Title: Integrability and Conformal Bootstrap: One Dimensional Defect CFT

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Series: Quantum Fields and Strings

Date: October 19, 2021 - 2:00 PM

URL: https://pirsa.org/21100006

Abstract: Thanks to integrability, we have access to a significant portion of the conformal data of planar N=4 SYM: in particular, the scaling dimensions of all single-trace operators at finite 't Hooft coupling. Can we solve the theory by using these data as an input for the conformal bootstrap?

We recently started to explore this question in a simplified setup: the one-dimensional CFT living on a supersymmetric Wilson line embedded in the 4D theory.

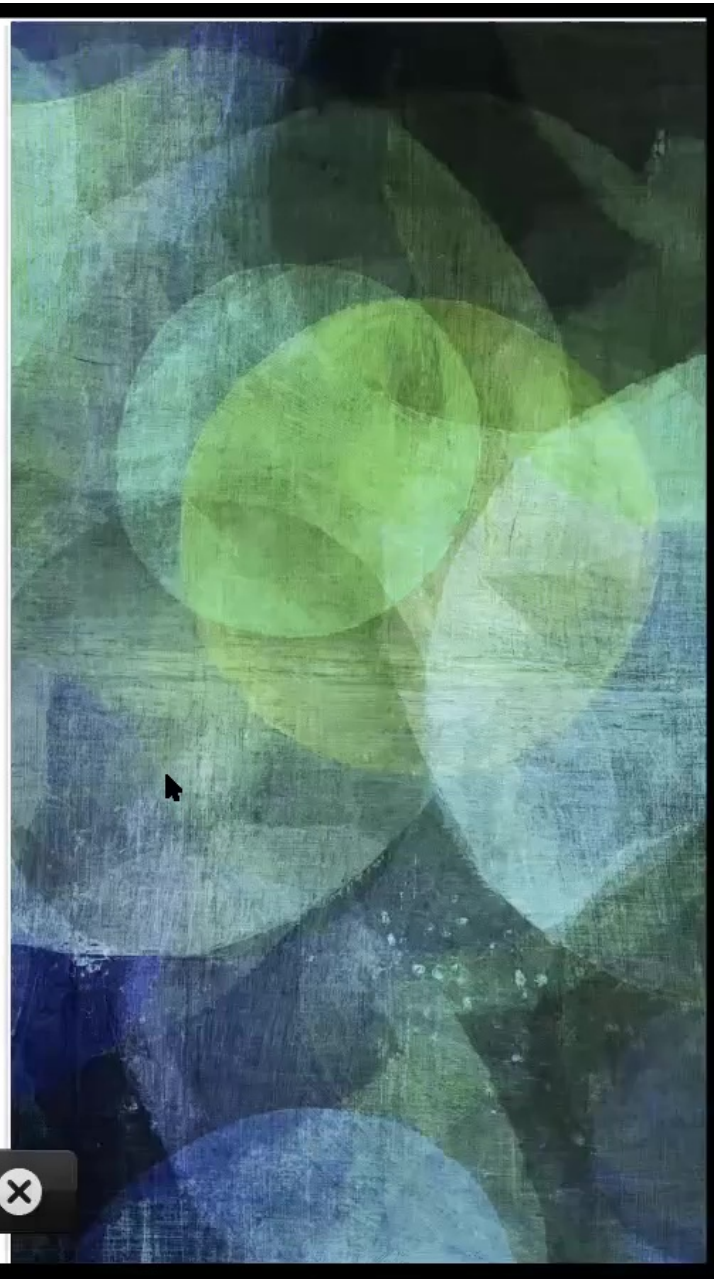
After reviewing how integrability describes the spectrum of this 1D CFT, I will discuss how, using the numerical conformal bootstrap with a minimal input from the spectral data, it is possible to compute with good precision a non-supersymmetric OPE coefficient at finite 't Hooft coupling. I will conclude discussing the open questions and future perspectives. Based on 2107.08510 with N.Gromov, J.Julius and M.Preti.

INTEGRABILITY & CONFORMAL BOOTSTRAP:
THE 1D DEFECT CFT

PERIMETER, OCT 2021

Andrea Cavaglià

Based on work with N. Gromov, J. Julius and M. Preti
hep-th/2107.08510



Can we combine Integrability (i.e., exact spectral data) with the Conformal Bootstrap to study $\mathcal{N} = 4$ SYM?

Similar ideas were used for 2D CFTs
[Picco,Ribault,Santachiara '16]
[He,Jacobsen,Saleur '20]

Motivations:

- study observables beyond the (present) reach of either method on its own

(e.g. the theory may lie deep inside the bootstrap bounds at small/finite λ)

[Beem, Rastelli, van Rees '13, '19]

- predictions for future integrability methods
- hope of exploring the non-planar regime
- new insights on how to solve the theory?

- Introduction
- Our setup: the Wilson line defect CFT
- Integrability for the spectrum
- Bootstrapping OPE coefficients
- Outlook

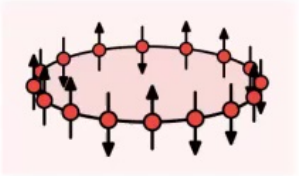
Integrability

SU(N) $\mathcal{N} = 4$ SYM , large N , finite $\lambda = N g_{YM}^2$

Single-trace operators $\simeq$ closed strings $\simeq$ spin chain states

$$\mathcal{O}(x) = Tr\left(\Phi_i \Phi_j \mathcal{F}_{\mu\nu} \dots\right)$$

$$\langle \mathcal{O}(x) \bar{\mathcal{O}}(y) \rangle \propto \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}(\lambda)}}$$



Quantum Spectral Curve

[Gromov,Kazakov.Leurent,Volin '13]

Numerical non-perturbative spectrum,
Complex spin (BFKL physics, Regge trajectories…),
Analytic weak coupling exp. (11 loops and more), ….
+ non-local operators, …

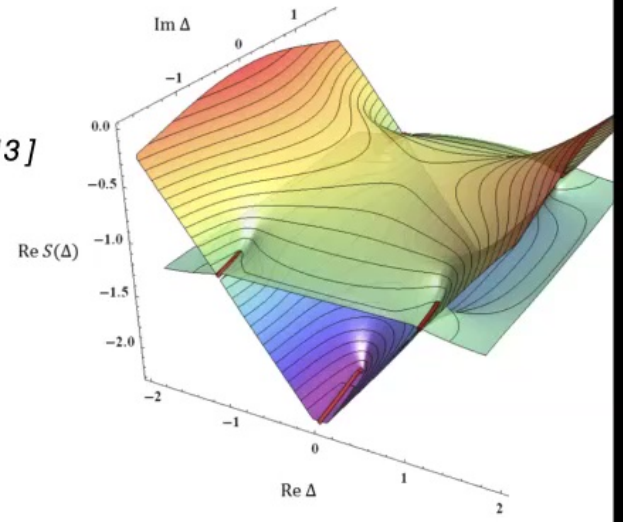


Fig: [Levkovich-Maskyuk,Gromov,Sizov '15]

There is much more:

All closed-strings worldsheets in AdS5 x S5 should be integrable

$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle$

single traces

should be solvable even at all orders in 1/N

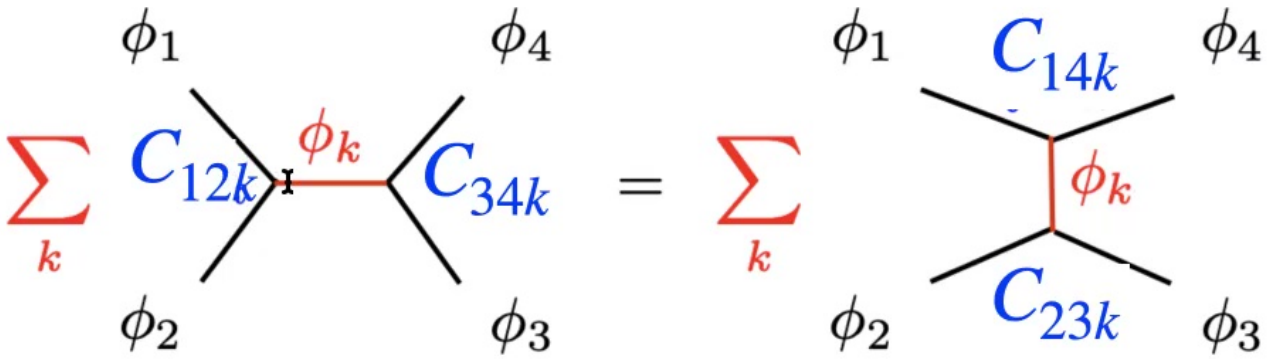
[Bargheer, Caetano,Fleury,Komatsu,Vieira '13]

(+ Wilson loops, amplitudes, D-branes…)

A lot of progress: hexagonalization methods,
Separation of Variables, correlators in the Fishnet theory, …

… but we are not there yet.

$C_{123}(\lambda)$ for 3 generic single-trace operators is still impossible to compute.



Naive idea: once spectrum is known,
crossing symmetry gives a linear relation on OPE coefficients

However the planar OPE contains also double-trace contributions

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \underbrace{\langle \mathcal{O}\mathcal{O} \rangle \langle \mathcal{O}\mathcal{O} \rangle}_{free,\ O(1)} + \underbrace{\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle_{connected}}_{O(1/N^2)}$$

OPE contains double trace operators

$$\mathcal{O}_d \sim : \mathcal{O}_1 \partial^{2n} \mathcal{O}_2 :$$

$$\Delta_{\mathcal{O}_d} = \Delta_{\mathcal{O}_1} + \Delta_{\mathcal{O}_2} + 2n + O(\tfrac{1}{N^2})$$

$$C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_d} = C^2_{free} + O(\tfrac{1}{N^2})$$

OPE contains
single-trace operators $\mathcal{O}_s = Tr(\dots)$

(with $C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_s} = O(\tfrac{1}{N^2})$)

\+ sub-leading double-trace data

We start by studying a simpler setup without this complication.

Setup 1/2 -BPS susy Wilson line in $\mathcal{N}=4$ SYM, with insertions

$$Tr\left[Pe^{\int_{-\infty}^{t_1} dt\left(iA_t+\Phi_{||}\right)} O_1(t_1)\, Pe^{\int_{t_1}^{t_2} dt\left(iA_t+\Phi_{||}\right)} O_2(t_2)\ldots O_n(t_n)\, Pe^{\int_{t_n}^{+\infty} dt\left(iA_t+\Phi_{||}\right)}\right]$$
$$\equiv \langle\langle O_1(t_1)O_2(t_2)\ldots O_n(t_n)\rangle\rangle$$

Insertions in adjoint of $SU(N_c)$,

e.g. $O=\Phi^i_\perp$,
$O=\Phi_{||}$,
$O=(\Phi^b F_{\mu\nu}\Phi^c D_\rho\Psi\ldots)$, ...

= infinitesimal bumps in the Wilson line
[Drukker,Kawamoto '06]

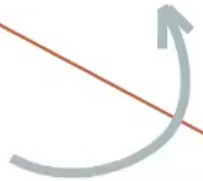
Dual perturbative 2d QFT at strong coupling *[Giombi,Roiban,Tseytlin '06]*

We will study these correlators + integrability

Symmetries

1D Conformal Symmetry:

$$SL(2,\mathbb{R}) = \{\mathbb{D}_t, \mathbb{P}_t, \mathbb{K}_t\}$$



Vacuum

$$Tr\, P e^{\int dt \left(iA_t + \Phi_{||} \right)}$$

Rotations of the orthogonal scalars $\Phi^i_\perp$: $SO(5)$

Rotations around the line: $SO(3)$

$$SL(2,\mathbb{R}) \times SO(3) \times SO(5) \subset OSp(2,2|4)$$

Important $\frac{1}{2}$-BPS multiplets for this talk:

$$\mathscr{B}_1 = \left\{ \Phi^i_\perp,\ i \leq 5\,; \ldots \right\}, \qquad \Delta_{\mathscr{B}_1} = 1\,,$$
$$\mathscr{B}_2 = \left\{ \Phi^{(i}_\perp \Phi^{j)}_\perp,\ i,j \leq 5\,; \ldots \right\}, \Delta_{\mathscr{B}_2} = 2$$

Simplest non protected operators:

$$O_{\Delta_1} = \Phi_{||} + \ldots,$$

$$\left\{ O_{\Delta_1} \ldots O_{\Delta_3} \right\} = \textbf{Span} \left\{ \Phi^2_{||},\ \sum_i^5 \Phi^i_\perp \Phi^i_\perp \right\} + \ldots$$

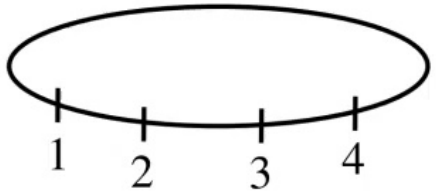
Bootstrap setup: the simplest 4-point function

Choose a primary in $\mathscr{B}_1$: $\Phi^1_\perp \equiv \Phi_\perp$

$$\langle\langle \Phi_\perp(x_1)\Phi_\perp(x_2)\Phi_\perp(x_3)\Phi_\perp(x_4)\rangle\rangle = \frac{1}{x_{12}^2}\frac{1}{x_{34}^2}\mathscr{A}(\chi)$$

Cross ratio: $\chi = \dfrac{x_{12}x_{34}}{x_{13}x_{24}}, \quad x_{ij} = x_i - x_j$

Crossing equation:



$$(1234) \simeq (4123)$$

$$\frac{1}{x_{12}^2\, x_{34}^2}\mathscr{A}(\chi) = \frac{1}{x_{14}^2\, x_{32}^2}\mathscr{A}(1-\chi)$$

[Liendo Meneghelli Mitev '17]

Constraints of superconformal symmetry

$$\mathcal{A}(\chi) = F\chi^2 + \mathcal{D}_\chi \circ f(\chi)$$

$\mathcal{D}_\chi =$ known diff. op.

Crossing:

$$(1-\chi)^2 f(\chi) + \chi^2 f(1-\chi) = 0$$

Operator product expansion

$$f(\chi) = f_I(\chi) + C^2_{BPS}(\lambda) f_{\mathcal{B}_2}(\chi) + \sum_\Delta C^2_\Delta f_\Delta(\chi)$$

singlets of $SO(5) \times SO(3)$, **nontrivial dimensions**

$$F = 1 + C^2_{1,1,2}$$

$$f_I(\chi) = \chi$$

$$f_{\mathcal{B}_2}(\chi) = \chi\left(1 - {}_2F_1(1,2,4;\chi)\right)$$

$$f_\Delta(\chi) = \frac{1}{1-\Delta}\left[\chi^{\Delta+1}\,{}_2F_1(\Delta+1,\Delta+2,2(\Delta+2);\chi)\right]$$

Study of a related topological observable gives

$$1 + C^2_{\text{BPS}} = \frac{3WW''}{(W')^2}$$

$$W = \frac{2I_1(\sqrt{\lambda})}{\sqrt{\lambda}}$$

Conformal bootstrap constraint

$$\mathcal{G}_\bullet(\chi) \equiv (1-\chi)^2 f_\bullet(\chi) + \chi^2 f_\bullet(1-\chi)$$

$$\underbrace{\mathcal{G}_I(\chi) + C^2_{BPS}(\lambda)\,\mathcal{G}_{\mathcal{B}_2}(\chi)}_{\text{explicit function}} + \sum\nolimits_\Delta C^2_{1,1,\Delta}\mathcal{G}_\Delta(\chi) = 0$$

$$\equiv \mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi)$$

$$\{\Delta\},\quad \{C^2_{1,1,\Delta}\}$$

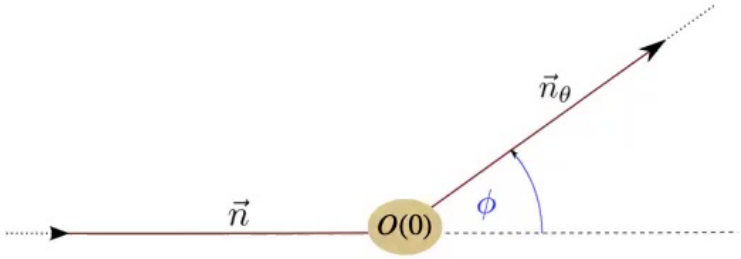
unknowns

Existing Results: Numerical bootstrap *[Liendo, Meneghelli, Mitev '17]*
Functional bootstrap at strong coupling *[Meneghelli Ferrero '21]*

$$\Delta_{\Phi_{||}} = 2 - \frac{5}{\sqrt{\lambda}} + \frac{295}{24}\frac{1}{\lambda} - \frac{305}{16}\frac{1}{\lambda^{3/2}} + \left(\frac{351845}{13824} - \frac{75}{2}\zeta(3)\right)\frac{1}{\lambda^2} + \dots$$

$$C^2_{1,1,\Delta_{\Phi_{||}}} = \frac{2}{5} - \frac{43}{30\sqrt{\lambda}} + \frac{5}{6\lambda} + \left(\frac{11195}{1728} + 4\zeta_3\right)\frac{1}{\lambda^{3/2}} - \left(\frac{1705}{96} + \frac{1613}{24}\zeta_3\right)\frac{1}{\lambda^2} + \dots$$

A useful generalisation



$$W = \mathrm{tr}\,\mathrm{P}\exp\left(\int_{-\infty}^{0} dt(iA\cdot\dot{x} + \vec{\Phi}\cdot\vec{n}|\dot{x}|)\right) \times O(0) \times \mathrm{P}\exp\left(\int_{0}^{\infty} dt(iA\cdot\dot{x}_\phi + \vec{\Phi}\cdot\vec{n}_\theta|\dot{x}_\phi|)\right)$$

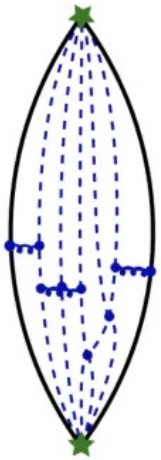


fig. from [Drukker '12]

Weak coupling/long op's picture: **open spin chain with integrable boundaries, Bethe ansatz**

For "orthogonal" insertions: [Correa Maldacena Sever '12] [Drukker '12]

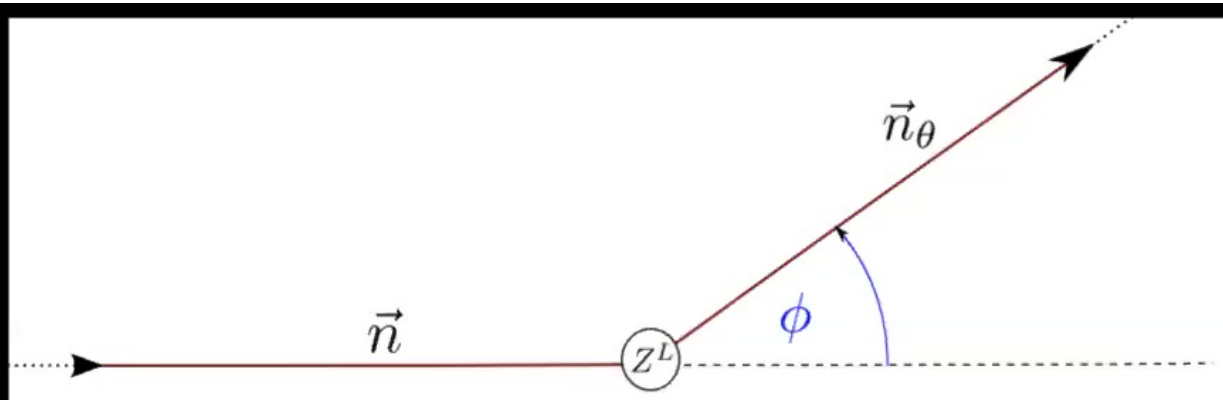
,

For "parallel" insertions: [Correa, Leoni, Luque '18] (1-loop, one sector)

Non-perturbative method:

boundary thermodynamic Bethe ansatz equations / QSC

[Drukker'12][Correa ... nov,Levkovich-Maslyuk'15]



From the QSC point of view, the neutral states are in the same sector as the supersymmetric "empty cusp"

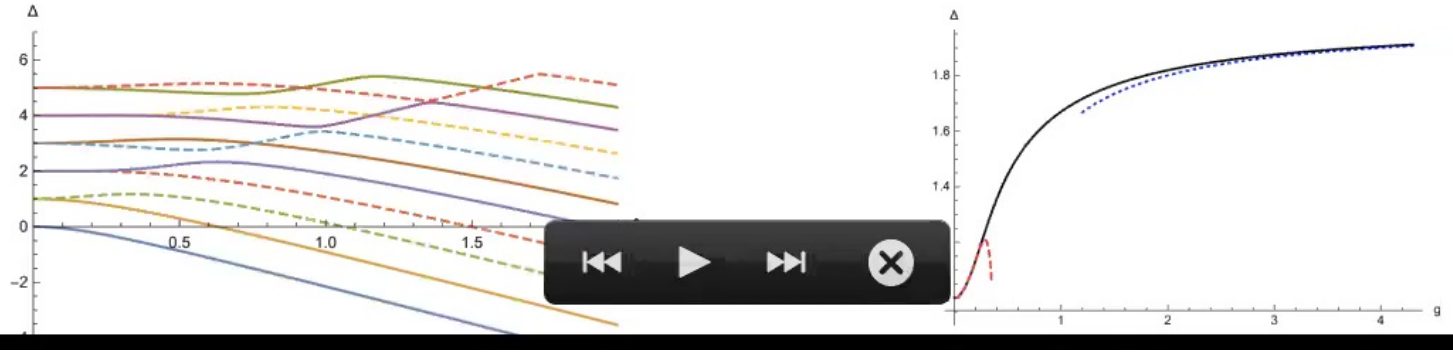
$$\phi \to 0\,,\ \theta \to 0\,,\ L = 0$$

They appear as excited states solving the same equations

[AC,Gromov,Levkovich-Maslyuk'18] (proved in ladders limit)

[Grabner,Gromov,Julius'20] (general)

[Julius, unpub.]



Quantum Spectral Curve:

Functional relations (algebra) + *Analytic properties for Q-functions*

Example: SU(2) invariant Heisenberg spin chain

SU(2) QQ-relations:

$$\begin{vmatrix} Q_1(u-i/2) & Q_1(u+i/2) \\ Q_2(u-i/2) & Q_2(u+i/2) \end{vmatrix} \propto Q_{12}(u)$$

Analytic properties:

$$Q_1(u) = e^{\phi u}\prod_{i=1}^{N_1}(u-u_i), \quad Q_2(u) = e^{-\phi u}\prod_{i-1}^{N_2}(u-w_i) \qquad Q_{12}(u) = -2i\sin\phi\, u^L$$

In the Heisenberg case, this implies the usual Bethe equations: $\left(\frac{u_k+i/2}{u_k-i/2}\right)^L = e^{-2i\phi}\prod_{j\neq k}^{N_1}\frac{u_k-u_j+i}{u_k-u_j-i}$,

Quantum Spectral Curve

QQ-relations
(reduction of PSU(2,2|4) system)

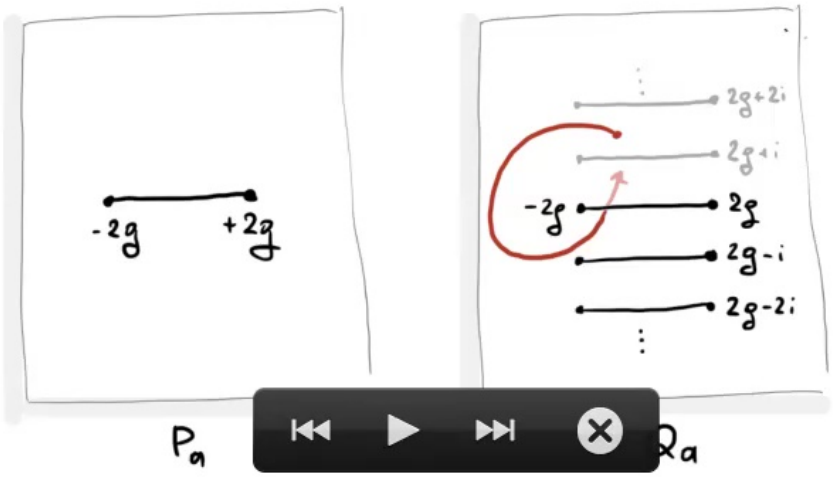
$$Q_{a|i}(u+\tfrac{i}{2}) - Q_{a|i}(u-\tfrac{i}{2}) = \mathbf{P}_a(u)\mathbf{Q}_i(u)$$

$$\mathbf{P}_a(u) = \mathbf{Q}^i(u)Q_{a|i}(u \pm \tfrac{i}{2}),$$

$$\mathbf{P}^a(u) = \chi^{ab}\mathbf{P}_b(-u), \qquad \mathbf{Q}^i(u) = \chi^{ij}\mathbf{Q}_j(-u)$$

$$\mathbf{Q}_i(u) = \mathbf{P}^a(u)Q_{a|i}(u \pm \tfrac{i}{2}),$$

Q-functions have cuts dictated by
the 't Hooft coupling



$$(16\pi^2\lambda = g^2)$$

QSC for the straight line, details

$$\mathbf{P}_1 \sim A_1 u^{3/2} f_1(u)\,, \quad \mathbf{Q}_1 \sim B_1 u^{3/2+\Delta} g_1(u)\,,$$
$$\mathbf{P}_2 \sim A_2 u^{-3/2} f_2(u)\,, \quad \mathbf{Q}_2 \sim B_2 u^{1/2+\Delta} g_2(u)\,,$$
$$\mathbf{P}_3 \sim A_3 u^{1/2} f_3(u)\,, \quad \mathbf{Q}_3 \sim B_3 u^{-3/2-\Delta} g_3(u)\,,$$
$$\mathbf{P}_4 \sim A_4 u^{-5/2} f_4(u)\,, \quad \mathbf{Q}_4 \sim B_4 u^{-5/2-\Delta} g_4(u)\,,$$

parameters

$$g_a(u) \simeq 1, \;\; u \to \infty$$

$$f_a(u) = \sum_{n=0}^{\infty} \frac{c_{a,n}}{x^n(u)}$$

$$x(u) = \frac{u}{2g} + \sqrt{\left(\frac{u}{2g}\right)^2 - 1}$$

With QQ-relations, we can get all Q-functions (numerically in terms of $\{c_{a,n}\}$

Fix parameters numerically by imposing gluing across the cut (-2g, 2g)

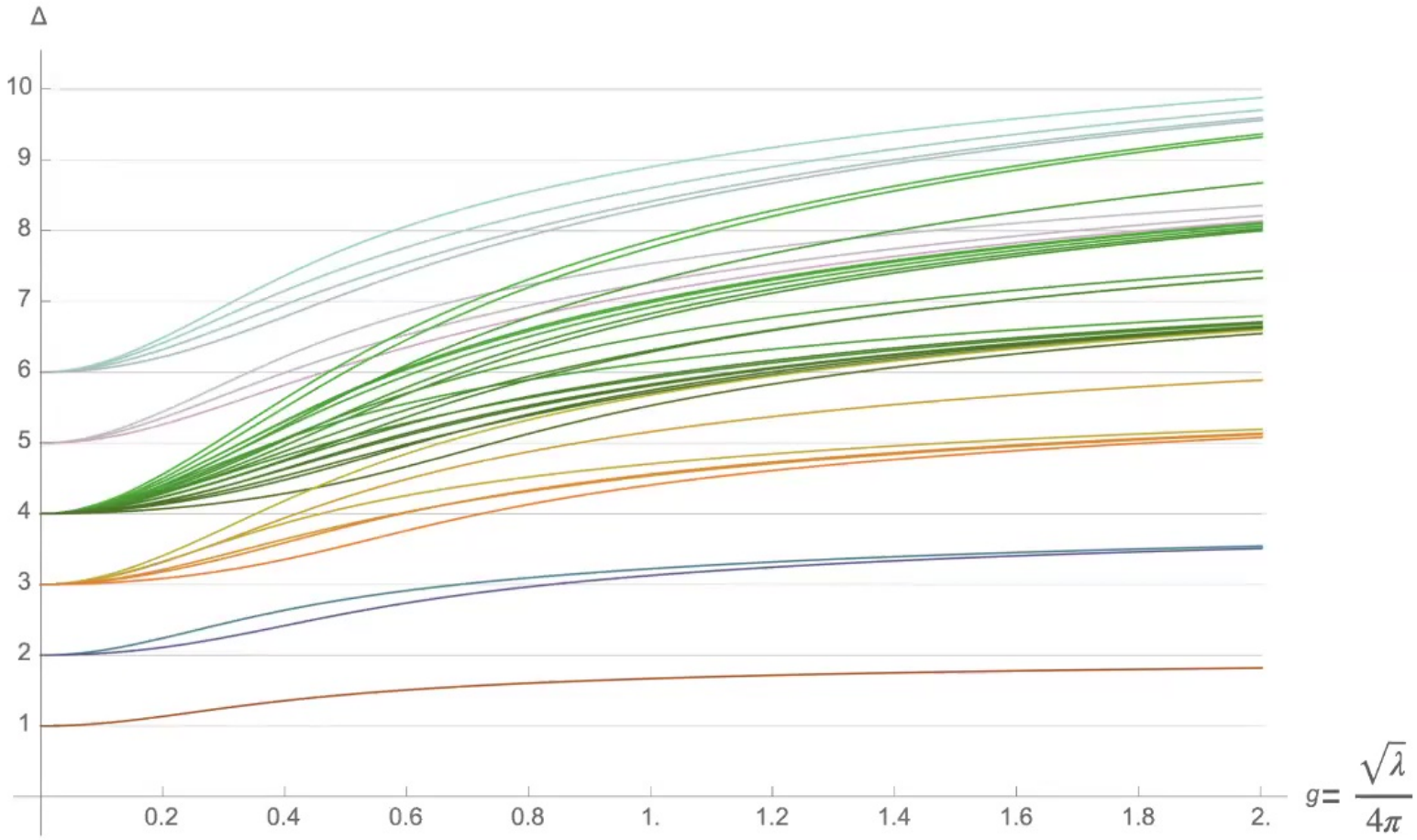
$$\begin{pmatrix} \tilde{\mathbf{q}}_1(u) \\ \tilde{\mathbf{q}}_2(u) \\ \tilde{\mathbf{q}}_3(u) \\ \tilde{\mathbf{q}}_4(u) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha \sinh(2\pi u) & 0 & 1 & 0 \\ 0 & -\alpha \sinh(2\pi u) & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{q}_1(-u) \\ \mathbf{q}_2(-u) \\ \mathbf{q}_3(-u) \\ \mathbf{q}_4(-u) \end{pmatrix}$$

$$\mathbf{q}_i(u) = \frac{\mathbf{Q}_i(u)}{u^{1/2}}$$

→ **many solutions**

Weak coupling solution
seful to find all states

[Julius, unpub.] + [AC,Gromov,Julius,Preti '21]



We computed 35 states flowing in the OPE

$$\mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi)+\sum_{\Delta_n} C^2_{1,1,\Delta_n}\mathcal{G}_{\Delta_n}(\chi)=0$$

First attempt

$$\mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi) + \sum_{\Delta_n} C^2_{1,1,\Delta_n}\mathcal{G}_{\Delta_n}(\chi) = 0$$

Truncate the equation $\sum_{\Delta} \to \sum_{\Delta \leq \Delta_N}$

Evaluate at (complex) values of the cross ratio χ_i, $1 \leq i \leq N$ → Finite linear system

→ Solve for $\left\{ C^2_{1,1,\Delta_i} \right\}_{i=1}^{N}$

Different choices of $\{\chi_i\}$ lead to a different solution.
Study many possibilities and do statistics. Estimate=average. *[Picco,Ribault,Santachiara '16]*

This gives poor results for a dense spectrum.

Without the special 2D techn *aleur '20]* ,
we can use this method **pling.**

More general approach: use linear functionals

$$\mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi)+\sum_{\Delta_n} C^2_{1,1,\Delta_n}\mathcal{G}_{\Delta_n}(\chi)=0$$

$$\rightarrow\quad \alpha\left[\mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi)\right]+\sum_{\Delta_n} C^2_{1,1,\Delta_n}\,\alpha\left[\mathcal{G}_{\Delta_n}\right]=0$$

$$\alpha\left[f\right]\equiv\sum_{l=0}^{\infty}\alpha_l\, f^{(l)}(\chi)\Big|_{\chi=\frac{1}{2}}$$

The dream functionals would satisfy:

$$\alpha^{dream}_m\left[\mathcal{G}_{\Delta_n}\right]=\delta_{m,n}\quad\rightarrow\quad C^2_{1,1\Delta_n}=-\,\alpha^{dream}_n\left[\mathcal{G}_{1+\mathcal{B}_2}\right]$$

(they are known for free spectra *[Mazac Paulos '18]*)

More pragmatically, we can try to $\left[\mathcal{G}_{\Delta}\right]$ is small for $\Delta>\Delta_*$

Positive semidefinite functionals and rigorous bounds

Our 1D CFT is unitary → $C^2_{1,1\Delta} \geq 0$.

Use inequalities to bound the allowed conformal data. *[El-Showk,Paulos,Poland,Rychkov, Simmons-Duffin,Vichi '12]*

General numerical bootstrap setup:

impose $\alpha[\mathcal{G}_\Delta] \geq 0$ above a gap threshold $\Delta > \Delta_*$ (with some extra conditions),

This can be done efficiently with Semi Definite Programming.
We use the powerful package SDPB *[Simmons-Duffin '15]*

Truncation is necessary, but the **bounds are rigorous**:

$$\alpha\left[f(\chi)\right] \sim \sum_{l=0}^{N_{der}} \alpha_l \, f^{(2l)}(\chi)\Big|_{\chi=\frac{1}{2}} \quad , \quad \text{bounds better and better for } N_{der} \to \infty.$$

Bounds for the first OPE coefficient $C^2_{1,1,\Delta_1}$

Bootstrap eq.: $\mathcal{G}_{1+\mathcal{B}_2}(\lambda,\chi) + C^2_{1,1,\Delta_1}\mathcal{G}_{\Delta_1}(\chi) + \sum_{n\geq 2} C^2_{1,1,\Delta_n}\mathcal{G}_{\Delta_n}(\chi) = 0$

Upper bound alg.:

Using SDPB, find the functional such that

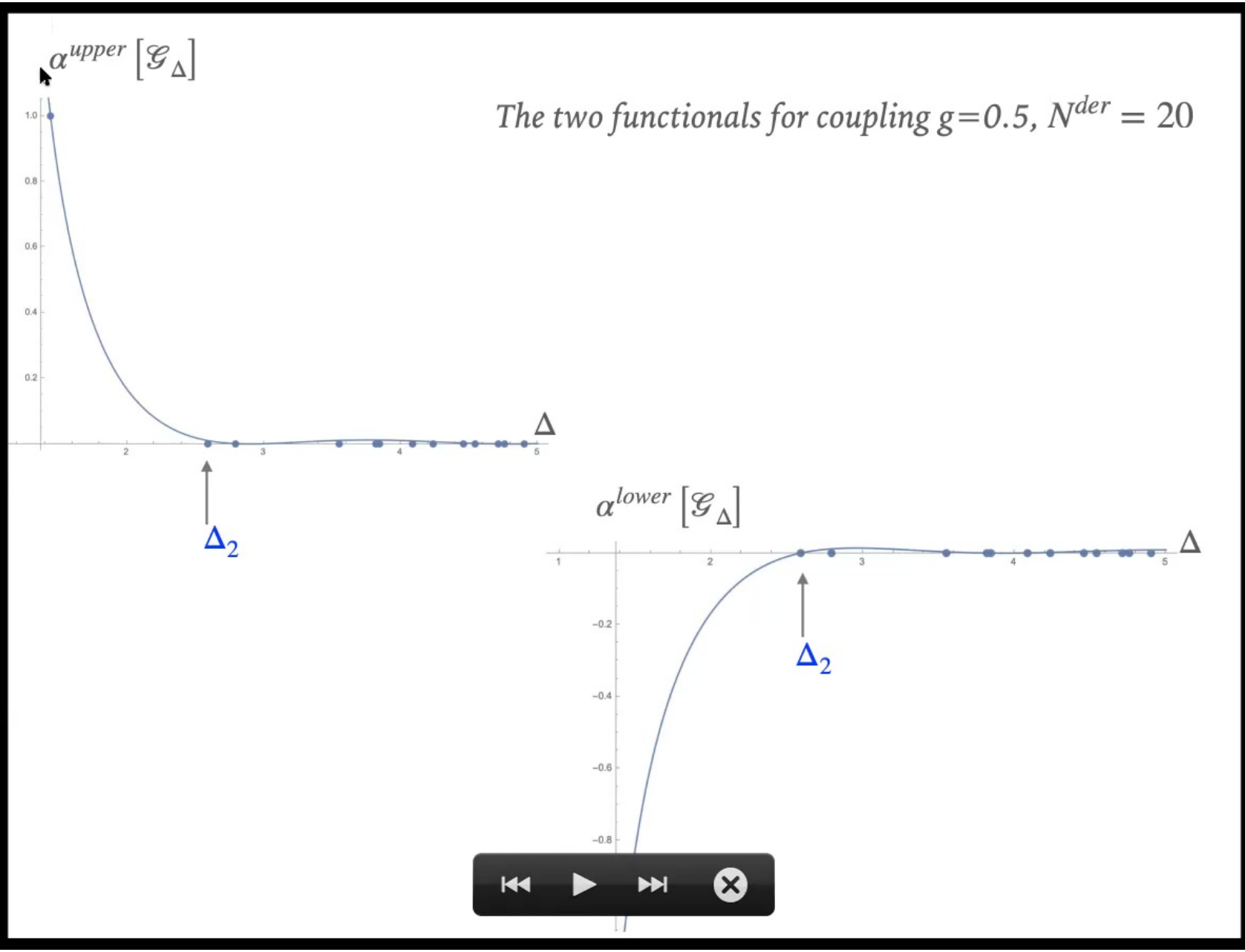
$\alpha^{upper}[\mathcal{G}_{\Delta_1}] = 1\,,$

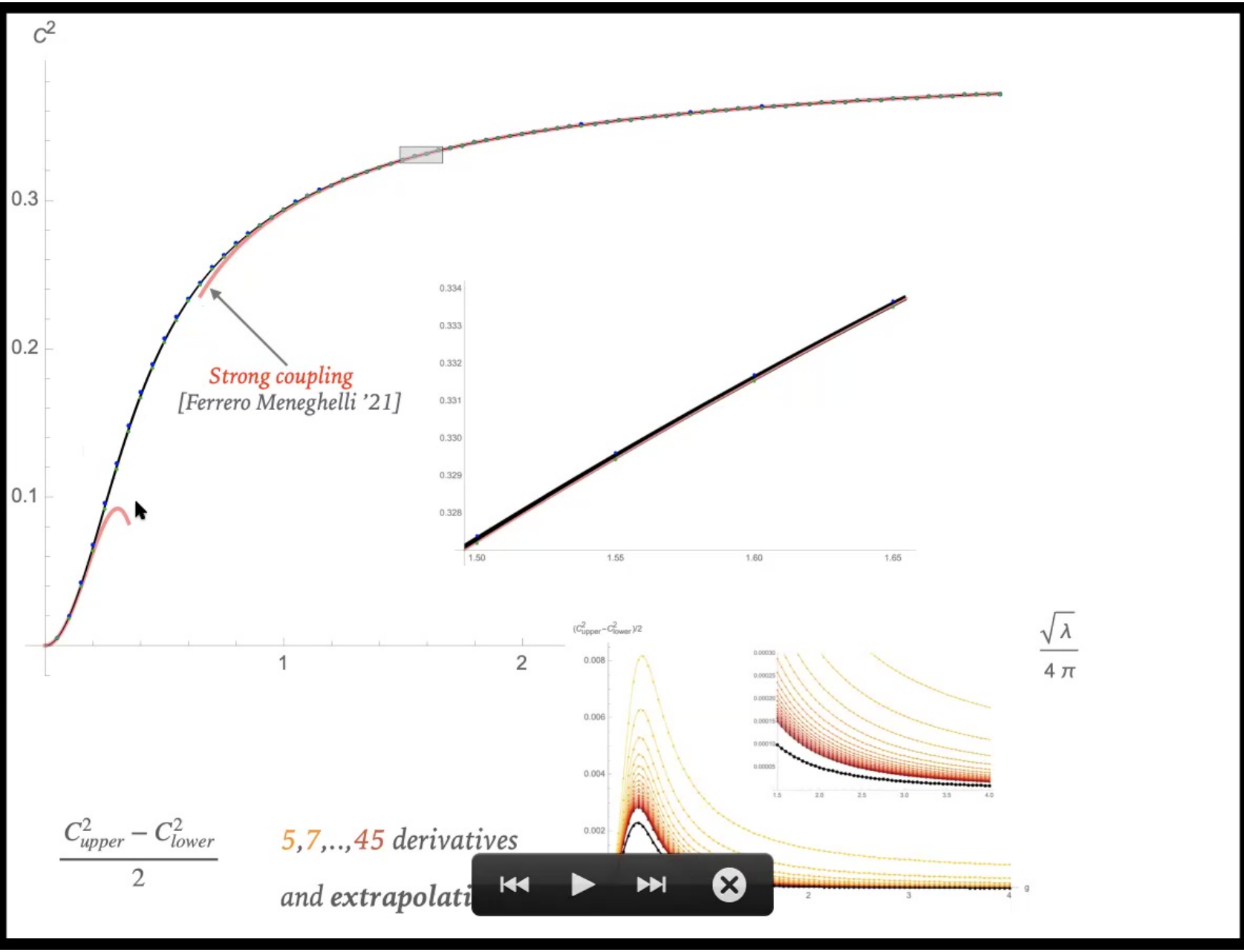
$\alpha^{upper}[\mathcal{G}_{\Delta}] \geq 0$ for $\Delta \geq \Delta_* \equiv \Delta_2$

$\alpha^{upper}[\mathcal{G}_{1+\mathcal{B}_2}]$ **is maximal** $\equiv -\mathbf{B}_{upper}$

$$\rightarrow \quad -\mathbf{B}_{upper} + C^2_{1,1,\Delta_1} + (\geq 0 \quad quantity) = 0$$

$$\rightarrow \quad C^2_{1,1,\Delta_1} \leq \mathbf{B}_{upper}$$


$\alpha^{upper}\left[\mathcal{G}_\Delta\right]$
The two functionals for coupling $g=0.5$, $N^{der}=20$
1.0
0.8
0.6
0.4
0.2
2
3
4
5
Δ
Δ_2
$\alpha^{lower}\left[\mathcal{G}_\Delta\right]$
1
2
3
4
5
Δ
Δ_2
−0.2
−0.4
−0.6
−0.8


C^2
0.3
0.2
0.1
1
2
Strong coupling
[Ferrero Meneghelli '21]
0.334
0.333
0.332
0.331
0.330
0.329
0.328
1.50
1.55
1.60
1.65
$\frac{\sqrt{\lambda}}{4\pi}$
$(C^2_{upper}-C^2_{lower})/2$
0.008
0.006
0.004
0.002
$\frac{C^2_{upper}-C^2_{lower}}{2}$
5,7,..,45 derivatives
and extrapolati

More experiments

Use $C^2_{1,1,\Delta_2}$, $C^2_{1,1,\Delta_3}$ as parameters

$$\underbrace{\mathcal{G}_{1+\mathcal{B}_2(\lambda,\chi)}+C^2_{\Delta_2}\mathcal{G}_{\Delta_2}+C^2_{\Delta_3}\mathcal{G}_{\Delta_3}}_{\equiv\mathcal{G}_{1+\mathcal{B}_2+(\Delta_2,\Delta_2)}}+C^2_{1,1,\Delta_1}\mathcal{G}_{\Delta_1}(\chi)+\sum_{n\geq 4}C^2_{1,1,\Delta_n}\mathcal{G}_{\Delta_n}(\chi)=0$$

Repeat the previous algorithm but with $\Delta_* = \Delta_4$, and

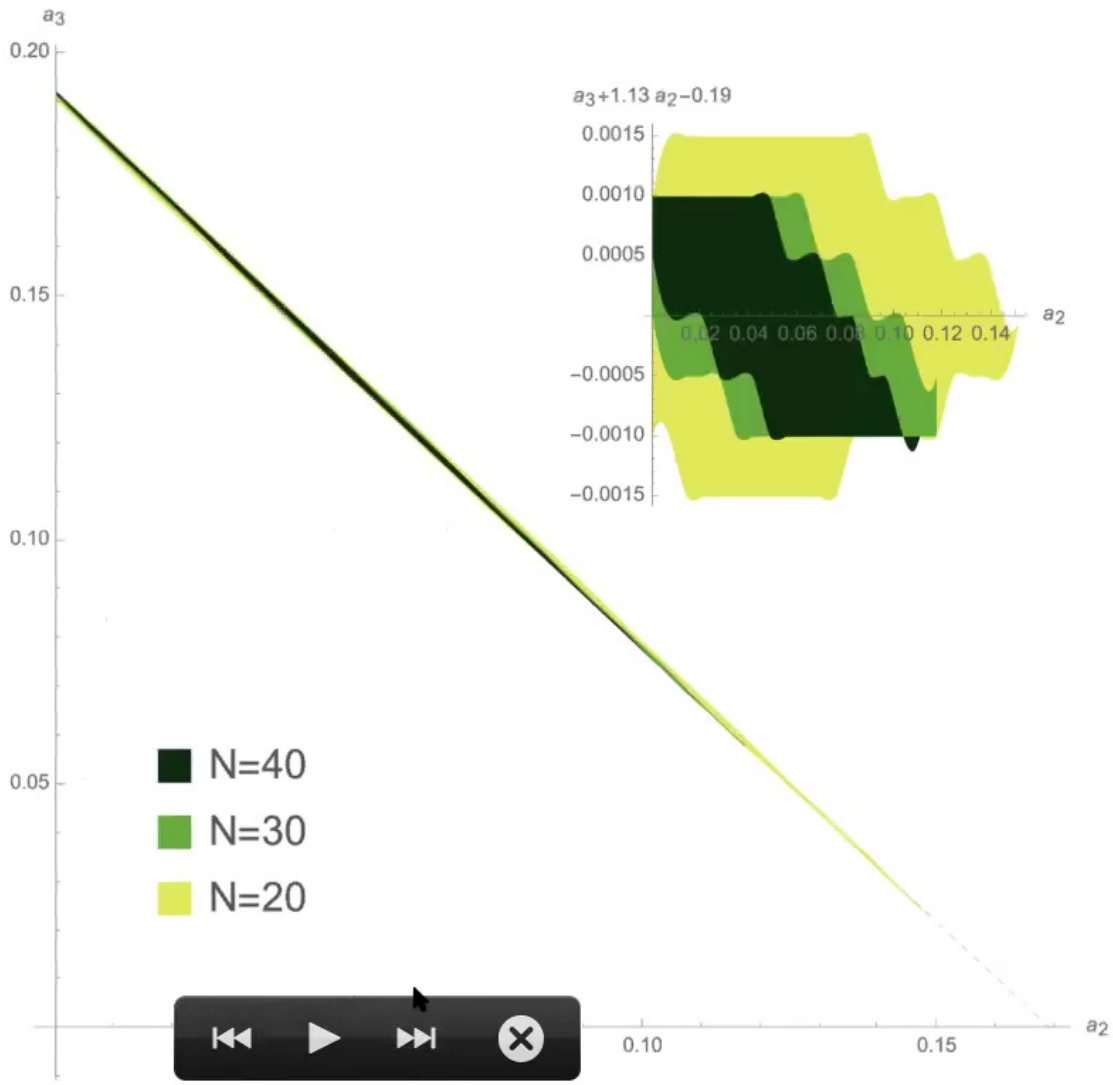
$\mathbf{B}_{lower}(C^2_{\Delta_2}, C^2_{\Delta_3})$ = **maximal value of** $\alpha[\mathcal{G}_{1+\mathcal{B}_2+(\Delta_2,\Delta_3)}]$, for $\alpha[\mathcal{G}_{\Delta_1}] = 1$

$\mathbf{B}_{upper}(C^2_{\Delta_2}, C^2_{\Delta_3}) = -$ **maximal value of** $\alpha[\mathcal{G}_{1+\mathcal{B}_2+(\Delta_2,\Delta_3)}]$, for $\alpha[\mathcal{G}_{\Delta_1}] = -1$

When do the bounds become in

Allowed island for

$a_2 = C_2^2 \, , a_3 = C_3^2$



Accurate data for an OPE coefficient combining QSC and numerical bootstrap.

So far, **finite but surprisingly good resolution**, **with input from only 2 states of the spectrum**.

Is it possible to shrink the bounds to zero?

Can we get C_n with good precision?

Can we determine mathematically a CFT just from the spectrum?

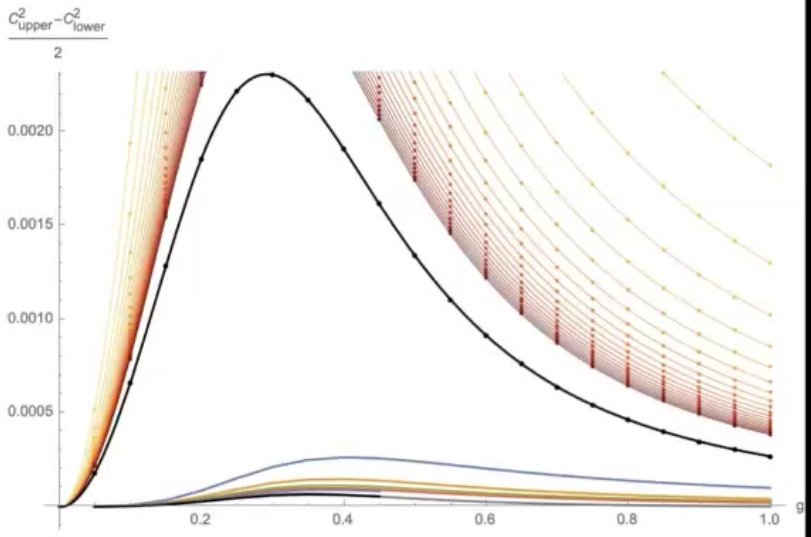
4pt of **local** operators?

Things to try to shrink the bounds:

More data from integrability

[AC, Gromov, Julius, Preti to appear]

(preliminarily, we see a gain of precision by a factor of 50!)



Multi-correlator bootstrap (as for 3D Ising)

New types of algorithms?
We can relax positivity between different states…

(preliminary: error goes down by another factor of ~ 2 at least)

Analytic functionals/Poly *[Mazac Paulos '18] [Mazac '18]*

Can we use the analytic lightcone bootstrap in 4D ?

[Alday Maldacena '07]
[Komargodski Zhiboedov '13]
[Fitzpatrick, Kaplan, Poland,Simmons-Duffin '13]

Can control infinite families of operators thanks to analyticity in spin

It works well at large N!

$$\tilde{c}(\ell,\Delta)=\frac{1+(-1)^{\ell}}{4}\tilde{\kappa}(\tfrac{\Delta+\ell}{2})\int_0^1\frac{dz}{z^2}\frac{d\bar{z}}{\bar{z}^2}\left(\frac{\bar{z}-z}{z\bar{z}}\right)^2\tilde{g}_{\ell+3,\Delta-3}(z,\bar{z})\,\mathrm{dDisc}\left[\mathcal{G}(z,\bar{z})\right]$$

[Caron-Huot '17]

Intriguing: dDisc is determined only by single-trace op's at large N!

Was used iteratively in 1/N in the sugra limit ($\lambda\to\infty$)

E.g. double-trace data at strong coupling:

$$\Delta_{0,2}=6-\frac{4}{N^2}-\frac{45}{N^4}+\cdots$$
$$\Delta_{0,4}=8-\frac{48}{25}\frac{1}{N^2}-\frac{12768}{3125}\frac{1}{N^4}+\cdots$$

[Aharony Alday Bissi Perlmutter '16]
[Alday, Bissi '17]

Can extend this to finite λ ?

Thank you!