Title: On generalized hyperpolygons, Higgs bundles and branes

Speakers: Laura Schaposnik

Series: Mathematical Physics

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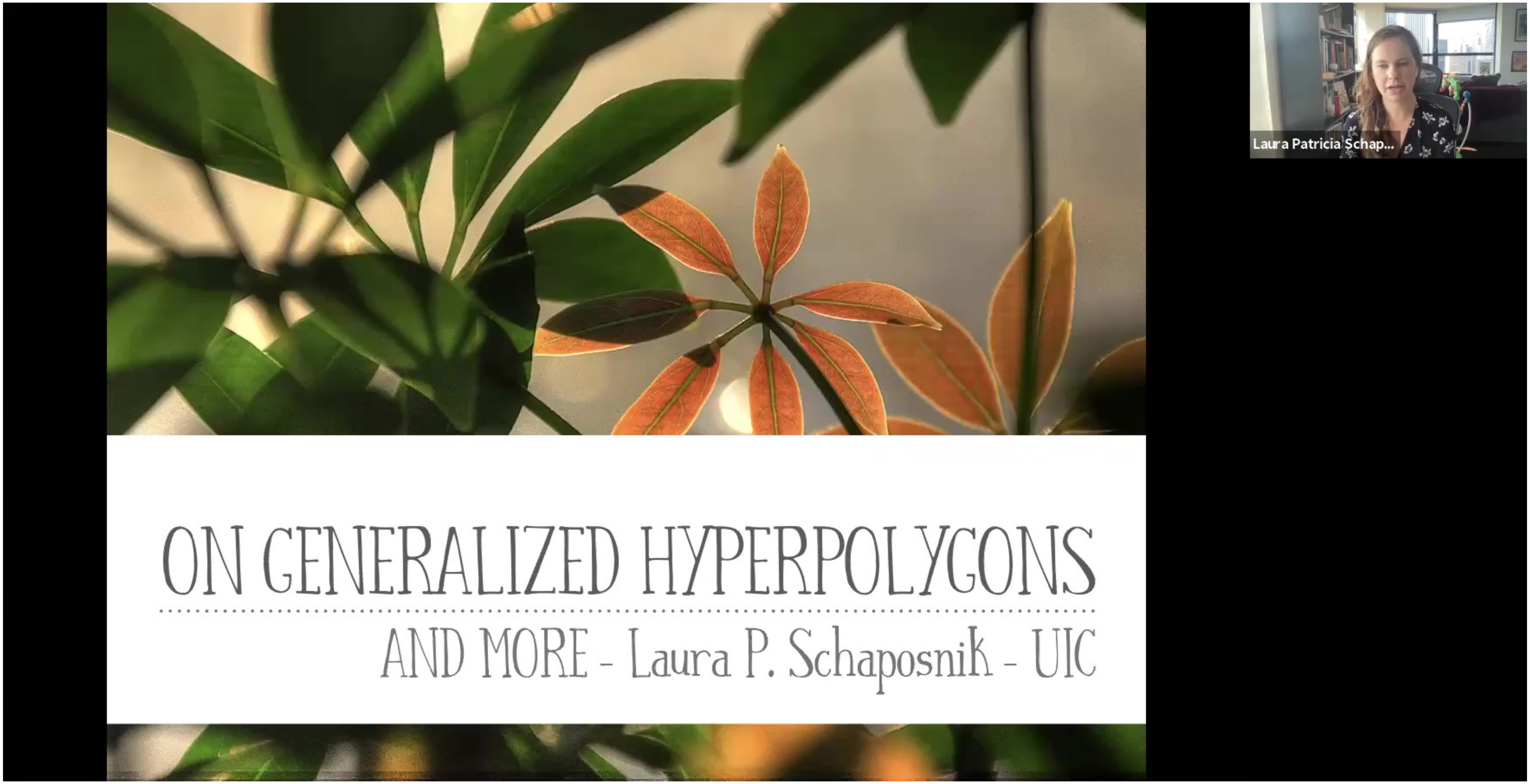
Abstract: In this talk we will introduce generalized hyperpolygons, which arise as Nakajima-type representations of a comet-shaped quiver, following recent work with Steven Rayan. After showing how to identify these representations with pairs of polygons, we shall associate to the data an explicit meromorphic Higgs bundle on a

genus-g Riemann surface, where g is the number of loops in the comet. We shall see that, under certain assumptions on flag types, the moduli space of generalized hyperpolygons admits the structure of a completely integrable Hamiltonian system. Finally, we shall look into the appearance of branes within the moduli space of generalized hyperpolygons as well as of Higgs bundles, and consider mirror symmetry for such branes. Time permitting, we will mention some other recent results in various areas of science.

Zoom Link: https://pitp.zoom.us/j/91592778202?pwd=WnM2VS9pS2c0QVIxVWFOdGhFMTdEdz09


ON GENERALIZED HYPERPOLYGONS
AND MORE - Laura P. Schaposnik - UIC
Laura Patricia Schap...

THE PLAN

1. (Generalized) Hyperpolygons
2. From Hyperpolygons to Higgs bundles
3. Integrable systems of hyperpolygons
4. Branes of Higgs bundles and Hyperpolygons
5. Introducing myself: Some other projects I have going on...

We will be using mostly...

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MODULI SPACES OF GENERALIZED HYPERPOLYGONS

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[Received 20 January 2020]

In memory of Sir Michael Atiyah (1929-2019), an inspiration to geometers the world round

Abstract

We introduce the notion of *generalized hyperpolygon*, which arises as a representation, in the sense of Nakajima, of a comet-shaped quiver. We identify these representations with rigid geometric figures, namely pairs of polygons: one in the Lie algebra of a compact group and the other in its complexification. To such data, we associate an explicit meromorphic Higgs bundle on a genus-g Riemann surface, where g is the number of loops in the comet, thereby embedding the Nakajima quiver variety into a Hitchin system on a punctured genus-g Riemann surface (generally with positive codimension). We show that, under certain assumptions on flag types, the space of generalized hyperpolygons admits the structure of a completely integrable Hamiltonian system of Gelfand–Tsetlin type, inherited from the reduction of partial flag varieties. In the case where all flags are complete, we present the Hamiltonians explicitly. We also remark upon the discretization of the Hitchin equations given by hyperpolygons, the construction of triple branes (in the sense of Kapustin–Witten mirror symmetry), and dualities between tame and wild Hitchin systems (in the sense of Painlevé transcendents).

1. Introduction

One constant theme in the work of Michael Atiyah has been the interplay of algebra, geometry and physics. The construction of complete, asymptotically locally Euclidean (ALE), hyperkähler 4-manifolds—in other words, of *gravitational instantons*—from a graph of Dynkin type is the capstone of a particular program for constructing Kähler–Einstein metrics, relevant to both geometry and physics and using only linear algebra. This construction is at once the geometric realization of the McKay correspondence for finite subgroups of SU(2) [38], a generalization of the Gibbons–Hawking ansatz [15], and the analogue of the Atiyah–Drinfel'd–Hitchin–Manin technique [2] for constructing Yang-Mills instantons. The construction completes a circle of ideas. First, an instanton

Higgs Bundles— Recent Applications

Laura P. Schaposnik

Introduction

This note is dedicated to introducing Higgs bundles and the Hitchin fibration, with a view towards their appearance within different branches of mathematics and physics, focusing in particular on the role played by the *integrable system* structure carried by their moduli spaces. On a compact Riemann surface Σ of genus $g \geq 2$, Higgs bundles are pairs (E, Φ) where

- E is a holomorphic vector bundle on Σ, and
- the Higgs field $\Phi : E \to E \otimes K$ is a holomorphic map for $K := T^*\Sigma$.

Since their origin in the late 1980s in work of Hitchin and Simpson, Higgs bundles manifest as fundamental ob-

- Via the nonabelian Hodge correspondence developed by Corlette, Donaldson, Simpson, and Hitchin and in the spirit of Uhlenbeck–Yau's work for compact groups, the moduli space is analytically isomorphic as a real manifold to the *de Rham moduli space* $\mathcal{M}_{dR}$ of flat connections on a smooth complex bundle.
- Via the Riemann–Hilbert correspondence there is a complex analytic isomorphism between the de Rham space and the *Betti moduli space* $\mathcal{M}_B$ of surface group representations $\pi_1(\Sigma) \to G_{\mathbb{C}}$.

Some prominent examples where these moduli spaces appear in mathematics and physics are:

WHAT IS... a Hyperpolygon?

Steven Rayan and Laura P. Schaposnik

Part of the spectacular success of the study of *quiver representations* is the realization that one may construct many interesting geometries—both old and new—from just the data of a directed graph. For us, a quiver will simply be a directed graph with nodes labelled by natural numbers and multiedges permitted. Familiar geometries such as those of projective space and Grassmannians can be constructed from a relatively simple graph, consisting of just two nodes and an arrow from one to the other, which is a so-called A_2 quiver. By increasing the complexity of the quiver, one can produce more interesting spaces.

Quivers and flag varieties. As suggested by the connection of projective space and Grassmannians to A-type quivers, the ADE Dynkin-type quivers play a fundamental role in the theory and capture many of its connections to geometry, representation theory, combinatorics, and physics.

$$r_m \leftarrow r_{m-1} \cdots r_3 \leftarrow r_2 \leftarrow r_1$$

which is subject to the conjugation action of the group $G = \prod_{i=1}^{m-1} \mathrm{GL}(r_i, \mathbb{C})$. Through a suitable notion of quotient furnished either by geometric invariant theory (GIT) or by symplectic reduction, one may restrict to a subvariety of V on which the group action is free. The result of restricting in this way and then quotienting is typically denoted by $V/\!/G$. The *quiver variety* $V/\!/G$ is an example of a *moduli space*, a space that keeps track of representations of the original quiver up to the equivalence furnished by G.

In the case of the A-type quiver, the quotient $V/\!/G$ is a *partial flag variety* $\mathcal{F}_{r_1,\dots,r_m}$, from which the Grassmannians are recovered as $\mathcal{F}_{r_1,r_2}$. In the case of projective space $\mathbb{P}^{r-1} = \mathcal{F}_{1,r}$, GIT issues the familiar instruction of deleting the origin from $\mathrm{Hom}(\mathbb{C}, \mathbb{C}^r)$ before taking the quotient. For economy, we will denote the tuple of labels by $\underline{r} = (r_1, \dots, r_m)$.

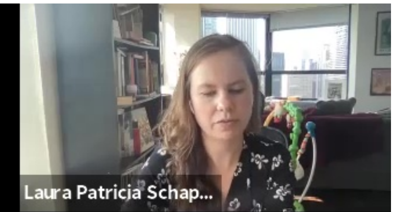
The partial flag varieties are prototypical examples of quiver varieties. One proceeds in essentially the same way for all quivers: an arrow between vertices labelled u and v

1. HYPERPOLYGONS

The moduli space of Hyperpolygons appears as the Nakajima variety of star-shaped quivers.

Since we will be generalizing this set up, let's do a quick recap of how this variety is built.

$$\mathrm{Rep}\left(\underset{u}{\overset{R_u}{\bullet}} \longrightarrow \underset{v}{\overset{R_v}{\bullet}} \right) = \underbrace{\mathrm{Hom}\left(\mathbb{C}^{R_u}, \mathbb{C}^{R_v}\right)}_{V}$$



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$$\mathrm{Rep}\left(\begin{tikzcd} \overset{R_u}{\underset{u}{\bullet}} \arrow[r, "x"] & \overset{R_v}{\underset{v}{\bullet}} \arrow[l, bend left, dashed, "y"] \end{tikzcd}\right) = T^*\underbrace{\mathrm{Hom}(\mathbb{C}^{R_u}, \mathbb{C}^{R_v})}_{T^*V} \cong \begin{array}{c} x \\ \mathrm{Hom}(\mathbb{C}^{R_u}, \mathbb{C}^{R_v}) \\ \oplus \\ \mathrm{Hom}(\mathbb{C}^{R_v}, \mathbb{C}^{R_u}) \\ y \end{array}$$

NAKAJIMA QUIVER

To construct moduli spaces, we consider symplectic reduction, since it will be the path to generalized Hyperpolygons.

To each quiver Q we can associate two functions, which can be built node by node, and which give us moment maps:

$$\mu : T^* \mathrm{Rep}(Q) \longrightarrow \bigoplus_v \mathfrak{u}(R_v)^*$$

$$\gamma : T^* \mathrm{Rep}(Q) \longrightarrow \bigoplus_v \mathfrak{gl}(R_v)^*$$

These moment maps are associated to the complex adjoint action of

$$G = \Big(\prod_{v \text{ non central}} U(R_v) \times SU(R) \Big) / \pm 1 \quad \text{on } \mathrm{Rep}(Q)$$

label of central node

for $U(R_h)$ and $U(R_v)$ acting by left and right multiplication on the arrow, and $SU(R)$ by multiplication on one side of arrow to/from center.

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$$\gamma : T^* \mathrm{Rep}(Q) \longrightarrow \bigoplus_{v} \mathfrak{gl}(R_v)^*$$

For
$$\begin{tikzcd} \underset{u}{\bullet} \arrow[r, "x"'] & \underset{v}{\bullet} \arrow[l, bend right, dashed, "y"'] \end{tikzcd}$$
we have

$$\mu_u(x, y) = \underbrace{x^* x - y y^*}_{\mathfrak{u}(R_u)^*}$$

$$\gamma_u(x, y) = \underbrace{x y}_{\mathfrak{gl}(R_u)^*}$$

The Nakajima quiver variety of a quiver Q is constructed from Q and its moment maps, and it is a HyperKähler variety. We first consider

$$\mathrm{Rep}(Q)/G \quad \text{for} \quad G=\prod_{v} GL(k_v,\mathbb{C})$$

The Nakajima quiver variety of a quiver Q is constructed from Q and its moment maps, and it is a Hyperkähler variety. We first consider

$$T^*\mathrm{Rep}(Q) /\!\!/_{\alpha} G \qquad \text{for} \qquad G = \prod_{v} GL(R_v, \mathbb{C})$$

To obtain the Nakajima quiver variety we restrict $T^*\mathrm{Rep}(Q)$ to the intersection of two level sets

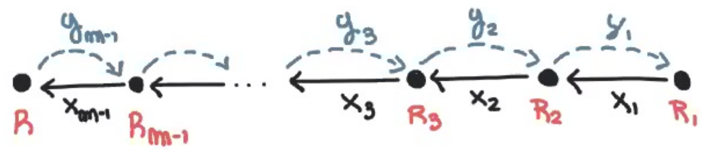
compact!

$$\mu^{-1}(\alpha) \cap \nu^{-1}(0) / G \qquad G = \prod_{v} U(R_v)/\pm 1$$

$$\alpha \in Z(\mathfrak{g}^*)$$

From work of Hitchin-Karlhede-Lindström-Roček, the quotient inherits quaternionically-commuting complex structures I, J, K and Riemannian metric g.

Our interest is on (generalized) Hyperpolygons. To build them, we start with an A-Type quiver



The corresponding Nakajima quiver variety is

$$\underbrace{T^*\mathcal{F}_{\underline{R}}}_{\text{Partial flag variety}} \qquad \underline{R} \subseteq [R] = \{1,2,\dots,R\} \qquad G = \prod_{\underline{R}/\{R\}} U(R_i)/\pm 1$$

$$\#(\underline{R}) = m$$

$$Z(\mathfrak{g}^*) = \bigoplus_{i=1}^{m-1} \mathfrak{u}(R_i)^*$$

$$(\alpha, 0, 0 \dots 0) \quad \alpha \in \mathbb{R}$$

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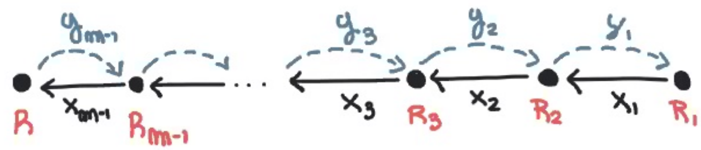


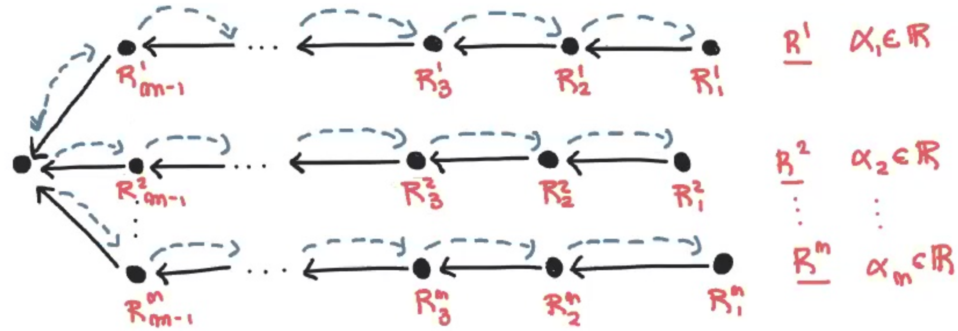
The corresponding Nakajima quiver variety is

$T^*\mathcal{F}_{\underline{R}}$ and it has a $U(R)$ action w/ complex moment map

$$\gamma_m(x_{m-1}, y_{m-1}) = x_{m-1}\, y_{m-1}$$

and we can use this to understand star-shaped quivers.

Our interest is on (generalized) Hyperpolygons, so consider now star-shaped quivers: interlocking n-many A-star quivers:



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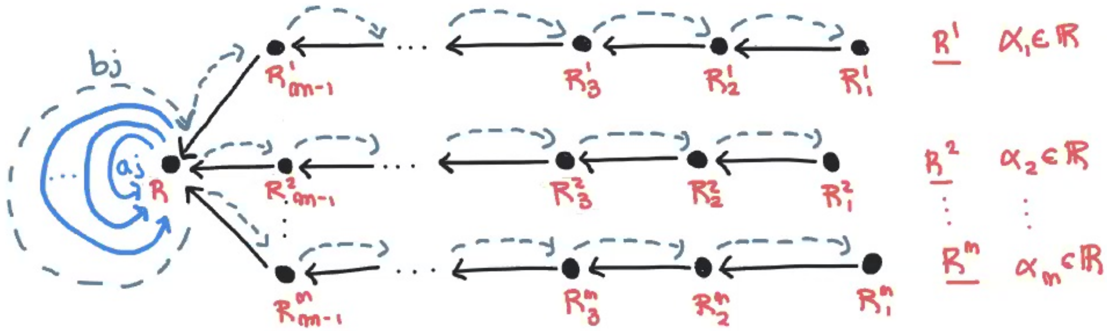
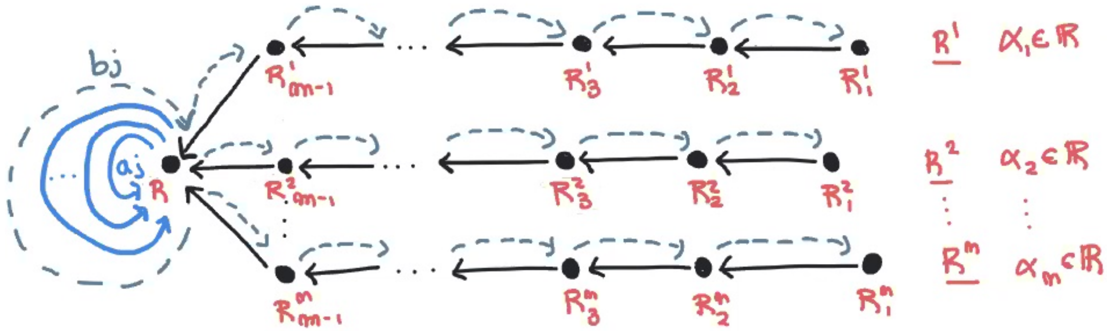


To build the space of generalized Hyperpolygons, we need first to add g loops to obtain a comet-shaped quiver.

We will consider its Nakajima quiver with the doubled arrows

At the central node R we restrict to the SU(R) action.

Our interest is on (generalized) Hyperpolygons, so consider now star-shaped quivers: interlocking n-many A-star quivers:

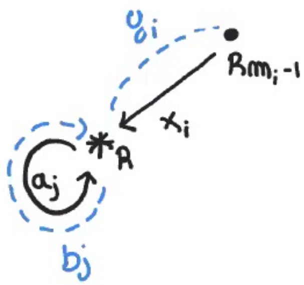


The Nakajima quiver variety of the above comet $\mathcal{Q}_n$ is

$$T^*\mathcal{F}_{\underline{R}^1} \times \dots \times T^*\mathcal{F}_{\underline{R}^n} \times T^*\mathfrak{sl}(R,\mathbb{C})^g /\!/\!/_{0}\, SU(R)$$

$$\underline{\alpha} = (\alpha_1, \dots, \alpha_n)$$

To build the appropriate moment maps, it is useful to consider how the loops appear. At the central node $*$ we can express the moment maps as follows:



$$\mu_*(x, y, a, b) = \sum_{i=1}^{m} \left(x_i x_i^* - y_i^* y_i\right)_0 + \sum_{j=1}^{g} [a_j, a_j^*] + [b_j, b_j^*] = 0$$

$$\gamma_*(x, y, a, b) = \sum_{i=1}^{m} \left(x_i y_i\right)_0 + \sum_{j=1}^{g} [a_j, b_j] = 0$$

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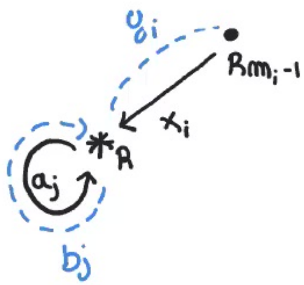


$$\mu_*(x, y, a, b) = \sum_{i=1}^{m} (x_i x_i^* - y_i^* y_i)_0 + \sum_{j=1}^{g} [a_j, a_j^*] + [b_j, b_j^*] = 0$$

$$\gamma_*(x, y, a, b) = \sum_{i=1}^{m} (x_i y_i)_0 + \sum_{j=1}^{g} [a_j, b_j] = 0$$

the quotient space

$$X^{g}_{\underline{R}^1, \underline{R}^2 \dots \underline{R}^n}(\alpha) = \frac{\mu_*^{-1}(0) \cap \gamma_*^{-1}(0)}{SU(R)}$$

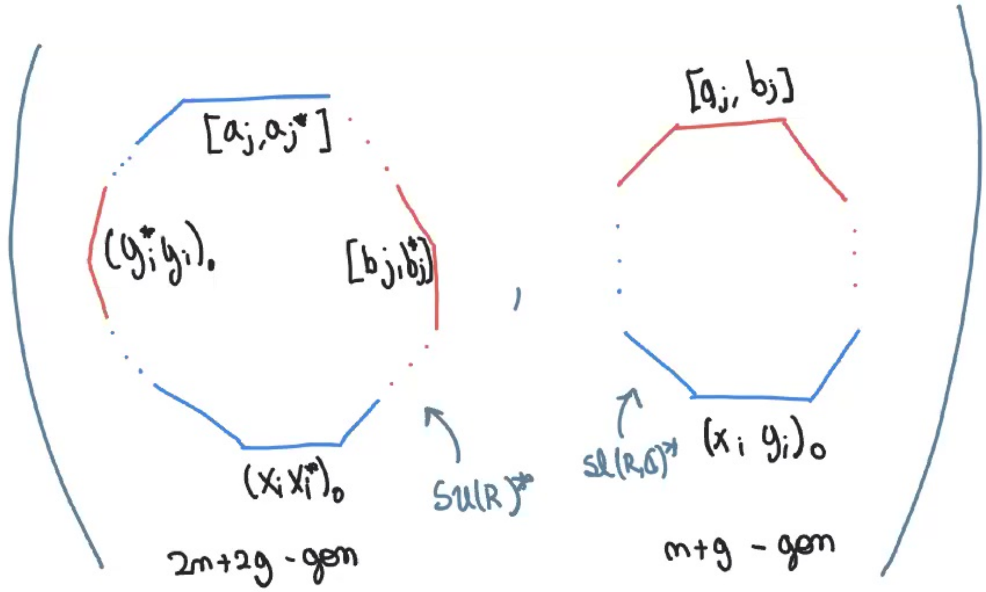
is a hyperkähler variety of complex dimension

$$2\left(\sum_{i=1}^{m} \dim \mathcal{F}_{\underline{R}^i} + (g-1)(R^2-1)\right)$$

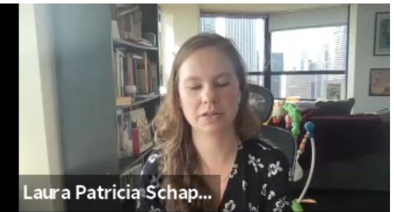
Eg. When all Flags are complete

$m R(R-1) + 2(g-1)(R^2-1)$

We can think of the objects in $X^{g}_{\underline{R}^1,\dots,\underline{R}^m}(\alpha)$ in the following way:



pairs of polygons with sides determined by the quantities in the moment maps.



The space $\mathcal{X}^{g}_{\underline{R}^1,\dots,\underline{R}^n}(\alpha)$ is the moduli space of (generalized) hyperpolygons of length α.

Hyperpolygons where first studied by Konno as a hyperkähler extension of the usual polygon space, and later by Harada-Proudfoot as a Nakajima quiver variety. They are closely related to Higgs bundles, as first seen by Godinho-Mandini for rank 2 and later extended to any rank by Fisher-Rayan.

With Steven Rayan we extended this to the setting of comet-shaped quivers, leading to the space of generalized hyperpolygons and we show they are also closely related to Higgs bundles and integrable systems.

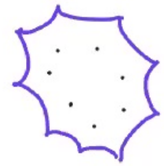
2. Generalized Hyperpolygons and Higgs Bundles

Nakajima quiver varieties can be seen as finite-dim analogs of Hitchin systems. Recall we have two moment map equations:

$$\mu_*(x,y,a,b)=\sum_{i=1}^{m}\left(x_i x_i^* - y_i^* y_i\right)_0 + \sum_{j=1}^{g}[a_j,a_j^*]+[b_j,b_j^*] = 0$$

$$\nu_*(x,y,a,b)=\sum_{i=1}^{m}\left(x_i y_i\right)_0 + \sum_{j=1}^{g}[a_j,b_j] = 0$$

We can associate a corresponding Higgs Field by considering a punctured surface:

Let $D=\sum_i z_i$ in a 4g-gon  $\subset \mathcal{H}$ ($\mathbb{C}$ if $g=0,1$}

Then, the Higgs Field can be defined as

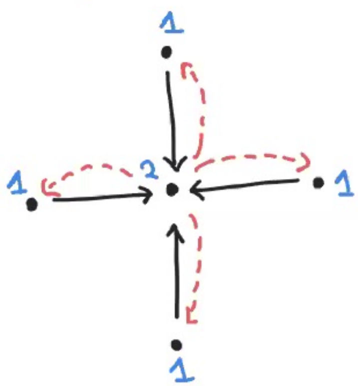
$$\phi(z)=\sum_{i=1}^{n}\frac{x_i\, y_i}{z-g_z(z_i)}\,dz$$

For the trivial rank R bundle on

$$X=\mathcal{H}/_{\Gamma} \qquad (\mathbb{C}/\Lambda \text{ for } g=1,\quad \mathbb{P}^1\ g=0)$$

Let's take a look at an example in the case of hyperpolygons

(not generalized $\Rightarrow$ no higher genus)



affine D_4 Dynkin diagram

each flag is of the form

$\underline{R}^i = (1,2) = [2]$

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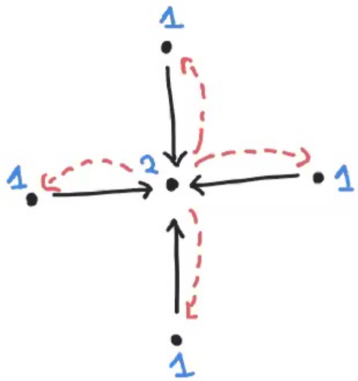

affine D_4 Dynkin diagram

each flag is of the form

$\underline{R}^i = (1,2) = [2]$

We consider $X_{[2],[2],[2],[2]}(\underline{\alpha})$

the Nakajima quiver variety for the affine Dynkin diagram D_4

- this is a K_3 surface with complete ALE metric
- Embeds into the Hitchin system on $\mathbb{P}^1 / \{z_1, z_2, z_3, z_4\}$. The associated Higgs bundles are parabolic of Rank 2 on $\mathbb{P}^1$ with 4 tame singularities.
- the embedding is not one of hyperkähler varieties: the difference between this hyperpolygon space and the parab. Higgs space is the Hitchin section.

3. INTEGRABLE SYSTEMS

Given the correspondence between Higgs bundles and (generalized) Hyperpolygons, we look into the appearance of integrable systems.

(back in '94 Nakajima mentioned that one would expect Nakajima quiver varieties to be completely integrable Hamiltonian systems)

Fisher-Rayan showed this is the case for

$$\mathcal{X}^{0}_{[1,R],\dots[1,R]}(\underline{\alpha}) \quad R \leq 3 \text{ and arbitrary } n$$

by embedding the space into certain space of tame parabolic Higgs bundles on the punctured sphere.

We can now extend this to generalized Hyperpolygons whose comets have complete or minimal arms.

Recall that to construct $\mathcal{X}^{g}_{\underline{R}^1,\dots,\underline{R}^n}(\alpha)$ we considered a quotient and a restriction via two moment maps.

$$\underbrace{T^*F_{\underline{R}^1}\times T^*F_{\underline{R}^2}\times\dots\times T^*F_{\underline{R}^n}}_{\text{GELFAND-TSETLIN-type}} \times \underbrace{T^*\mathfrak{sl}(r,\mathbb{C})^g}_{\text{LIE-POISSON-TYPE}}$$

all these have are hyperkähler varieties

$$\mathcal{X}^{g}_{\underline{R}^1,\underline{R}^2,\dots\underline{R}^n}(\underline{\alpha})$$

inherits its symplectic structure.

Do these integrable systems descend with the symplectic structure?

Recall that to construct $\mathcal{X}^{\mathfrak{g}}_{\underline{R}^1,\dots,\underline{R}^n}(\alpha)$ we considered a quotient and a restriction via two moment maps.

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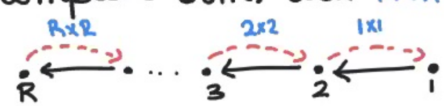
Do these integrable systems descend with the symplectic structure?

$$\mathcal{X}^{\mathfrak{g}}_{\underline{R}^1,\underline{R}^2,\dots,\underline{R}^n}(\underline{\alpha})$$

inherits its symplectic structure.

To study this, first assume c complete arms in the comet and $n-c$ minimal arms (this is, with tables $(1,R)$).

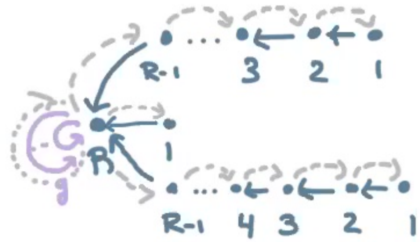
The complete arms each $k\times k$ block contributes k invariants.



Hence, we have the following invariants: (for c the number of complete ones in the comet

- $c(1+2+\ldots+(R-1)) = \dfrac{c(R-1)R}{2}$

natural invariants from c complete flag which come from the Gelfand-Tsetlin integrable systems



- $(m-c)(R-1)$ from the minimal flags; and
- $g(R^2-1)$ From the loops $b_j \in sl(R,\mathbb{C})$

For each R, m, g, c, the Hyperkähler reduction by $SU(R)$ fixes $N(R,m,g,c)$ of the invariants, and we show that

$$\frac{c(R-1)R}{2} + (m-c)(R-1) + g(R^2-1) - N(R,m,g,c) = \frac{1}{2}\dim \mathcal{X}^{g}_{\underline{R}^1,\ldots,\underline{R}^m}(\alpha)$$

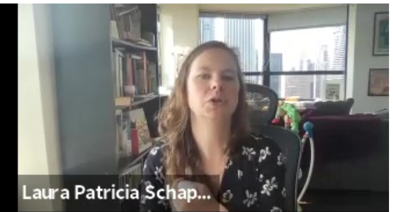
Theorem (w/ Rayan): the space $\mathcal{X}^{g}_{\underline{R}^1,\ldots,\underline{R}^m}(\alpha)$ is a completely integrable Hamiltonian system of Gelfand-Tsetlin Type.

In particular, this shows the existence of

sub-integrable systems

in meromorphic Hitchin systems that do not see the complex geometry of the algebraic curve.

Moreover, through the Hyperkähler structure, we may ask about the appearance of Lagrangians on complex holomorphic subspaces of those sub-integrable systems.



4. BRANES OF HYPERPOLYGONS

Recall that given a hyperkähler variety with quaternions I, J, K and corresponding symplectic forms $\omega_I, \omega_J, \omega_K$ we can consider compatibility of subvarieties with these structures.

A complex subvariety with respect to one of the complex structures is a B-Brane

A lagrangian subvariety with respect to one of the symplectic forms is an A-Brane

When considering the whole hyperkähler space, for the given ordering (I, J, K), one may then have branes of types

$(A, B, A) \qquad (A, A, B) \qquad (B, A, A) \qquad (B, B, B)$ } TRIPLE BRANES.

BRANES - QUESTIONS WE WANT TO ASK

$\mathcal{M}_{GC}$

BRANES - QUESTIONS WE WANT TO ASK

WHAT ARE THOSE FAMILIES OF BRANES IN TERMS OF REPRESENTATIONS?

$$\begin{tikzcd}
\mathrm{Hom}^+(\pi_1(\Sigma) \to G_{\mathbb{C}}) \arrow[d, phantom, "\wr"] & & \\
\mathcal{M}_{G_{\mathbb{C}}} \arrow[rr, leftrightarrow, "SYZ"] \arrow[dr, "h"'] & & \mathcal{M}_{{}^L G_{\mathbb{C}}} \arrow[dl, "{}^L h"] \\
& \mathcal{A}_{G_{\mathbb{C}}} & \\
& \mathcal{M}_? \arrow[u] &
\end{tikzcd}$$

HOW TO CONSTRUCT FAMILIES OF ALL TYPES OF BRANES?

WHAT ARE THE DUAL OBJECTS?

HOW DO THE BRANES LIE WITHIN THE HITCHIN FIBRATION?
NEW ABELIAN / NON-ABELIAN SPACES?

HOMOLOGICAL MIRROR SYMMETRY RELATES THE FUKAYA CATEGORY OF ONE SPACE (A BRANES) TO THE DERIVED CATEGORY OF COHERENT SHEAVES OF THE OTHER (B-BRANES)

HIGGS BUNDLES – THE HITCHIN FIBRATION

$$\begin{tikzcd} \mathcal{M}_{G_\mathbb{C}} \arrow[r] \arrow[d, leftrightarrow, "SYZ"'] & \mathcal{A}_{G_\mathbb{C}} \arrow[d, phantom, "\cong"] \\ \mathcal{M}_{^LG_\mathbb{C}} \arrow[r] & \mathcal{A}_{^LG_\mathbb{C}} \end{tikzcd}$$

Langlands dual group

$Sp(2n,\mathbb{C}) \sim SO(2n+1,\mathbb{C})$

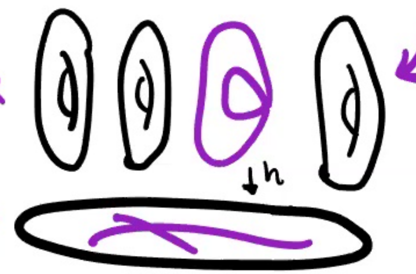
$SL(n,\mathbb{C}) \sim PGL(n,\mathbb{C})$

MIRROR SYMMETRY

DUALITY BETWEEN CALABI-YAU MANIFOLDS

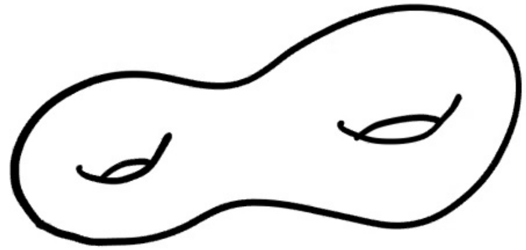
$\mathcal{M}_{G_\mathbb{C}}$ and $\mathcal{M}_{^LG_\mathbb{C}}$ are mirror partners and (generic) fibres are dual

WHAT ABOUT THE SINGULAR FIBRES?



- For Higgs bundles without punctures, in terms of sigma models first considered by Kapustin-Witten '06.
- Many new types of triple branes of Higgs bundles can be constructed via involutions (w/ Baraglia '13, '14) and finite group actions (w/ Heller '18, w/ Heller-Biswas '20).

BRANES – CONSTRUCTION OF BRANES

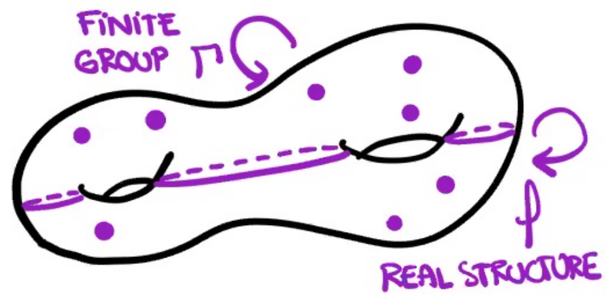
$G_{\mathbb{C}}$-Higgs bundle $\rightarrow$ 

(A, B, A) (B, B, B)

(B, A, A) (A, A, B)

BRANES – CONSTRUCTION OF BRANES

$G_{\mathbb{C}}$-Higgs bundle $\longrightarrow$



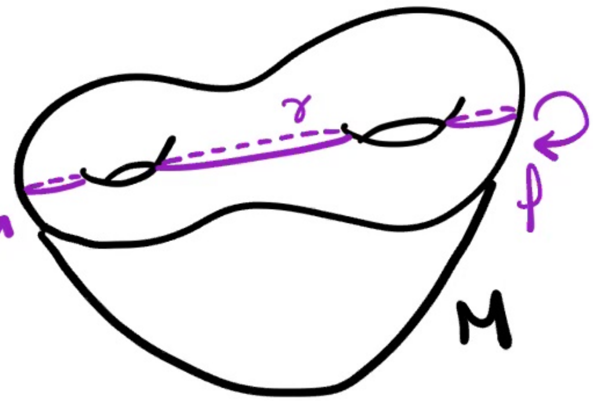
↺ τ REAL FORM, FIXING G

(A, B, A) (B, B, B)

(B, A, A) (A, A, B)

APPLICATIONS – (A,B,A)-BRANES

ASK WHICH REPRESENTATIONS EXTEND FROM $\pi_1(\Sigma)$ TO $\pi_1(M)$?

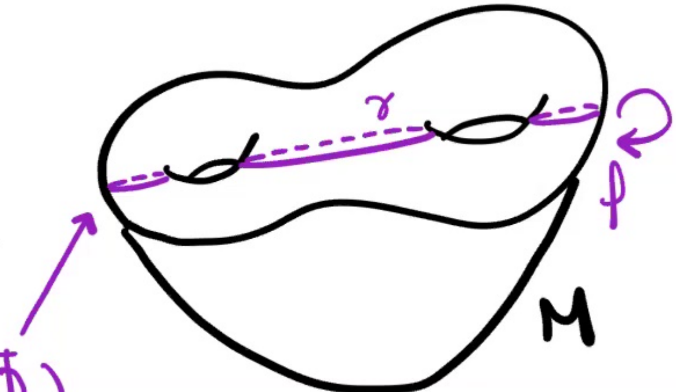


(E, Φ)

FOR EACH CIRCLE γ, THERE IS A CHOICE OF ± 1 FOR EACH DIM. OF $E|_\gamma$, DEFINING $[E] \in K_{\mathbb{Z}_2}(\Sigma)$

APPLICATIONS – (A,B,A)-BRANES

ASK WHICH REPRESENTATIONS EXTEND FROM $\pi_1(\Sigma)$ TO $\pi_1(M)$?



(E, Φ)

FOR EACH CIRCLE γ, THERE IS A CHOICE OF ± 1 FOR EACH DIM. OF $E|_\gamma$, DEFINING $[E] \in K_{\mathbb{Z}_2}(\Sigma)$

THEOREM (W/ BARAGLIA)

THE REPRESENTATIONS THAT EXTEND CORRESPOND TO HIGGS BUNDLES (E, Φ) IN THE (A,B,A)-BRANE FOR WHICH $[E] \in K_{\mathbb{Z}_2}(\Sigma)$ IS TRIVIAL.

- For Higgs bundles without punctures, in terms of sigma models, first considered by Kapustin - Witten '06.
- Many new types of triple branes of Higgs bundles can be constructed via involutions (w/ Baraglia '13, '14) and finite group actions (w/ Heller '18, w/ Heller-Biswas '20).

ASK: Which triple branes can be constructed in (generalized) hyperpolygon space? What are their dual spaces?

- Expanding on involutions, for $R=2$ $g=0$ branes were constructed as examples by Hoskins-Schaffhauser '19.

We can begin the study of triple branes of generalized hyperpolygons by considering involutions which <u>do not</u> satisfy the requirements of HS'19.

An Example

Consider the involution of $\mathcal{X}^{g}_{\underline{R}^1 \dots \underline{R}^n}(\underline{\alpha})$ which negates cotangent directions. This is:

$$\sigma : [x, y, a, b] \longmapsto [x, -y, a, -b]$$

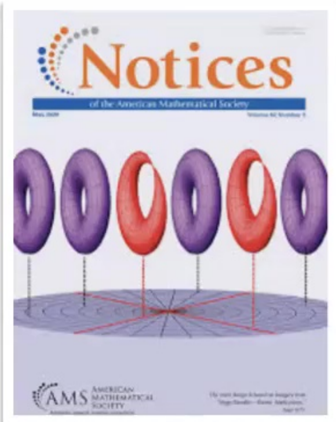
The fixed point set is given by hyperpolygons $[x, 0, a, 0]$ which is the generalized polygon space

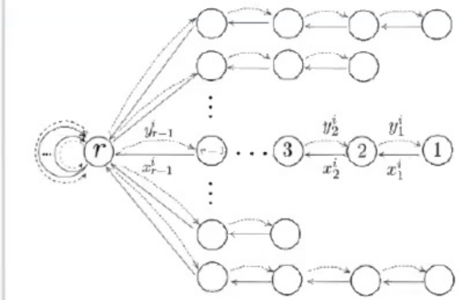
$$\mathcal{P}^{g}_{\underline{R}^1, \dots, \underline{R}^n}(\underline{\alpha})$$

We can show that this is a (B,A,A)-Brane inside the hyperpolygon space $\mathcal{X}^{g}_{\underline{R}^1, \dots \underline{R}^n}(\underline{\alpha})$.

In our current project, we are building novel types of generalized hyperpolygon branes, considering their mirror objects as well as other dualities.

SOME OF MY RECENT PROJECTS ON HIGGS BUNDLES AND HITCHIN SYSTEMS...





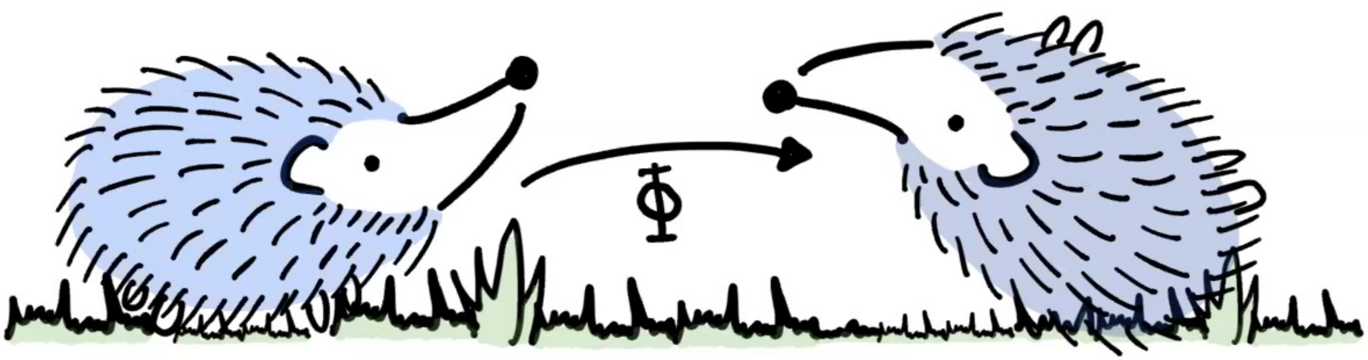
Snapshots of modern mathematics
from Oberwolfach

№ 8/2020

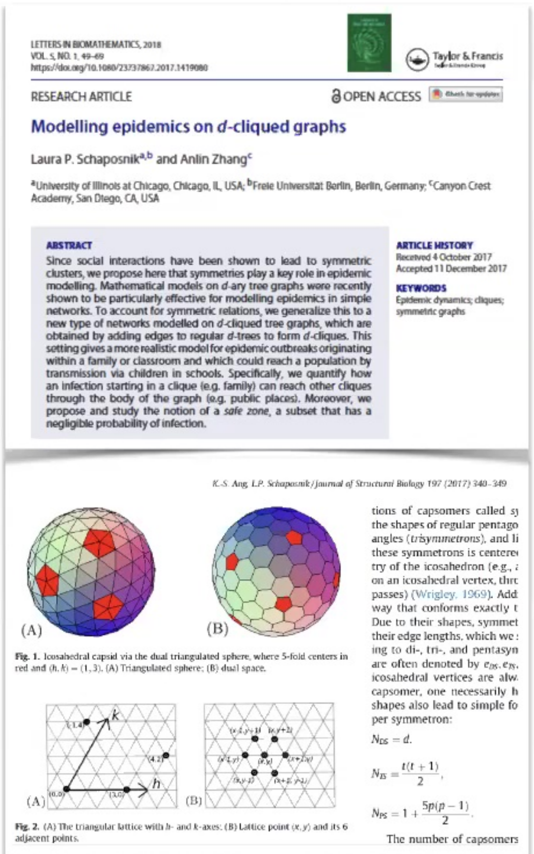
Higgs bundles without geometry

Steven Rayan [1] • Laura P. Schaposnik [2]

Higgs bundles appeared a few decades ago as solutions to certain equations from physics and have attracted much attention in geometry as well as other areas of mathematics and physics. Here, we take a very informal stroll through some aspects of linear algebra that anticipate the deeper structure in the moduli space of Higgs bundles.



SOME OF MY PROJECTS WITH STUDENTS...



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RESEARCH ARTICLE

OPEN ACCESS

Modelling epidemics on *d*-cliqued graphs

Laura P. Schaposnik[a,b] and Anlin Zhang[c]

[a]University of Illinois at Chicago, Chicago, IL, USA; [b]Freie Universität Berlin, Berlin, Germany; [c]Canyon Crest Academy, San Diego, CA, USA

ABSTRACT

Since social interactions have been shown to lead to symmetric clusters, we propose here that symmetries play a key role in epidemic modelling. Mathematical models on *d*-ary tree graphs were recently shown to be particularly effective for modelling epidemics in simple networks. To account for symmetric relations, we generalize this to a new type of networks modelled on *d*-cliqued tree graphs, which are obtained by adding edges to regular *d*-trees to form *d*-cliques. This setting gives a more realistic model for epidemic outbreaks originating within a family or classroom and which could reach a population by transmission via children in schools. Specifically, we quantify how an infection starting in a clique (e.g. family) can reach other cliques through the body of the graph (e.g. public places). Moreover, we propose and study the notion of a *safe zone*, a subset that has a negligible probability of infection.

ARTICLE HISTORY
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Accepted 11 December 2017

KEYWORDS
Epidemic dynamics; cliques; symmetric graphs

K.-S. Ang, L.P. Schaposnik / Journal of Structural Biology 197 (2017) 340–349

Fig. 1. Icosahedral capsid via the dual triangulated sphere, where 5-fold centers in red and $(h,k) = (1,3)$. (A) Triangulated sphere; (B) dual space.

Fig. 2. (A) The triangular lattice with h- and k-axes; (B) Lattice point (x,y) and its 6 adjacent points.

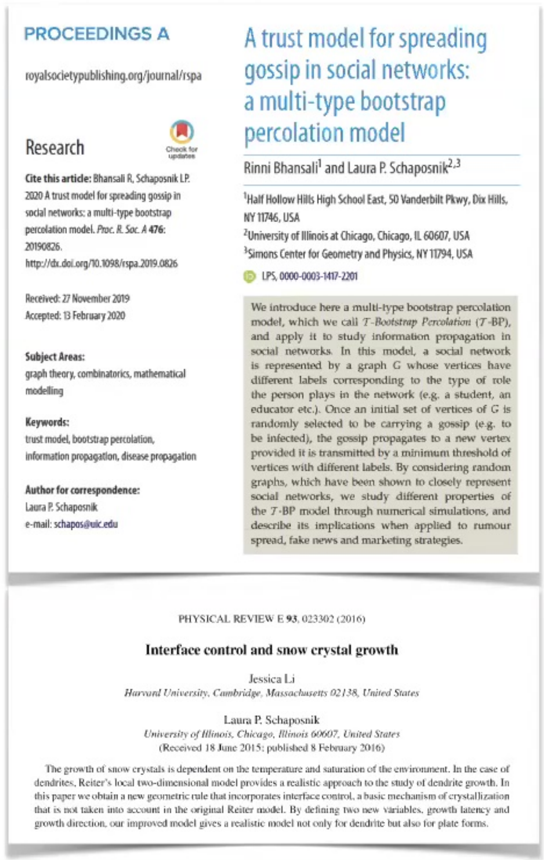
tions of capsomers called sy
the shapes of regular pentago
angles (*trisymmetrons*), and li
these symmetrons is centere
try of the icosahedron (e.g., a
on an icosahedral vertex, thre
passes) (Wrigley, 1969). Add
way that conforms exactly t
Due to their shapes, symmet
their edge lengths, which we
ing to di-, tri-, and pentasym
are often denoted by $e_{DS}, e_{TS},$
icosahedral vertices are alw
capsomer, one necessarily h
shapes also lead to simple fo
per symmetron:

$$N_{DS} = d,$$

$$N_{TS} = \frac{t(t+1)}{2},$$

$$N_{PS} = 1 + \frac{5p(p-1)}{2}.$$

The number of capsomers



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Research

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http://dx.doi.org/10.1098/rspa.2019.0826

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Subject Areas:
graph theory, combinatorics, mathematical modelling

Keywords:
trust model, bootstrap percolation, information propagation, disease propagation

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A trust model for spreading gossip in social networks: a multi-type bootstrap percolation model

Rinni Bhansali[1] and Laura P. Schaposnik[2,3]

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[2]University of Illinois at Chicago, Chicago, IL 60607, USA
[3]Simons Center for Geometry and Physics, NY 11794, USA

LPS, 0000-0003-1417-2201

We introduce here a multi-type bootstrap percolation model, which we call *T-Bootstrap Percolation* (*T*-BP), and apply it to study information propagation in social networks. In this model, a social network is represented by a graph G whose vertices have different labels corresponding to the type of role the person plays in the network (e.g. a student, an educator etc.). Once an initial set of vertices of G is randomly selected to be carrying a gossip (e.g. to be infected), the gossip propagates to a new vertex provided it is transmitted by a minimum threshold of vertices with different labels. By considering random graphs, which have been shown to closely represent social networks, we study different properties of the *T*-BP model through numerical simulations, and describe its implications when applied to rumour spread, fake news and marketing strategies.

PHYSICAL REVIEW E **93**, 023302 (2016)

Interface control and snow crystal growth

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Harvard University, Cambridge, Massachusetts 02138, United States

Laura P. Schaposnik
University of Illinois, Chicago, Illinois 60607, United States
(Received 18 June 2015; published 8 February 2016)

The growth of snow crystals is dependent on the temperature and saturation of the environment. In the case of dendrites, Reiter's local two-dimensional model provides a realistic approach to the study of dendrite growth. In this paper we obtain a new geometric rule that incorporates interface control, a basic mechanism of crystallization that is not taken into account in the original Reiter model. By defining two new variables, growth latency and growth direction, our improved model gives a realistic model not only for dendrite but also for plate forms.

SOME OF MY PANDEMIC PROJECTS WITH STUDENTS...

PHYSICAL REVIEW RESEARCH **2**, 033350 (2020)

Extrapolating continuous color emotions through deep learning

Vishaal Ram,[1] Laura P. Schaposnik,[2,*] Nikos Konstantinou,[3] Eliz Volkan,[4] Marietta Papadatou-Pastou,[5] Banu Manav,[6] Domicele Jonauskaite,[7] and Christine Mohr[7]

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[2] *Department of Mathematics, Statistics and Computer Science, University of Illinois, Chicago, Illinois 60607, USA*
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[6] *Kadir Has University, Faculty of Art and Design, Department of Interior Architecture and Environmental Design, Kadir Has Caddesi 34083 Cibali-İstanbul*
[7] *Institute of Psychology, University of Lausanne, Lausanne 1015, Switzerland*

(Received 6 June 2020; accepted 29 July 2020; published 2 September 2020)

By means of an experimental dataset, we use deep learning to implement an RGB (red, green, and blue) extrapolation of emotions associated to color, and do a mathematical study of the results obtained through this neural network. In particular, we see that males (type-*m* individuals) typically associate a given emotion with darker colors, while females (type-*f* individuals) associate it with brighter colors. A similar trend was observed with older people and associations to lighter colors. Moreover, through our classification matrix, we identify which colors have weak associations to emotions and which colors are typically confused with other colors.

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A modified age-structured SIR model for COVID-19 type viruses

Vishaal Ram[1] & Laura P. Schaposnik[2]

We present a modified age-structured SIR model based on known patterns of social contact and distancing measures within Washington, USA. We find that population age-distribution has a significant effect on disease spread and mortality rate, and contribute to the efficacy of age-specific contact and treatment measures. We consider the effect of relaxing restrictions across less vulnerable age-brackets, comparing results across selected groups of varying population parameters. Moreover, we analyze the mitigating effects of vaccinations and examine the effectiveness of age-targeted distributions. Lastly, we explore how our model can applied to other states to reflect social-distancing policy based on different parameters and metrics.

Cell fusion through slime mold network dynamics

Sheryl Hsu[a] and Laura P. Schaposnik[*,b]
(⋆) *Corresponding author: schapos@uic.edu*

Physarum Polycephalum is a unicellular slime mold that has been intensely studied due to its ability to solve mazes, find shortest paths, generate Steiner trees, share knowledge, remember past events, and the implied applications to unconventional computing. The CELL model is a unicellular automaton introduced in [4] that models *Physarum*'s amoeboid motion, tentacle formation, maze solving, and network creation. In the present paper, we extend the CELL model by spawning multiple CELLs, allowing us to understand the interactions between multiple cells, and in particular, their mobility, merge speed, and cytoplasm mixing. We conclude the paper with some notes about applications of our work to modeling the rise of present day civilization from the early nomadic humans and the spread of trends and information around the world. Our study of the interactions of this unicellular organism should further the understanding of how *Physarum Polycephalum* communicates and shares information.

Keywords: Cell fusion, network dynamics, slime mold

The Power of Many: A *Physarum* Swarm Steiner Tree Algorithm

Sheryl Hsu [a], Fidel I. Schaposnik Massolo[b] and Laura P. Schaposnik[*,c]
(⋆) *Corresponding author: schapos@uic.edu*

We create the first *Physarum swarm algorithm*, which can be used to solve the Euclidean Steiner tree problem. *Physarum* is a unicellular slime mold with the ability to form networks and fuse with other *Physarum* organisms. We use the simplicity and fusion of *Physarum* to create large swarms which independently operate to solve the Steiner problem. The *Physarum* Steiner tree algorithm then utilizes a swarm of *Physarum* organisms which gradually find terminals and fuse with each other, sharing intelligence. The algorithm is also highly capable of solving the obstacle avoidance Steiner tree problem and is a strong alternative to the current leading algorithm. The algorithm is of particular interest due to its novel approach, time complexity, rectilinear properties, and ability to run on varying shapes and topological surfaces.

Keywords: Steiner tree, slime mold, cell fusion, networks, swam algorithm, obstacle avoidance

SOME OF MY PROJECTS IN OTHER TOPICS...

Working from Home. ~~2 Months~~ 4 Months and Still Counting…

Alessandra Frabetti (Université de Lyon, Villeurbanne, France), Vladimir Salnikov (La Rochelle Université, France) and Laura Schaposnik (University of Illinois, Chicago, USA)

Dear friends and colleagues,

These days most of us are working from home and this seems to be a unique experience. We thought of writing an article about this topic for the Newsletter of the European Mathematical Society.

So a couple of questions for you:

One way or another, some of us replace school teachers for our kids from kindergarten through to university (tough job, right?) You have probably invented some nice recipes for how to explain mathematics and science in general to non-specialists that you are willing to share. Or you have found some resources that you have used successfully and would like to advertise. Please do!

The other way around: your nearest and dearest can now see you working. Maybe you have finally taken the time to explain what your job really is? What kind of science you do and why. Maybe even convinced someone that watching a seminar on your TV in the living room is more interesting than cartoons? Or just some interesting discussion happened unexpectedly? Or even better, a funny episode? Please share!

Impatiently waiting for your stories.
Yours, Vladimir

My letter, 23/04/2020 (Vladimir Salnikov)

This is the e-mail that I wrote to a rather large list of people to whom I usually make scientific announcements. It sort of speaks for itself: I realised that we were (and still are) crossing some historic period, and I was curious about how other people are handling it.

I received several replies, from which we chose some that people were ready to share with a broader audience, hence this article.

Actually, I received a lot of material, so we will continue in the next issue of the EMS Newsletter. And since we now have no idea how the situation will evolve, I will be attaching dates to be able to fit the stories into the context. In this issue we will address mostly the family aspect of the story, and the second part will be about everything "online". I do hope it will not become a regular section, though. And I will start with my own short anecdote.

My son Miron has just turned 4 years old, and he actually motivated me to write the above letter by showing me how I work. He glued together two sheets of A4 paper, drew some sort of rectangle on one of them and small rectangles on the other one. You have guessed it: he said it was his laptop. Then the following happened (I just watched, with no interference). He sat on the sofa in the living room, opened his "laptop" and said, "I am going to work now". He pressed the "buttons" for a couple of minutes, then said "No, it is too noisy here", he closed the "laptop" and went to his bedroom. He closed the door and stayed there for about 3 minutes (I do not know what he was doing, but from the sound I guess he was still typing). He came back, his "laptop" under his arm and said, "I was working, but now I am finished", and put it aside. Well, I recognised myself … did you?

Laura Schaposnik, letter from 28/04/2020

Being pregnant and thus at high risk, we started self-isolating before any official advice was given here in Chicago. Hence, now being in week 6 of house confinement, we've developed some favourite ways in which we're teaching our 20 month-old son about numbers and letters.

Our day starts at 7 am, and after some light reading we offer little Nikolay some "pasas de uva": that's when he runs to the table, knowing there will be 5 raisins ready for him, displayed in a line which will get shorter as he eats. Each time one is gone, we count backwards… by now, the little habit has grown into him tidying all his food into lines and making as if he were counting the objects. Maybe he's counting them in his head – the sounds he makes, albeit not English or Spanish numbers, are certainly precise and get repeated every time he is counting!

Whilst most of our books are not about counting – in fact, we don't have any about it! – we have turned most of them into letters and counting games… this is how

BRILL Behaviour (2018) DOI:10.1163/1568539X-00003496 Behaviour brill.com/beh

The phone walkers: a study of human dependence on inactive mobile devices

Laura P. Schaposnik * and James Unwin
University of Illinois at Chicago, Chicago, IL 60647, USA
*Corresponding author's e-mail address: schapos@uic.edu

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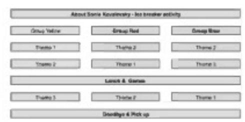
Sonia Kovalevsky Days: The Potential to Inspire

Laura P. Schaposnik and James Unwin, University of Illinois at Chicago

Sonia Kovalevsky overcame adversity to become the first woman to receive a PhD in mathematics and has since become a role model for young women interested in math and science. AWM members have organized Sonia Kovalevsky (SK) Days at colleges and universities throughout the country for almost three decades: the days typically consist of a program of workshops, talks, and problem solving sessions designed for students aged 7–17. The activities of the SK Days are intended to encourage young women to continue their studies in mathematics and to help them learn about educational possibilities.

With some guidance, organizing these events can be done very smoothly, and the whole experience can be extremely rewarding not only for the students, but also for the organizers and volunteer helpers. Indeed, witnessing the enthusiasm of the students for learning new mathematics is priceless. In what follows we would like to give some advice on how to make these events successful and encourage colleagues to host similar series of events at their institutions.

Most of what we will explain here is based on our experience running the SK Days at the University of Illinois experience has shown that groups of about 12–15 people are best, with one volunteer for every 5 or 6 students. Hence, depending on the number of girls registered, one can have 1, 2 or 3 parallel sessions.

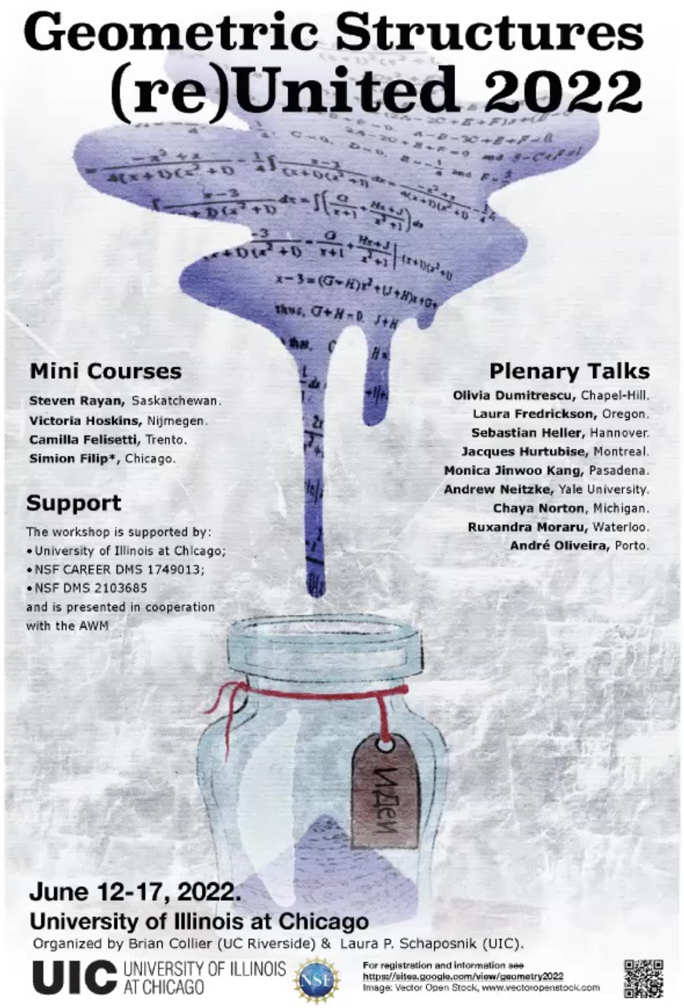


After a few years of modifying the schedule, it seems that sessions of 45 minutes work best, with 5 to 10 minutes breaks in between, and an hour lunch break (typically pizza!). We've usually run the program on Saturdays from 10:30 a.m. to 3 p.m., to accommodate families coming from outer Chicago; in a smaller city, the program could be longer.

Before the event. There are five main tasks to take care of before the event, which can be done as little as a month in advance. In order of priority these are:

I. Set the date and book rooms.

The most successful days will be Saturdays that don't coincide with holidays or school activities. Contacting a few



Geometric Structures (re)United 2022
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Victoria Hoskins, Nijmegen.
Camilla Felisetti, Trento.
Simion Filip*, Chicago.
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and is presented in cooperation with the AWM
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Laura Patricia Schap...