Title: Many-Body Quantum Chaos and Spectral Form Factor

Speakers: Amos Chan

Series: Quantum Matter

Date: October 26, 2021 - 3:30 PM

URL: https://pirsa.org/21100008

Abstract: The study of spectral statistics is of importance due to its universality and utility as a robust diagnostic of quantum chaos. For closed many-body quantum chaotic systems, I will present two results: (i) a quantum-classical mapping that connects the Thouless time, which characterizes the onset of RMT of the spectral form factor (SFF); and the spectral gap of a dual classical stochastic system; (ii) a set of Lyapunov exponents which characterize the spectral statistics in the thermodynamic limit. For open quantum systems with complex spectra, I will propose and analyze a generalized SFF, and show that dissipative quantum chaotic systems display a "dip-ramp-plateau" behaviour with a quadratic ramp.

Many-body Quantum Chaos & Spectral Form Factor

Amos Chan
Princeton Center for Theoretical Science

Perimeter Institute
October, 2021

Overview

Goal: To find universal signatures of many-body quantum chaos

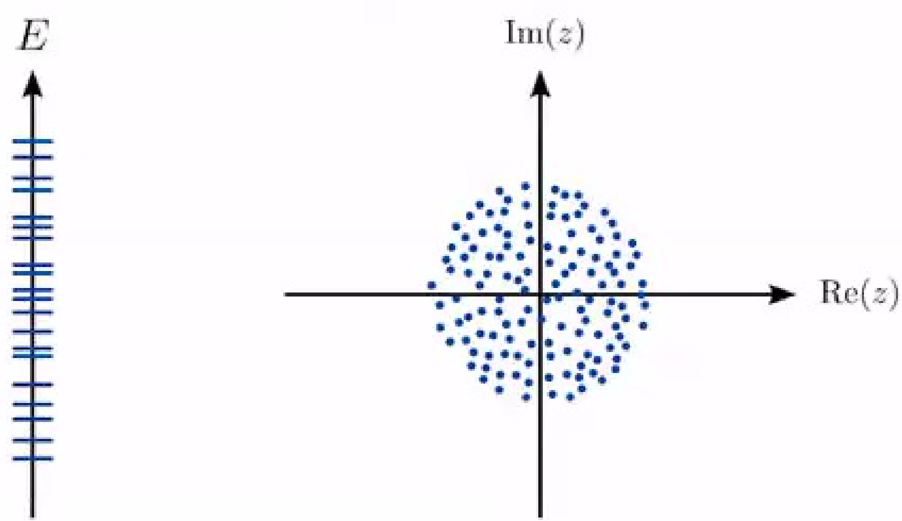
Part I: Closed MBQC systems & SFF

[AC, De Luca, Chalker, PRX '18]
[AC, De Luca, Chalker, PRL '18]
[Moudgalya, Prem, Huse, AC, PRR '21]
[AC, Shivam, Huse, De Luca, 2109.04475]
and more upcoming works

Part II: Open MBQC systems & SFF

[Li, Prosen, AC, PRL '21]
[Li, Yan, Prosen, AC, forthcoming]

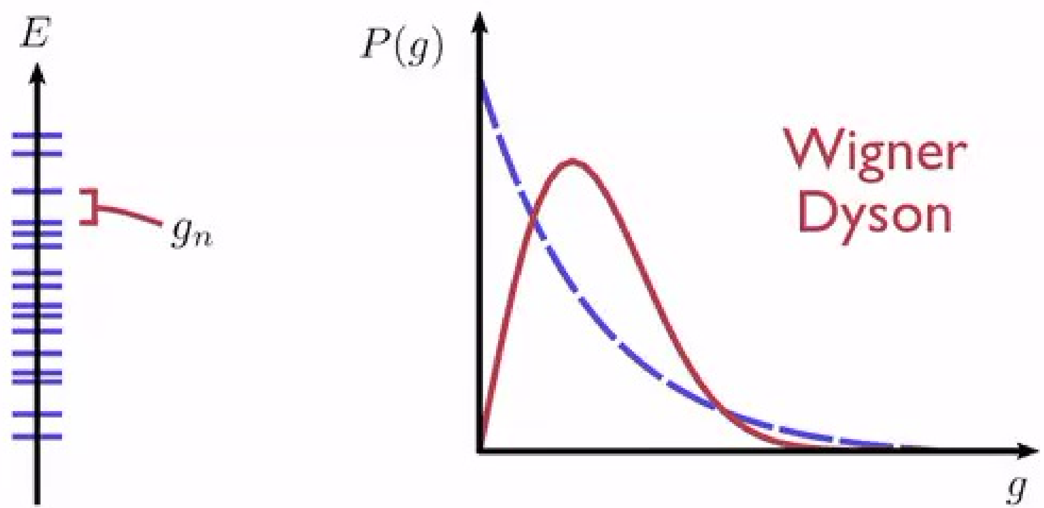
Spectral Statistics



Hermitian Non-Hermitian

Q: Universal signatures from eigenlevel correlation in generic quantum many-body systems?

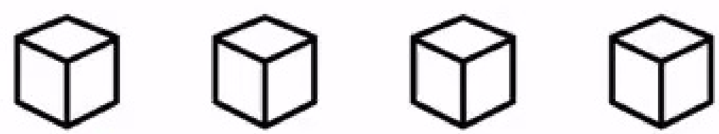
Quantum Chaos



$$\boxed{\text{BGS Conjecture: } P_{\rm sys}(g) \approx P_{\rm RMT}(g) \implies \text{Quantum chaos}}$$

[Wigner, '55, Dyson, '62]
[Bohigas, Giannoni, Schmit, '84]

Random matrix theory

Ensemble 

$H \in \mathrm{GUE}$ or $U \in \mathrm{CUE}$

- Universality $\Rightarrow$ BGS conjecture
- Analytical tractability

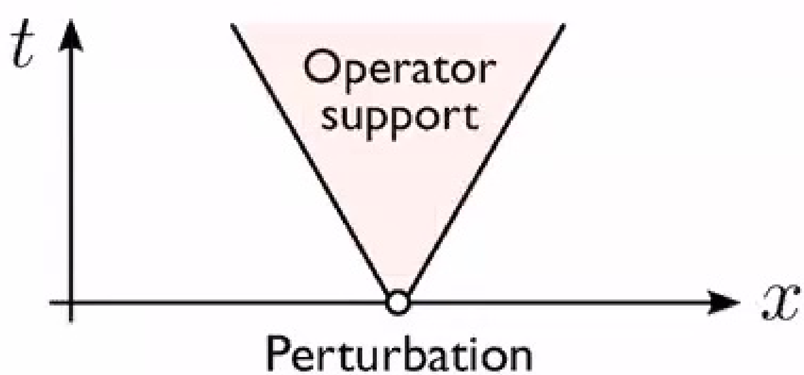
[Wigner, '55, Dyson, '62]
[Bohigas, Giannoni, Schmit, '84]

Many-Body Quantum Chaos?

- RMT: No preferred basis

No locality

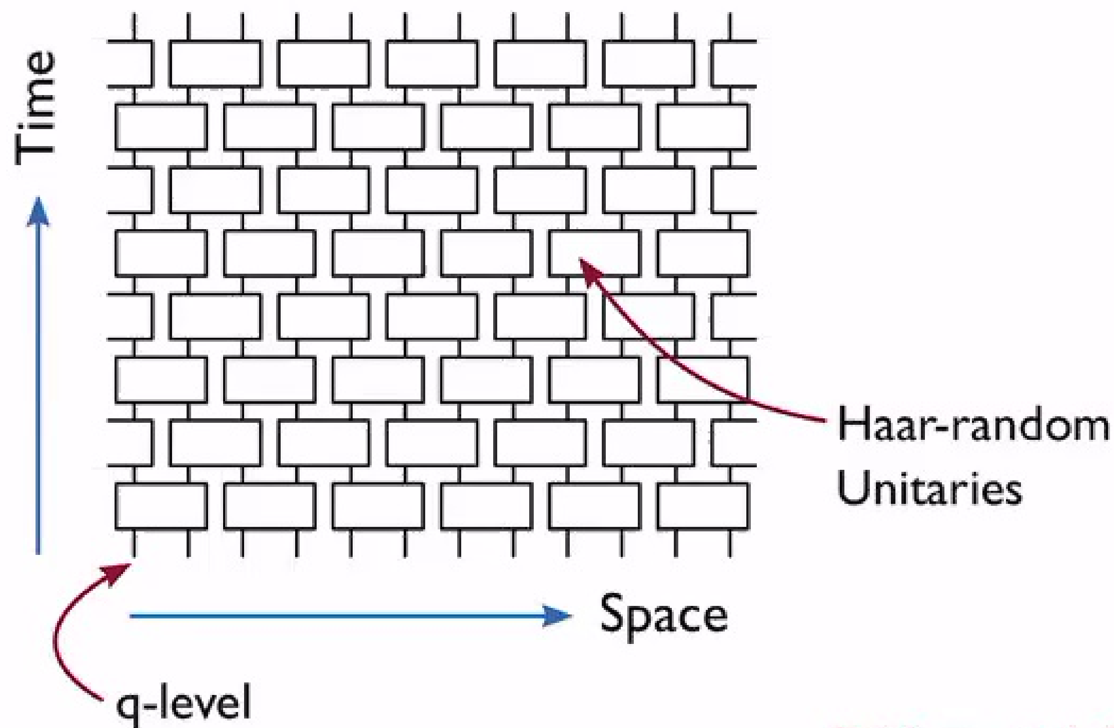
- MBQ: Lieb-Robinson bound



Locality

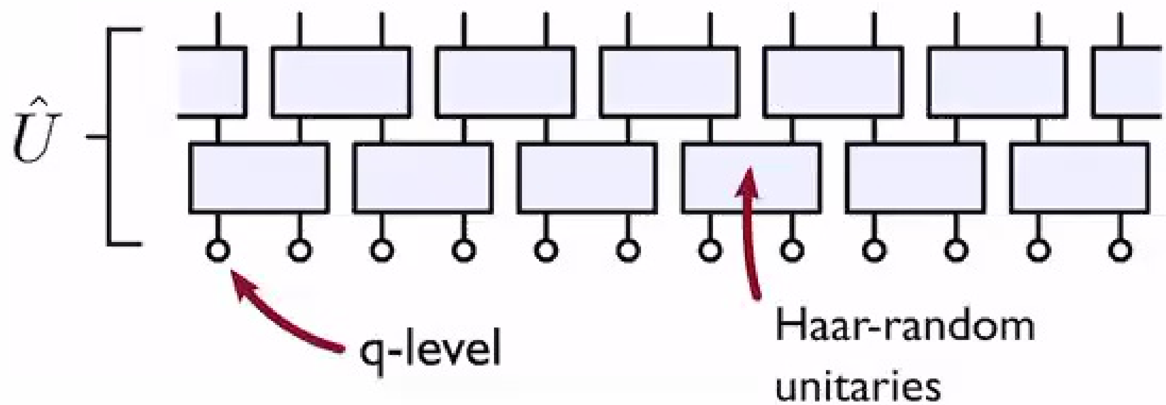
[Lieb, Robinson, '72]

$$\boxed{\text{Random Quantum Circuits} \sim \text{RMT} \times \text{locality}}$$



[Nahum et al. '17]

Floquet RQC

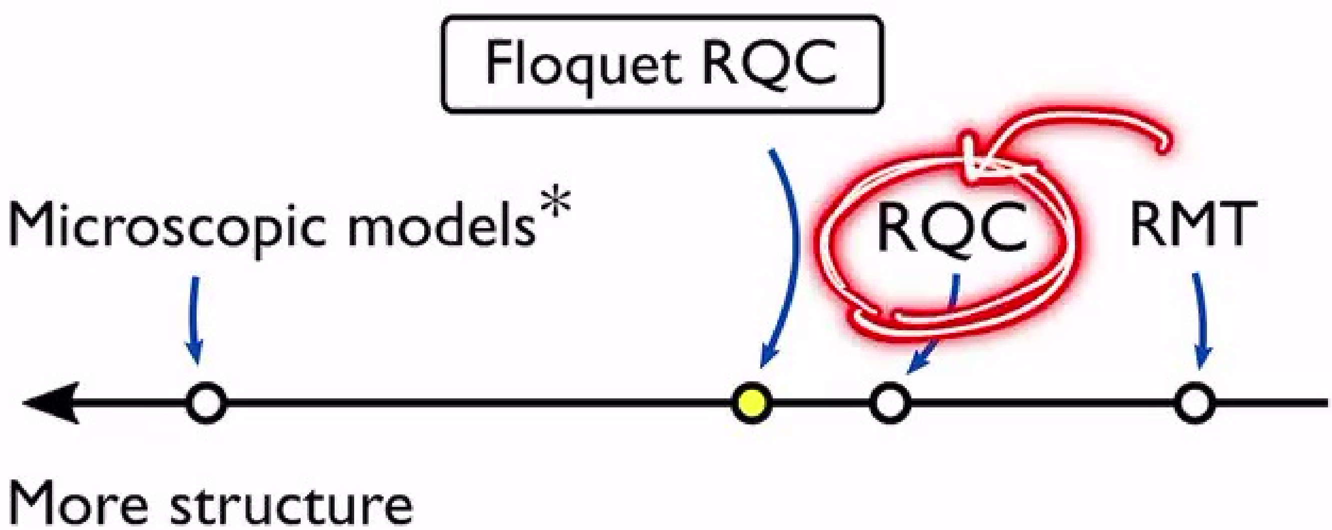


$$\hat{U} = \hat{U}_2 \hat{U}_1 \; , \qquad \hat{U}_1 = \bigotimes_{i \in \mathrm{odd}} u_{i,i+1} \; , \qquad \hat{U}_2 = \bigotimes_{i \in \mathrm{even}} u_{i,i+1}$$

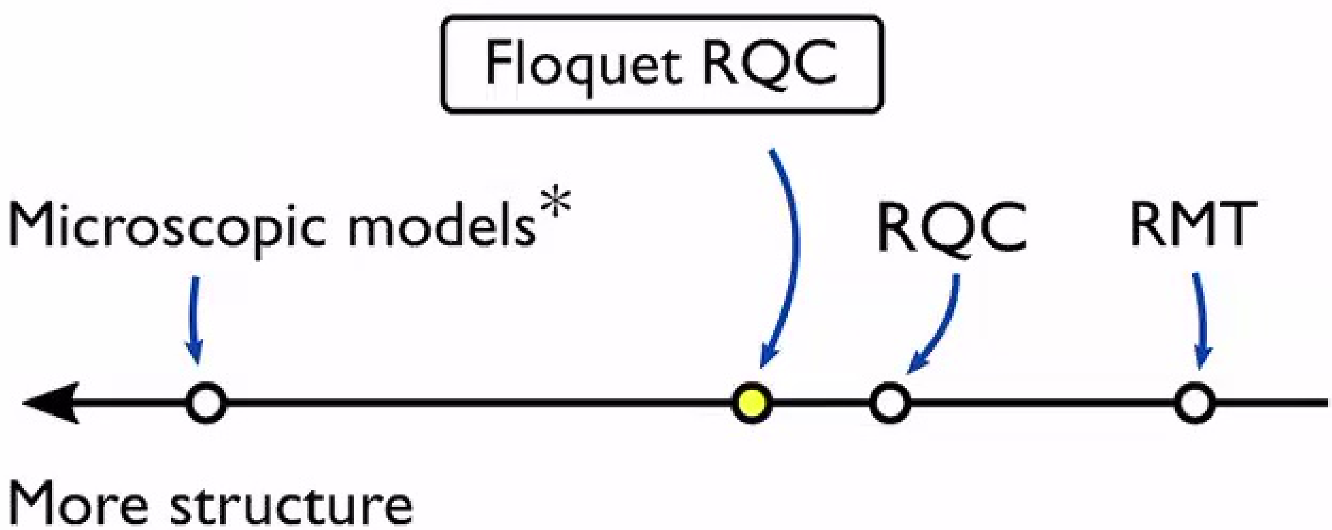
$\boxed{\text{Floquet operator } \hat{U} \implies \text{Spectral statistics}}$

[AC, De Luca, Chalker '18]

Theory Space



Theory Space



Spectral Form Factor

$$R(\omega) = \langle \rho(E)\rho(E+\omega)\rangle$$

eigenvalues

F.T. $\Updownarrow$ d.o.s. $\rho(E) = \frac{1}{N}\sum_n \delta(E - E_n)$

$$\boxed{K(t) = \sum_{m,n} \left\langle e^{i(E_m - E_n)t} \right\rangle = \left\langle \mathrm{Tr}[U(t)]\mathrm{Tr}[U^\dagger(t)] \right\rangle}$$

SFF in HEP
Shenker et al.
SYK

JHEP

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Black holes and random matrices

Jordan S. Cotler,[a] Guy Gur-Ari,[a] Masanori Hanada,[a,b,c] Joseph Polchinski,[d,e] Phil Saad,[a] Stephen H. Shenker,[a] Douglas Stanford,[f] Alexandre Streicher[a,d] and Masaki Tezuka[g]

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JHEP

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Accepted: July 13, 2018
Published: July 18, 2018

Onset of random matrix behavior in scrambling systems

Hrant Gharibyan,[a] Masanori Hanada,[a,b,c,d] Stephen H. Shenker[a] and Masaki Tezuka[e]

[a]Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA 94305, U.S.A.

A semiclassical ramp in SYK and in gravity

Phil Saad,[a] Stephen H. Shenker,[a] and Douglas Stanford[b]

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AC, De Luca, Chalker
Floquet RQC

Bertini, Kos, Ljubotina, Prosen
Dual unitary QC

PHYSICAL REVIEW X 8, 041019 (2018)

Solution of a Minimal Model for Many-Body Quantum Chaos

Amos Chan, Andrea De Luca, and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom

(Received 20 December 2017; published 8 November 2018)

PHYSICAL REVIEW LETTERS 121, 060601 (2018)

Spectral Statistics in Spatially Extended Chaotic Quantum Many-Body Systems

Amos Chan, Andrea De Luca, and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom

(Received 6 April 2018; published 7 August 2018)

PHYSICAL REVIEW X 8, 021062 (2018)

Many-Body Quantum Chaos: Analytic Connection to Random Matrix Theory

Pavel Kos, Marko Ljubotina, and Tomaž Prosen*
Physics Department, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

(Received 5 February 2018; revised manuscript received 12 April 2018; published 8 June 2018)

PHYSICAL REVIEW LETTERS 121, 264101 (2018)

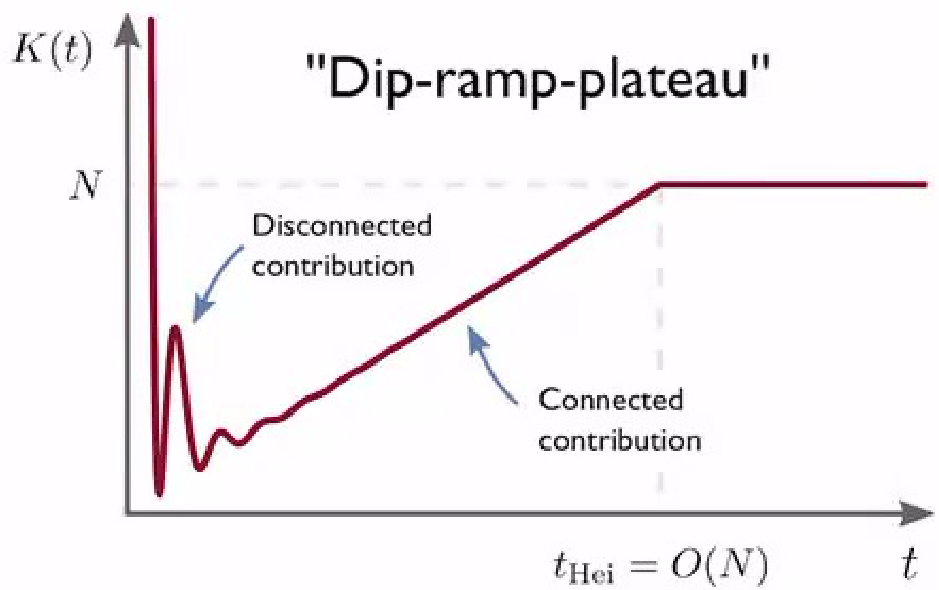
Exact Spectral Form Factor in a Minimal Model of Many-Body Quantum Chaos

Bruno Bertini, Pavel Kos, and Tomaž Prosen
Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

(Received 22 May 2018; published 27 December 2018)

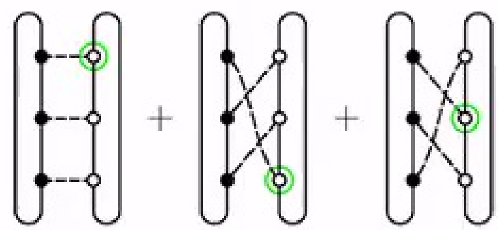
SFF for GUE

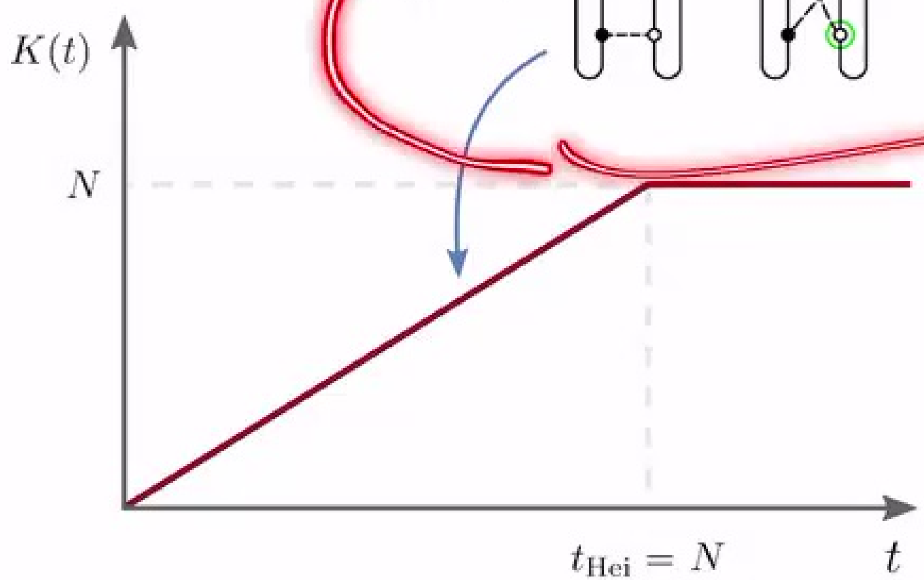
$$K(t) = \left\langle \sum_{m,n} e^{-i(E_m - E_n)t} \right\rangle$$



$$K_{\mathrm{GUE}}(t) = \begin{cases} N^2 \dfrac{J_1(2t)^2}{t^2} + \dfrac{t}{2}, & t \leq 2N \\ N^2 \dfrac{J_1(2t)^2}{t^2} + N, & t > 2N \end{cases}$$

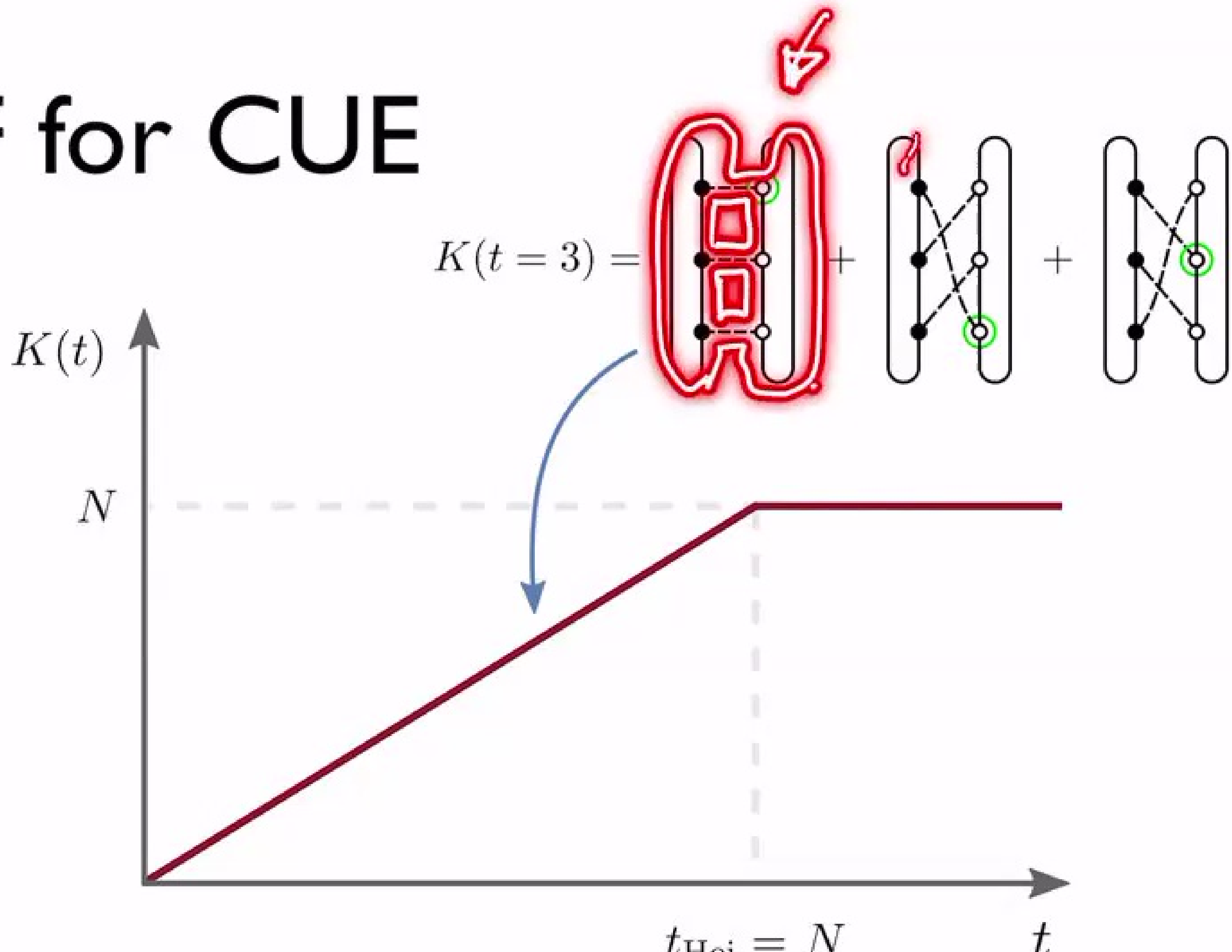
SFF for CUE

$K(t=3) =$ 

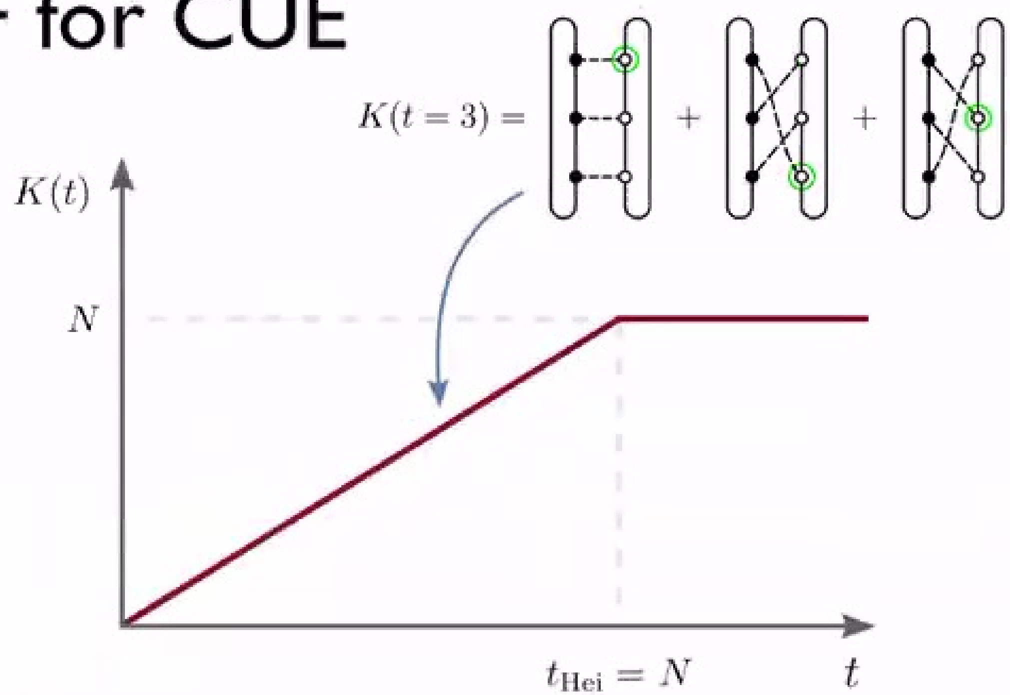


$$K(t) = \langle \mathrm{Tr}[U(t)]\mathrm{Tr}[U^\dagger(t)] \rangle = t \, , \qquad t \leq t_{\mathrm{Hei}}$$

SFF for CUE

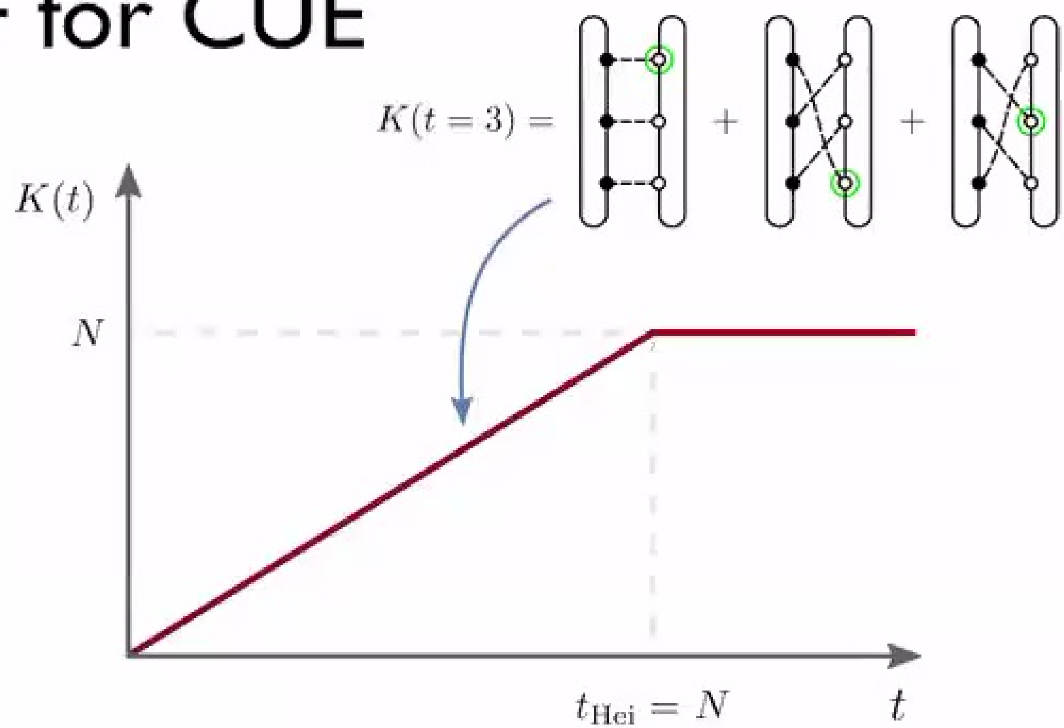


SFF for CUE



$$\boxed{K(t) = \langle \mathrm{Tr}[U(t)]\mathrm{Tr}[U^\dagger(t)] \rangle = t\ , \qquad t \leq t_{\mathrm{Hei}}}$$

SFF for CUE

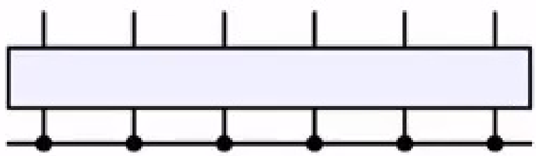


$$\boxed{K(t) = \langle \mathrm{Tr}[U(t)]\mathrm{Tr}[U^\dagger(t)] \rangle = t \,, \qquad t \le t_{\mathrm{Hei}}}$$

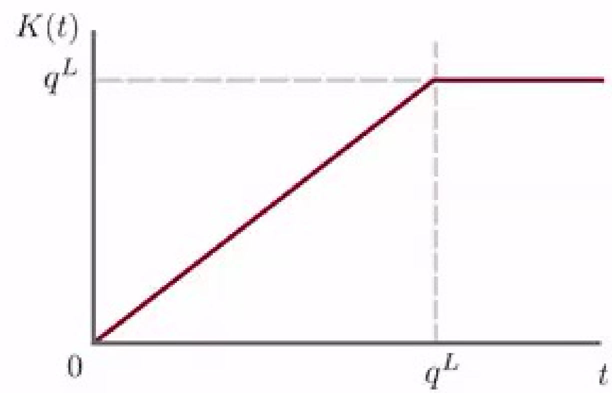
Regimes

$$K(t) = \sum_{m,n} \langle e^{i(\theta_m - \theta_n)t} \rangle$$

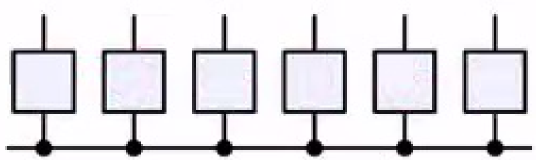
- Fully-coupled system:



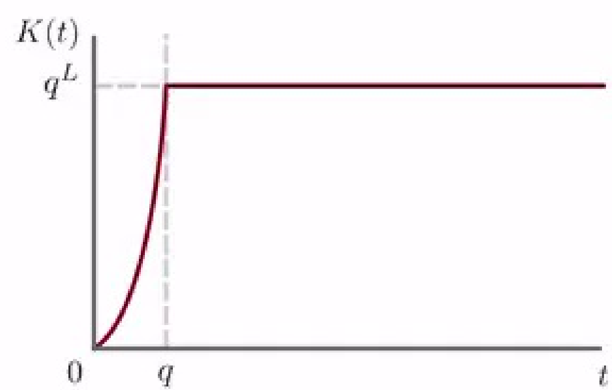
$$K(t) = t, \quad t \leq q^L$$



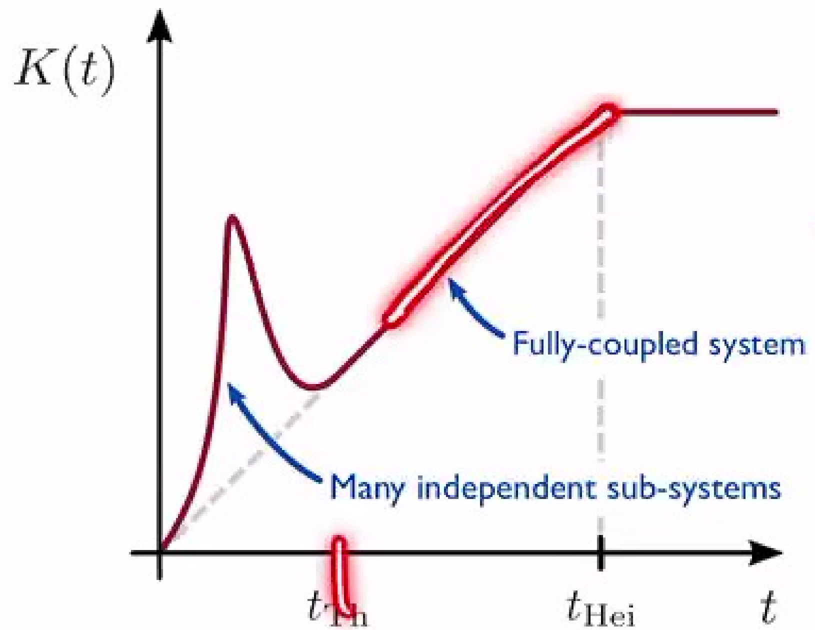
- Un-coupled subsystems:



$$K(t) = t^L, \quad t \leq q$$

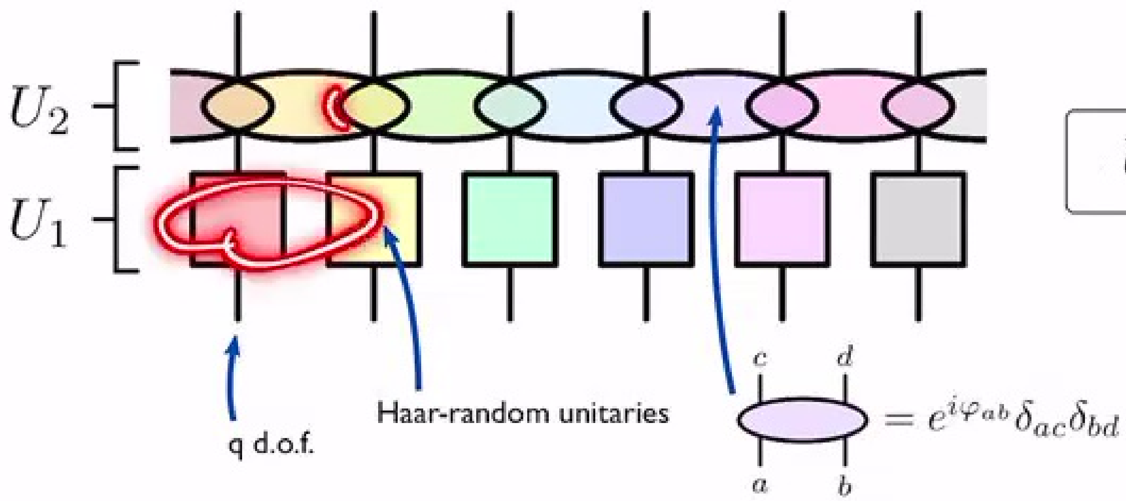


MBQC: Onset of RMT



t_{Th} = the time after which $K(t) \sim K_{\mathrm{RMT}}$

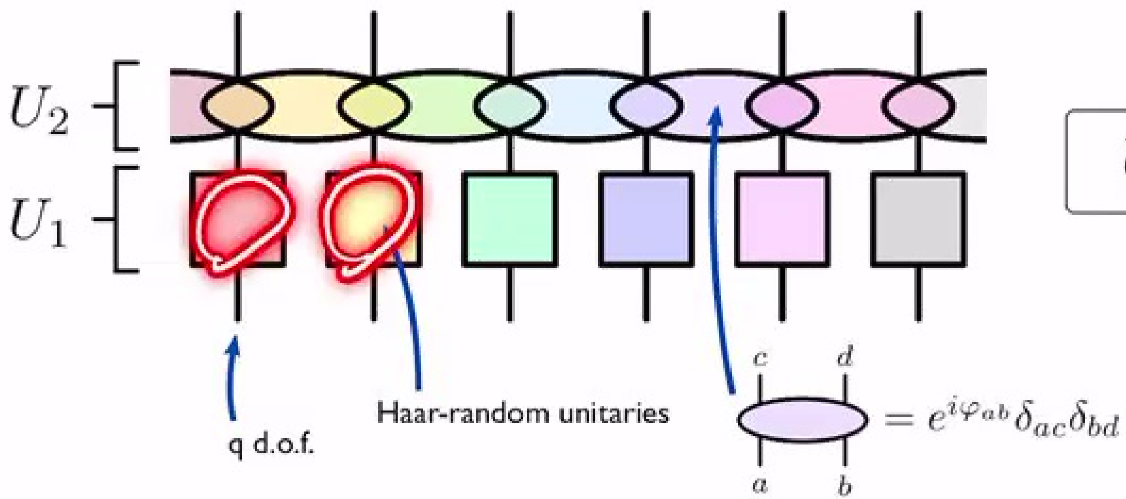
Random Phase Model



$$\boxed{U = U_2 \cdot U_1}$$

[AC, De Luca, Chalker, PRL '18]
[AC, De Luca, Chalker, PRR '21]
[AC, Shivam, Huse, De Luca, 2109.04475]

Random Phase Model



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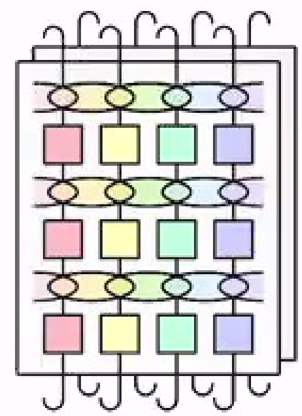
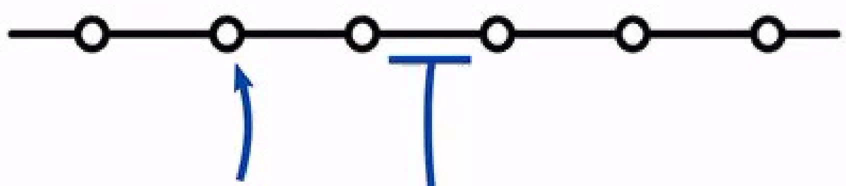
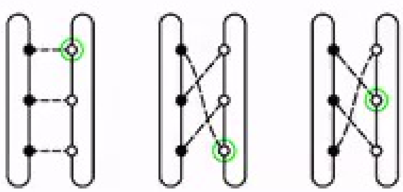
Quantum-Classical Mapping I

$K(t) = \langle \mathrm{Tr}[U(t)]\mathrm{Tr}[U^\dagger(t)] \rangle$





$\sigma_i = 1, 2, \ldots, t$



$$\text{Boltzmann Weight} = \begin{cases} 1 & \text{if} \quad \sigma_i = \sigma_{i+1} \\ e^{-\epsilon t} & \text{if} \quad \sigma_i \neq \sigma_{i+1} \end{cases}$$

$$\boxed{\lim_{q\to\infty} K(t) = Z_{\mathrm{t-state-Potts}}}$$

[AC, De Luca, Chalker, PRL '18]

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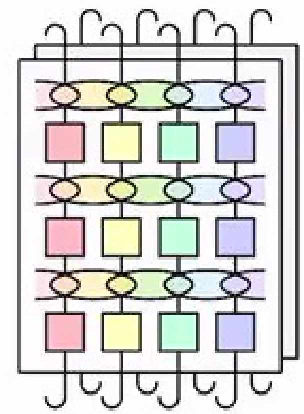

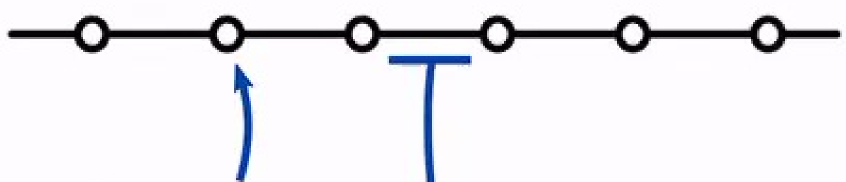

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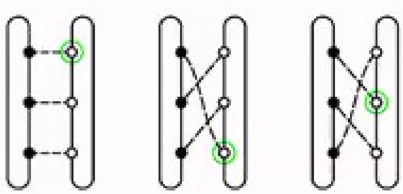

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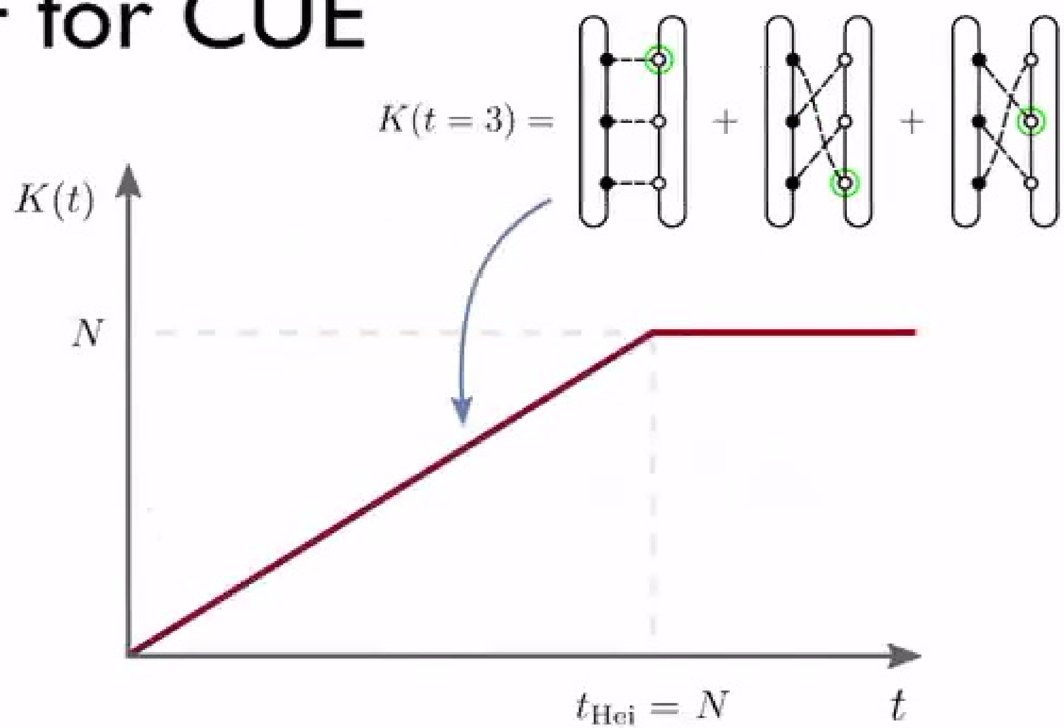

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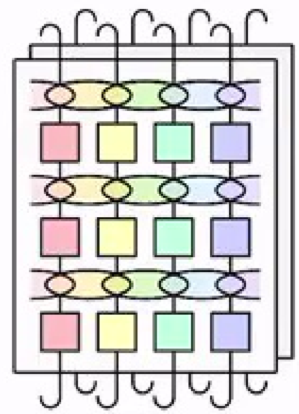
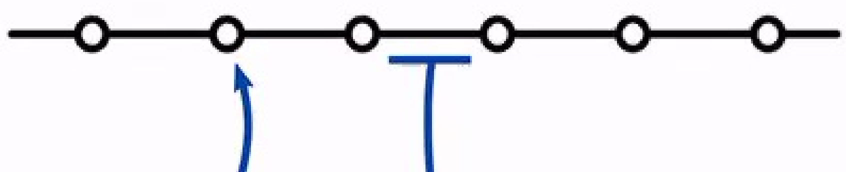
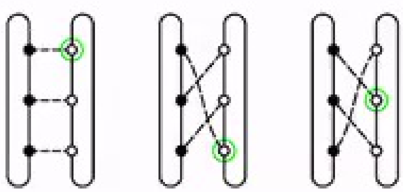




$$\sigma_i = 1, 2, \ldots, t$$



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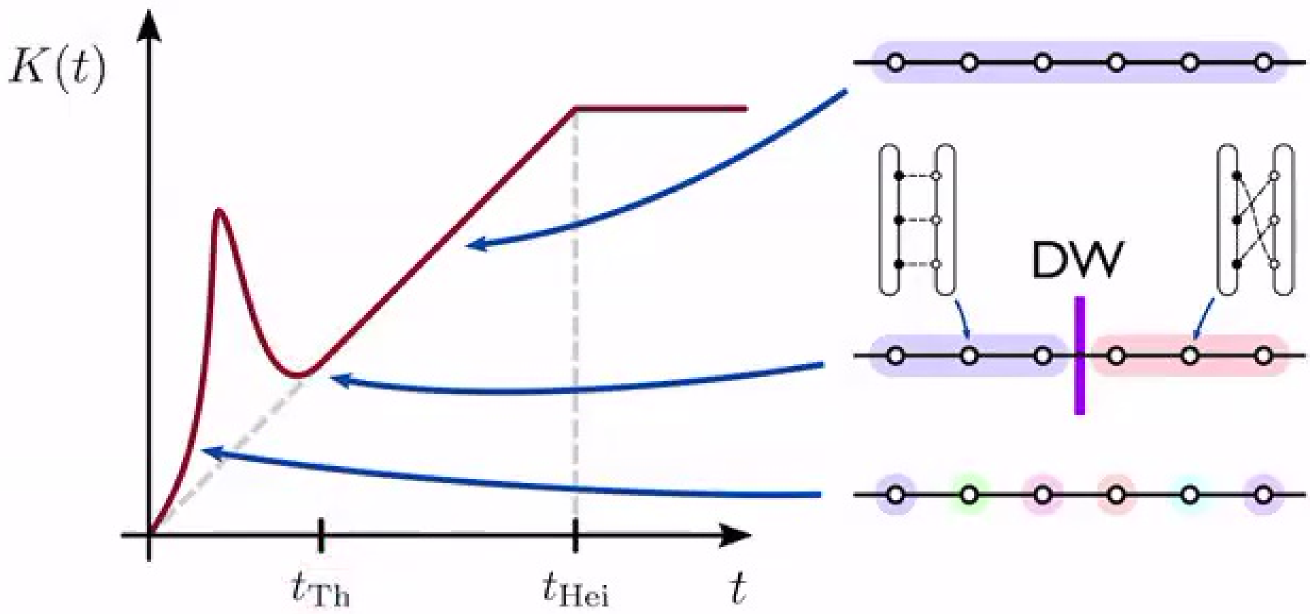
$$\boxed{\lim_{q\to\infty} K(t) = Z_{t-\text{state}-\text{Potts}}}$$

[AC, De Luca, Chalker, PRL '18]
[AC, Shivam, Huse, De Luca, 2109.04475]

Domain Walls

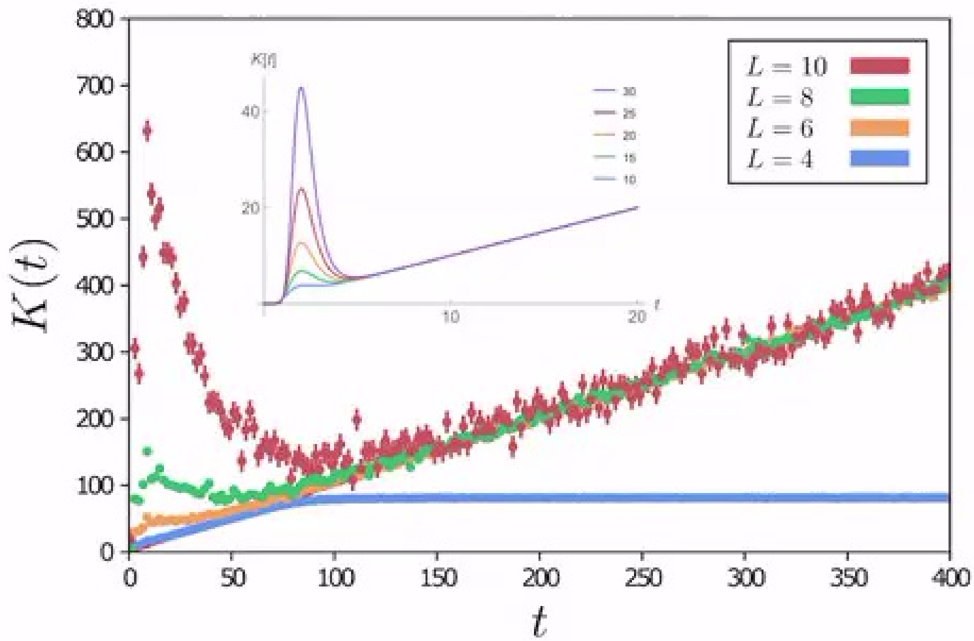


$$\boxed{t_{\mathrm{Th}} \sim \log L} \qquad \Longrightarrow \qquad t_{\mathrm{Th}} \sim L^2$$

[AC, De Luca, Chalker, PRL '18]
[Garratt, Chalker, PRX '21]
[AC, Shivam, Huse, De Luca, 2109.04475]

[Friedman, AC, De Luca, Chalker, PRL '19]
[Moudgalya, Prem, Huse, AC, PRR '21]

Results



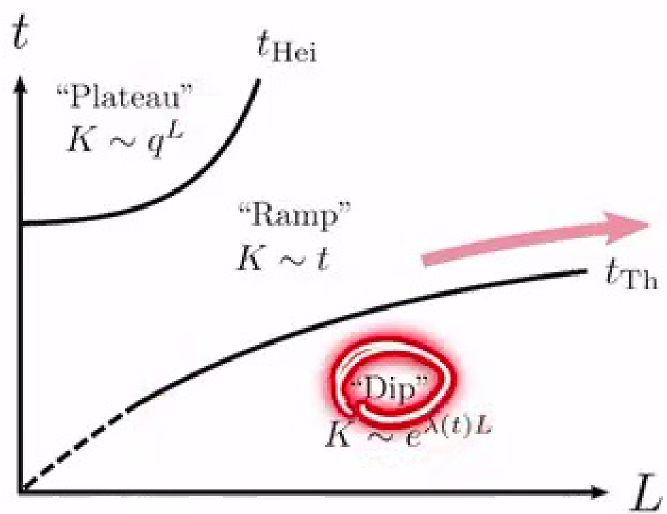
$$\boxed{K(t,L) = (t-1)(1-e^{-\epsilon t})^L + [1+(t-1)e^{-\epsilon t}]^L}$$

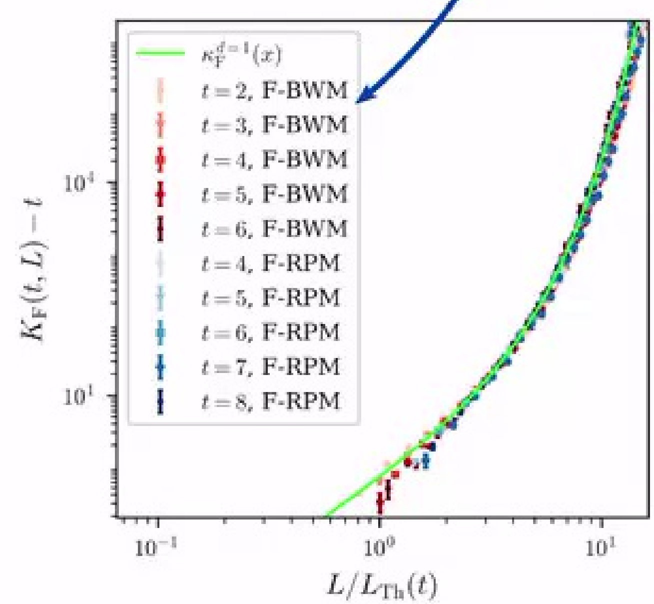
[AC, De Luca, Chalker, PRL '18]

[AC, Shivam, Huse, De Luca, 2109.04475]

Universality

2 Different Models!





$$\boxed{\lim_{\substack{L,t\to\infty \\ L/L_{\mathrm{Th}}(t)=x}} K(t,L) - t = e^{x} - x - 1 \equiv \kappa(x)}$$

$$x = L/L_{\mathrm{Th}}(t)$$

[AC, De Luca, Chalker, PRL '18]
[AC, Shivam, Huse, De Luca, 2109.04475]

Quantum-Classical Mapping I

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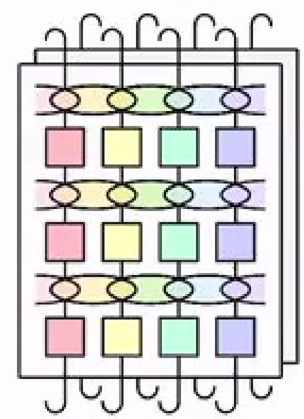

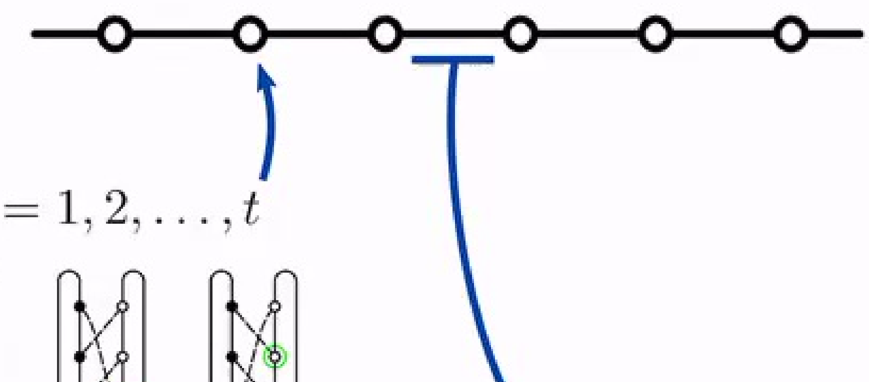

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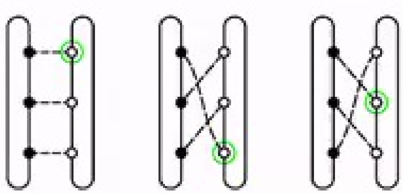

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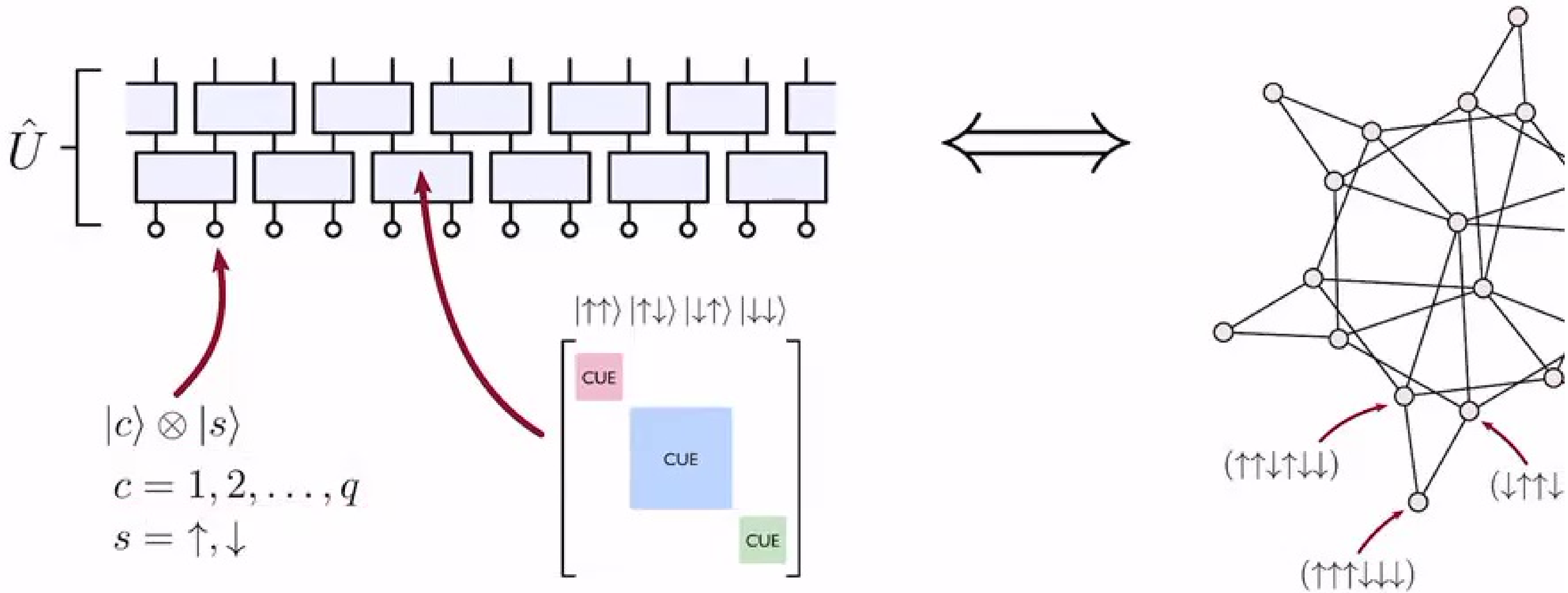
[AC, De Luca, Chalker, PRL '18]
[AC, Shivam, Huse, De Luca, 2109.04475]

Quantum-Classical Mapping



$$\boxed{K(t;Q) \overset{q\gg1}{=} |t|\,\mathrm{Tr}_Q[M^t] \overset{t\gg1}{=} |t|\,\mathrm{Tr}_Q\left[e^{-H_{\mathrm{RK}}t}\right]}$$

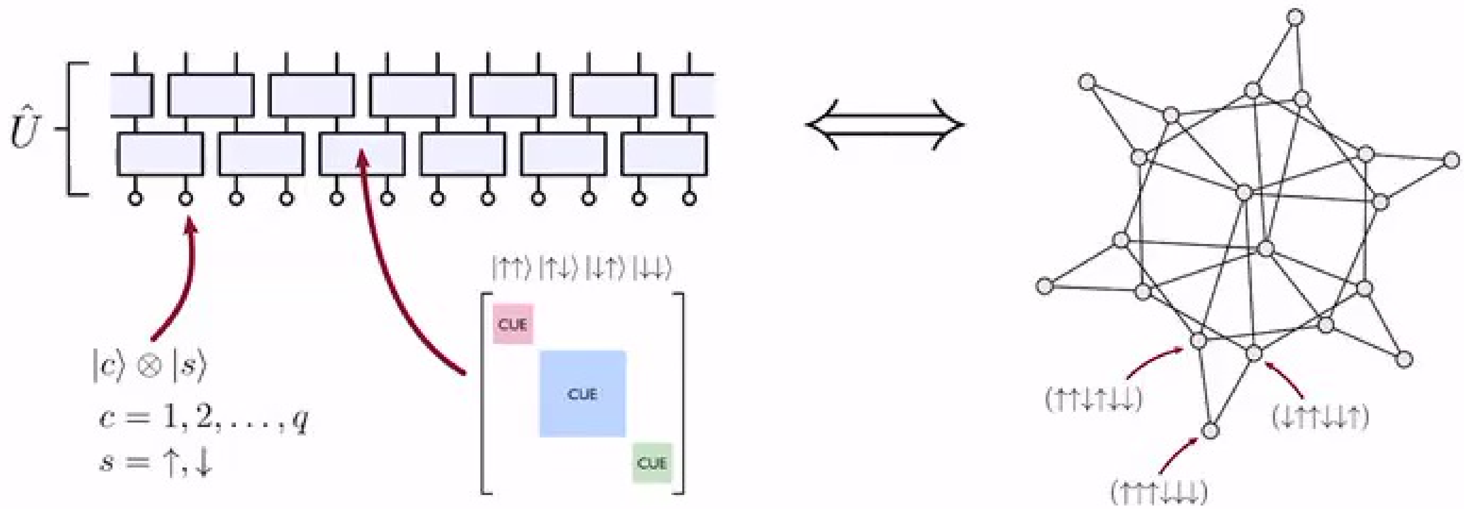
Thouless time t of Floquet random quantum circuit

16:16 Tue 26 Oct

2021_10_26_PI_seminar

× Embeddable Log... × Embeddable Log... × Stochastic Hamilt... × Lie Algebra Marko... × BCH Formula Opti... × 2021_10_26_semi... × 2021_10_26_P...

Quantum-Classical Mapping II



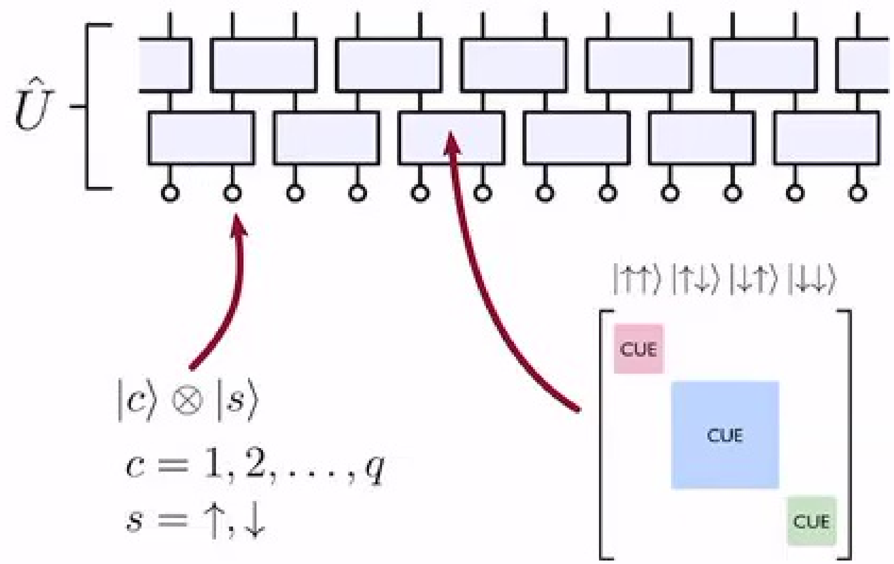
$$K(t;Q) \overset{q\gg 1}{=} |t|\,\mathrm{Tr}_Q[M^t] \overset{t\gg 1}{=} |t|\,\mathrm{Tr}_Q[e^{-H_{\mathrm{RK}}t}]$$

Thouless time t_{Th} of Floquet random quantum circuit

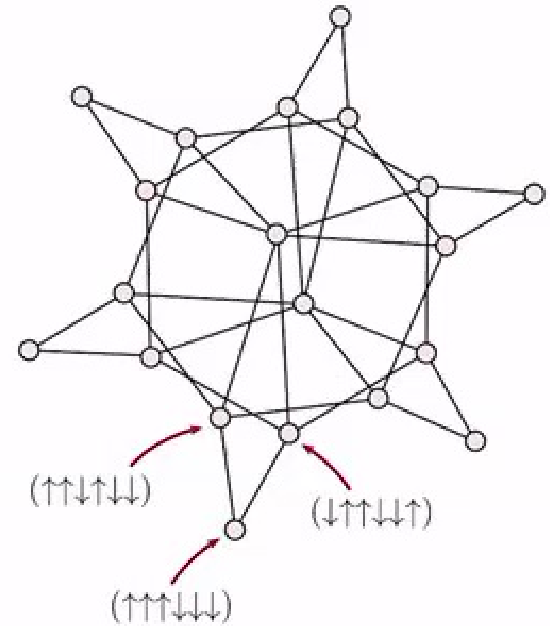
$\Updownarrow$

Spectral gap Δ of Markov chain and Rokhsar-Kivelson Hamiltonian

[Friedman, AC, De Luca, Chalker PRL '19] [Moudgalya, Prem, Huse, AC, PRB '21]



$\Longleftrightarrow$



$$\boxed{K(t;Q) \overset{q\gg 1}{=} |t|\,\mathrm{Tr}_Q[M^t] \overset{t\gg 1}{=} |t|\,\mathrm{Tr}_Q\left[e^{-H_{\mathrm{RK}}t}\right]}$$

Thouless time t_{Th} of Floquet random quantum circuit

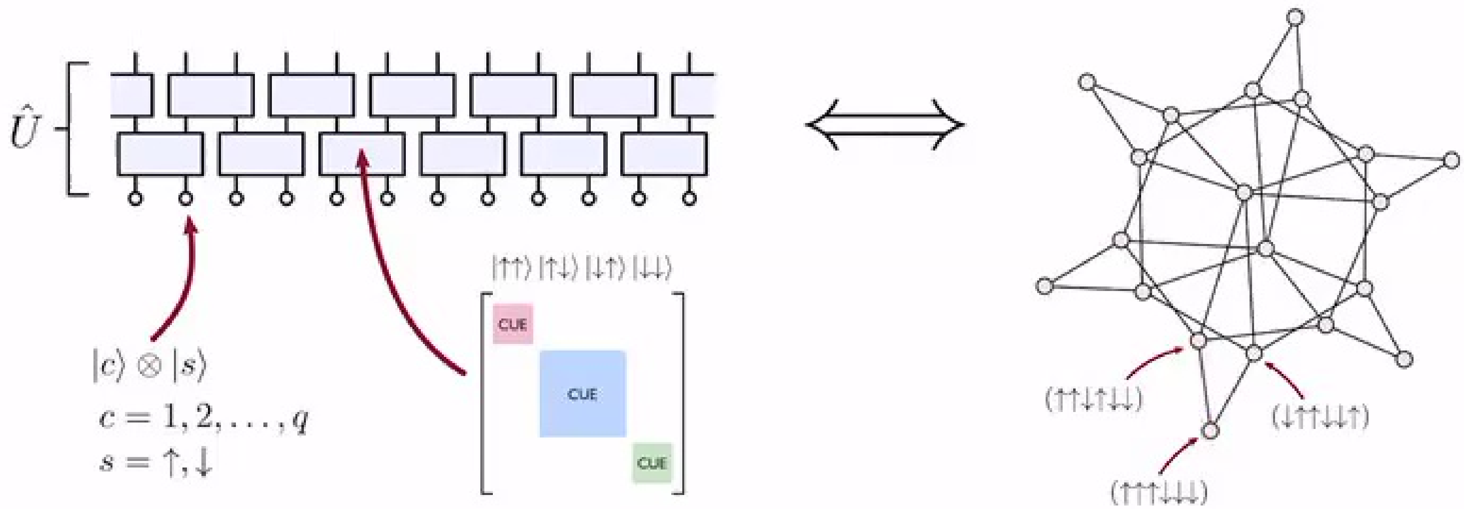
$\Updownarrow$

Spectral gap Δ of Markov chain and Rokhsar-Kivelson Hamiltonian

[Friedman, AC, De Luca, Chalker PRL '19]

[Moudgalya, Prem, Huse, AC, PRR '21]

Quantum-Classical Mapping II



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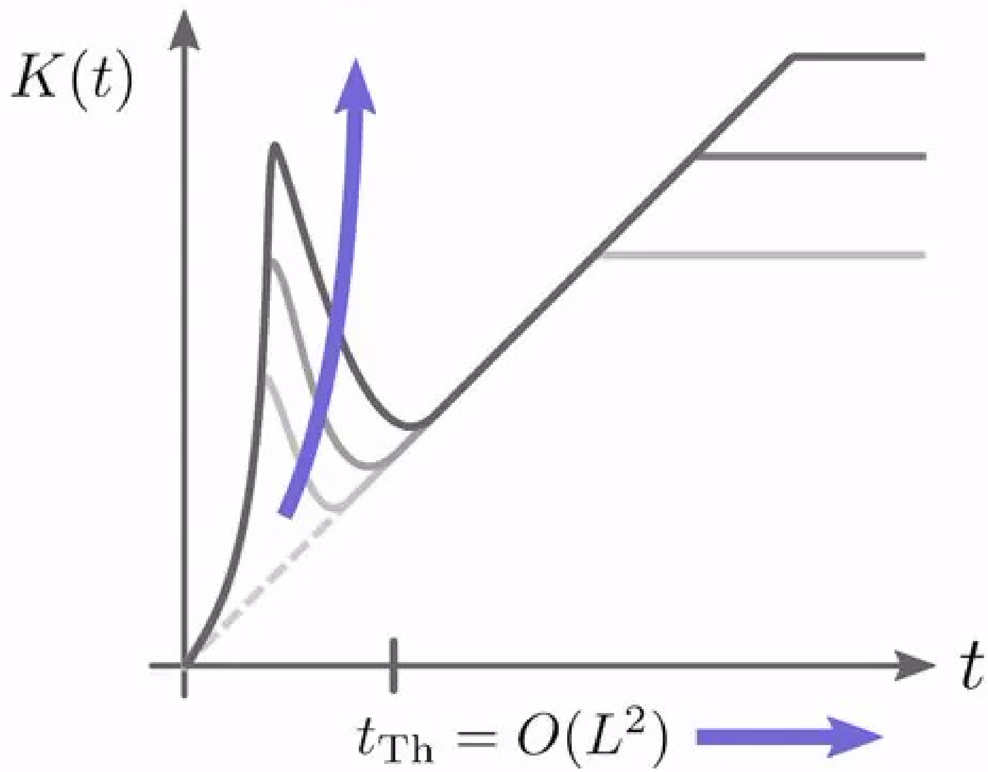
[Friedman, AC, De Luca, Chalker PRL '19] [Moudgalya, Prem, Huse, AC, PRB '21]

$L \to \infty$? Spectral Lyapunovs

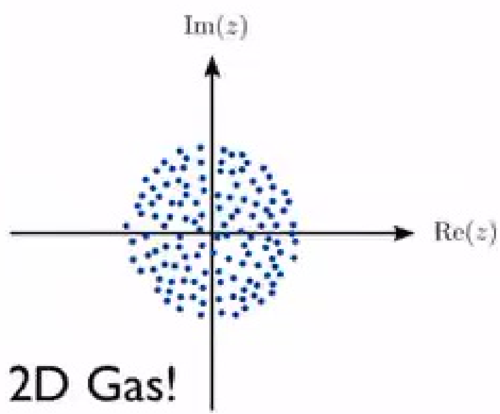


[AC, De Luca, Chalker, PRR '21]

Dissipative SFF

$$K(t) = \left\langle \sum_{m,n} e^{i(z_m - z_n)t} \right\rangle$$

$$z_m = x_m + i y_m$$



2D Gas!

$$R(\omega, \omega') = \langle \rho(x,y)\rho(x+\omega, y+\omega') \rangle$$

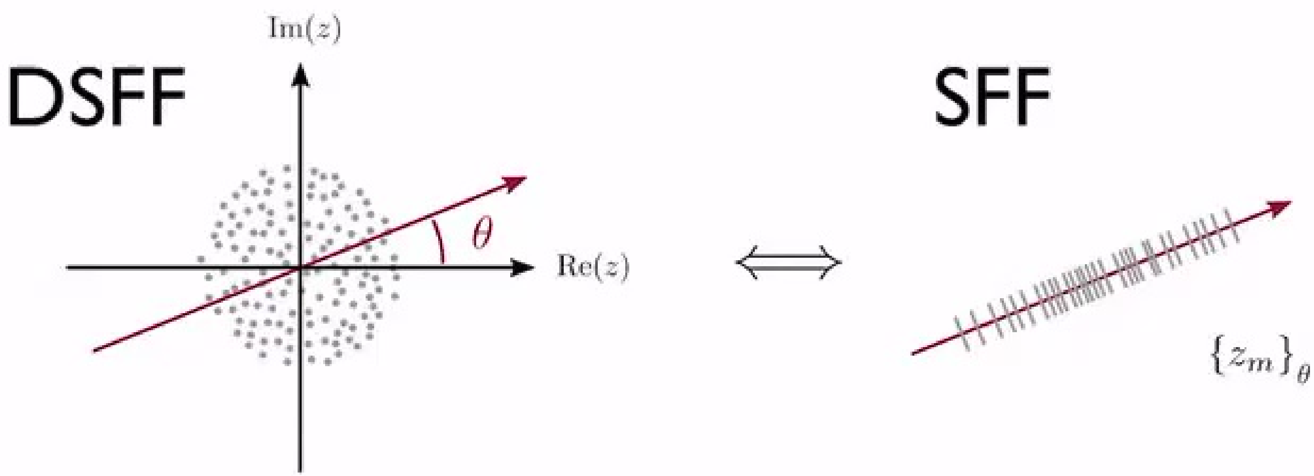
$\Updownarrow$ F.T.

$$\boxed{K(t,s) = \left\langle \sum_{m,n} e^{i(x_m - x_n)t + i(y_m - y_n)s} \right\rangle}$$

[Li, Prosen, AC, PRL '21]

DSFF Interpretation

$$K(t,s) = \left\langle \sum_{m,n} e^{i(x_m - x_n)t + i(y_m - y_n)s} \right\rangle \qquad \tau = t + is = |\tau|\cos\theta + i|\tau|\sin\theta$$

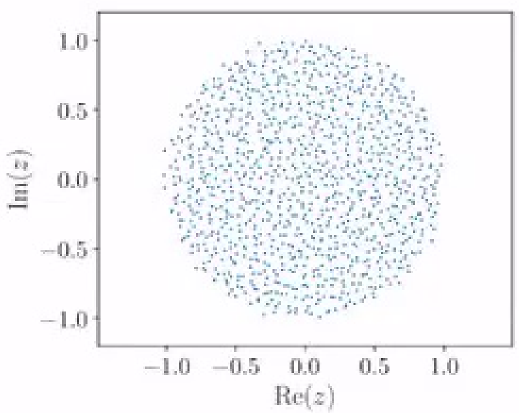


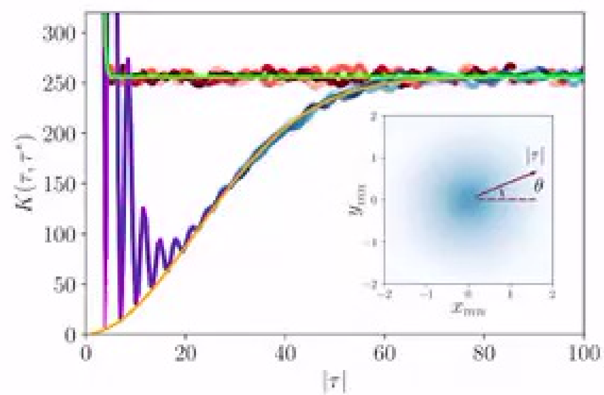
$$\boxed{\begin{array}{c} \text{DSFF } K(|\tau|,\theta) \text{ of } \{z_m\} \text{ at fixed } \theta \\ \Updownarrow \\ \text{SFF } K(|\tau|) \text{ of } \{z_m\} \text{ projected along } \theta\text{-axis} \end{array}}$$

DSFF × Dissipative Quantum Chaos

$$K_{\mathrm{GinUE}}(\tau,\tau^*) = N + N^2\frac{4J_1(|\tau|)^2}{|\tau|^2} - N\exp\left(-\frac{|\tau|^2}{4N}\right)$$

$$K_{\mathrm{Poi}}(\tau,\tau^*) = N + N(N-1)e^{-|\tau|^2}$$





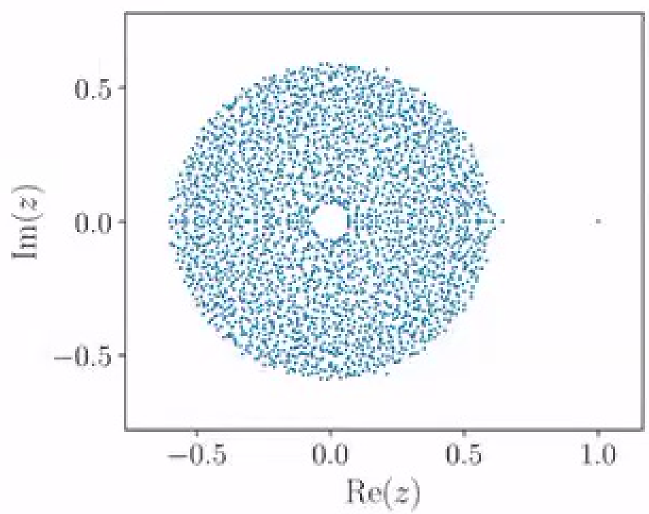
Features:
- Symmetries
- Dip-ramp-plateau
- Quadratic "ramp"!!
- Time scales
- Degeneracies
- Hyperuniformity

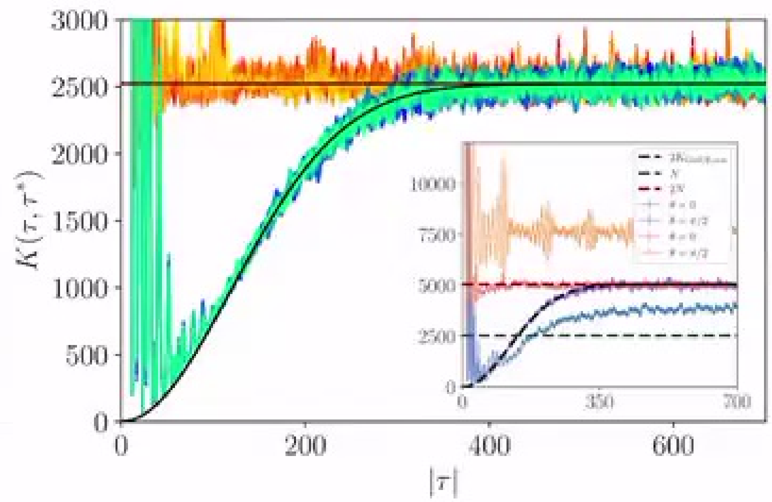
DMBQC Conjecture: $\tau_{\mathrm{Th}} \le |\tau| \implies K_{\mathrm{DQC}} \approx K_{\mathrm{GinUE}}$

Quantum Kicked Top

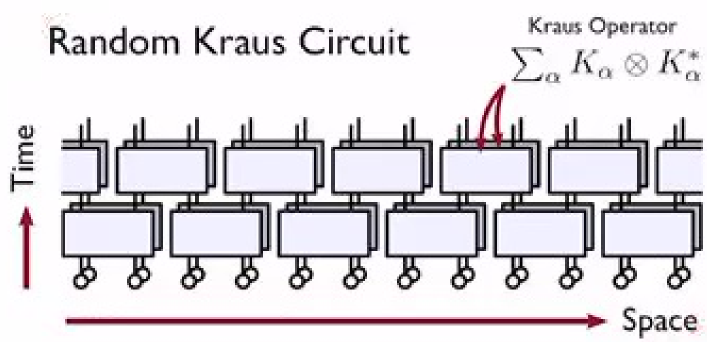
$$H(t) = pJ_z + \frac{k_0}{2j} J_z^2 + \frac{k_1}{2j} J_y^2 \sum_{n=-\infty}^{\infty} \delta(t-n)$$

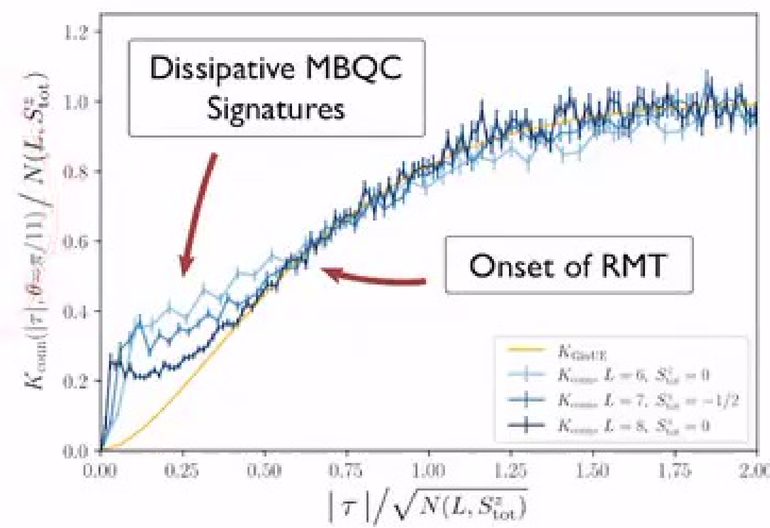
$$\Phi = \sum_a (K_a \otimes K_a^*)(U \otimes U^*), \quad K_{1,2} = \frac{(J_x \pm i J_y)}{\sqrt{j(j+1)}}, \quad K_3 = \frac{\sqrt{2} J_z}{\sqrt{j(j+1)}}$$





Open *Many-Body* Quantum Chaos





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Conclusions

Part I: Closed MBQC systems & SFF

- Quantum-classical mappings
- RK Hamiltonians
- Domain walls
- Thouless time
- Scaling forms etc.

[AC, De Luca, Chalker, PRX '18]
[AC, De Luca, Chalker, PRL '18]
[Moudgalya, Prem, Huse, AC, PRR '21]
[AC, Shivam, Huse, De Luca, 2109.04475]
and more upcoming works

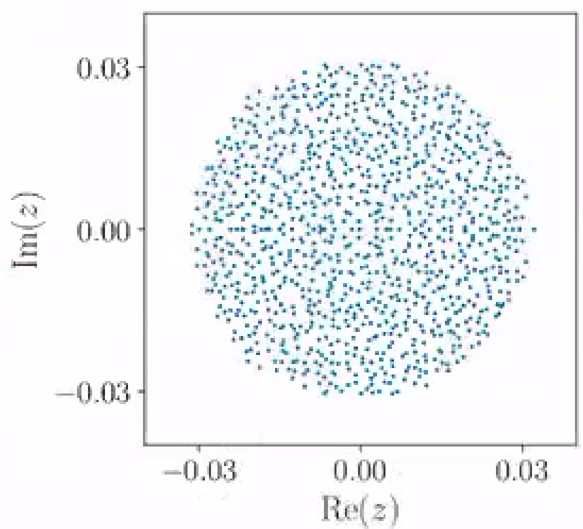
Part II: Open MBQC systems & SFF

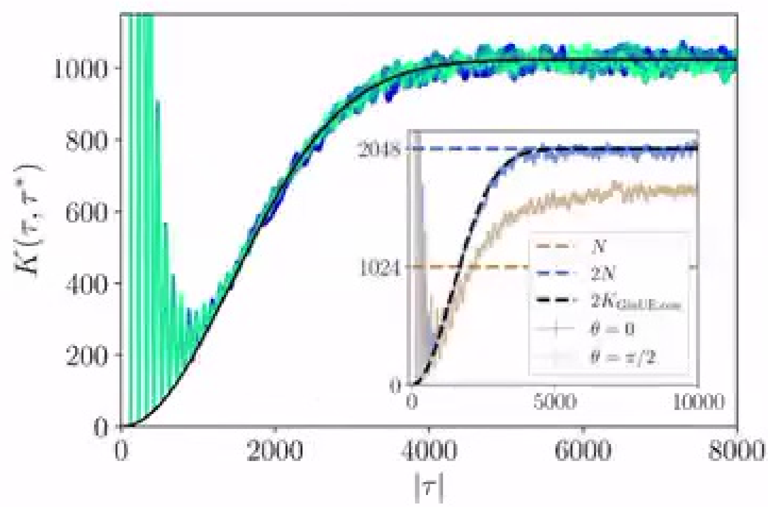
- DSFF & universality
- Dip-ramp-plateau w/ quadratic ramp
- DMBQC & the bump etc.

[Li, Prosen, AC, PRL '21]
[Li, Yan, Prosen, AC, forthcoming]

Classical Stochastic Maps

$$S_{ij} = \frac{|M_{ij}|^2}{\sum_i |M_{ij}|^2}, \quad M \in \text{CUE, GinUE}$$





DSFF $\times$ Dissipative Quantum Chaos

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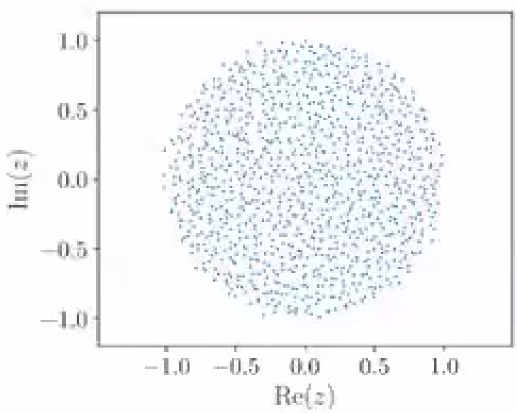

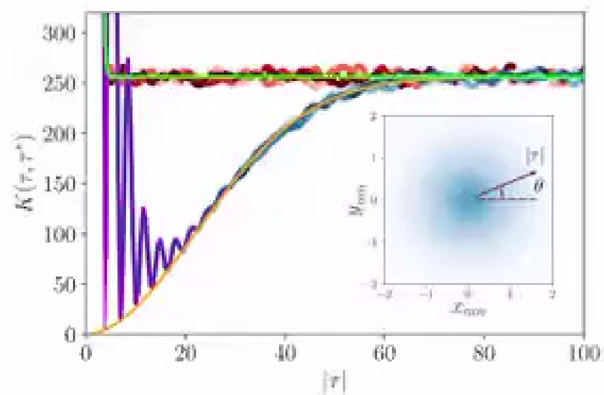

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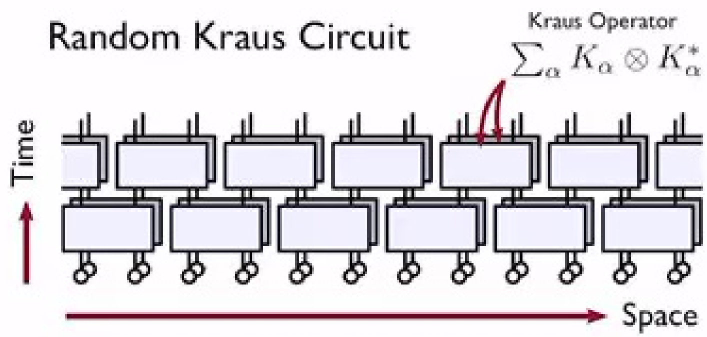
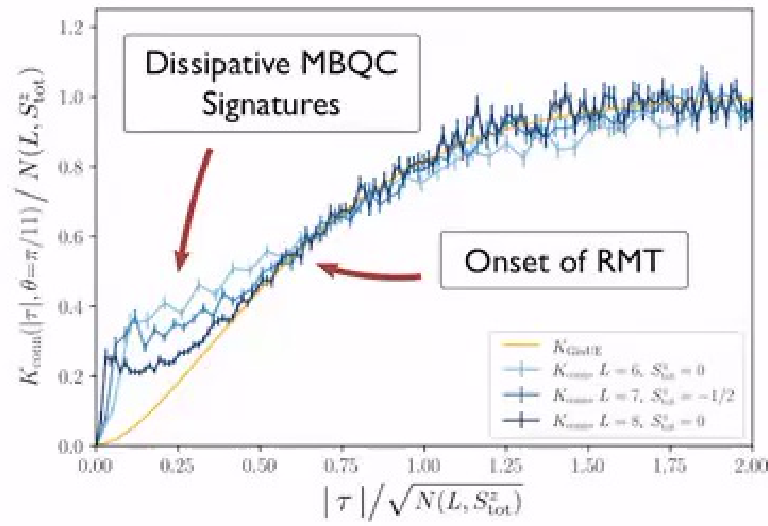
Open *Many-Body* Quantum Chaos





F.T. $\Updownarrow$

t Δ

eigenvalu

d.o.s. $\rho(E$

$$(t) = \sum_{m,n} \langle e^{i(E_m - E_n)t} \rangle = \langle \mathrm{Tr}[U(t)]$$

δ_{mn}

$\frac{1}{\Delta}$

$\sum_m = N$

SFF Results

Models	t_{Th}	Quantum-classical	Approximations	References
Floquet RQC w/o symmetries	$O(1)$	/	Large-q	AC, De Luca, Chalker, PRX 2018
	$\log L$	Potts chain		AC, De Luca, Chalker, PRL 2018
Floquet RQC w/ translational inv.	$\log L$	Model of permutations		AC, Shivam, Huse, De Luca, 2109.04475
Floquet RQC w/ conserved charge	L^2	Heisenberg chain		Friedman, AC, De Luca, Chalker, PRL 2019
Floquet RQC w/ conserved multipoles	L^{2m}	Rokshar-Kivelson		Moudgalya, Prem, Huse, AC, PRR 2021
Kicked Ising w/ dual unitarity	$O(1)$	/	Dual unitarity	Bertini, Kos, Prosen, PRL 2018; 2012.12254
Kicked Ising w/ long range int.	$\log L$	/	Long range int.	Kos, Ljubotina, Prosen, PRX 2018 Roy, Prosen, 2005.10489
SYK	$\log N$	/	Large-N All-to-all int.	Saad, Shenker, Stanford, 1806.06840 Gharibyan et al. JHEP 2018 Cotler et al. JHEP 2016

Related works: Garcia-Garcia, Verbaarschot, PRD 2016, Altland, Bagrets, Nuc Phys B 2017, Schiulaz, Torres-Herrera, Santos PRB 2019, Khramtsova, Lanina, JHEP 21
Liao, Vikram, Galitski, PRL 2020, Winer, Jian, Swingle, PRL 2020, Nivedita, Shackleton, Sachdev, PRE 2020, Liao, Galitski, arXiv 2021, and more!