Title: Perverse sheaves and relative Langlands duality

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Series: Mathematical Physics

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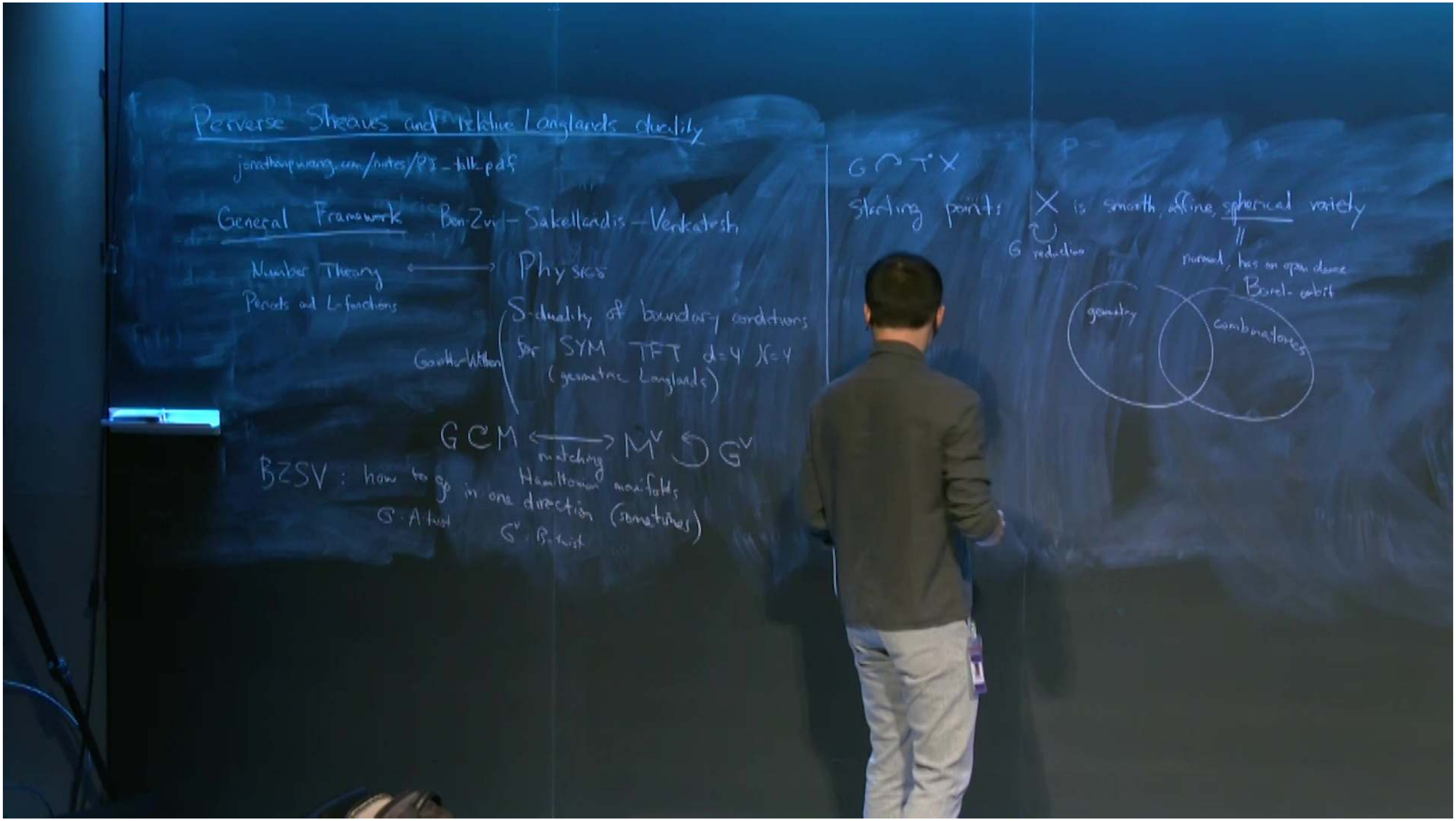
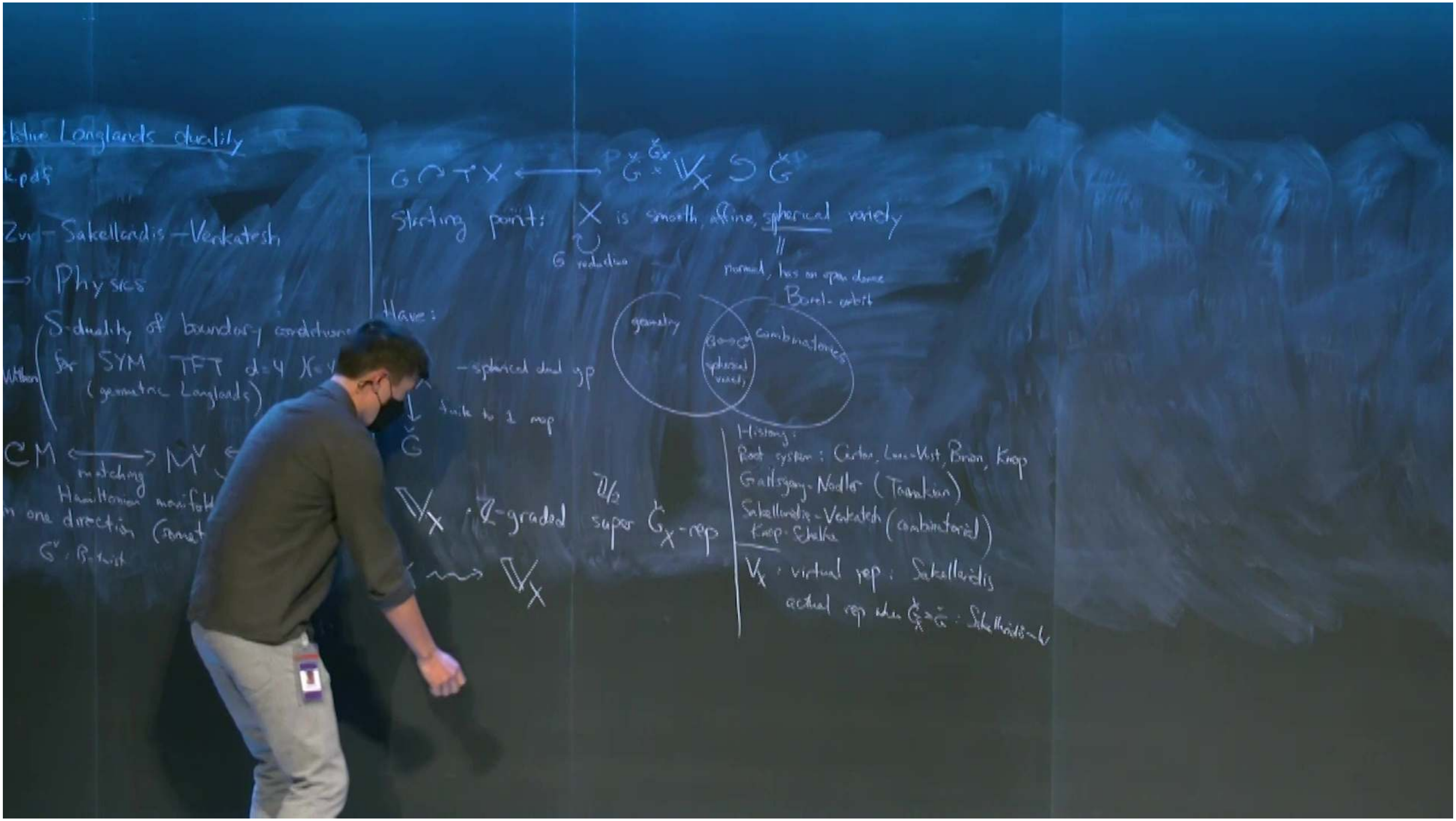
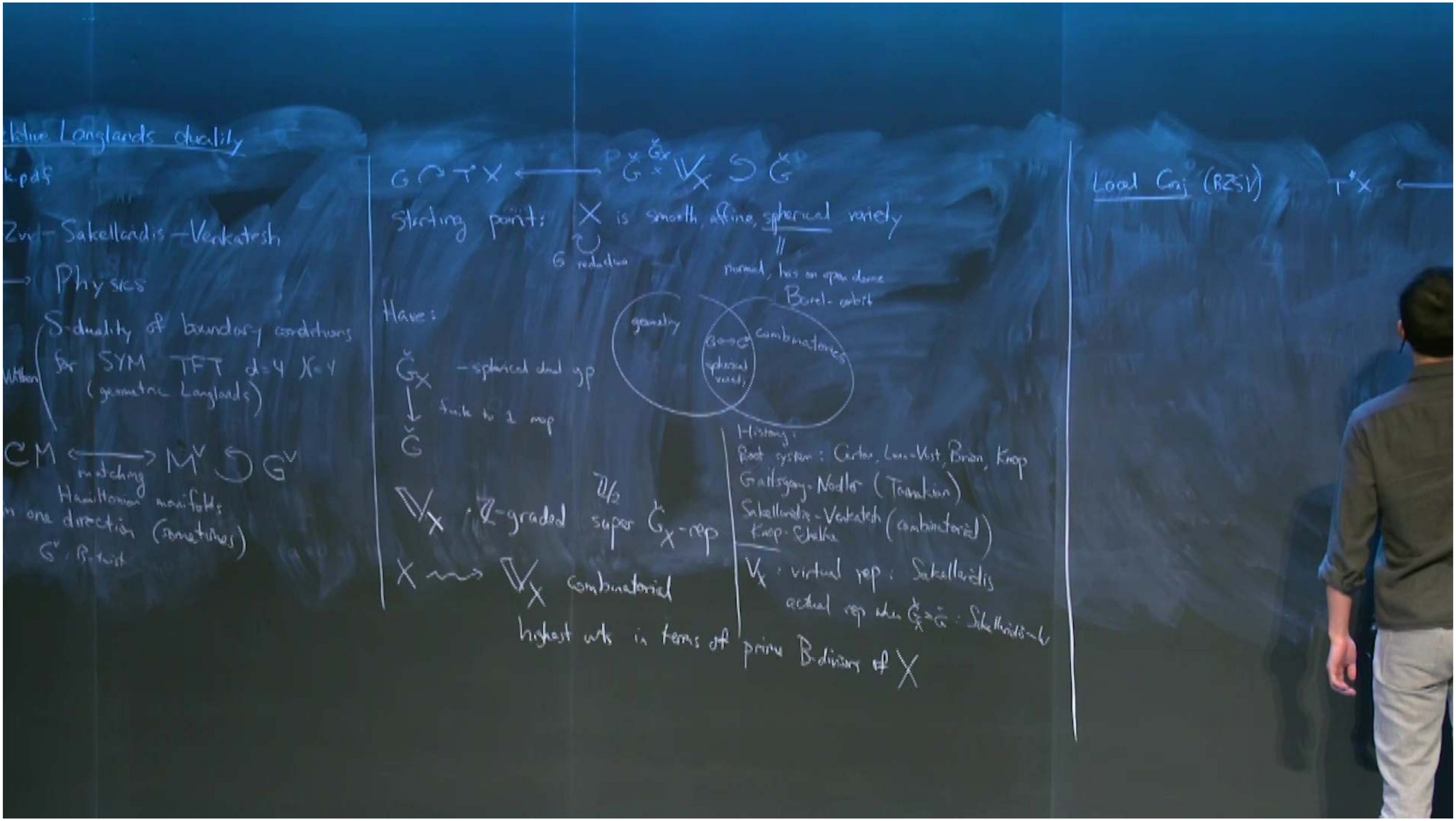
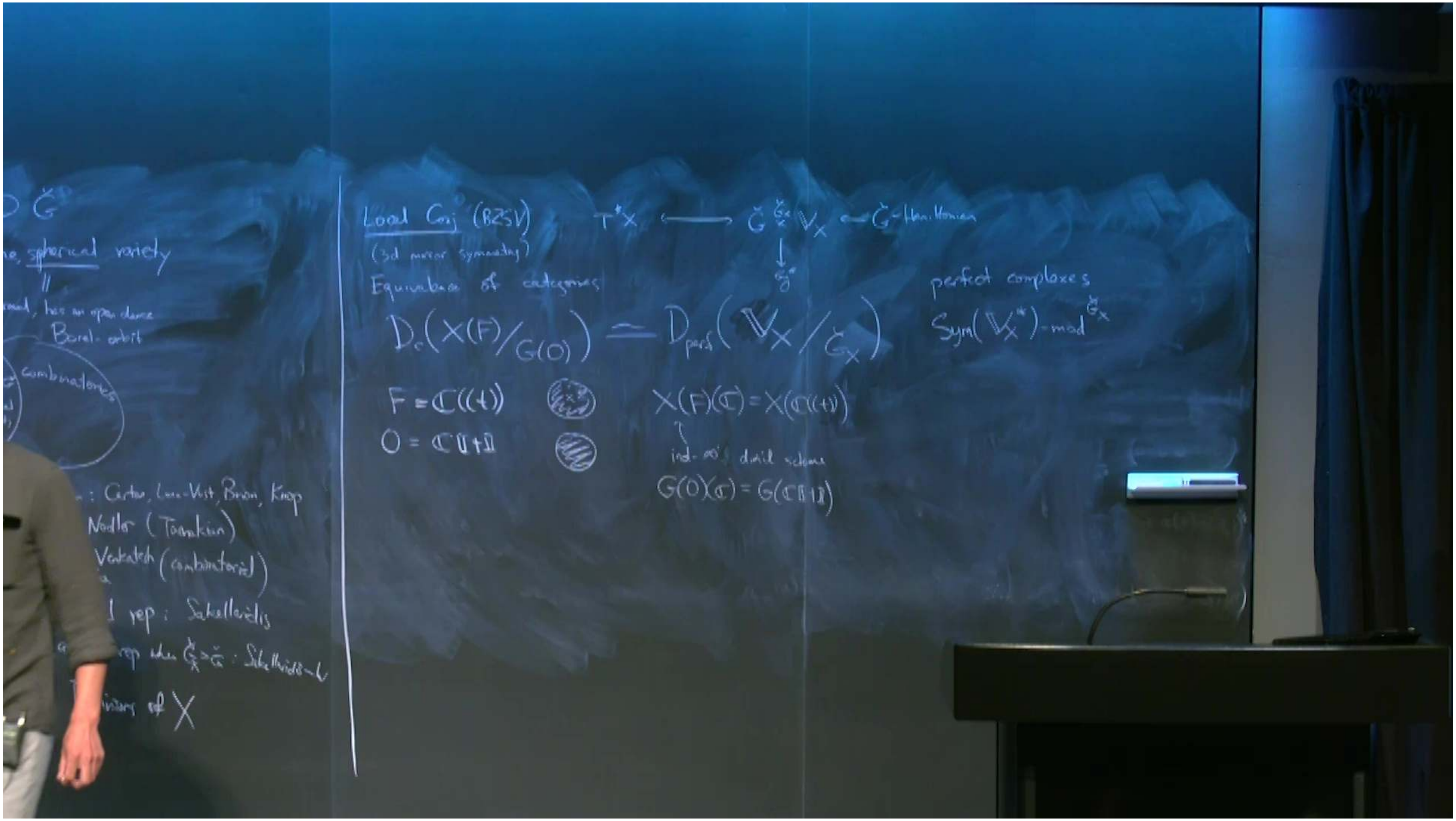
Abstract: The program of Ben-Zvi--Sakellaridis--Venkatesh connects the construction of L-functions in number theory with S-duality of boundary conditions in 4d. In particular this predicts certain equivalences of categories between equivariant D-modules on the formal loop space of a smooth variety X and equivariant quasi-coherent sheaves on a Hamiltonian manifold. I discuss an extension of this conjecture to certain singular varieties X and the possibility of quantizing the equivalence. I will explain joint work with Yiannis Sakellaridis on computing a certain factorization algebra which plays a role in the story.

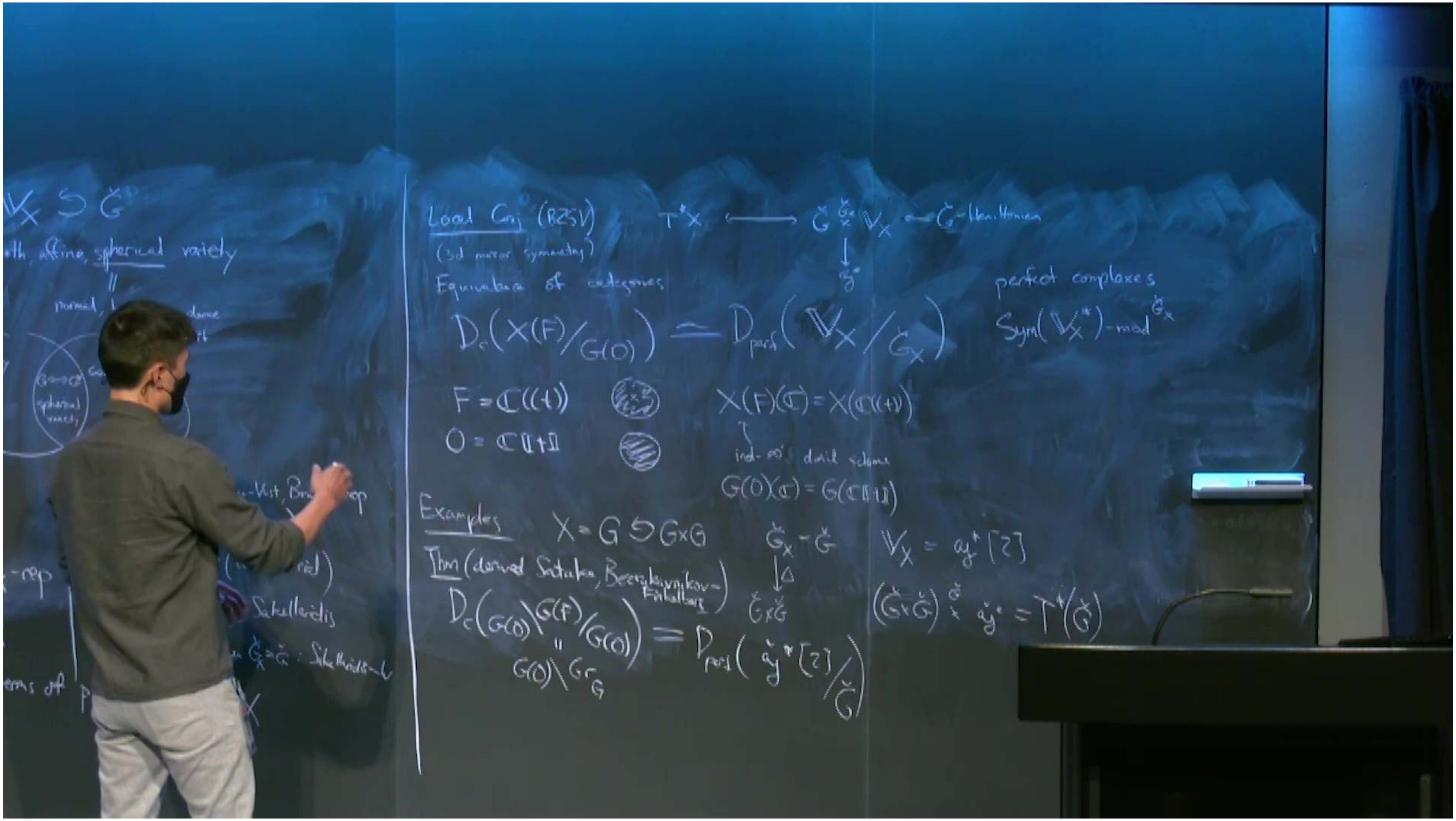
Zoom Link: https://pitp.zoom.us/j/95543248994?pwd=bmZIRnEyLzZnNmlEWW5oNTEwaEhNUT09

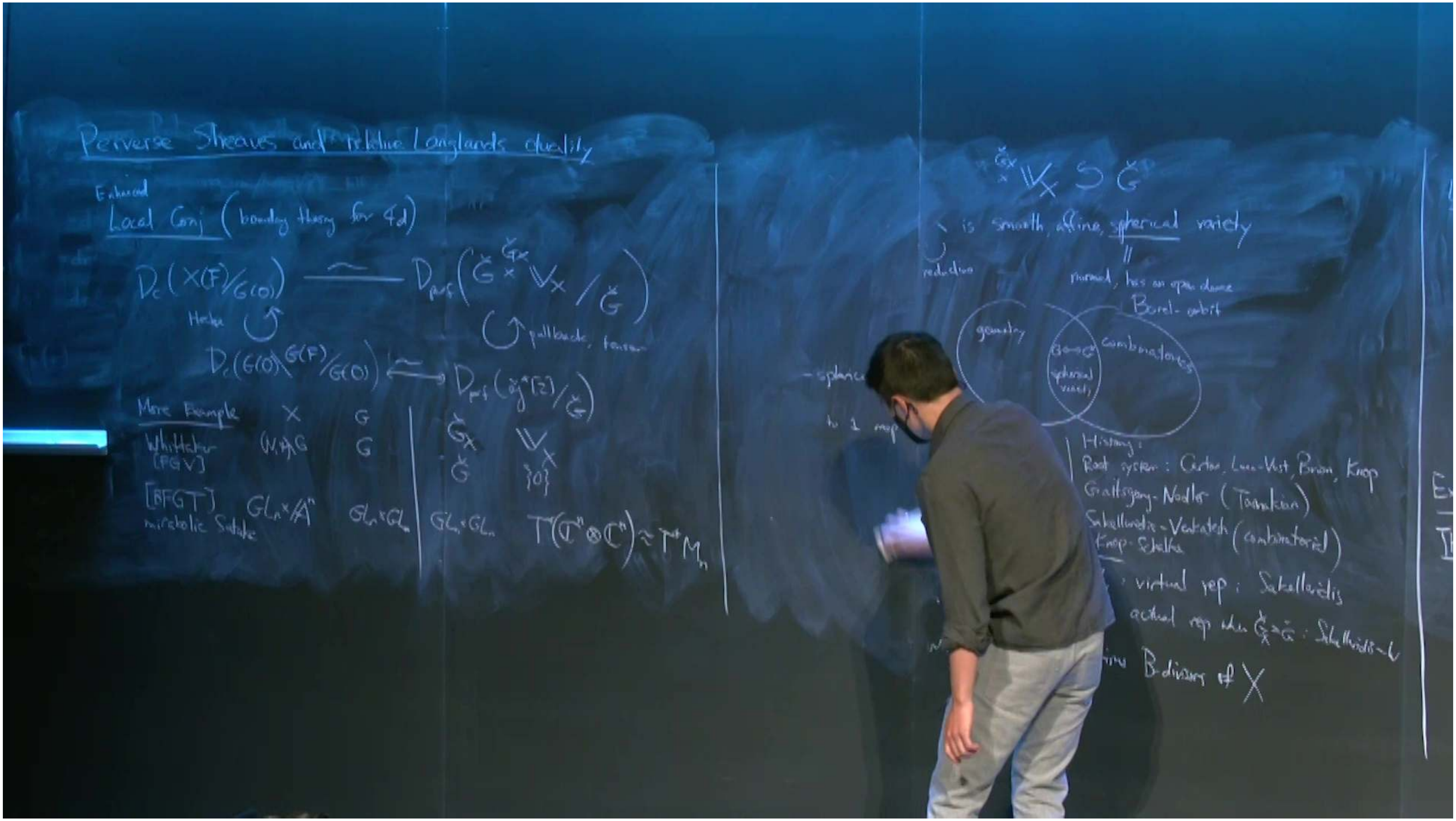

Perverse Sheaves and relative Langlands duality
General Framework Ben-Zvi - Sakellaridis - Venkatesh
Number Theory
Periods and L-functions
Physics
S-duality of boundary conditions
for SYM TFT d=4 N=4
(geometric Langlands)
Gaiotto-Witten
GCM
matching
Hamiltonian manifolds
BZSV: how to go in one direction (sometimes)
G: A-twist
G∨: B-twist
Starting points
X is smooth, affine, spherical variety
G reductive
normal, has an open dense
Borel-orbit
geometry
combinatorics

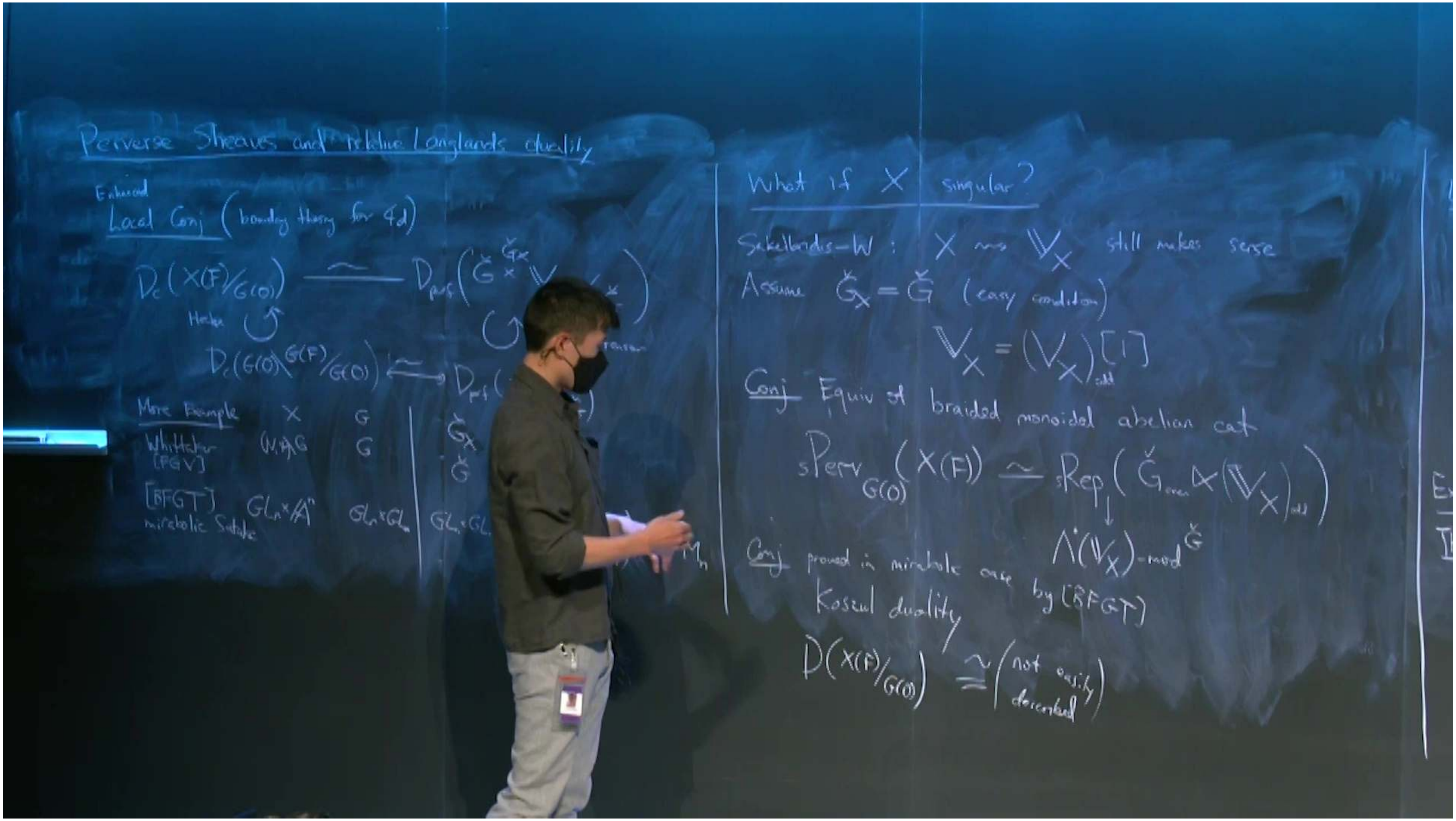

Langlands duality
Sakellaridis – Venkatesh
Physics
S-duality of boundary conditions
for SYM TFT d=4
(geometric Langlands)
matching
Hamiltonian manifold
Starting point: X is smooth, affine, spherical variety
normal, has an open dense
Borel-orbit
Have:
– spherical dual gp
geometry
combinatorics
History:
Root system: Carter, Luna-Vust, Brion, Knop
Gaitsgory-Nadler (Tannakian)
Sakellaridis-Venkatesh (combinatorial)
Knop-Schalke
virtual rep: Sakellaridis
Z-graded super rep

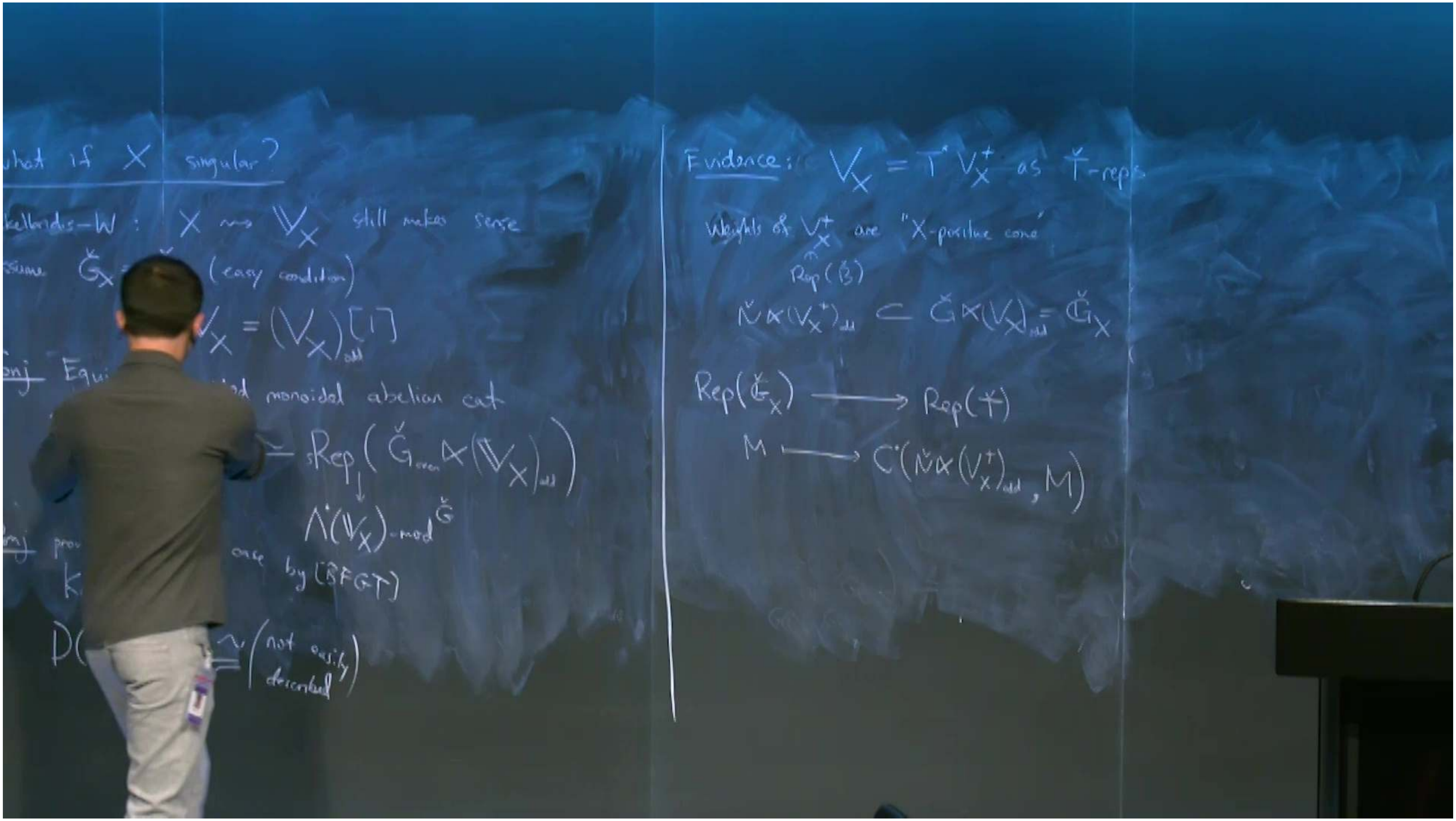

Langlands duality
Sakellaridis - Venkatesh
Physics
S-duality of boundary conditions
for SYM TFT d=4
(geometric Langlands)
matching
Hamiltonian manifolds
one direction (sometimes)
G ↷ T*X ⟵⟶ $\check{G}\times\check{G}_X$ × $\mathbb{V}_X$ ↺ $\check{G}$
Starting point: X is smooth, affine, spherical variety
G reductive
normal, has an open dense Borel-orbit
Have:
$\check{G}_X$ - spherical dual gp
finite to 1 map
$\check{G}$
$\mathbb{V}_X$: Z-graded super $\check{G}_X$-rep
Z/2
X ⟿ $\mathbb{V}_X$ combinatorial
highest wts in terms of prime B-divisors of X
geometry
combinatorics
spherical variety
History:
Root system: Cartan, Luna-Vust, Brion, Knop
Gaitsgory-Nadler (Tannakian)
Sakellaridis-Venkatesh (combinatorial)
Knop-Schalke
$\mathbb{V}_X$: virtual rep: Sakellaridis
actual rep when $\check{G}_X = \check{G}$: Sakellaridis
Local Conj (BZSV)
T*X


Local Conj (BZSV)
(3d mirror symmetry)
Equivalence of categories
perfect complexes
F = ℂ((t))
O = ℂ[[t]]
ind-∞ dim'l scheme
spherical variety
Borel-orbit
combinatorics
Nadler (Tannakian)
Venkatesh (combinatorial)
rep : Sakellaridis

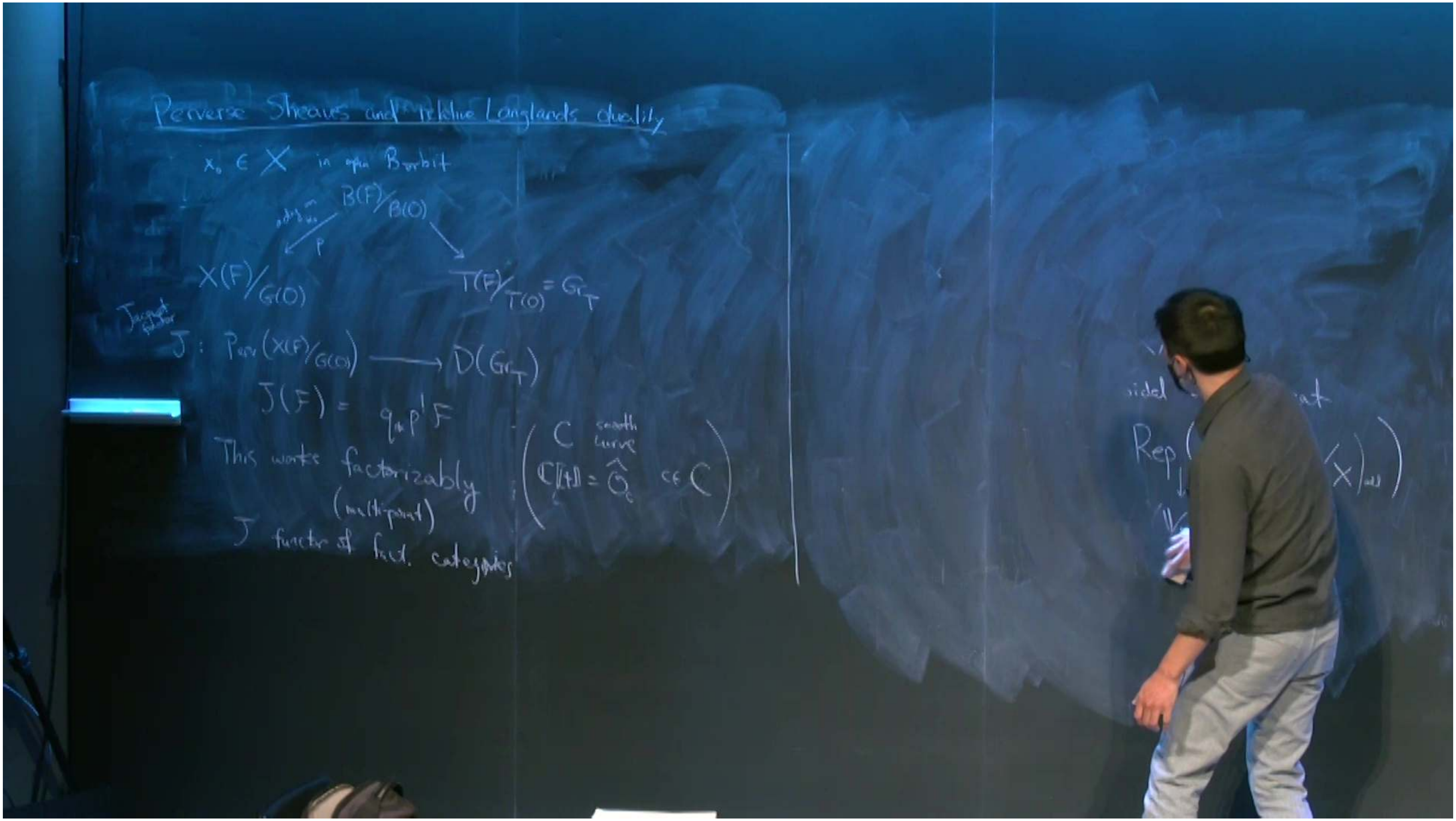

Local Conj (BZSV)
(3d mirror symmetry)
Equivalence of categories
$T^*X \longleftrightarrow \check{G} \overset{\check{G}_X}{\times} \mathbb{V}_X \longleftarrow \check{G}$-Hamiltonian
perfect complexes
$D_c(X(F)/G(O)) \simeq D_{perf}(\mathbb{V}_X/\check{G}_X)$
$\mathrm{Sym}(\mathbb{V}_X^*)\text{-mod}^{\check{G}_X}$
$F = \mathbb{C}((t))$
$O = \mathbb{C}[[t]]$
$X(F)(\mathbb{C}) = X(\mathbb{C}((t)))$
ind-∞ dimensional scheme
$G(O)(\mathbb{C}) = G(\mathbb{C}[[t]])$
Examples
$X = G \circlearrowleft G \times G$
$\check{G}_X = \check{G}$
$\mathbb{V}_X = \check{\mathfrak{g}}^*[2]$
Thm (derived Satake, Bezrukavnikov-Finkelberg)
$D_c(G(O)\backslash G(F)/G(O)) = D_{perf}(\check{\mathfrak{g}}^*[2]/\check{G})$
$G(O)\backslash Gr_G$
$(\check{G} \times \check{G}) \overset{\check{G}}{\times} \check{\mathfrak{g}}^* = T^*(\check{G})$
$\mathbb{V}_X \circlearrowleft \check{G}$
smooth, affine, spherical variety
Sakellaridis

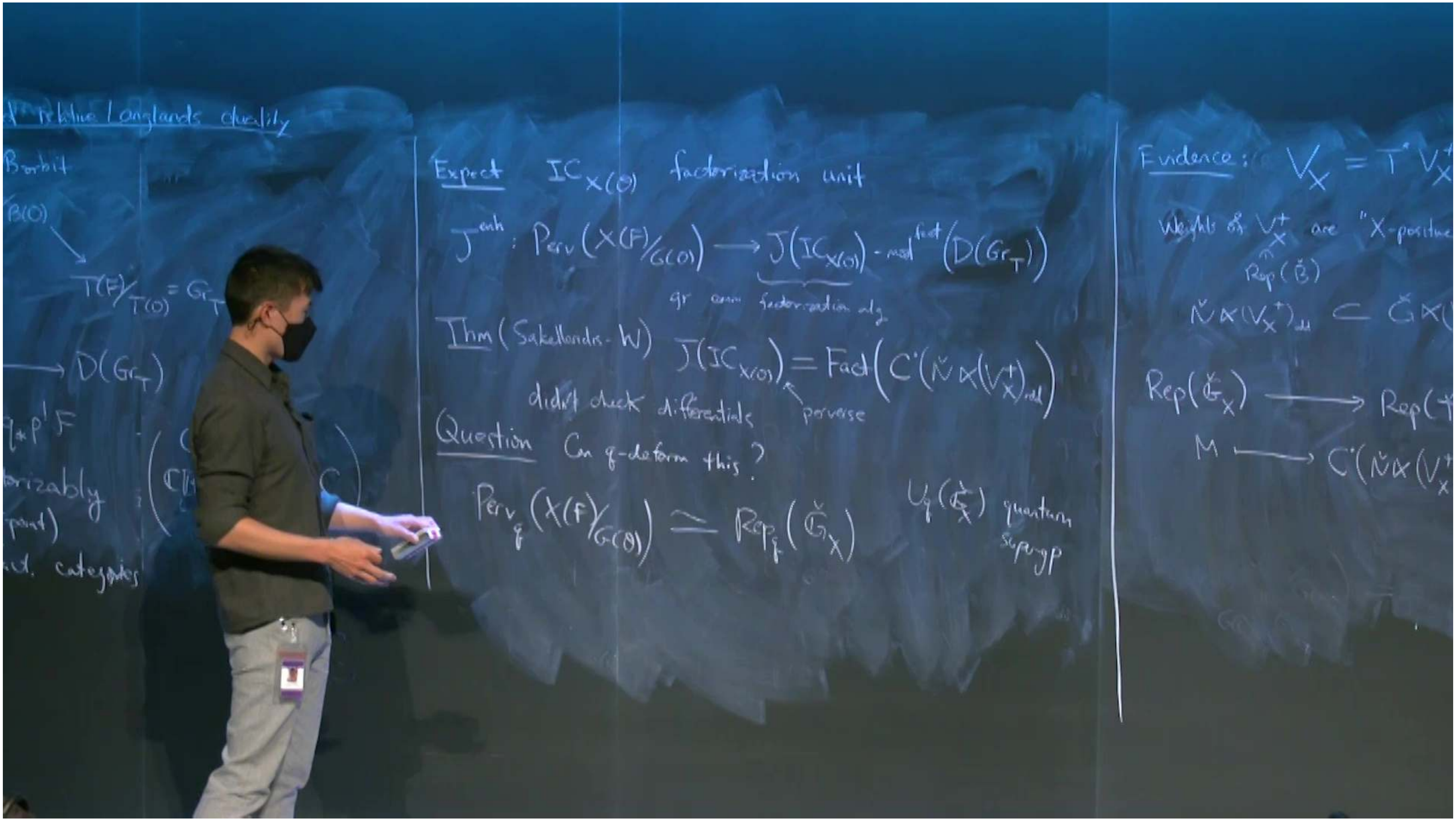

Perverse Sheaves and relative Langlands duality
Local Conj (boundary theory for 4d)
Hecke
More Example
Whittaker
[FGV]
[BFGT]
mirabolic Satake
is smooth, affine, spherical variety
normal, has an open dense
Borel-orbit
geometry
combinatorics
spherical variety
History:
Root system: Cartan, Luna-Vust, Brion, Knop
Gaitsgory-Nadler (Tannakian)
Sakellaridis-Venkatesh (combinatorial)
Knop-Schalke
virtual rep: Sakellaridis
B-divisors of X


Perverse Sheaves and relative Langlands duality
Enhanced
Local Conj (boundary theory for 4d)
More Example
Whittaker
[FGV]
[BFGT]
mirabolic Satake
What if X singular?
Sakellaridis-W : X ⇝ 𝕍_X still makes sense
Assume Ǧ_X = Ǧ (easy condition)
𝕍_X = (𝕍_X)_odd [1]
Conj Equiv of braided monoidal abelian cat
sPerv_G(O)(X(F)) ≃ sRep(Ǧ_even ⋉ (𝕍_X)_odd)
Λ˙(𝕍_X)-mod^Ǧ
Conj proved in mirabolic case by [BFGT]
Koszul duality
D(X(F)/G(O)) ≅ (not easily described)



What if X singular?
X ⇝ V_X still makes sense
(easy condition)
monoidal abelian cat
Rep
(not easily described)
by [BFGT]
Evidence: V_X = T* V_X^+ as Ť-reps
Weights of V_X^+ are "X-positive cone"
Rep(Ǧ_X) ⟶ Rep(Ť)


Perverse Sheaves and relative Langlands duality
in open B-orbit
This works factorizably
(multipoint)
smooth curve
Rep


Relative Langlands duality
Expect IC_X(O) factorization unit
J^enh : Perv(X(F)/G(O)) → J(IC_X(O))-mod^fact(D(Gr_T))
gr comm factorization alg
Thm (Sakellaridis-W) J(IC_X(O)) = Fact(C^•(Ň ⋉ (V_X^+)_ad))
didn't check differentials
perverse
Question Can q-deform this?
Perv_q(X(F)/G(O)) ≃ Rep_q(Ǧ_X)
U_q(Ǧ_X) quantum supergp
Evidence:
Weights of V_X^+ are "X-positive"
Rep(Ǧ_X)
M
categories