Title: Non-relativistic physics in AdS and its CFT dual

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Abstract: In this talk we discuss aspects of the non-relativistic two-body problems in AdS spacetime and their CFT duals. Specifically, we focus on understanding the spectrum of double-twist operators in the dual CFT as well as realizing the flat space scattering and bound states from the CFT correlator.

To understand the double-twist operator spectrum we use quantum perturbation theory in the bulk. We then match the result with the inversion formula for consistency. Next, we show how to obtain the flat space scattering from the correlator by using Euclidean time evolution to construct the scattering states; finally we demonstrate how to get the bound states through the WKB approximation. Time permitting, we will also discuss how to obtain physical quantities such as precession of the near-circular orbits in AdS from the data of the double twist operators lying on the first two Regge trajectories.

Non-relativistic physics in AdS and its CFT dual

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McGill University
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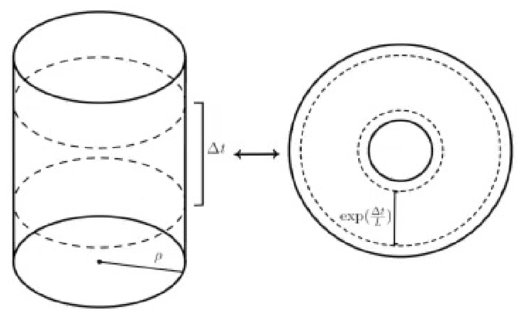
Motivation

The study of the correspondence between AdS spacetime and CFT has started more than 20 years ago with Maldacena 98' and Witten 98':



Theory of quantum gravity in d+1-dim AdS spacetime $\Longleftrightarrow$ d-dim CFT

Motivation

Even though this correspondence has been studied for 20 years, there are much more physical insights and intuition left to be understood. In this direction we aim to study the duality in the well-known Non-relativistic limit.

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Motivation

Even though this correspondence has been studied for 20 years, there are much more physical insights and intuition left to be understood. In this direction we aim to study the duality in the well-known Non-relativistic limit.

In particular some questions we focus on are as follows:

- Can we understand the CFT scaling dimension and OPE spectrum (Regge trajectories) in the NR limit?
  - Can we understand physical properties of two-body orbits such as precession in AdS from properties of this spectrum?
- Can we understand non-relativistic Flat space physics from the CFT correlator?

  Expected when we take the AdS radius to infinity.

The full relativistic version of these questions have been of interest for many years and have been massively studied in different limit:

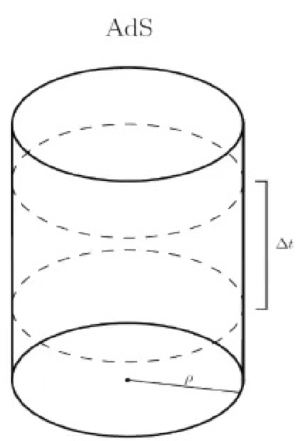
Polchinski, Giddings, Gary, Penedones, Kaplan, Fitzpatrick, Katz, Poland, Simmons-Duffin, Raju, Hijano, van Rees, Caron-huot,...

Non-relativistic two-body problem in AdS

- The metric of global AdS is

$$ds^2 = -c^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-1}$$

where $f(r) = \frac{r^2}{L^2} + 1$.
Global AdS can be thought as a cylinder. We can see that by defining a ρ such that $\tan\rho \equiv \frac{r}{L}$. So pictorially we get:



We are interested in non-relativistic dynamics so if τ is the proper time of a geodesic in this background we get $\frac{dx^i}{d\tau} \ll \frac{dt}{d\tau}$. This in turn means we can identify the time component of the metric as follows:

$$g_{tt} = -(c^2 + 2\Phi)$$

Where Φ is the gravitational potential given as:

$$\Phi = \frac{1}{2}\omega^2 r^2$$

with $\omega = \frac{c}{L}$. This potential confines the dynamics of non-relativistic particles to $r \ll L$.

We can now write the Hamiltonian of the NR particles in AdS:

$$H = \sum_{i=1}^{2} \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega^2 r_i^2 \right) + V(r)$$

Outline

1. Symmetry of The Problem
2. Extracting Data about the Two particles in AdS
   - CFT double-twist data from quantum theory in the bulk
   - CFT data from cross-channel CFT
   - Precession of the near-circular orbit from the Regge Trajectories
3. Finding an scattering amplitude from a CFT correlation function
   - CFT correlation Function
   - Preparing the states through Euclidean time Evolution
   - Flat Space Amplitude
4. Summary and Outlook

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Conformal Symmetry

We start by studying the symmetry algebra in order to find out which state representation trivializes them and hence are simpler.
The symmetry algebra of AdS_{d+1} is SO(d,2) which is spanned by the dilatation operator D, momentum P_i, special conformal transformation K_i and rotation $M_{ij} = -M_{ji}$. We have:

$$D^\dagger = D, \quad M_{ij}^\dagger = M_{ij}, \quad K_i^\dagger = P_i$$

The algebra is given by:

$$[D, P_i] = P_i, \quad [D, K_i] = -K_i, \quad [K_i, P_j] = 2\delta_{ij} D - 2iM_{ij}$$

Along with the $SO(d)$ algebra of rotation for M_{ij}. Also K_i and P_i transform like vectors under rotation.

Taking Non-relativistic limit of the Symmetry Group

The dilation operator is the time translate operator in global coordinates in AdS so, $D = i\partial_t$. So it is our Hamiltonian. In the NR limit for particles of total mass M, this will become the rest-energy Mc^2 plus the non-relativistic Hamiltonian:

$$D \sim M + H_{NR}$$

The conformal algebra then reduces to:

$$[H_{NR}, P_i] = P_i, \quad [H_{NR}, K_i] = -K_i, \quad [K_i, P_j] = 2\delta_{ij} M.$$

Expressing the action of P_i in global coordinate we have:

$$P_i \sim (mx_i - ip_i)$$

P_i acts linearly in mx_i and p_i thus it acts on the center of mass wavefunction

Non-relativistic Symmetry Group of AdS

P_i looks familiar from quantum mechanics! It is creation operator forthe harmonic oscillator, $P_i \sim A_i^\dagger$. We can then rewrite the algebra:

$$[H_{NR}, A_i^\dagger] = A_i^\dagger, \quad [H_{NR}, A_i] = -A_i, \quad [A_i, A_j^\dagger] = \delta_{ij}$$

We have indeed recovered the algebra of the Harmonic oscillator as expected with $A_i^\dagger$ the creation operator acting on the center of mass wavefunction.
From this we we can rewrite H_{NR}:

$$H_{NR} = (A_i^\dagger \cdot A_i + \frac{d}{2}) + H_{\text{Relative}}.$$

We focus on the primary states by considering states annihilated by K_i (or equivalently A_i). This trivialized the symmetry and correspond to the ground state of the harmonic oscillator.

The energy shift of the primary two-particle states $[OO]_{E,\ell}$ comes from the H_{relative}

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CFT data from quantum theory in the bulk

As argued any information about the energy shifts of a primary two particle-state can be read form the relative Hamiltonian. The non-relativistic potential for the two body problem is given as:

$$H_{\text{relative}} = \frac{p^2}{2\mu} + \frac{1}{2}\mu r^2 + V(r)$$

We can use effective quantum theory to obtain the energy shift due to a potential V(r) when $|V(r)| \ll \frac{1}{2}\mu\omega r^2$:

$$\delta E_{\ell,n} = \langle n\ell|V|n\ell\rangle + \sum_{m\neq n}^{\infty} \frac{|\langle m\ell|V|n\ell\rangle|^2}{E_{n,\ell} - E_{m,\ell}} + \dots$$

Here $|n\ell\rangle$ is the solution of the unperturbed Hamiltonian. Since the dilation operator is the Hamiltonian in AdS we have :

$$\delta E_{\ell,n} = \gamma_{n,\ell}$$

where $\gamma_{n,\ell}$ is the anomalous dimension of the double twist operator $[O_1 O_2]_{n,\ell}$ with $\Delta_{1,2} \gg 1$

Kepler Potential in d=4

Focusing on the $V(r) = \frac{G_N m_1 m_2}{r}$ we can separate the energy shifts of different order in G_N as follows:

$$\gamma^{(1)}_{n,\ell} = \delta E^{(1)}_{\ell,n} = \int_{\text{AdS}} \phi_{n\ell}(r)\phi^*_{n\ell}(r)V(r)d\text{Vol}$$

$$\gamma^{(2)}_{n,\ell} = \delta E^{(2)}_{\ell,n} = \sum_{m=0}^{\infty} \frac{1}{E_{\ell,n} - E_{\ell,m}} \left| \int_{\text{AdS}_4} \phi_{n\ell}(r)\phi^*_{n\ell}(r)V(r)d\text{Vol}\right|^2$$

where $\langle r|n\ell\rangle = \phi_{n,\ell}(r)$ and dVol is the volume element of AdS at large L_{AdS}.

Solving the unperturbed Hamiltonian

We can solve the unperturbed Hamiltonian $V(r) = 0$ to obtain $\phi_{n,\ell}(r) = \langle r | n\ell \rangle$

$$\phi_{n,\ell}(r) = \mathcal{N}_{n,l} \frac{e^{-\frac{1}{2}\mu r^2}}{\sqrt{r}} (\mu r^2)^{\frac{l}{2}+\frac{d}{4}} {}_1F_1(-n, l + \frac{d}{2}; \mu r^2)$$

Note: this solution can be obtained by taking the NR limit of the solution to the AdS Laplacian ($\Delta \gg 1$ and $\frac{r}{L_{\text{AdS}}} \ll 1$ with $\Delta \frac{r}{L_{\text{AdS}}}$ fixed).

Kepler Potential in d=4

Anomalous Dimension

We can then calculate the integral to obtain the result for all n!

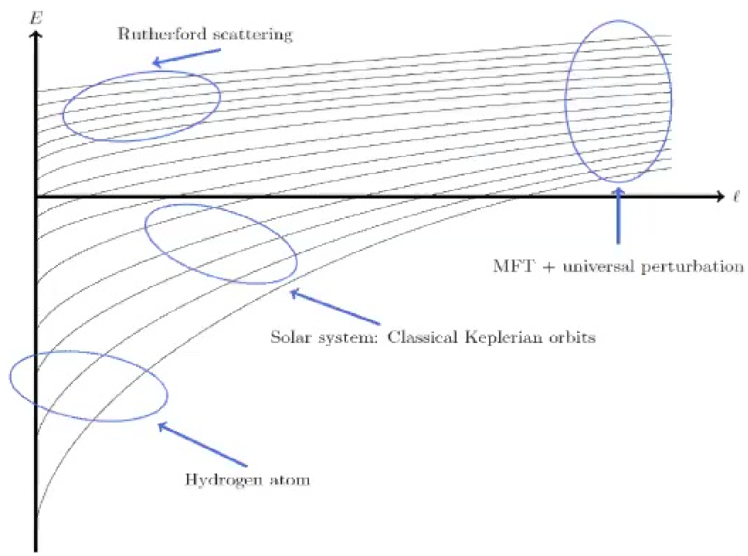
$$\gamma^{(1)}_{n,\ell} = G_N m_1 m_2 \sqrt{\mu} \frac{\left(\frac{1}{2}\right)_n \Gamma(\ell+1)}{n! \Gamma(\ell+\frac{3}{2})} {}_3F_2(-n, \ell+1, \frac{1}{2}; \ell+\frac{3}{2}, -n+\frac{1}{2}; 1)$$

It would be illuminating to look at different limits of this formula

- Flat space: large n, large ℓ, $n \gg \ell$ we get $\gamma^{(1)}_{n,\ell} \sim -\frac{G_N m_1 m_2}{\pi} \sqrt{\frac{\mu}{n}} \log(\frac{n}{\ell})$.
- Large ℓ limit: $\gamma^{(1)}_{n,\ell} \sim -G_N m_1 m_2 \sqrt{\frac{\mu}{\ell}}$.

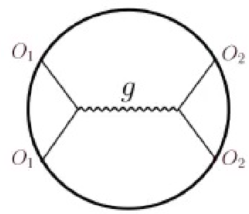
Regge Trajectory

With the above analysis we can then understand the Regge Trajectory:



Diagrammatic Interpretation in the cross-channel

The perturbation discussed above has a nice interpretation in terms of the number of graviton exchanged in the cross-channel:



$+ \ldots$

In the AdS/CFT correspondence this then maps to exchange of stress-tensor and its multi-twist operators in the CFT.
We can then use Lorentzian inversion formula, a formula developed by Caron-huot, 2017', to extract the scaling dimension and OPE coefficient of the operators in CFT .
Note that this analysis is fully relativistic.

CFT correlation Function

The information about the two-body dynamics discussed above can be extracted from correlation function of 4 scalar primary operators in CFT. This is given as

$$\langle \phi_1^\dagger(x_1)\phi_2^\dagger(x_2)\phi_1(x_3)\phi_2(x_4)\rangle = \frac{1}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_1+\Delta_2}{2}}}\mathcal{G}(z,\bar{z}),$$

where $x_{ij} = x_i - x_j$ and the conformal cross-ratios z, $\bar{z}$ are defined as

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad (1-z)(1-\bar{z}) = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}\,.$$

In the NR limit we have $\Delta_1, \Delta_2 \gg 1$ The operator dimension is related to the masses of the dual particles as $\Delta \sim m + O(1)$.

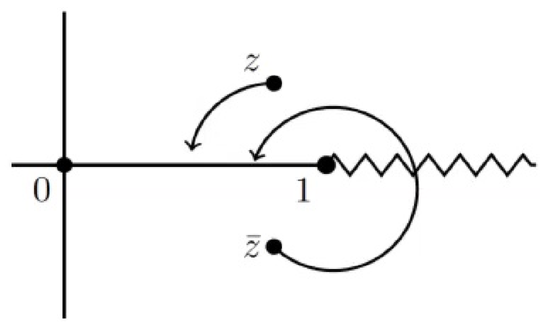
Getting the Spectrum From CFT

- We can use Lorentzian Inversion Formula to get the double-twist spectrum order by order in G_N as well.
  - Simple at order G_N for the first few trajectories

Lorentzian inversion formula generates the CFT data from the double discontinuity of the correlator

$$\mathrm{dDisc}[\mathcal{G}(z,\bar{z})] = \mathcal{G}(z,\bar{z}) - \frac{1}{2}(\mathcal{G}^{\circlearrowleft}(z,\bar{z}) + \mathcal{G}^{\circlearrowright}(z,\bar{z}))$$

We build $\mathcal{G}^{\circlearrowleft}(z,\bar{z})$ by analytically continuing the correlator from Euclidean region $\bar{z} = z^*$ around its branch cut at $\bar{z} = 1$



Simpler than the correlator itself!

Lorentzian Inversion Formula: Quick Review

The generating function for the Lorentzian inversion formula is:

$$C^t(z,\beta) = \int_z^1 d\bar{z}(\ldots) \times [\text{Inverse Block}] \times \text{dDisc}_t[\mathcal{G}(z,\bar{z})]$$

where $\beta = \Delta + J$

Inverse Block : $\quad G_{\Delta,J} \to G_{J+d-1,\Delta-d+1}$

$$\text{dDisc}_t[\mathcal{G}(z,\bar{z}] = \sum_{t-\text{OPE}} f_{23O} f_{14O} \text{dDisc}[(\ldots)(z,\bar{z}) G_{\Delta_O,l_O}(1-z,1-\bar{z})]$$

Added to u-channel contribution, this gives the full contribution:

$$C(z,\beta) = C^t(z,\beta) + (-1)^J C^u(z,\beta)$$

where C contains the CFT data through its power and its coefficient:

$$C(z,\beta) = \sum_m C_m(\beta) z^{\frac{\tau_m}{2}}$$

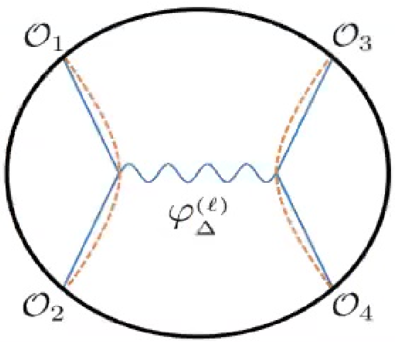
m labels the m+1th Regge trajectory.

Integral Representations for Blocks

- Geodesic Witten Diagram: start with the Witten diagram for a specific exchanged operator with dimension Δ and l and replace the integration over the bulk with integration along the geodesics

This reproduces the conformal block for dimension Δ and spin l proved by [Hijano, kraus, Perlmutter, Snively, 2015]:

For the pairwise identical operators [1]



$$G_{\Delta,\ell} \propto \int_{\gamma_{12}} d\lambda \int_{\gamma_{34}} d\lambda' \phi_\Delta^{(\ell)}(y(\lambda), y(\lambda'); \Delta, \ell)$$

[1] picture taken from GWD paper

Evaluate the Inversion Formula

- Expand the inverse block and the dDisc in power of z to read the spectrum for each Regge trajectories
  - Advantage: at order G_N only identity and stress-tensor contributes to the double discontinuity of the correlator.
  - Disadvantage: The double expansion of the dDisc and the inverse block in power of z gets complicated except for $n \gg 1$ and the first few $n = 0, 1, 2$ in odd dimensions

Obstruction: The blocks do not admit closed form in odd dimensions.

- Solution: use Geodesic Witten diagram to get integral representation for the stress-tensor block and use collinear expansion $(z \to 0)$ for the inverse block

Geodesic Witten Diagram

$\phi_{\Delta}^{(l)}(y(\lambda), y(\lambda'); \Delta, \ell)$ is the spin-ℓ propagator pulled-back on the two geodesics.

- Using symmetry in the global coordinate we can fix one geodesic to lie at the spatial origin $r = 0$. The integral then reduces to:

$$G_{\Delta,\ell} \propto \int_{\gamma} d\lambda h_{a_1 \ldots a_\ell}(y(\lambda)) \frac{dy^{a_1}}{d\lambda} \cdots \frac{dy^{a_\ell}}{d\lambda}$$

where $h_{a_1 \ldots a_\ell}$ is transverse traceless symmetric spin-ℓ tensor solution to wave equation:

$$h_{a_1 \ldots a_\ell} = (\Delta(\Delta - d) - \ell) h_{a_1 \ldots a_\ell}$$

Using this we can then calculate the stress-tensor block in any dimension:

$$G_{\Delta=d,\ell=2} = \mathcal{N}_d \int_{-\infty}^{\infty} d\lambda \frac{1}{r(\lambda)^{d-2}} \left(\dot{t}(\lambda)^2 - \frac{\dot{r}(\lambda)^2}{(1 + r(\lambda)^2)^2} \right)$$

Lorentzian Inversion Formula: Stress-tensor Exchange

With the above analysis the inversion formula can then be analytically performed to give the anomalous dimension in the large Δ limit is:

$$\gamma^{(1)}_{n=0,\ell} = -2 f_T^2 \frac{128}{3\pi} \sqrt{\frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}} \frac{\Gamma(\ell+1)}{\Gamma(\ell+\frac{3}{2})}$$

$$\gamma^{(1)}_{n=1,\ell} = -2 f_T^2 \frac{128}{3\pi} \sqrt{\frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}} \frac{(4\ell+5)\Gamma(\ell+1)}{4\sqrt{2}\Gamma(\ell+\frac{5}{2})}$$

Stress-tensor OPE

We check that matching these with the bulk calculation (${}_3F_2$) upon realizing $\mu = \sqrt{\frac{\Delta_1\Delta_2}{\Delta_1+\Delta_2}}$ correctly identifies f_T:

$$\frac{f_T^2}{\Delta_1 \Delta_2} = \left(\frac{3}{2}\right)^2 c_T^{-1} \qquad c_T = 96 \frac{L^2_{\text{AdS}}}{2\pi G_N}$$

Highlights

- Repeating the calculation for the conserved current gives the same result up to a general normalization. Indeed exchanging any conserved operator gives the same result in the large Δ_i limit. This is expected because in the bulk they all correspond to $\frac{1}{r}$ potential.
- All of the calculations above can be done for general d. However here we focused on d=3
- We can also do the calculation with a scalar exchange with large dimension. $\Delta_{\text{Exch}} \sim \Delta_{\text{ext}}$ and obtain the result for the Yukawa potential in the bulk.

Next Order

- The next order, G_N^2 the direct calculation in CFT becomes too tedious: need to sum over all the double traces of $[TT]_{n,l}$.
- The calculation in the bulk involves the following infinite sum:

$$\gamma_{n,\ell}^{(2)} = \delta E_{\ell,n}^{(2)} = \sum_{m=0}^{\infty} \frac{1}{E_{\ell,n} - E_{\ell,m}} \left| \int_{\mathrm{AdS}_4} \phi_{n\ell}(r)\phi_{n\ell}^*(r)V(r)d\mathrm{Vol}\right|^2$$

This can be performed for $n = 0$ and $n = 1$ to give the near-circular precession at order G_N^2

$$\gamma_{0,\ell}^{(2)} = -G_N^2(\ldots)(\ell,d)_4F_3(\ldots,\frac{d}{2}+\ell+1;1)$$

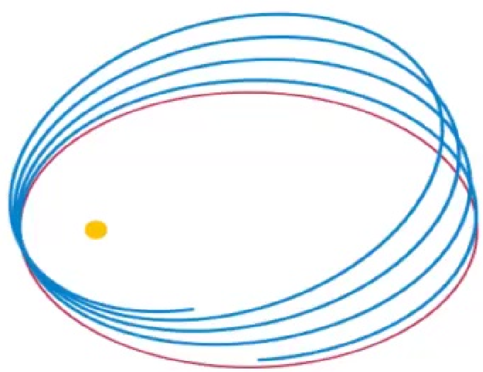
$$\gamma_{1,\ell}^{(2)} = -G_N^2((\ldots)_4F_3(\ldots,\frac{d}{2}+\ell+2;1)$$

$$+((\ldots)_4F_3(\ldots,\frac{d}{2}+\ell+2;1)+((\ldots)_4F_3(\ldots,\frac{d}{2}+\ell+2;1))$$

Remember at any order in the coupling we must get the same result for any $V(r) \propto 1/r$ up to normalization! This is very nontrivial from the CFT viewpoint!

Precession of near-circular orbits

- circular orbit in AdS is the lowest energy orbits for a given angular momentum: this means it contributes to leading Regge trajectory by definition. The period is determined by frequency ω_ϕ
- perturbing away from exactly circular, we get linearised radial motion with $\omega_r = \sqrt{\frac{V''(r)}{m}}$
- The ratio of these then gives the precession $\frac{\omega_r}{\omega_\phi}$: If the ratio is an integer the orbit does not precess.
  - Kepler: $\frac{\omega_r}{\omega_\phi} = 1$
  - Harmonic oscillator: $\frac{\omega_r}{\omega_\phi} = 2$



Precession from the anomalous dimension

- for any given l, we have one-dim motion in the effective potential $\Delta_n E_{n,\ell} = \hbar\omega_r$
- for any given n, we have $\Delta_\ell E_{n,\ell} = \hbar\omega_\phi$

The precession of the near circular-orbit is then the ratio of the gap between the first two Regge trajectories and their slope which in the regime of our perturbation theory is:

$$\frac{\omega_r}{\omega_\phi} = 2 + G_N \frac{32\left(4\left(\sqrt{2}-2\right)l + 5\sqrt{2} - 12\right)\Gamma(l+1)}{3\pi\Gamma\left(l+\frac{5}{2}\right)} + O(G_N^2)$$

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CFT correlation Function

We now aim to build a flat space scattering amplitude which is equivalent to the CFT correlator :

$$\langle \phi_1^\dagger(x_1)\phi_2^\dagger(x_2)\phi_1(x_3)\phi_2(x_4)\rangle = \frac{1}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_1+\Delta_2}{2}}}\mathcal{G}(z,\bar{z}),$$

The relevant regime for us is the NR limit $\Delta_1, \Delta_2 \gg 1$ with $z\Delta_i$ and $\bar{z}\Delta_i$ fixed.

Preparing Single-Particle state

- Primary state $|O\rangle$ or ground state of HO: operator inserted at distant Euclidean past $t_E = -\infty$ on the cylinder (or at origin in the y-plane)
- Apply the translation operator $e^{-i\vec{y}\cdot\vec{P}}$ to the ground state to create a single state operator at any other point at Euclidean time and angle:

$$|O\rangle \propto e^{-iy\cdot\vec{P}}|O\rangle$$
$$\sim e^{\sqrt{2m}\vec{y}\cdot\vec{A}^{\dagger}}|0\rangle$$

We see that we get a coherent state: a Gaussian wavefunction with the same width as the ground state but center offset from the origin

Preparing Two-Particles state

- Non-interacting two particle state: tensor product of two coherent states defined above. Splitting this into center of mass and separation of the particles is simple:
  - Split the center of mass special conformal operator as a sum:

$$\vec{K}_1 + \vec{K}_2 = -i\sqrt{2m_1}\vec{a}_1 - i\sqrt{2m_2}\vec{a}_2$$

    The tensor product state is then eigenstate of $\vec{K}$ with eigenvalues $-2i(m_1\vec{y}_1 + m_2\vec{y}_2)$
  - We then choose $m_1\vec{y}_1 + m_2\vec{y}_2 = 0$ so the product state is annihilated by $\vec{K}$ and is a primary state.

Preparing Two-Particles state

- This motivates us to consider the 'Euclidean scattering states $|\psi(\tau_{in}, \Omega_{in})\rangle$ defined by:

$$|O_1 O_2\rangle \propto |\psi\rangle \otimes |0\rangle_{\text{COM}}$$

where

$$|\psi\rangle \sim e^{\vec{\alpha}_{in} \cdot \vec{a}^{\dagger}} |0\rangle$$

Here $\vec{a}^{\dagger}$ are creation operator built from the separation variables.

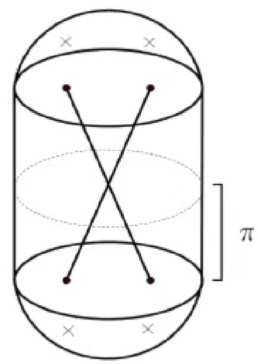
Flat Space Amplitude

We build the flat space scattering amplitude by inserting a real time $\frac{\pi}{\omega}$ evolution operator between the prepared initial and final states:

$$G^{\text{scat}}(E,\theta) = \langle \psi_{\text{in}} | e^{-i\frac{\pi}{\omega}H} | \psi_{\text{out}} \rangle$$

where from the Euclidean insertion point we can find out the dependence of the correlator on z and $\bar{z}$. Pictorially we have:

$\mathcal{G}(z,\bar{z}) =$



see [Hijano 2019] for similar construction

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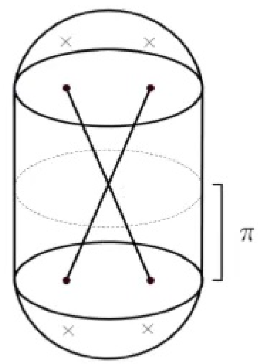

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Summary & Conclusion

In this talk we analyzed the non-relativistic two-body problem in AdS. Specifically in this limit:

- As a first step we have exploited perturbation theory in order to extract data about the CFT spectrum.
  - We find that we can get leading and subleading (only for the first few n) anomalous dimension of double twist operators $[O_1 O_2]_{n,\ell}$ when $\Delta_i \gg 1$.
  - We then used CFT tools to obtain the anomalous dimension for $n = 0$ and $n = 1$ and finding perfect agreeement after taking $\Delta_i \gg 1$
  - We used the obtained the CFT data to obtain the precession of AdS orbit in the regime that our perturbation is valid.

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- As a first step we have exploited perturbation theory in order to extract data about the CFT spectrum.
  - We find that we can get leading and subleading (only for the first few n) anomalous dimension of double twist operators $[O_1 O_2]_{n,\ell}$ when $\Delta_i \gg 1$.
  - We then used CFT tools to obtain the anomalous dimension for $n = 0$ and $n = 1$ and finding perfect agreeement after taking $\Delta_i \gg 1$
  - We used the obtained the CFT data to obtain the precession of AdS orbit in the regime that our perturbation is valid.
- We have introduced a physical way to extract the flat space bulk s-matrix from the AdS/CFT correspondence. This has a few advantages compared to the constructions available in the literature:
  - Since we used Euclidean time evolution in order to prepare the states we did not need to smear in and out states over real time
  - We did not had to treat $V(r)$ as a small correction. We split (mass) + (COM) + (relative), where mass is large but the interaction can still be strong

Future Direction

- NR dynamics turns out to be an easy toy model in which we can prepare the states do the partial wave decomposition and much more. It is thus the perfect playground to study the strongly coupled bulk physics from the CFT
- Higher order in perturbation theory. This is interesting from the CFT perspective as it gives the contribution of the resummation of all multi twist operator at a single order.
- Precession of orbits in all regime: where the Kepler potential dominates and etc
- Of course getting the OPE coefficient perturbatively using this method would be of interest as well.
- One can also try other central potentials to read CFT data in other regime or exchange of other particles.