Big\}$$

Notable early works by **Jack & Jones**

$$\beta_G = -\frac{19}{24\pi}G^2$$

and by **Kawai, Ninomiya, Aida, Kitazawa et al.**

$$\beta_G = -\frac{25}{24\pi}G^2$$

What is going on?

Symmetry

Symmetries of $S[g_{\mu\nu}]$ are *Diff*, generated

$$\delta g_{\mu\nu} = (\mathcal{L}_\xi g)_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$$

Rewrite $g_{\mu\nu} = e^{\frac{2}{d-2}\varphi}\tilde{g}_{\mu\nu}$ for $\tilde{g}_{\mu\nu}$ unimodular metric (e.g. fixed determinant)

$$\delta^* \tilde{g}_{\mu\nu} = 2\tilde{g}_{\rho(\mu}\tilde{\nabla}_{\nu)}\xi^\rho - \frac{2}{d}\tilde{g}_{\mu\nu}\tilde{\nabla}_\rho\xi^\rho \qquad \delta^*\varphi = \xi^\mu\partial_\mu\varphi + \frac{d-2}{2d}\varphi\tilde{\nabla}_\rho\xi^\rho$$

Another realization $\mathit{Diff}^* \simeq \mathit{Diff}$ "Unimodular Dilaton Gravity"

$$\mathit{Diff} \ltimes \mathit{Weyl} \xrightarrow{\delta^*\sqrt{\tilde{g}}=0} \mathit{Diff}^*$$

19 VERSUS 25 (WAY TOO MANY WORDS AGAIN, MY BAD)

Path-integral with *Diff* produces 19, however J&J noticed:

- Beyond 1-loop we have $\frac{1}{d-2}$ poles of kinematical origin
  This is related to the discontinuity of the number of degrees of freedom for $d \to 2$
- Naive subtraction of kinematical $\frac{1}{d-2}$ leads to inconsistency with dimreg

Let me add that $R = 0$ on-shell, so a divergence $\frac{1}{\epsilon} R$ is unphysical per se

Path-integral with *Diff** produces 25, notice:

- Relation with strings for which conformal mode is not quantized
  Numerology: $2d$-gravity equals bc-ghosts plus $\varphi \implies -25 = -26 + 1$
- Preserving *Weyl* requires classical topological charge (string's dilaton) in $2d$

Unclear generalization for $d \neq 2$

When starting this project I was very much biased towards 25 but open to change

Recipe for d-dependent counterterms

Three-steps strategy:

- Regularize/subtract Feynman diagrams with dimensional regularization $d = 2 - \zeta$ where it makes sense ($\zeta = -\epsilon > 0$ for conformal mode stability)

$$d^2x \to \mu^\epsilon d^d x$$

- Treat any *other* dependence on d as parametric (like N in $SU(N)$ gauge theories)

$$\text{e.g.} \qquad g^\mu{}_\mu = d \neq 2 + \epsilon$$

- Continue to $\epsilon = -\zeta > 0$ (crossing fingers and hoping for the best) with

$$G \to G\mu^\epsilon$$

Using the *Diff* realization

Computation using covariant heat kernel methods

$$\Gamma_{\rm div} = \frac{\mu^{-\zeta}}{4\pi\zeta}\int \mathrm{d}^d x\sqrt{g}\Big\{\frac{g_0}{g_1}\Big[-\frac{1}{2}d(d+1) + \frac{d(d^2-d-4)\lambda}{4(d-2)} - \delta\beta\Big(d + \frac{d\lambda}{(d-2)}\Big)\Big] + R\Big[\frac{5d^2-3d+24}{12} + \frac{1}{4}(-d^2+d+4)\lambda + \delta\beta(d+\lambda-2)\Big]\Big\}$$

Highlights: kinematical poles, parametric and gauge mixed dependences
$\Longrightarrow$ Naive dependence on beta functions

Isolate on-shell using EOMs with cosmological constant

$$\Gamma_{\rm div} = \frac{\mu^{-\zeta}}{\zeta}\int \mathrm{d}^d x\sqrt{g}\Big\{AR + J_{\mu\nu}\Big(G^{\mu\nu} + \frac{g_0}{2g_1}g^{\mu\nu}\Big)\Big\}$$

Diff on-shell RG

Going on-shell factorizes a "source" for the EOMs

$$\Gamma_{\rm div} = \frac{\mu^{-\zeta}}{\zeta}\int \mathrm{d}^d x\sqrt{g}\left\{AR + J_{\mu\nu}\left(G^{\mu\nu} + \frac{g_0}{2g_1}g^{\mu\nu}\right)\right\}$$

$$A = \frac{36+3d-d^2}{48\pi}$$

$$J_{\mu\nu} = \frac{g_{\mu\nu}}{4\pi}\left\{\frac{d^2-d-4}{2(d-2)}\lambda - \delta\beta\left(2+\frac{2\lambda}{(d-2)}\right) - d - 1\right\}$$

Subtraction on-shell introduces

$$\beta_G = \epsilon G - \frac{36+3d-d^2}{48\pi}G^2 \quad \xrightarrow{d\to 2} \quad \beta_G = -\frac{19}{24\pi}G^2$$

UV fixed point for a "conformal window" $0 < d < 7.685$ (previously found by **Falls**)

Diff* ON-SHELL RG

We checked that using *Diff** realization gives the same on-shell for $d \neq 2$

However only in $d = 2$ gauge-dependence drops anyway

Furthermore, we can introduce topological charge q to break *Weyl* classically
q does not receive radiative corrections only in $d = 2$ (q is a free parameter)

$$S_{\mathrm{top}} = q \int \mathrm{d}^2 x \sqrt{\tilde{g}}\, \varphi\, \tilde{R}$$

So we can set q to cancel the Weyl anomaly $\langle T \rangle$ in $d = 2$
this results in $\beta_G = -\frac{25}{24\pi} G^2$ which is consistent with String Theory

PERSPECTIVES

Diff realization:

- Extension to two loops $\Longrightarrow$ corrections to conformal window

Diff* realization:

- Unimodular dilaton higher derivative gravity $\Longrightarrow$ cancellation of $4d$ anomaly

In general, lessons for the nonperturbative functional methods

- Necessary to go on-shell to have physical gauge independent results
  $\Longrightarrow$ We need to understand how with functional methods
- Independence on parametrization
  $\Longrightarrow$ Even milder dependence would become acceptable

Universal models of quantum gravity in other dimensions

Based on power counting we have bare actions for any $d = 2m$ and $m \in \mathbb{N}$

Higher derivative gravity in $d = 4$ (well-explored)

$$S[g] = \int \mathrm{d}^4 x \sqrt{g} \left\{ \frac{1}{2\lambda} C^2 - \frac{1}{\rho} E_4 + \frac{1}{\xi} R^2 \right\}$$

Higher derivative gravity in $d = 6 - \epsilon$ (finally recently explored by **Knorr**)

$$S[g] = \int \mathrm{d}^6 x \sqrt{g} \Big\{ a_1 R \Box R + a_2 R_{\mu\nu} \Box R_{\mu\nu} + a_3 R^3 + a_4 R R_{\mu\nu} R^{\mu\nu} + a_5 R_\mu{}^\nu R_\nu{}^\alpha R_\alpha{}^\mu$$
$$+ a_6 R_{\mu\nu} R_{\alpha\beta} C^{\mu\nu\alpha\beta} + a_7 R C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} + a_8 R^{\mu\nu} C_{\mu\alpha\beta\gamma} C^{\nu\alpha\beta\gamma}$$
$$+ a_9 C_{\mu\nu}{}^{\alpha\beta} C_{\alpha\beta}{}^{\rho\theta} C_{\rho\theta}{}^{\mu\nu} + a_{10} C^\mu{}_\alpha{}^\nu{}_\beta C^\alpha{}_\rho{}^\beta{}_\theta C^\rho{}_\mu{}^\theta{}_\nu \Big\}$$

Thank you