Title: A quantum prediction as a collection of epistemically restricted classical predictions

Speakers: William Braasch

Series: Quantum Foundations

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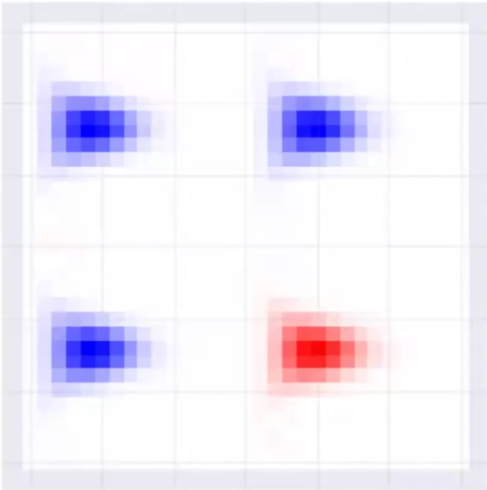
Abstract: A toy model due to Spekkens is constructed by applying an epistemic restriction to a classical theory but reproduces a host of phenomena that appear in quantum theory. The model advances the position that the quantum state may be interpreted as a reflection of an agent's knowledge. However, the model fails to capture all quantum phenomena because it is non-contextual. Here we show how a theory similar to the one Spekkens proposes requires only a single augmentation to give quantum theory for certain systems. Specifically, one must combine all possible epistemically restricted classical accounts of a quantum experiment. The rule for combination is simple: sum the nonrandom parts of all classical predictions to arrive at the nonrandom part of the quantum prediction.

A quantum prediction as a collection of epistemically restricted classical predictions



William Braasch Jr.
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Williams College

Overview

<u>Initial goal:</u> To express quantum theory in terms of phase space probability distributions.

<u>What we actually have:</u> A formulation of quantum theory in terms of phase space probability distributions for *prime power dimensions*.

<u>Main message:</u> There exists a way to combine the predictions of epistemically restricted observers to reproduce quantum predictions.

<u>Our main equation:</u> $$\Delta P(E|\mathcal{E},\rho) = \frac{1}{\mathcal{Z}} \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E},\rho)$$

Outline

- Motivation and epistemically restricted theories
- Discrete phase space and discrete Wigner functions
- Quasiprobabilistic quantum theory: states, channels, and measurements
- Classical frameworks and epistemically restricted states (borrowing tools from tomography)
- Quantum theory from collection of epistemically restricted theories
- Conclusions

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Motivation

Do quantum states represent **physical reality** or an **agent's knowledge** of physical reality?

Are quantum states **ontic** or **epistemic** objects?

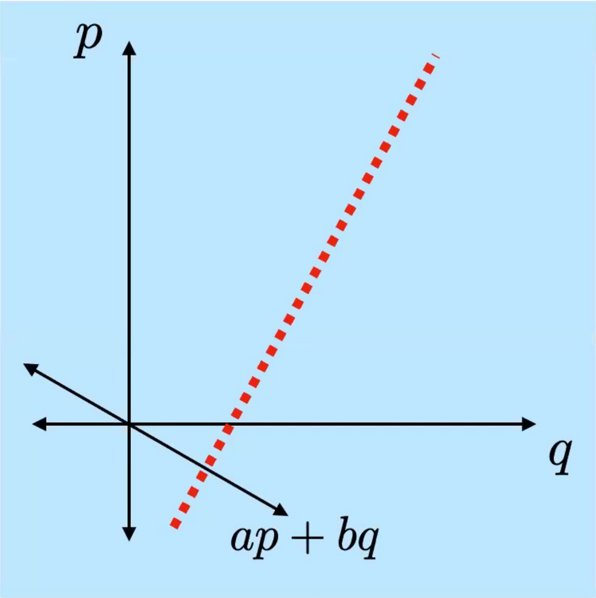
Spekkens and collaborators have developed a number of theories that demonstrate how a **classical ontological theory** subject to an **epistemic constraint** can reproduce substantial aspects of quantum theory.

RW Spekkens, Physical Review A 75 (3), 032110 (2007).
SD Bartlett, T Rudolph, RW Spekkens, Physical Review A 86 (1), 012103 (2012).
RW Spekkens, Quantum Theory: Informational Foundations and Foils, 83-135 (2016).

Epistemically restricted classical theories

Sketch of the idea:

- Start with a classical ontological theory that describes the kinematics and dynamics.
- Epistemic states are classical statistical distributions over ontic states.
- Specify criteria that determine the form of the “legal” epistemic states. (I.e. That which is knowable due to some principle.)



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Epistemically restricted classical theories

Phenomena arising in epistricted theories	Phenomena not arising in epistricted theories
Noncommutativity	Bell inequality violations
Coherent superposition	Noncontextuality inequality violations
Collapse	Computational speed-up (if it exists)
Complementarity	Certain aspects of items on the left
No-cloning	
No-broadcasting	
Interference	
Teleportation	
Remote steering	
Key distribution	
Dense coding	
Entanglement	
Monogamy of entanglement	
Choi-Jamiolkowski isomorphism	
Naimark extension	
Stinespring dilation	
Ambiguity of mixtures	
Locally immeasurable product bases	
Unextendible product bases	
Pre and post-selection effects	
Quantum eraser	
And many others...	

TABLE II: Categorization of quantum phenomena.

RW Spekkens, Quantum Theory: Informational Foundations and Foils, 83-135 (2016).

Epistemically restricted classical theories

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TABLE II: Categorization of quantum phenomena.

Our question: is there a mathematical way to get from epistemically restricted classical theories to quantum theory?

RW Spekkens, Quantum Theory: Informational Foundations and Foils, 83-135 (2016).

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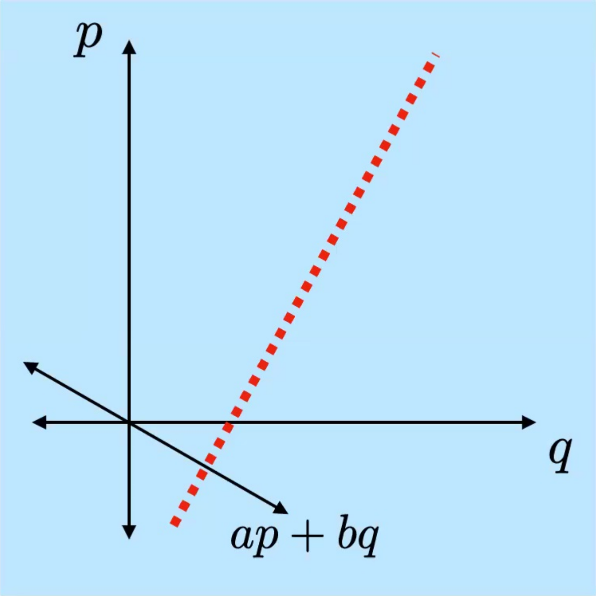

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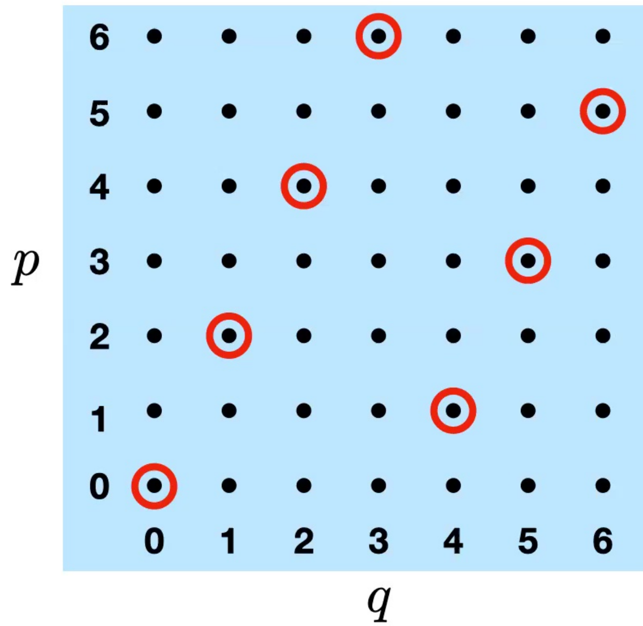
Epistemically restricted classical theories

$$5q + p = 0$$



Sketch of the idea:

- Start with a classical ontological theory that describes the kinematics and dynamics.
- Epistemic states are classical statistical distributions over ontic states.
- Specify criteria that determine the form of the "legal" epistemic states. (I.e. That which is knowable due to some principle.)

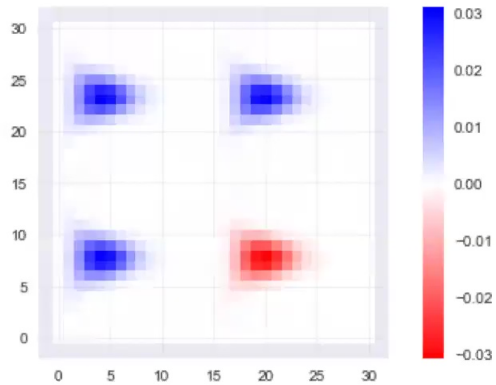
Are there any good reasons to use this set for the ontic states of discrete classical systems?

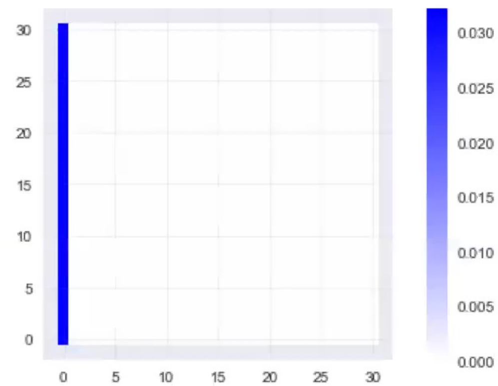
(I'm now going to focus on quantum systems with **finite**—and eventually prime—dimensional Hilbert spaces in this talk.)

Epistemically restricted classical theories

One can represent quantum states with Wigner functions.

They exists for both discrete and continuous systems.





For the stabilizer subtheory of quantum mechanics, Wigner functions can be used as a hidden variable model (HVM).

This motivates the definition of the classical theory of d-level systems with this discrete phase space.

Next step is to find an epistemic restriction that yields the HVM.

Epistemically restricted classical theories

"Guiding analogy:

A set of observables is *jointly measurable* if and only if it is commuting relative to the matrix commutator.

A set of variables is *jointly knowable* if and only if it is commuting relative to the Poisson bracket."

[RW Spekkens, Quantum Theory: Informational Foundations and Foils, 83-135 (2016).]

Epistemically restricted classical theories

The epistemic restriction:

- Knowable variables can only be linear combinations of position and momentum or **quadrature variables**:

$$aq + bp \quad \text{where} \quad a, b \in \mathbb{Z}_d$$

- Jointly knowable variables must commute via poisson bracket (PB): $[f, g]_{PB} = 0$

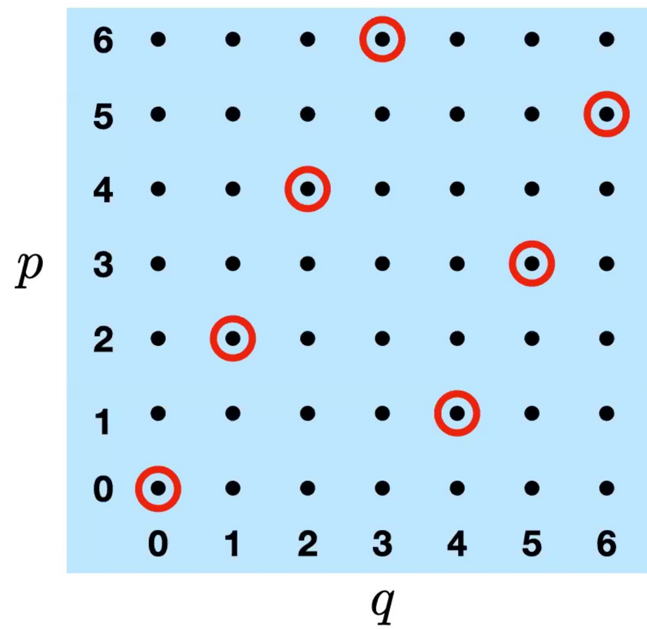
There exists a reasonable discrete analogue of the PB using finite differences.

The value of the discrete PB for two quadrature variables is equal to the symplectic inner produce of the two variables.

$$[f, g]_{PB} = \langle f, g \rangle = f^T J g \quad \text{where} \quad J \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & \dots \\ -1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & \\ 0 & 0 & -1 & 0 & \\ \vdots & & & & \ddots \end{pmatrix}$$

Epistemically restricted classical theories

- For the systems we are considering, we only need two dimensional phase space, i.e. one position and one momentum variable.
- States of maximal knowledge are uniform distributions over **lines** in phase space.



$$5q + p = 0$$

Epistemically restricted classical theories

"… why bother with the symplectic stuff in this talk?"

The epistemic restriction must be preserved by the dynamics.

Affine symplectic transformations map the set of quadrature variables to itself.

(These are symplectic transformations followed by a displacement across phase space.)

These are the "legal" transformations for the epistemically restricted theory.

Epistemically restricted classical theories

Main messages:

For systems with a single degree of freedom, the ontic states are given by a single position and momentum coordinate in phase space.

The epistemic states are lines representing a uniform probability distribution over certain ontic states.

The legal transformations are affine symplectic transformations.

Our question:

How can finite-dimensional quantum systems be represented in terms of such a structure?

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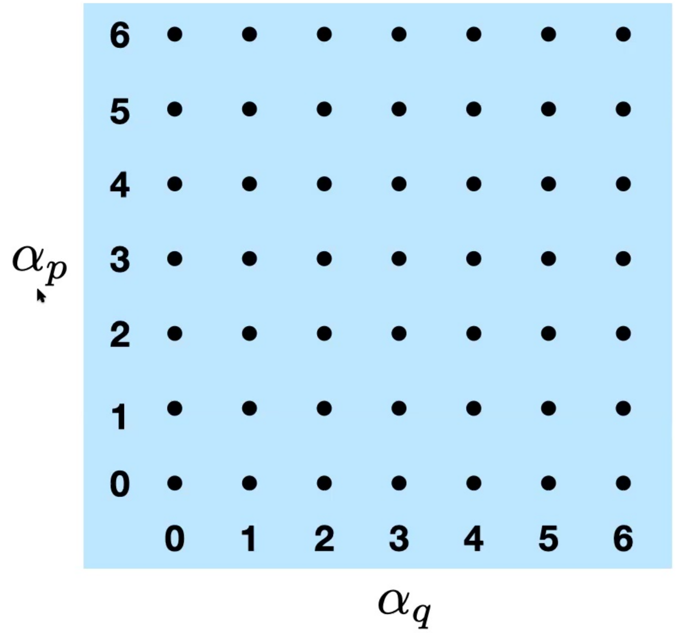
Discrete phase space

Assume the Hilbert space dimension, d , is prime.

Here we have a discrete phase space when $d = 7$.

Arithmetic performed on such a space will be modulo d .



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Phase-point operators

- A complete, Hermitian operator basis indexed by phase space points: A_α

$$\rho = \sum_\alpha Q(\alpha|\rho) A_\alpha$$

- The A_α's are orthogonal relative to the Hilbert-Schmidt inner product:

$$\mathrm{Tr}[A_\alpha A_\beta] = d\delta_{\alpha\beta}$$

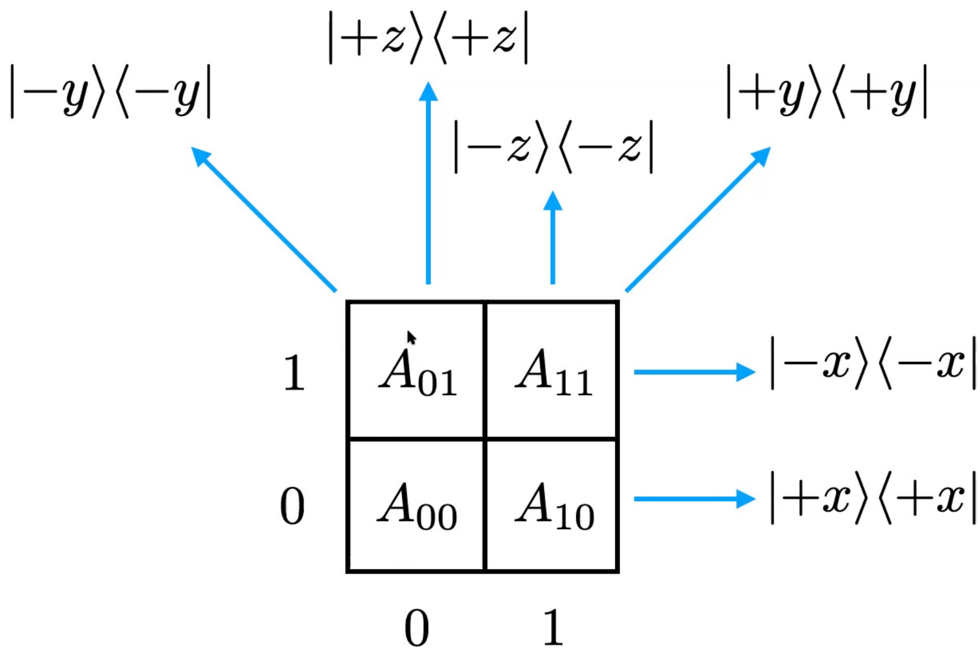
- Key property: the average of phase-point operators along any line gives a projector onto a **pure state**.

- A complete set of parallel lines (a striation) defines an **orthonormal basis** for the Hilbert Space.

- Bases associated with different striations are **mutually unbiased**.

W. K. Wootters, Ann. Phys. 176, 1 (1987).
K. S. Gibbons, M. J. Hoffman and W. K. Wootters, Phys. Rev. A 70, 062101 (2004).
D. Gross, J. Math. Phys. 47, 122107 (2006).

Phase-point operators for a qubit



An arrow means "average."

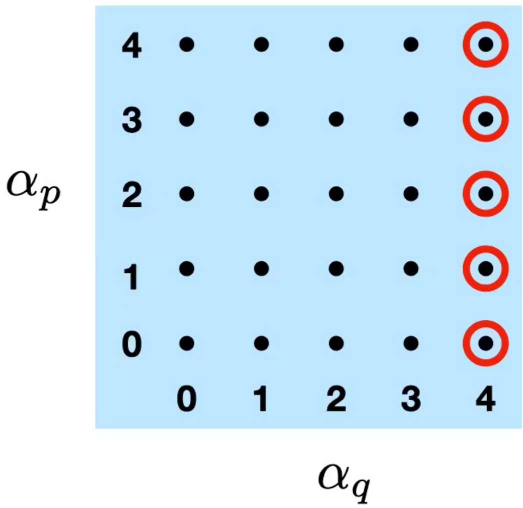
Discrete Wigner functions

- The expansion coefficients for density matrices over the phase-point operators:

$$\rho = \sum_\alpha Q(\alpha|\rho) A_\alpha, \qquad Q(\alpha|\rho) = \frac{1}{d}\mathrm{Tr}[\rho A_\alpha]$$

- Remember: the average of phase-point operators along any line gives a projector onto a **pure state**.

- Discrete Wigner functions **give proper marginal distributions.**



$$\sum_{\alpha\in\lambda} Q(\alpha|\rho) = \sum_{\alpha\in\lambda}\frac{1}{d}\mathrm{Tr}[A_\alpha \rho]$$
$$= \mathrm{Tr}\left[\frac{1}{d}\sum_{\alpha\in\lambda} A_\alpha \rho\right]$$
$$= \mathrm{Tr}\left[|4\rangle\langle 4|\rho\right]$$
$$= P(4|\rho)$$

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Discrete Wigner functions

$$\rho = \sum_{\alpha} Q(\alpha|\rho) A_\alpha, \qquad Q(\alpha|\rho) = \frac{1}{d}\mathrm{Tr}[\rho A_\alpha]$$

Properties of $Q(\alpha|\rho)$:

- Real distribution on phase space.
- Normalized to 1.
- Sum over any line is the probability of the state associated with the line.
- Can have **negative** values.

Discrete Wigner function examples

$$|0\rangle \longrightarrow \begin{array}{|c|c|}\hline \frac{1}{2} & 0 \\ \hline \frac{1}{2} & 0 \\ \hline \end{array}$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \longrightarrow \begin{array}{|c|c|}\hline 0 & 0 \\ \hline \frac{1}{2} & \frac{1}{2} \\ \hline \end{array}$$

$$\frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle \longrightarrow \frac{1}{8} \times \begin{array}{|c|c|}\hline 3-\sqrt{3} & 1+\sqrt{3} \\ \hline 3+\sqrt{3} & 1-\sqrt{3} \\ \hline \end{array}$$

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Quasiprobabilistic representation of quantum theory

- Preparation ρ:
  Wigner function $Q(\alpha|\rho)$
- Channel $\mathcal{E}$:
  Transition quasiprobabilities $Q_{\mathcal{E}}(\beta|\alpha)$
- Measurement outcome E:
  Response quasiprobability $Q(E|\beta)$

$$P(E|\mathcal{E},\rho) = \sum_{\beta,\alpha} Q(E|\beta)Q_{\mathcal{E}}(\beta|\alpha)Q(\alpha|\rho)$$

Channel: example of transition quasiprobabilities

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$$

Rotation of a qubit by 90° around the +y axis.

$Q_U(\beta|\alpha)$

β \ α	00	01	10	11
00	1/2	1/2	-1/2	1/2
01	-1/2	1/2	1/2	1/2
10	1/2	1/2	1/2	-1/2
11	1/2	-1/2	1/2	1/2

	0	1
1	0	0
0	1	0

$\xrightarrow{U}$

	0	1
1	-1/2	1/2
0	1/2	1/2

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Measurement: examples of response quasiprobabilities

Quasiprobability of getting the outcome:

$$|0\rangle \longrightarrow \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 1 & 0 \\ \hline\end{array}$$

Normalization differs from that of Wigner functions.

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \longrightarrow \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 1 \\ \hline\end{array}$$

$$\frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle \longrightarrow \frac{1}{4} \times \begin{array}{|c|c|}\hline 3-\sqrt{3} & 1+\sqrt{3} \\ \hline 3+\sqrt{3} & 1-\sqrt{3} \\ \hline\end{array}$$

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Quasiprobabilistic representation of quantum theory

$$P(E|\mathcal{E},\rho) = \sum_{\beta,\alpha} Q(E|\beta)Q_{\mathcal{E}}(\beta|\alpha)Q(\alpha|\rho)$$

Prep: $|+x\rangle\langle+x| =$

1	0	0
0	1/2	1/2
	0	1

$=$

00	1/2
01	0
10	1/2
11	0

Channel:

$U =$

00	1/2	1/2	-1/2	1/2
01	-1/2	1/2	1/2	1/2
10	1/2	1/2	1/2	-1/2
11	1/2	-1/2	1/2	1/2
	00	01	10	11

Meas:

$|+x\rangle\langle+x| =$

1	0	1	0
00	01	10	11

Quasiprobabilistic representation of quantum theory

$$P(E|\mathcal{E},\rho) = \sum_{\beta,\alpha} Q(E|\beta)Q_{\mathcal{E}}(\beta|\alpha)Q(\alpha|\rho)$$

$$P\big(|+x\rangle \big| U, |+x\rangle\big) =$$

1	0	1	0

1/2	1/2	-1/2	1/2
-1/2	1/2	1/2	1/2
1/2	1/2	1/2	-1/2
1/2	-1/2	1/2	1/2

1/2
0
1/2
0

$$= 1/2$$

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Epistemically restricted probability distributions

"To specify an epistemic state on must specify:

1. the set of quadrature variables that are known to that agent and

2. the values of these variables."

RW Spekkens, Quantum Theory: Informational Foundations and Foils, 83-135 (2016).

Epistemically restricted probability distributions

For each component of an experiment
(preparation, channel, measurement outcome),

1. Choose a **"framework"** that imposes a certain form of the probability distribution.

2. Construct a **restricted** probability distribution R that conforms to the framework applied to Q.

Each distribution R is a **non-negative** probability distribution.
The set of all R's for **all possible** frameworks contains the same information as Q.

Epistemically restricted states

Framework choice is a **striation** (B).

Get R^B by averaging the Wigner function over each line.

All other marginals will then be uniform.

$Q(\alpha|\rho)$: $\frac{1}{8} \times$

$3-\sqrt{3}$	$1+\sqrt{3}$
$3+\sqrt{3}$	$1-\sqrt{3}$

$\longleftarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$

Slope: ∞ $\quad 1 \quad 0$

Slope ∞ — $R^B(\alpha|\rho)$: $\frac{1}{8} \times$

3	1
3	1

Slope 1 — $\frac{1}{8} \times$

$2-\sqrt{3}$	$2+\sqrt{3}$
$2+\sqrt{3}$	$2-\sqrt{3}$

Slope 0 — $\frac{1}{8} \times$

2	2
2	2

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Epistemically restricted states

Recover Q from the R's : $\Delta Q(\alpha|\rho) = \sum_B \Delta R^B(\alpha|\rho)$

$Q(\alpha|\rho)$: $\frac{1}{8} \times$

$3-\sqrt{3}$	$1+\sqrt{3}$
$3+\sqrt{3}$	$1-\sqrt{3}$

$\leftarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$

Slope: ∞ $\quad 1 \quad 0$

$R^B(\alpha|\rho)$: $\frac{1}{8} \times$

3	1
3	1

$\frac{1}{8} \times$

$2-\sqrt{3}$	$2+\sqrt{3}$
$2+\sqrt{3}$	$2-\sqrt{3}$

$\frac{1}{8} \times$

2	2
2	2

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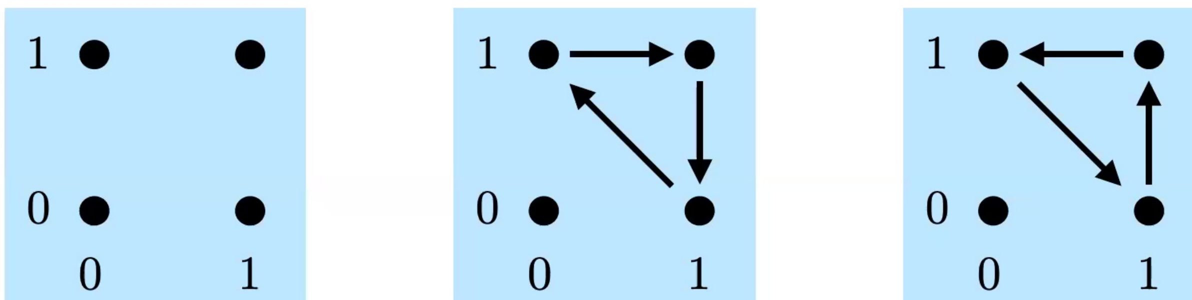
Epistemically restricted channels

- Framework choice is a symplectic matrix

- These are the “legal” symplectic matrices for a qubit

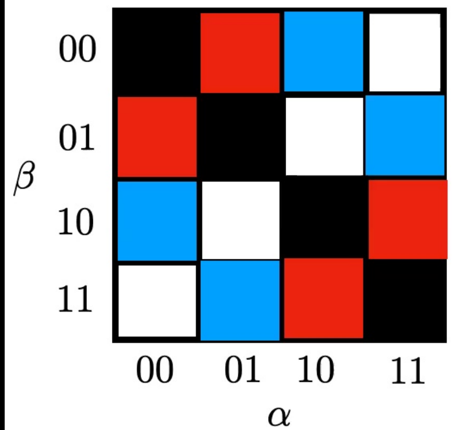
$$\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathcal{R} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad \mathcal{L} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

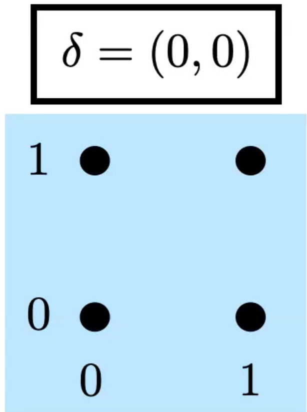


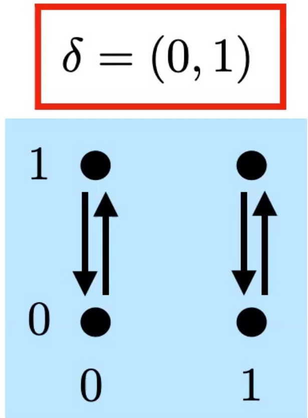
Epistemically restricted channels

- "Displacement classes" associated with a symplectic matrix S
- Different displacement classes are labeled with displacements δ
- A set of pairs of points (α, β) such that $\beta = S\alpha + \delta$

$$\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$







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Epistemically restricted channels

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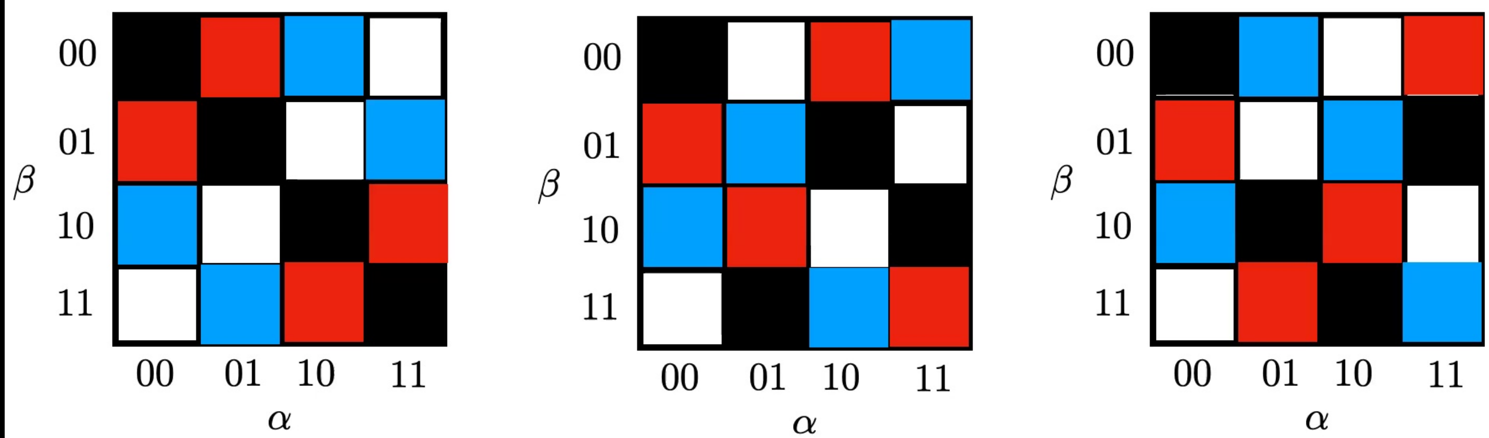
- A set of pairs of points (α, β) such that $\beta = S\alpha + \delta$

$$\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathcal{R} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad \mathcal{L} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

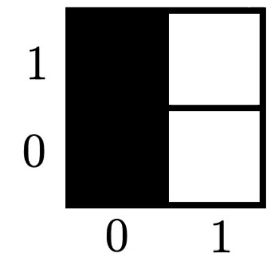


Displacement classes

Analogy between states and channels

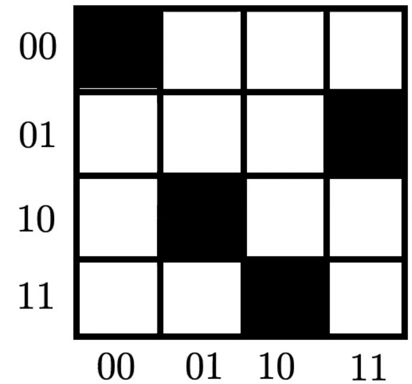
States:

1. Average quasiprobabilities of a line
2. Set value at all points in the line to this *probability*



Channels:

1. Average transition quasiprobabilities corresponding to those of an affine symplectic transformation
2. Set value at all points in the displacement class to this *probability*



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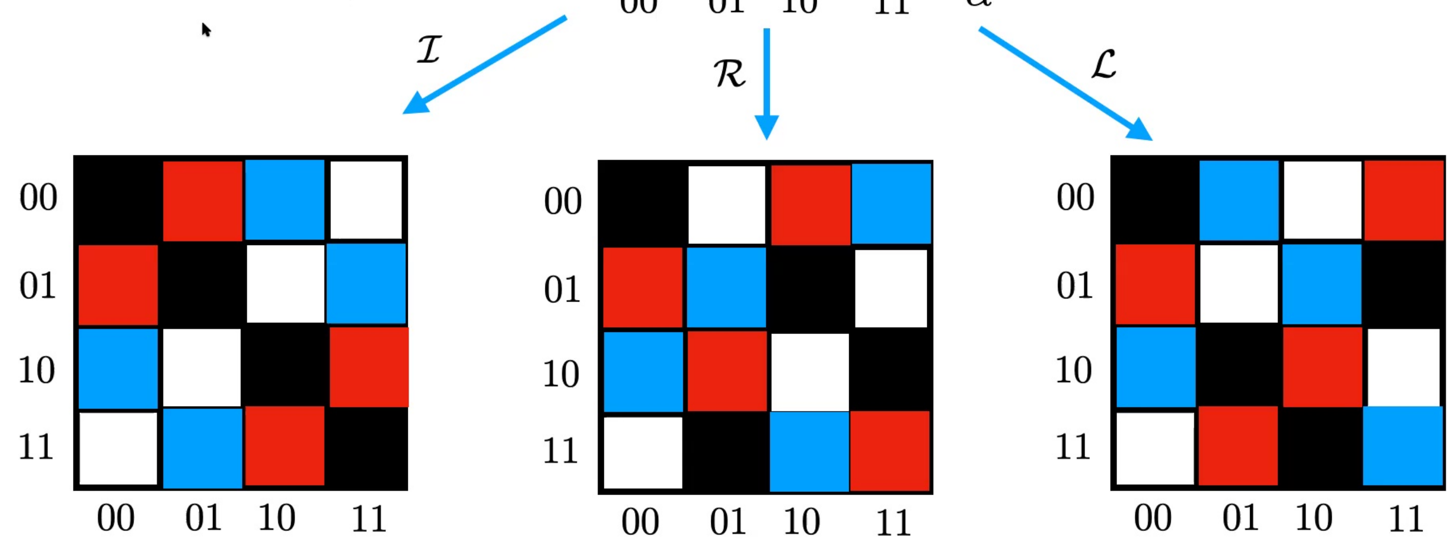
Epistemically restricted channels

Get $R^S(\beta|\alpha)$ by averaging $Q(\beta|\alpha)$ over each displacement class.

$Q_U(\beta|\alpha)$

β \ α	00	01	10	11
00	$1/2$	$1/2$	$-1/2$	$1/2$
01	$-1/2$	$1/2$	$1/2$	$1/2$
10	$1/2$	$1/2$	$1/2$	$-1/2$
11	$1/2$	$-1/2$	$1/2$	$1/2$



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Epistemically restricted channels

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$Q_U(\beta|\alpha)$:

β \ α	00	01	10	11
00	$1/2$	$1/2$	$-1/2$	$1/2$
01	$-1/2$	$1/2$	$1/2$	$1/2$
10	$1/2$	$1/2$	$1/2$	$-1/2$
11	$1/2$	$-1/2$	$1/2$	$1/2$

$R^S(\beta|\alpha)$

$\mathcal{I}$

β \ α	00	01	10	11
00	$1/2$	0	0	$1/2$
01	0	$1/2$	$1/2$	0
10	0	$1/2$	$1/2$	0
11	$1/2$	0	0	$1/2$

$\mathcal{R}$

β \ α	00	01	10	11
00	0	$1/2$	0	$1/2$
01	0	$1/2$	0	$1/2$
10	$1/2$	0	$1/2$	0
11	$1/2$	0	$1/2$	0

$\mathcal{L}$

β \ α	00	01	10	11
00	$1/2$	$1/2$	0	0
01	0	0	$1/2$	$1/2$
10	$1/2$	$1/2$	0	0
11	0	0	$1/2$	$1/2$

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Epistemically restricted channels

Recover $Q(\beta|\alpha)$ from the R's : $\Delta Q_{\mathcal{E}}(\beta|\alpha) = \sum_S \Delta R^S_{\mathcal{E}}(\beta|\alpha)$

(Channels must be unital.)

$Q_U(\beta|\alpha)$:

β \ α	00	01	10	11
00	1/2	1/2	−1/2	1/2
01	−1/2	1/2	1/2	1/2
10	1/2	1/2	1/2	−1/2
11	1/2	−1/2	1/2	1/2

$R^S(\beta|\alpha)$

$\mathcal{I}$

	00	01	10	11
00	1/2	0	0	1/2
01	0	1/2	1/2	0
10	0	1/2	1/2	0
11	1/2	0	0	1/2

$\mathcal{R}$

	00	01	10	11
00	0	1/2	0	1/2
01	0	1/2	0	1/2
10	1/2	0	1/2	0
11	1/2	0	1/2	0

$\mathcal{L}$

	00	01	10	11
00	1/2	1/2	0	0
01	0	0	1/2	1/2
10	1/2	1/2	0	0
11	0	0	1/2	1/2

Epistemically restricted measurements

Framework choice is a **striation** (B').

Get $R^{B'}(E|\beta)$ by averaging $Q(E|\beta)$ over each line.

$Q(E|\beta)$: $\frac{1}{4} \times$

$3-\sqrt{3}$	$1+\sqrt{3}$
$3+\sqrt{3}$	$1-\sqrt{3}$

$\longleftarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$

Slope: ∞

$R^{B'}(E|\beta)$: $\frac{1}{4} \times$

3	1
3	1

1

$\frac{1}{4} \times$

$2-\sqrt{3}$	$2+\sqrt{3}$
$2+\sqrt{3}$	$2-\sqrt{3}$

0

$\frac{1}{4} \times$

2	2
2	2

Epistemically restricted measurements

Recover Q from the $R's$: $\Delta Q(E|\beta) = \sum_{B'} \Delta R^{B'}(E|\beta)$

$Q(E|\beta)$: $\frac{1}{4} \times$

$3-\sqrt{3}$	$1+\sqrt{3}$
$3+\sqrt{3}$	$1-\sqrt{3}$

$\longleftarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$

Slope: ∞ 1 0

$R^{B'}(E|\beta)$: $\frac{1}{4} \times$

3	1
3	1

$\frac{1}{4} \times$

$2-\sqrt{3}$	$2+\sqrt{3}$
$2+\sqrt{3}$	$2-\sqrt{3}$

$\frac{1}{4} \times$

2	2
2	2

Prediction of an epistemically restricted observer

$$R^{\mathcal{F}}(E|\mathcal{E},\rho) = \sum_{\beta,\alpha} R^{B'}(E|\beta) R^{S}_{\mathcal{E}}(\beta|\alpha) R^{B}(\alpha|\rho)$$

$$\mathcal{F} = (B', S, B)$$

It is not the quantum mechanical probability.

However, the probability $R^{\mathcal{F}}(E|\mathcal{E},\rho)$ is a reasonable number between 0 and 1.

It is usually a **bad** prediction.

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Prediction of an epistemically restricted observer

$$R^{\mathcal{F}}(E|\mathcal{E},\rho) = \sum_{\beta,\alpha} R^{B'}(E|\beta) R^{S}_{\mathcal{E}}(\beta|\alpha) R^{B}(\alpha|\rho)$$

$$\mathcal{F} = (B', S, B)$$

For **most** values of B', S, and B, this prediction is what we would expect if ρ were the completely mixed state.

In our final equation with the Δ's, most of the terms in the sum will be equal to zero.

To avoid this, we need to set $B' = SB$.
Then we call the framework "coherent."

Outline

- Motivation and epistemically restricted theories
- Discrete phase space and discrete Wigner functions
- Quasiprobabilistic quantum theory: states, channels, and measurements
- Classical frameworks and epistemically restricted states (borrowing tools from tomography)
- → Quantum theory from collection of epistemically restricted theories
- Conclusions

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Collecting all the classical predictions

Can we get the quantum prediction by combining the "nonrandom parts" of all the classical predictions?

$$\Delta P(E|\mathcal{E},\rho) = \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E},\rho)$$

Yes, we need a "minimal reconstructing set" of symplectic matrices. This is a set of $d^2 - 1$ symplectic matrices such that the **difference** between any two of them has **nonzero determinant**.

Such a set exists if $d = 2, 3, 5, 7, 11.$

We do not know whether such a set exists for any other dimension.

[H. F. Chau, IEEE Trans. Inf. Theory 51, 1451 (2005).]

Collecting all the classical predictions

Can we get the quantum prediction by combining the "nonrandom parts" of all the classical predictions?

$$\Delta P(E|\mathcal{E}, \rho) = \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E}, \rho)$$

For any odd prime—and we can generalize to odd prime powers—if we use **all** symplectic matrices, we have

$$\Delta P(E|\mathcal{E}, \rho) = \frac{1}{d} \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E}, \rho)$$

The $1/d$ factor accounts for a **redundancy** introduced when all symplectic matrices are used.

Collecting all the classical predictions

General equation: $$\Delta P(E|\mathcal{E},\rho) = \frac{1}{\mathcal{Z}} \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E},\rho)$$

When using a minimal reconstructing set, $\mathcal{Z} = 1$.

When using all the symplectic matrices, $\mathcal{Z} = d$.

Collecting all the classical predictions

General equation: $$\Delta P(E|\mathcal{E},\rho) = \frac{1}{\mathcal{Z}} \sum_{\mathcal{F}} \Delta R^{\mathcal{F}}(E|\mathcal{E},\rho)$$

Is it surprising that we can get the quantum prediction?

Not really. We started with quantum theory via the Wigner function.

In order to start with just the R's, we need constraints that enforce the "legality" of the various probability distributions.

What does it mean?

- This mathematical method of combination does not connect with common notions in probability theory.
- There is no obvious state of affairs or ontology that presents itself.
- To bridge the “gap” between epistemically restricted theories and full quantum theory, one is forced to make a mathematical step that does not easily fit into a principled framework.
- To be clear, this does not imply that an epistemic interpretation of the quantum state must be abandoned. We’ve just looked at one specific approach.

Future research directions

- Search for a “native” description of the global constraint on the R’s.
- Work on extending to composite numbers.
- Engage with QBists.
- Explain how contextuality is restored.
- Explore connections with decoherence.

Conclusion

Epistemically restricted classical theories present a compelling argument for an epistemic interpretation of the quantum state.

One way to reach full quantum theory from such classical theories requires a simple—yet unusual—combination of the classical predictions.

This can be thought of as one way in which quantum theory requires us to go beyond classical probability theory.

arXiv:2107.02728 [quant-ph]

Conclusion

Epistemically restricted classical theories present a compelling argument for an epistemic interpretation of the quantum state.

One way to reach full quantum theory from such classical theories requires a simple—yet unusual—combination of the classical predictions.

This can be thought of as one way in which quantum theory requires us to go beyond classical probability theory.

Thank you for your attention!

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