Title: Analyticity and Unitarity for Cosmological Correlators

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Series: Quantum Fields and Strings

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Abstract: We consider quantum field theory on a rigid de Sitter space. We show that the perturbative expansion of late-time correlation functions to all orders can be equivalently generated by a non-unitary Lagrangian on a Euclidean AdS geometry. We use this to infer the analytic structure of the spectral density that captures the conformal partial wave expansion of a late-time four-point function, to derive an OPE expansion, and to constrain the operator spectrum. Generically, dimensions and OPE coefficients do not obey the usual CFT notion of unitarity. Instead, unitarity of the de Sitter theory manifests itself as the positivity of the spectral density. We illustrate and check these properties by explicit calculations in a scalar theory by computing first tree-level, and then full one-loop-resummed exchange diagrams.

Analiticity and Unitarity for Cosmological correlators

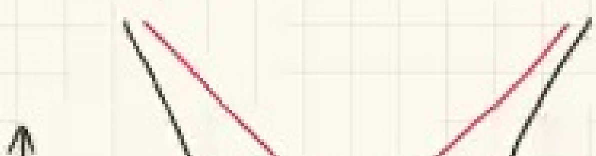
based on 2108.01695 w/ V. Gorbenko & S. Komatsu

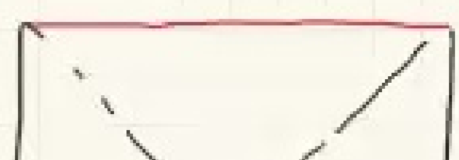
QFT on rigid dS background

= no dynamical gravity

dS: maximally symmetric space-time

exponentially expanding universe





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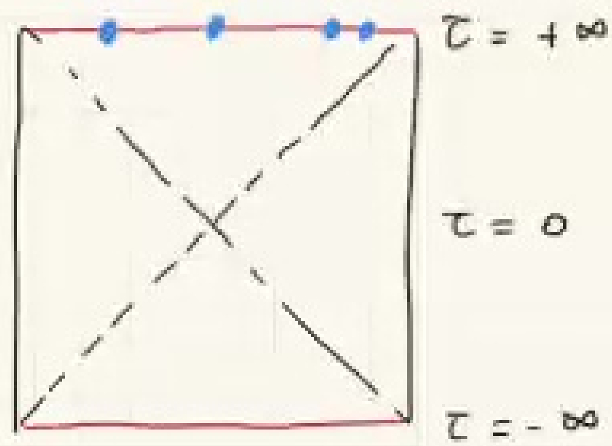
Perimeter

Acts as Euclidean Conformal group on

late-time correlators



$$\lim_{\tau \to +\infty} \tau^{\#} \langle O_1(\tau, \vec{x}_1) \dots O_n(\tau, \vec{x}_n) \rangle$$

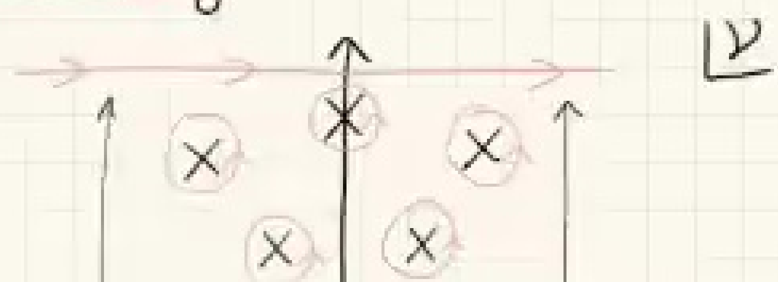
action of $SO(1, d+1)$

Summary of results

$$\langle O(\vec{x}_1)\, O(\vec{x}_2)\, O(\vec{x}_3)\, O(\vec{x}_4) \rangle$$

$$= \frac{1}{|\vec{x}_{12}|^{2\Delta}\,|\vec{x}_{34}|^{2\Delta}} \sum_J \int_0^\infty d\nu\, \boxed{\rho_J(\nu)}\, \underbrace{F_{\nu,J}(z,\bar{z})}_{\text{Conformal Partial Wave (CPW)}}$$

* **Analiticity:**



$\approx$ OPE

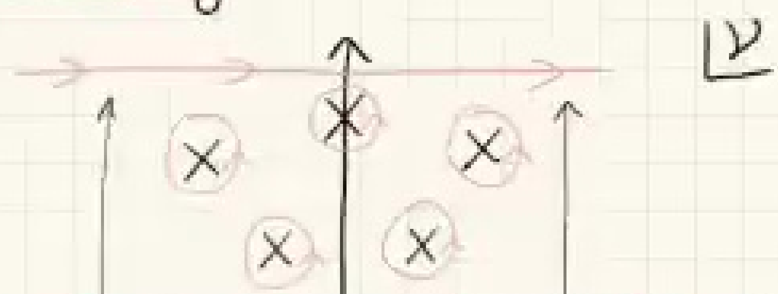
Summary of results

$$\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\,\mathcal{O}(\vec{x}_3)\,\mathcal{O}(\vec{x}_4) \rangle$$

$$= \frac{1}{|\vec{x}_{12}|^{2\Delta}\,|\vec{x}_{34}|^{2\Delta}} \sum_J \int_0^\infty d\nu\, \boxed{\rho_J(\nu)}\, \underline{F_{\nu,J}(z,\bar{z})}$$

Conformal Partial Wave (CPW)

* Analiticity:

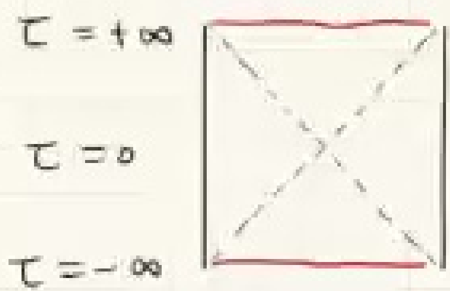


$\approx$ OPE

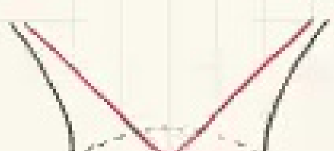
Motivations

* Observables in dS :

dynamical gravity $\Rightarrow$ asymptotic observables (e.g. AdS/CFT)



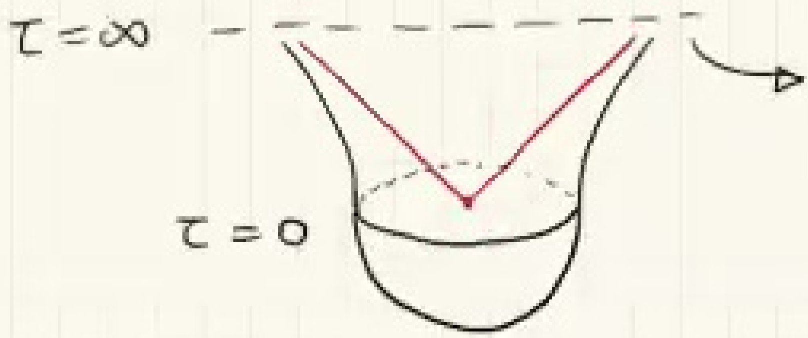
in dS only past & future space-like boundary



Hartle-Hawking state : only future boundary

(Parenthesis:

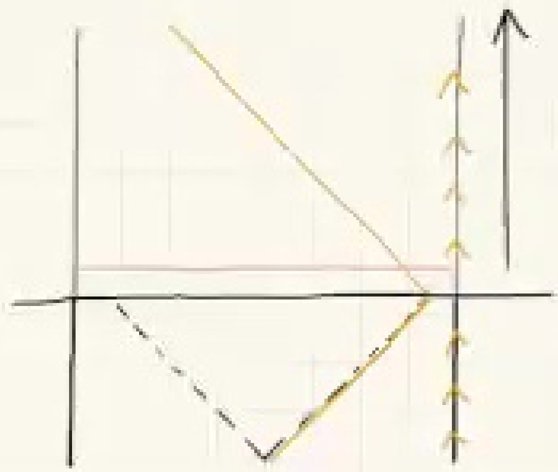
Other object of interest : $\Psi_{HH}[\phi]$



wave function on future ∞

$$\Psi_{HH}[\phi] \approx \sum_n \int_1 \cdots \int_n \underbrace{G_n(\vec{x}_1 \cdots \vec{x}_n)}_{n\text{-point correlator in CFT}} \phi(\vec{x}_1) \cdots \phi(\vec{x}_n)$$

* Cosmology :



flat space

cut @ large τ_c

end of Inflation (cartoon)

future observer: measures late-time correlators

CFT @ $\tau = +\infty$: 0^{th} order approximation

for small $\dot{\phi}$

Perturbation theory

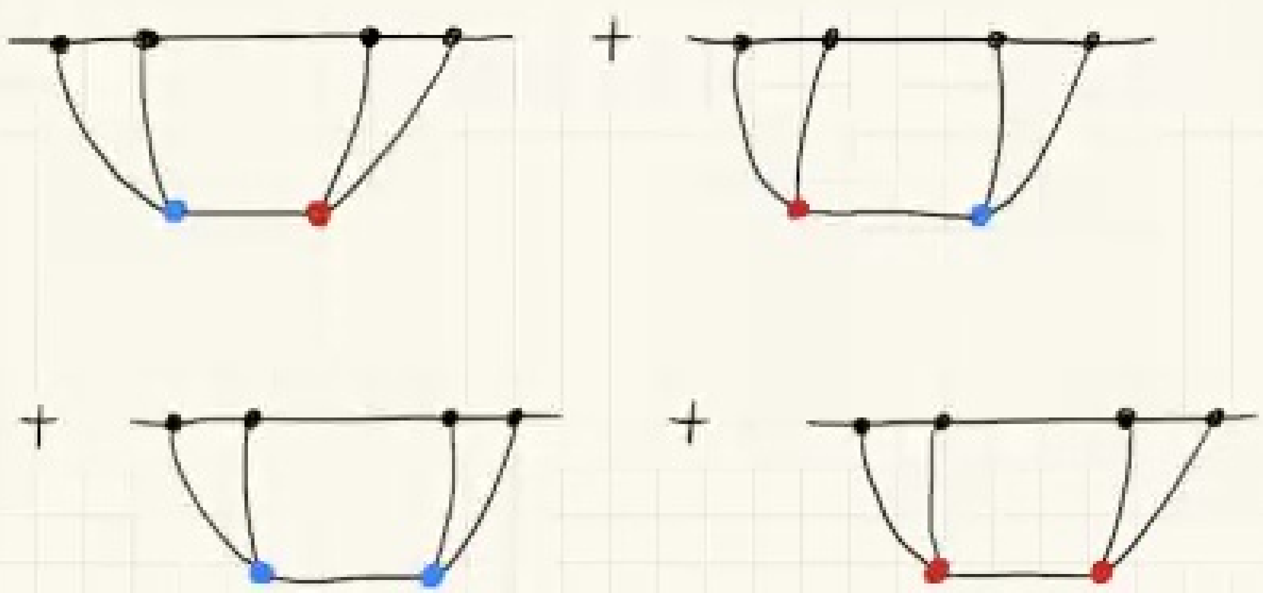
$$\lim_{t \to +\infty} t^{\#} \left(\langle \Omega | \, \phi(t, \vec{x}_1) \cdots \phi(t, \vec{x}_n) \, | \Omega \rangle \right)$$

HH vacuum

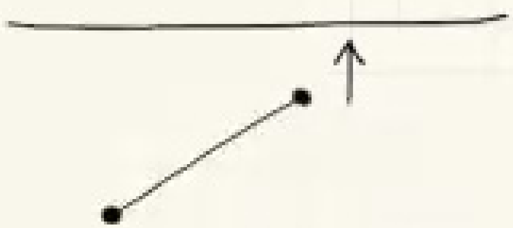
$$\frac{\langle 0 | \bar{T}\left(e^{i \int_{-\infty(1+i\epsilon)}^{t} H_{int}} \right) \phi^{(0)}(t, \vec{x}_1) \cdots \phi^{(0)}(t, \vec{x}_n) \, T\left(e^{-i \int_{-\infty(1-i\epsilon)}^{t} H_{int}} \right) | 0 \rangle}{\langle 0 | 0 \rangle}$$

in-in contour:

Diagrams (e.g. ϕ^3):



Different propagators:



$$|s| \longrightarrow +\infty$$

$$W_\nu(s) \sim \frac{C_\nu}{(-2s)^{\frac{d}{2}-i\nu}} + \frac{C_{-\nu}}{(-2s)^{\frac{d}{2}+i\nu}}$$

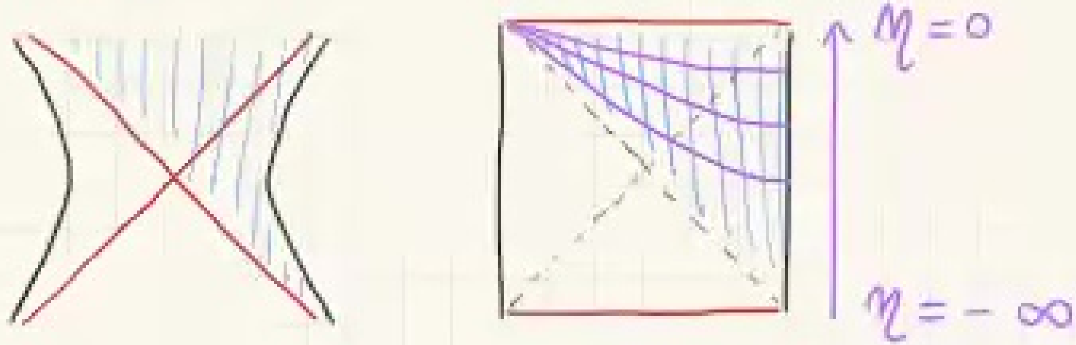
$m^2 < \frac{d^2}{4}\,,\quad 0 < i\nu < \frac{d}{2}$: light particles

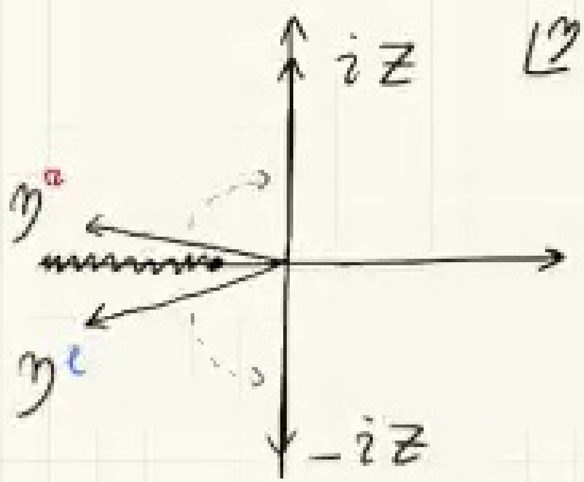
$\frac{d}{2} - i\nu$ dominates

$m^2 > \frac{d^2}{4}\,,\quad \nu \in \mathbb{R}$: heavy particles

Rotation to EAdS

Poincaré patch:





$$ds^2 = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2} \dashrightarrow -ds^2_{EAdS} = -\left(\frac{dz^2 + d\vec{x}^2}{z^2}\right)$$

$\Rightarrow$ any diagram in dS

$=$ linear combination of $\underbrace{\text{diagrams in EAdS}}_{\text{lots of technology}}$

* EAdS Lagrangian

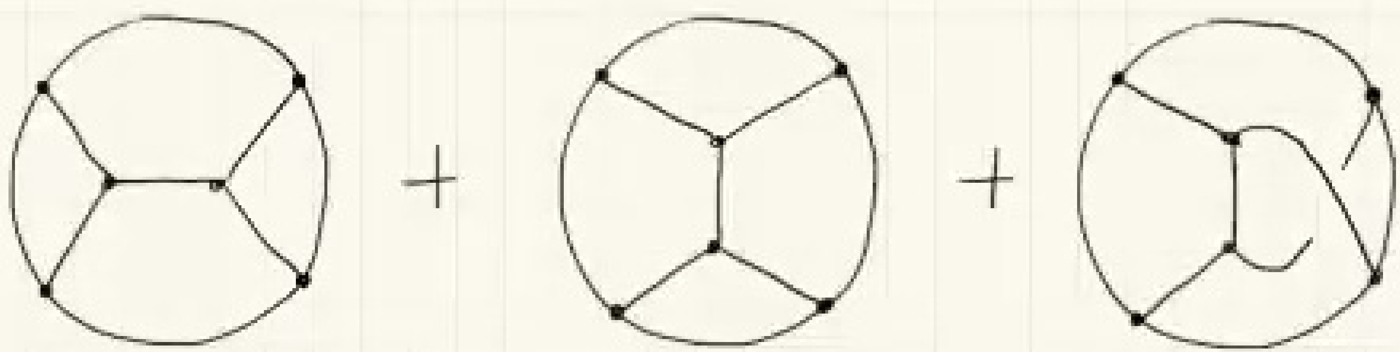
dS: $\qquad \mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - V(\phi)$

$\phi \longrightarrow \phi^{\ell}_{+},\ \phi^{\ell}_{-},\ \phi^{r}_{+},\ \phi^{r}_{-}$ on EAdS

$\langle \phi^{\ell}\ \phi^{r} \rangle = G^{AdS}$ part of $\ell\, r$ prop.

Analiticity

EAdS bdry 4pt function: e.g.

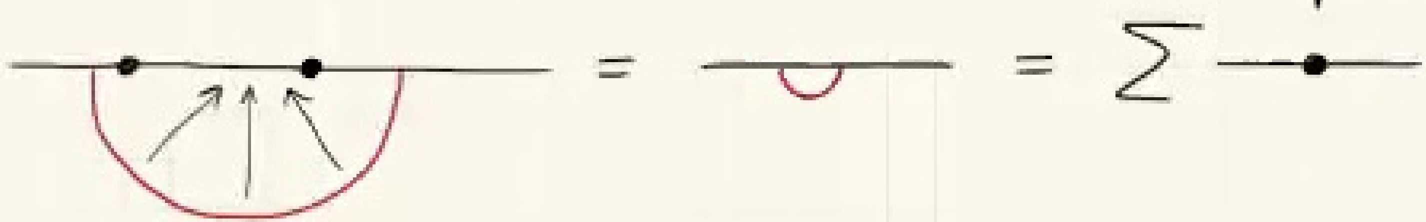


$$\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\,\mathcal{O}(\vec{x}_3)\,\mathcal{O}(\vec{x}_4)\rangle$$

$$= \frac{1}{|\vec{x}_{12}|^{2\Delta}\,|\vec{x}_{34}|^{2\Delta}} \sum_J \int_{-\infty}^{+\infty} d\nu\; \rho_J(\nu)\; F_{\nu,J}(z,\bar{z})$$

ρ with poles $\Longleftrightarrow$ (discrete) OPE expansion

Even beyond perturbation theory: state/operator correspondence



dS perturbative expansion = non-unitary EAdS theory

inherits

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Perimeter

Unitarity

EAdS unitarity :

$\longrightarrow$ real $\Delta \geq \Delta_{\text{bound}}$

$\longrightarrow$ positive $\left(C_{\mathcal{O}\mathcal{O}\mathcal{O}'_J}\right)^2 \propto \mathrm{Res}\left[\rho_J(\nu)\right]_{\nu_* = i\left(\frac{d}{2}-\Delta'\right)}$

dS late-time CFT : complex Δ

$\Rightarrow$ not unitary , no natural rotation to Minkowski or Hilbert state

Consequences of bulk unitarity ?

Idea: insert complete set of bulk states
and try to get $| \cdot |^2$

Warm-up: bulk 2pt function

$$\langle \Omega | \, \phi(t_1, \vec{x}_1) \, \phi(t_2, \vec{x}_2) | \Omega \rangle$$

$$\mathbb{1} = \underbrace{\int_0^{+\infty} d\nu \, |\nu\rangle\langle\nu|}_{\text{principal series}} + \dots$$

Positivity for 4pt function

$$\langle \Omega | O(\vec{x}_1)\, O(\vec{x}_2)\, O(\vec{x}_3)\, O(\vec{x}_4) | \Omega \rangle$$

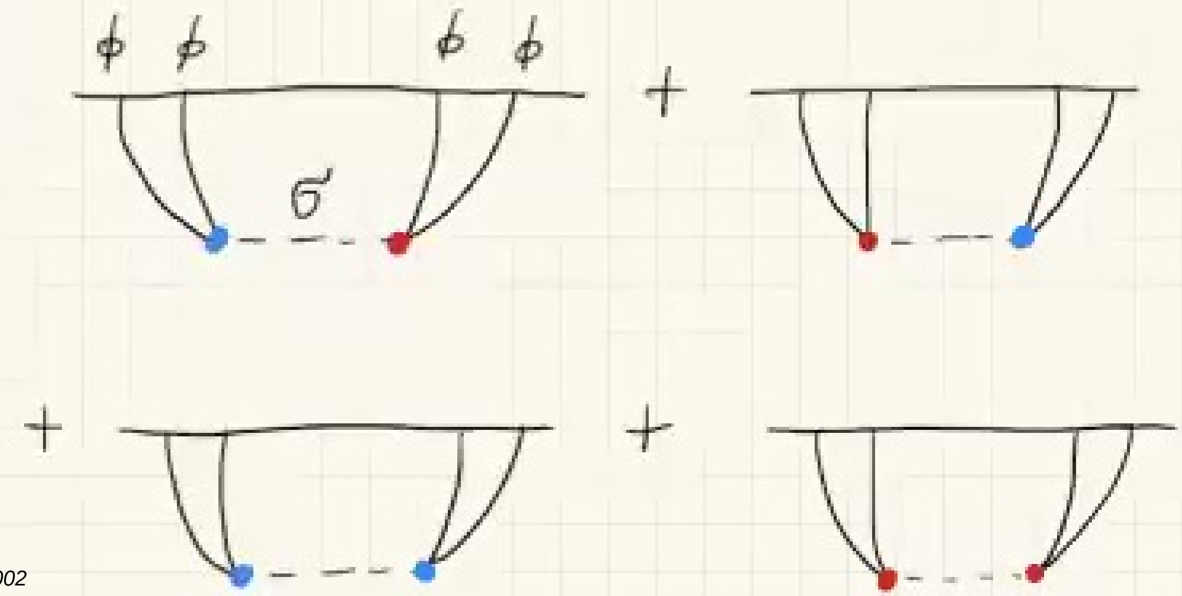
$$\mathbb{1} = \sum_J \int_0^\infty d\nu\, |\nu, J\rangle\langle \nu, J| + \dots$$

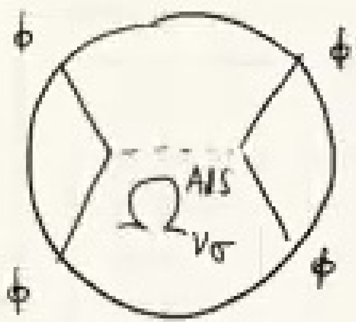
$$= \sum_J \int_0^\infty d\nu\, \langle \Omega | 12 | \nu\, J\rangle\langle \nu, J | 34 | \Omega \rangle$$

$$\mathcal{C}_{12}(\dots) = -m^2 + J(J+d-2)$$

Example

Exchange diagram in $g\, \underbrace{\phi^2}_{\text{light}}\, \underbrace{\sigma}_{\text{heavy}}$





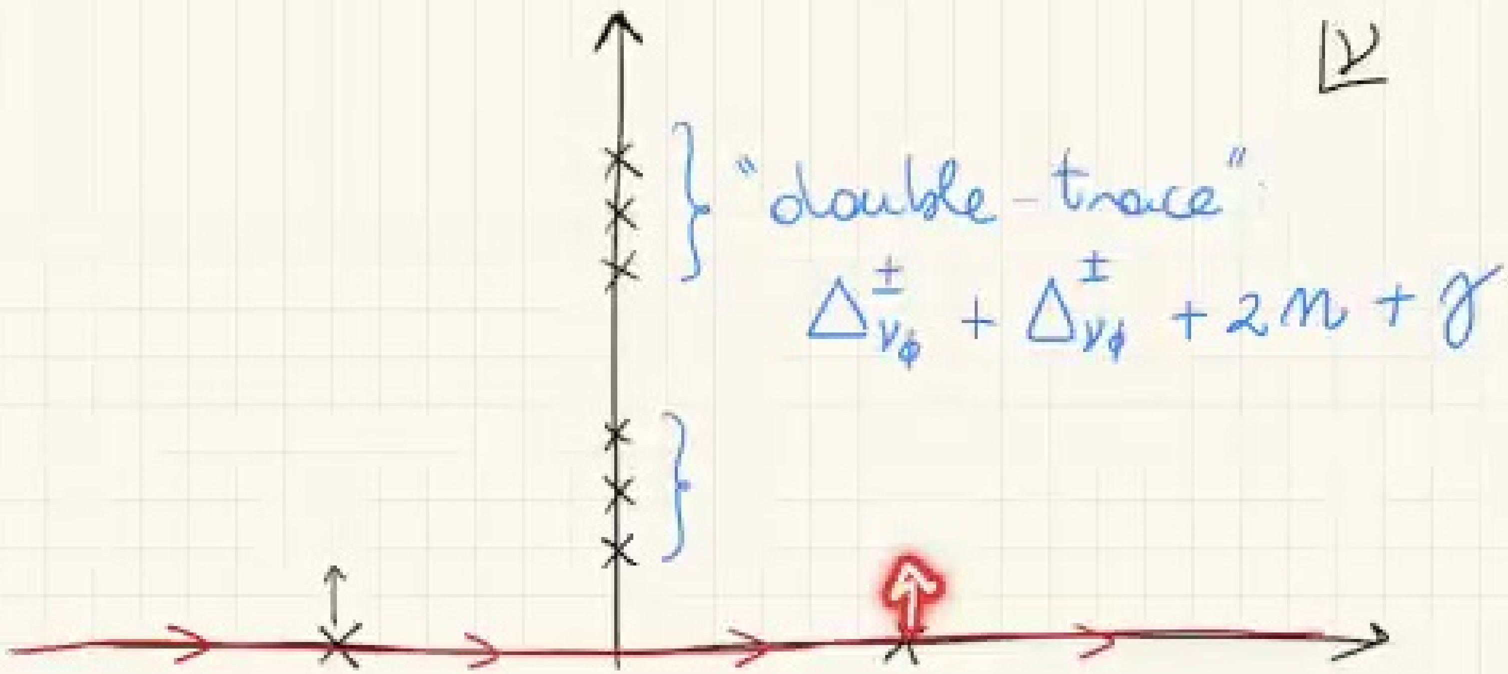
$$\propto \quad \mathcal{F}_{\nu_\sigma}$$

$$\propto \int_{-\infty}^{+\infty} d\nu \, \frac{1}{\nu^2-\nu_\sigma^2} \, \mathcal{F}_\nu$$

pole on the contour

Positivity of 2pt function $\langle \Omega | \phi^2 \phi^2 | \Omega \rangle$

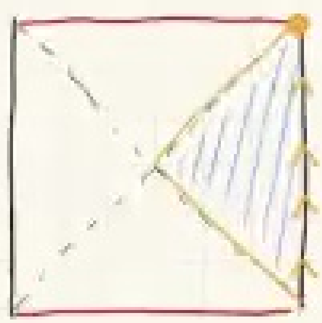
$$\Longrightarrow \quad \mathrm{Im}\, B_{\nu_\phi}(\nu) \geq 0$$



Comments on OPE and QNM

Physical origin of OPE ?

Maybe static patch:



* causal past of 1 point
* discrete set of complex frequencies : QNM

→ they control the behavior at late time of bulk correlators

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Perimeter

Future

* Beyond weak coupling:
  $O(N)$ $\lambda\phi^4$ @ large N

* Relation between CFT_φ
  and $CFT_{\text{correlators}}$

* Add $\dot{\phi}$: inflationary correlators