Title: Black Hole interior: Symmetries and regularization

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Series: Quantum Gravity

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Abstract: The spacetime in the interior of a black hole can be described by an homogeneous line element, for which the Einstein-Hilbert action reduces to a one-dimensional mechanical model. We have shown that this model exhibits a symmetry under the (2+1)-dimensional Poincare? group. The existence of this symmetry provides a powerful criterion to discriminate between different regularization and quantization schemes. It also unravels new aspects of symmetry for black holes, and opens the way towards a rigorous group quantization of the interior. Remarkably, the physical ISO(2,1) symmetry can be seen as a broken infinite-dimensional symmetry. This is done by reinterpreting the action for the model as a geometric action for the BMS3 group, where the configuration space variables are elements of the algebra bms3 and the equations of motion transform as coadjoint vectors.

Zoom Link: https://pitp.zoom.us/j/99646733321?pwd=U05kYU85V0Q4VCtrZ1BNV2JZbE1DUT09

Black Hole interior: Symmetries and regularization

Francesco Sartini

based on

Geiller, Livine, Sartini: arXiv:gr-qc/2010.07059
arXiv:gr-qc/2107.03878

September 9th 2021

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Motivations

Symmetries: Gauge vs Physical

- Play a central role in quantum theories
- GR rests solely on gauge symmetries, diffeomorphisms
- Symmetries interplay with boundary conditions, gauge can become physical

- Understand classical and quantum symmetry-reduced models in GR
  - Lots of things known about FLRW cosmologies [Ashtekar, Bojowald, Pawlowski, Singh, ...]
  - Recent work on BH interior [Ashtekar, Bodendorfer, Bojowald, Campiglia, Corichi, Gambini, Mele, Modesto, Munch, Olmedo, Pullin, Singh, ...]
  - Recent work on $\mathrm{SL}(2,\mathbb{R})$ symmetry in FLRW [Ben Achour, Livine]
  - We want to extend this to the BH interior

Roadmap

- Start from minisuperspace BH interior
- Reveal $\mathfrak{iso}(2,1)$ symmetry on phase space
- Relate this symmetry to a "hidden" symmetry of the action
- Embed the Poincaré group into BMS_3
- Reinterpret the previous results in terms of BMS_3 representations
- Opens many doors (quantization, generalization, inhomogeneous case, relation to boundary symmetries, mass evolution,...)

Classical Theory

Schwarzschild BH interior

- Radial coordinate becomes *time-like*.

$$\mathrm{d}s^2 = -\left(\frac{2M}{T} - 1\right)^{-1} \mathrm{d}T^2 + \left(\frac{2M}{T} - 1\right) \mathrm{d}r^2 + T^2 \mathrm{d}\Omega^2 \,,$$

- Described by Kantowski-Sachs cosmology:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \frac{8V_2}{V_1} \mathrm{d}x^2 + V_1 \mathrm{d}\Omega^2 \,,$$

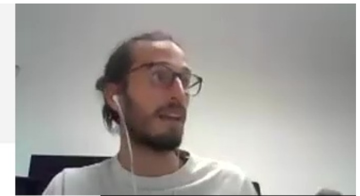
  - homogeneous as in LQC minisuperspace

Spatial integration cutoff: $x \in [0, L_0]$

- Einstein-Hilbert action reduces to a 1D mechanical model, invariant under time reparametrization

Lapse and cutoff

Regulator or energy level?



Time diffeomorphism allows for a (field dependent) lapse redefinition

$$N := L_0 N_{\mathrm{p}} \sqrt{\frac{V_1}{2V_2}}\,, \qquad \mathrm{d}s^2 = -L_0^2 N_{\mathrm{p}}^2 \frac{V_1}{2V_2}\mathrm{d}t^2 + \frac{8V_2}{V_1}\mathrm{d}x^2 + V_1 \mathrm{d}\Omega^2\,,$$

$$\mathcal{S}^{(t)}_{\mathrm{EH}}[N_{\mathrm{p}}, V_i] = \int \mathrm{d}t \left[N_{\mathrm{p}} L_0^2 + \frac{V_1'(V_2 V_1' - 2V_1 V_2')}{2N_{\mathrm{p}} V_1^2} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2N_{\mathrm{p}} V_1} (V_1 V_2)' \right) \right]\,,$$

we introduce the *proper time* gauge ($\mathrm{d}\tau = N_{\mathrm{p}}\mathrm{d}t$) and drop the boundary term,

$$\mathcal{S}^{(\tau)}[V_i] = \int \mathrm{d}\tau \left[L_0^2 + \frac{\dot{V}_1(V_2 \dot{V}_1 - 2V_1 \dot{V}_2)}{2V_1^2} \right]\,.$$

Lapse and cutoff

Regulator or energy level?

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⚠ How can the IR scale disappear? The scalar constraint imposed by the lapse translates into the fact that the Hamiltonian for τ (H) is L_0^2

$$\frac{\delta \mathcal{S}^{(t)}_\mathrm{EH}}{\delta N_\mathrm{p}} = 0 \quad \Leftrightarrow \quad H = L_0^2\,.$$

We need to remember this while inserting the solution into the metric

Hamiltonian setup

- Equation of motion for the lapse

$$\frac{\delta \mathcal{S}^{(t)}_{\text{EH}}}{\delta N_{\text{p}}} = 0 \qquad \Leftrightarrow \qquad L_0^2 = \frac{1}{N_{\text{p}}^2}\left[\frac{V_2 V_1'^2}{2V_1^2} - \frac{V_1' V_2'}{V_1}\right].$$

- We perform a Legendre transform of $\mathcal{S}^{(\tau)}$:

$$\mathcal{S}^{(\tau)} = \int \mathrm{d}\tau \left(P_i \dot{V}_i - H\right), \qquad \left| \begin{array}{lcl} P_1 & = & \frac{V_2 \dot{V}_1 - V_1 \dot{V}_2}{V_1^2} \\ P_2 & = & -\frac{\dot{V}_1}{V_1} \end{array} \right.$$

$$H = -P_2\left(V_1 P_1 - \frac{1}{2} V_2 P_2\right) = \frac{V_2 \dot{V}_1^2}{2V_1^2} - \frac{\dot{V}_1 \dot{V}_2}{V_1} \approx L_0^2 .$$

- 4-dimensional phase space with the Poisson brackets $\{V_i, P_j\} = \delta_{ij}$
- Time evolution: $\dot{\mathcal{O}} := \mathrm{d}_\tau \mathcal{O} = \{\mathcal{O}, H\} + \partial_\tau \mathcal{O}$

Classical solution

Solutions

$$V_1 = \frac{A}{2}(\tau - \tau_0)^2 \,,$$

$$P_1 = \frac{2B}{A(\tau - \tau_0)^2} \,,$$

$$V_2 = B(\tau - \tau_0) - \frac{L_0^2}{2}(\tau - \tau_0)^2 \,,$$

$$P_2 = -\frac{2}{\tau - \tau_0} \,.$$

First Integrals

$$A = \frac{V_1 P_2^2}{2} \,, \qquad B = V_1 P_1 \,.$$

Inserting these solutions in the metric, and changing the variables as:

$$\tau - \tau_0 = \sqrt{\frac{2}{A}}\, T \,, \qquad x = \frac{1}{2L_0}\sqrt{\frac{A}{2}}\, r \,,$$

we recover the standard Schwarzschild BH interior metric with mass:

$$M = \frac{B\sqrt{A}}{\sqrt{2}L_0^2} \,.$$

B/L_0 is the Komar charge associated to the Killing vector ∂_x.

$\mathfrak{iso}(2,1)$ algebra

- Check if and at which stage the Poisson brackets of V_i and its derivative form a closed algebra:
- V_2: $\mathfrak{sl}(2,\mathbb{R})$:

$$\dot{V}_2 = \{V_2, H\} = -V_1 P_1 - V_2 P_2 := C \qquad \dot{C} = \{C, H\} = H\,.$$

C is the generator of isotropic dilations of the phase space:

$$e^{\{\eta\, C,\cdot\}} \triangleright P_i = e^{-\eta} P_i\,, \qquad e^{\{\eta\, C,\cdot\}} \triangleright V_i = e^{\eta} V_i\,, \qquad \forall\, i \in \{1,2\}\,.$$

The brackets are:

$$\{C, V_2\} = V_2\,, \qquad \{V_2, H\} = C\,, \qquad \{C, H\} = -H\,.$$

The Casimir is:

$$\mathcal{C}_{\mathfrak{sl}(2,\mathbb{R})} = -C^2 - 2HV_2 = -B^2 < 0\,.$$

$\mathfrak{iso}(2,1)$ algebra

- Check if and at which stage the Poisson brackets of V_i and its derivative form a closed algebra:
- V_2: $\mathfrak{sl}(2,\mathbb{R})$ CVH
- V_1: extends to $\mathfrak{iso}(2,1)$

$$\dot{V}_1 = \{V_1, H\} = -V_1 P_2 := -D \qquad \dot{D} = \{D, H\} := A\,.$$

$$\begin{aligned} j_z &= \frac{1}{\sqrt{2}}(V_2 - H)\,, & k_x &= \frac{1}{\sqrt{2}}(V_2 + H)\,, & k_y &= C\,, \\ \Pi_x &= D\,, & \Pi_y &= \frac{1}{\sqrt{2}}(V_1 - A)\,, & \Pi_0 &= \frac{1}{\sqrt{2}}(V_1 + A)\,, \end{aligned}$$

that correspond to the generators of (2+1) Poincaré group. The two Casimirs are given by

$$\mathcal{C}_1 = \Pi_0^2 - \Pi_x^2 - \Pi_y^2 \approx 0\,, \qquad \mathcal{C}_2 = j_z\Pi_0 + k_y\Pi_x - \Pi_y k_x \approx 0\,.$$

- The Casimir conditions reduce the 6-dimensional Lie algebra back to the original four dimensional phase space
- The Black hole interior carries massless representation of (2+1) Poincaré group

Symmetries

The $\mathfrak{iso}(2,1)$ algebra encoding the dynamics is linked to an invariance of $\mathcal{S}$ under $\mathrm{ISO}(2,1) = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^3$:

- Möbius transformation on proper time

$$\tau \mapsto \tilde{\tau} = \frac{a\tau + b}{c\tau + d} \qquad \text{with} \qquad ad - bc = 1\,,$$
$$V_i \mapsto \widetilde{V}_i(\tilde{\tau}) = \frac{V_i(\tau)}{(c\tau + d)^2}\,.$$

- Abelian transformation on V_2

$$\tau \mapsto \tilde{\tau} = \tau\,,$$
$$V_1 \mapsto \widetilde{V}_1(\tau) = V_1\,,$$
$$V_2 \mapsto \widetilde{V}_2(\tau) = V_2 + (\alpha + \beta\tau + \gamma\tau^2)\dot{V}_1 - (\beta + 2\gamma\tau)V_1\,,$$

The induced variation of the action yields a total derivative as

$$\Delta\mathcal{S} = \int \mathrm{d}\tau \left[\frac{\mathrm{d}}{\mathrm{d}\tau}\left(2\gamma V_1 - (\alpha + \beta\tau + \gamma\tau^2)\frac{\dot{V}_1^2}{2V_1}\right)\right],$$

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- Abelian transformation on V_2

These are NOT residual diffeomorphism: they map a solution to a different one (e.g by changing the mass)

Möbius

$$A \mapsto \widetilde{A} = (d + c\tau_0)^2 A\,,$$
$$B \mapsto \widetilde{B} = B\,,$$
$$L_0^2 \mapsto \widetilde{L}_0^2 = (d + c\tau_0)\left(2cB + (d + c\tau_0)L_0^2\right)\,.$$

$$M_{\mathrm{BH}} \mapsto \frac{L_0^2(d + c\tau_0)}{\widetilde{L}_0^2} M_{\mathrm{BH}}\,.$$

Abelian

$$A \mapsto \widetilde{A} = A\,,$$
$$B \mapsto \widetilde{B} = B + A(\alpha + \beta\tau_0 + \gamma\tau_0^2)\,,$$
$$L_0^2 \mapsto \widetilde{L}_0^2 = L_0^2 - (\beta + 2\gamma\tau_0)A\,,$$

$$M_{\mathrm{BH}} \mapsto \frac{L_0^2 M_{\mathrm{BH}} + A^{3/2}(\alpha + \beta\tau_0 + \gamma\tau_0^2)/\sqrt{2}}{\widetilde{L}_0^2}\,.$$

Symmetries

Charges

- According to Noether's theorem we compute the conserved charges
- We translate the time derivative $\dot{V}_i$ to the momenta P_i

$\mathrm{SL}(2,\mathbb{R})$ charges

$$Q_- = H\,, \qquad Q_0 = C + \tau H\,, \qquad Q_+ = 2V_2 - 2\tau C - \tau^2 H\,.$$

that respectively generate the translation, dilation and special conformal transformation on τ

$\mathbb{R}^3$ charges

$$P_- = A\,, \qquad P_0 = D + \tau A\,, \qquad P_+ = -2V_1 - 2\tau D - \tau^2 A\,,$$

that corresponds to the different coefficient of the polynomial in τ

- These represent the initial condition of the $\mathfrak{iso}(2,1)$ generators presented before.
- They satisfy the same algebra

BMS extension

- In general relativity Poincaré group can be enhanced to its infinite dimensional extension
- Consider more general transformations:

$$\mathrm{SL}(2,\mathbb{R}) : \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = \frac{a\tau+b}{c\tau+d} \, , \\ V_i & \mapsto & \widetilde{V}_i(\tilde{\tau}) = \frac{V_i(\tau)}{(c\tau+d)^2} \, , \end{array} \right. \qquad \mathbb{R}^3 : \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = \tau \, , \\ V_1 & \mapsto & \widetilde{V}_1(\tau) = V_1 \, , \\ V_2 & \mapsto & \widetilde{V}_2(\tau) = V_2 + (\alpha + \beta\tau + \gamma\tau^2)\dot{V}_1 \\ & & \qquad - (\beta + 2\gamma\tau) V_1 \, . \end{array} \right.$$

$$D_f : \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = f(\tau) \, , \\ V_i & \mapsto & \widetilde{V}_i(\tilde{\tau}) = \dot{f}(\tau) \, V_i(\tau) \, , \end{array} \right. \qquad T_g : \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = \tau \, , \\ V_1 & \mapsto & \widetilde{V}_1(\tau) = V_1 \, , \\ V_2 & \mapsto & \widetilde{V}_2(\tau) = V_2 + g\dot{V}_1 - V_1 \dot{g} \, . \end{array} \right.$$

BMS extension

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They transform the action functional as:

$$\Delta_f \mathcal{S} = \int \mathrm{d}\tau \left[\mathrm{Sch}[f] V_2 - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\ddot{f}}{\dot{f}} V_2 \right) \right],$$

$$\Delta_g \mathcal{S} = \int \mathrm{d}\tau \left[-g^{(3)} V_1 + \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\ddot{g} V_1 - \frac{g \dot{V}_1^2}{2 V_1} \right) \right].$$

BMS extension

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- The theory is not invariant under BMS!
- The extra terms, proportional to V_i also appear when introducing a
  - cosmological *constant*: ΛV_1
  - kinetic term of an homogeneous scalar field: $\dot{\phi}^2 V_2$
- Extended concept of symmetry: moves into a family of different theories

BMS extension

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- The transformation $\mathrm{Ad}_{f,g} := T_g \circ D_f$ has the same composition and inverse law of the group
$$\mathrm{BMS}_3 = \mathrm{Diff}(S^1) \ltimes_{\mathrm{Ad}} \mathrm{Vect}(S^1)_{\mathrm{ab}}$$
- Moreover V_i transform exactly as the algebra element
$$\mathrm{Ad}_{f,g}(X;\alpha) = (\mathrm{Ad}_f X; \mathrm{Ad}_f \alpha + [\mathrm{Ad}_f X, g])\,, \qquad \mathrm{Ad}_f X = (\dot{f} X) \circ f^{-1}\,.$$

The V_i's belongs to the adjoint representation of the BMS_3 group:

$$\mathfrak{bms}_3 = \mathsf{Vect}(S^1) \oplus_{\mathsf{ad}} \mathsf{Vect}(S^1)_{\mathsf{ab}} \ni (V_1, V_2)\,,$$

BMS extension

Hamiltonian generators

- Even if the BMS transformations are not symmetries, they have an integrable generator

$$\mathcal{D}_X = -H\,X - C\,\dot{X} + V_2\,\ddot{X}\,,$$
$$\mathcal{T}_\alpha = A\,\alpha + D\,\dot{\alpha} + V_1\,\ddot{\alpha}\,.$$

- These are the generalisation of the $\mathrm{ISO}(2,1)$ charges presented before
- But their Poisson algebra is not a central extension of $\mathfrak{bms}$

$$\{\mathcal{D}_X, \mathcal{D}_Y\} = -\mathcal{D}_{[X,Y]} + \left(XY^{(3)} - YX^{(3)}\right)V_2\,,$$
$$\{\mathcal{T}_\alpha, \mathcal{T}_\beta\} = 0\,,$$
$$\{\mathcal{D}_X, \mathcal{T}_\alpha\} = -\mathcal{T}_{[X,\alpha]} + \left(\alpha X^{(3)} - X\alpha^{(3)}\right)V_1\,,$$

BMS extension
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$$\{\mathcal{D}_X, \mathcal{T}_\alpha\} = -\mathcal{T}_{[X,\alpha]} + \left(\alpha X^{(3)} - X\alpha^{(3)}\right) V_1\,,$$

⚠ The Hamiltonian do not belong to the abelian subgroup of generators

$$H = \mathcal{D}_{X(\tau)=-1}$$

Coadjoint representation

- The coadjoint action of a group G is defined on the dual Lie algebra $\mathfrak{g}^*$, here represented by *two forms* on the (decompactified) circle:

$$(J,P) \in \mathfrak{bms}^* \qquad \langle (J,P)|(X,\alpha)\rangle = \int \mathrm{d}\tau (JX + P\alpha)\,,$$

$$\langle \mathrm{Ad}^*_{f,g}(J,P)|(X,\alpha)\rangle := \langle (J,P)|\mathrm{Ad}_{(f,g)^{-1}}(X,\alpha)\rangle\ .$$

- Given an element p_0 in $\mathfrak{g}^*$ its coadjoint orbit $\mathcal{O}_{p_0}$ is

$$\mathcal{O}_{p_0} := \left\{ p = \mathrm{Ad}^*_g p_0 | \forall g \in G \right\}$$

- Classification of the orbits by their little group naturally leads to irreps of the full group

BMS extension
Coadjoint representation

- We take as covectors the equation of motion:

$$\delta\mathcal{S} = \int \mathrm{d}\tau [J\delta V_1 + P\delta V_2 + \mathrm{d}_\tau \theta]$$

  - Adding a boundary term the action can be written as pairing between algebra elements and their dual:

$$\mathcal{S} = \int \mathrm{d}\tau (JV_1 + PV_2)$$

    It has the same form of corner *charges* of 3D gravity

  - (J, P) transform as in the centrally-extended coadjoint representation of BMS:

$$P \xrightarrow[\mathrm{Ad}^*_{f^{-1},g}]{} \dot{f}^2(P \circ f) - c_2\,\mathsf{Sch}[f]\,,$$

$$J \xrightarrow[\mathrm{Ad}^*_{f^{-1},g}]{} \dot{f}^2\left(J + g\dot{P} + 2\dot{g}P - c_2 g^{(3)}\right) \circ f - c_1 \mathrm{Sch}[f]\,.$$

BMS extension
Coadjoint representation

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  - The central charges are $c_1 = 0, c_2 = 1$
- The little group of the orbit starting from $J = 0, P = 0$ is known to be $\mathrm{ISO}(2,1)$, the symmetry group of our theory

Perspective

Comments

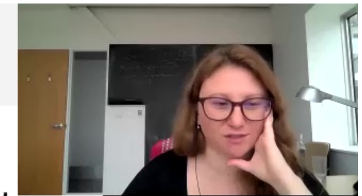
We have studied the symmetries of the phase space containing classical Schwarzschild interior solutions

- $\mathfrak{iso}(2,1)$ encoding the dynamic of phase space
- Lifted to a Lagrangian symmetry
- It descends from a BMS_3 structure
  - EOMs as coadjoint vectors with central extension
  - Stabilizer of the orbit as symmetry group
  - Action as bilinear form $\langle \mathfrak{g}^* | \mathfrak{g} \rangle$

What comes next?

- Group quantization of $\mathrm{ISO}(2,1)$
- We can describe mass evolution in terms of group flow
- Why BMS_3?
  - Role of boundaries/asymptotic symmetries
  - Broken symmetry?
  - Other GR systems have similar properties?

Perspective



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Thank you for your attention!