

Title: Analyticity and Unitarity for Cosmological Correlators

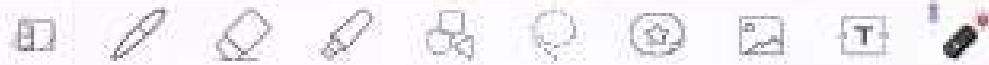
Speakers: Lorenzo Di Pietro

Series: Quantum Fields and Strings

Date: September 07, 2021 - 2:00 PM

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Abstract: We consider quantum field theory on a rigid de Sitter space. We show that the perturbative expansion of late-time correlation functions to all orders can be equivalently generated by a non-unitary Lagrangian on a Euclidean AdS geometry. We use this to infer the analytic structure of the spectral density that captures the conformal partial wave expansion of a late-time four-point function, to derive an OPE expansion, and to constrain the operator spectrum. Generically, dimensions and OPE coefficients do not obey the usual CFT notion of unitarity. Instead, unitarity of the de Sitter theory manifests itself as the positivity of the spectral density. We illustrate and check these properties by explicit calculations in a scalar theory by computing first tree-level, and then full one-loop-resummed exchange diagrams.



Analyticity and Unitarity for Cosmological correlators

based on 2108.01695 w/ V. Gorbenko
& S. Komatsu

QFT on rigid dS background
= no dynamical gravity

dS: maximally symmetric space-time
exponentially expanding universe





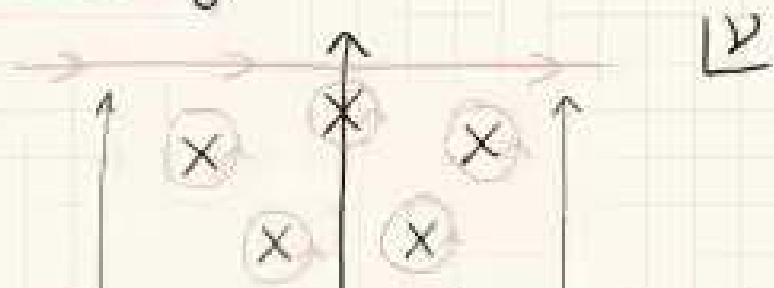
Summary of results

$$\langle O(\vec{x}_1) O(\vec{x}_2) O(\vec{x}_3) O(\vec{x}_4) \rangle$$

$$= \frac{1}{|\vec{x}_{12}|^{2\Delta} |\vec{x}_{34}|^{2\Delta}} \sum_J \int_0^\infty d\nu \rho_J(\nu) \underbrace{F_{\nu, J}(z, \bar{z})}_{\text{Conformal Partial Wave (CPW)}}$$

Conformal Partial Wave (CPW)

* Analyticity:



↙

≈ OPE



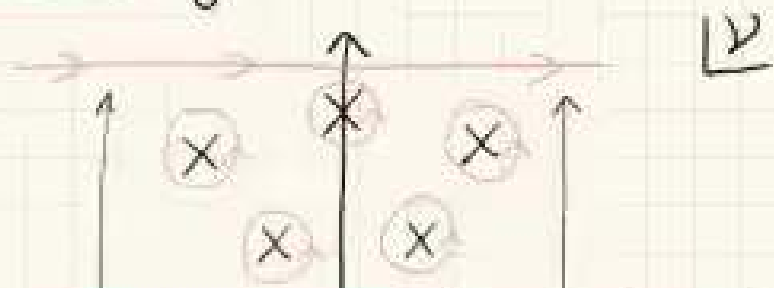
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Conformal Partial Wave (CPW)

* Analyticity:



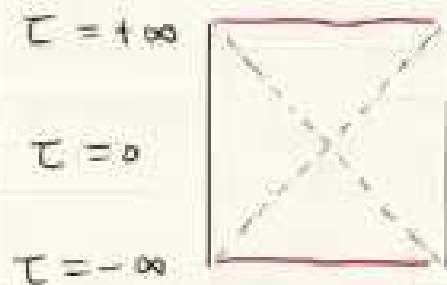
\approx OPE



Motivations

* Observables in dS:

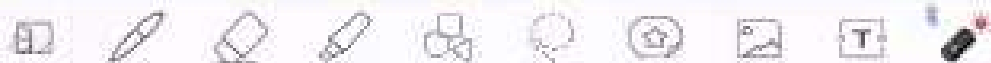
dynamical gravity \Rightarrow asymptotic observables
(e.g. AdS/CFT)



in dS only past & future
space-like boundary

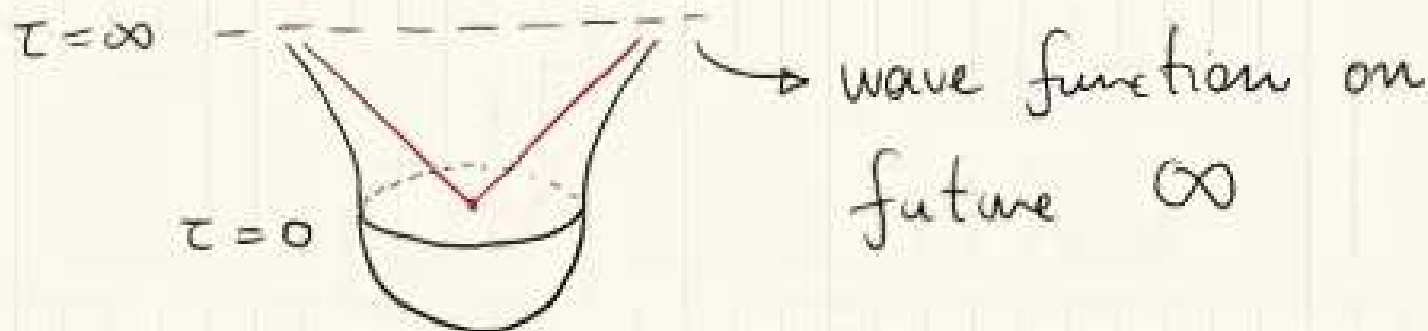


Hartle-Hawking state:
only future boundary



(Parenthesis:

Other object of interest : $\mathcal{U}_{HH}[\phi]$

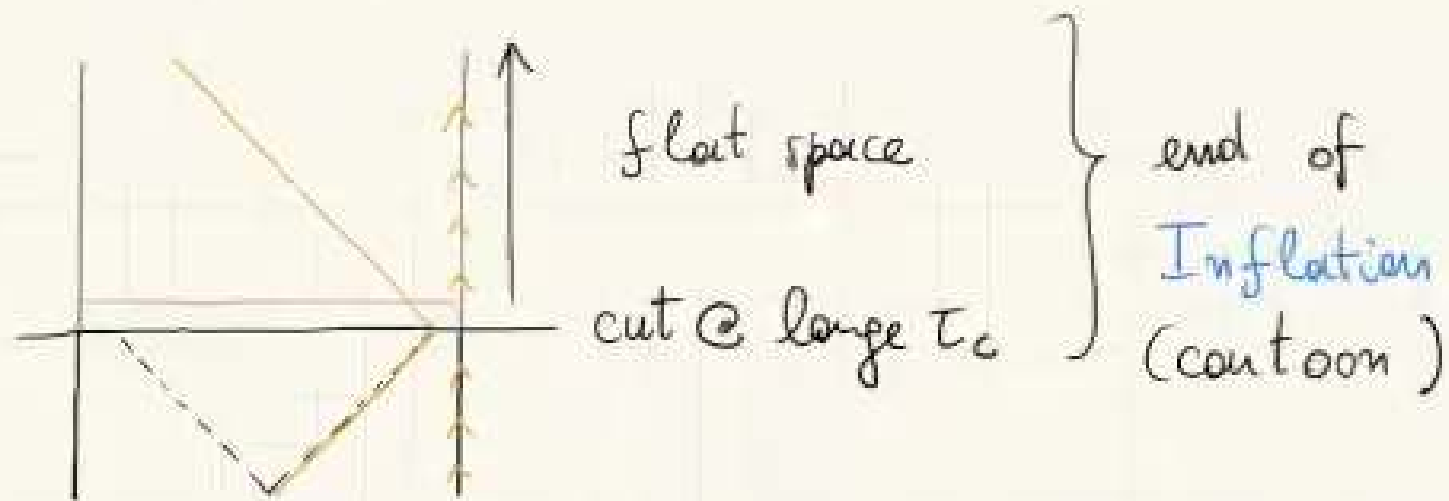


$$\mathcal{U}_{HH}[\phi] \approx \sum_m \int_1 \int_m \underbrace{G_m(\bar{x}_1 \dots \bar{x}_m)}_{m\text{-point correlator in CFT}} \phi(\bar{x}_1) \dots \phi(\bar{x}_m)$$

m -point correlator in CFT



* Cosmology:



future observer: measures late-time correlators

CFT @ $\tau = +\infty$: 0^{th} order approximation

for small ϕ

Perturbation theory

$$\lim_{t \rightarrow +\infty} t^{\#} \left(\langle \Omega | \phi(t, \vec{x}_1) \dots \phi(t, \vec{x}_n) | \Omega \rangle \right)$$

HH vacuum

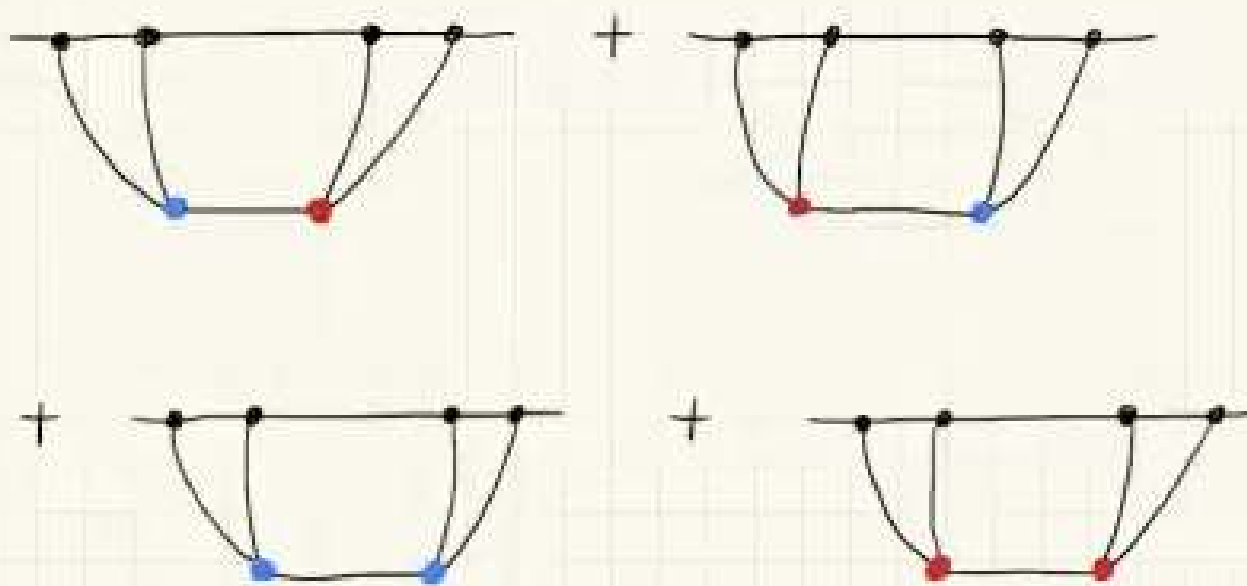
$$\langle 0 | \overline{T} \left(e^{i \int_{-\infty(1-i\epsilon)}^t H_{int}} \right) \phi^{(0)}(t, \vec{x}_1) \dots \phi^{(0)}(t, \vec{x}_n) T \left(e^{-i \int_{-\infty(1-i\epsilon)}^t H_{int}} \right) | 0 \rangle$$

$$\langle 0 | 0 \rangle$$

in-in contour:



Diagrams (e.g. ϕ^3):



Different propagators:



$$|S| \rightarrow +\infty$$

$$W_\nu(s) \sim \frac{C_\nu}{(-2s)^{\frac{d}{2}-i\nu}} + \frac{C_{-\nu}}{(-2s)^{\frac{d}{2}+i\nu}}$$

$$m^2 < \frac{d^2}{4}, \quad 0 < i\nu < \frac{d}{2} \quad \therefore \text{light particles}$$

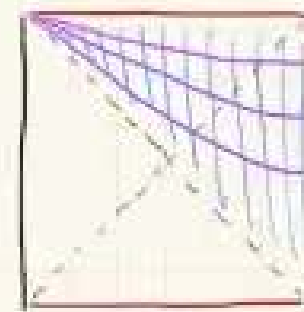
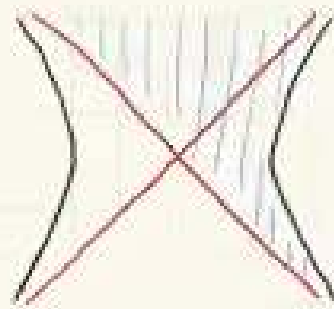
$\frac{d}{2} - i\nu$ dominates

$$m^2 \geq \frac{d^2}{4}, \quad \nu \in \mathbb{R} \quad \therefore \text{heavy particles}$$



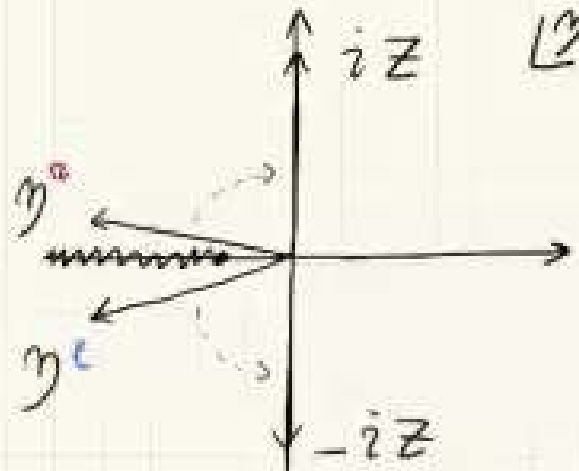
Rotation to EAdS

Poincaré patch:



$\eta = 0$

$\eta = -\infty$



$$ds^2 = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2} \longrightarrow -ds_{EAdS}^2 = -\left(\frac{dz^2 + d\vec{x}^2}{z^2}\right)$$



\Rightarrow any diagram in dS

= linear combination of diagrams in EAdS

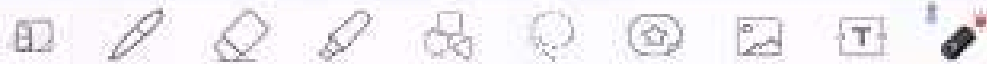
lots of technology

* EAdS Lagrangian

$$dS: \quad \mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - V(\phi)$$

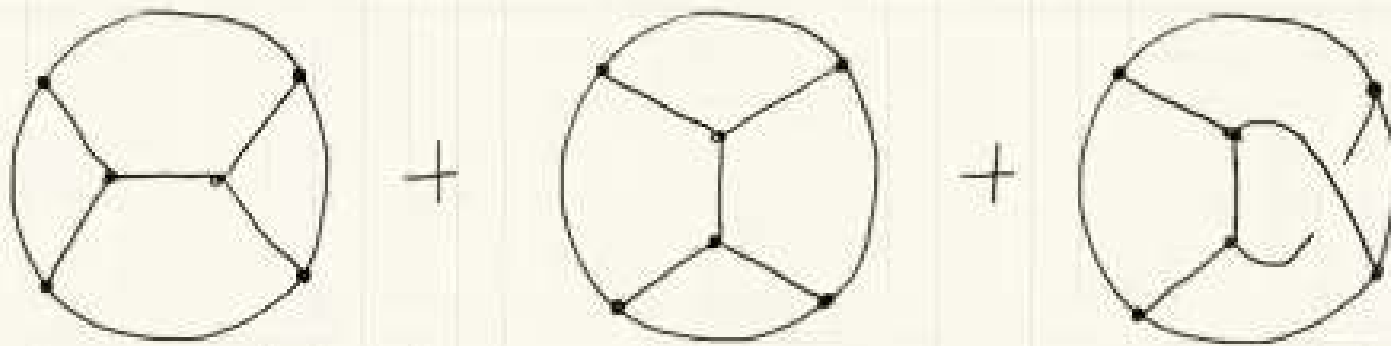
$\phi \rightarrow \phi_+^{\ell}, \phi_-^{\ell}, \phi_+^{\pi}, \phi_-^{\pi}$ on EAdS

$\langle \phi_+^{\ell} \phi_-^{\pi} \rangle = G_{\ell\pi}^{\text{AdS}}$ part of $\ell\pi$ prop.



Analyticity

EAdS boundary 4 pt function: e.g.



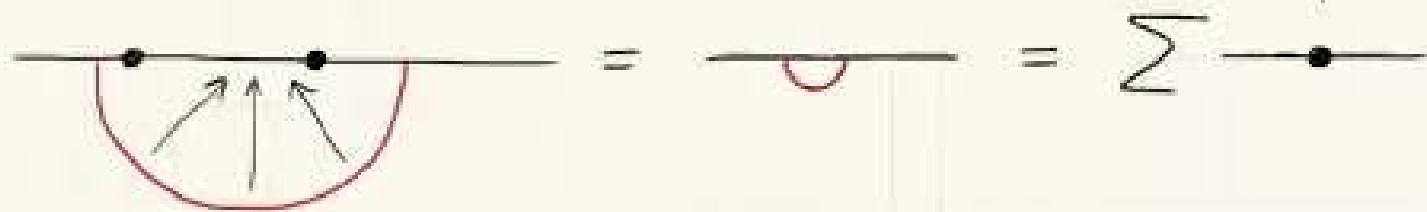
$$\langle \mathcal{O}(\vec{x}_1) \mathcal{O}(\vec{x}_2) \mathcal{O}(\vec{x}_3) \mathcal{O}(\vec{x}_4) \rangle$$

$$= \frac{1}{|\vec{x}_{12}|^{2\Delta} |\vec{x}_{34}|^{2\Delta}} \sum_{\mathcal{J}} \int_{-\infty}^{+\infty} d\nu \underbrace{\rho_{\mathcal{J}}(\nu)} \underbrace{F_{\nu, \mathcal{J}}(z, \bar{z})}$$



ρ with poles \iff (discrete) OPE expansion

Even beyond perturbation theory: state/operator correspondence



dS perturbative expansion = non-unitary EAdS theory



inherits



Unitarity

EAdS unitarity:

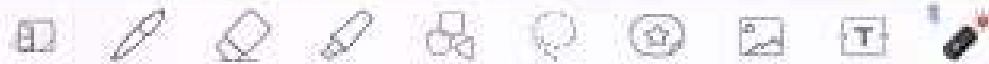
→ real $\Delta \geq \Delta_{\text{bound}}$

→ positive $(C_{000j'})^2 \propto \text{Res}[p_j(\nu)]_{\nu_* = i(\frac{d}{2} - \Delta')}$

dS late-time CFT: complex Δ

⇒ not unitary, no natural rotation to Minkowski or Hilbert state

Consequences of bulk unitarity?



Idea: insert complete set of bulk states
and try to get $1 \cdot 1^2$

Warm-up: bulk 2pt function

$$\langle \Omega | \phi(t_1, \vec{x}_1) \phi(t_2, \vec{x}_2) | \Omega \rangle$$



$$\mathbb{1} = \int_0^{+\infty} d\nu |\nu\rangle \langle \nu| + \dots$$

principal series



Positivity for 4pt function

$$\langle \Omega | \underbrace{O(\vec{x}_1) O(\vec{x}_2) O(\vec{x}_3) O(\vec{x}_4)} | \Omega \rangle$$

$$\mathbb{1} = \sum_J \int_0^\infty d\nu | \nu, J \rangle \langle \nu, J | + \dots$$

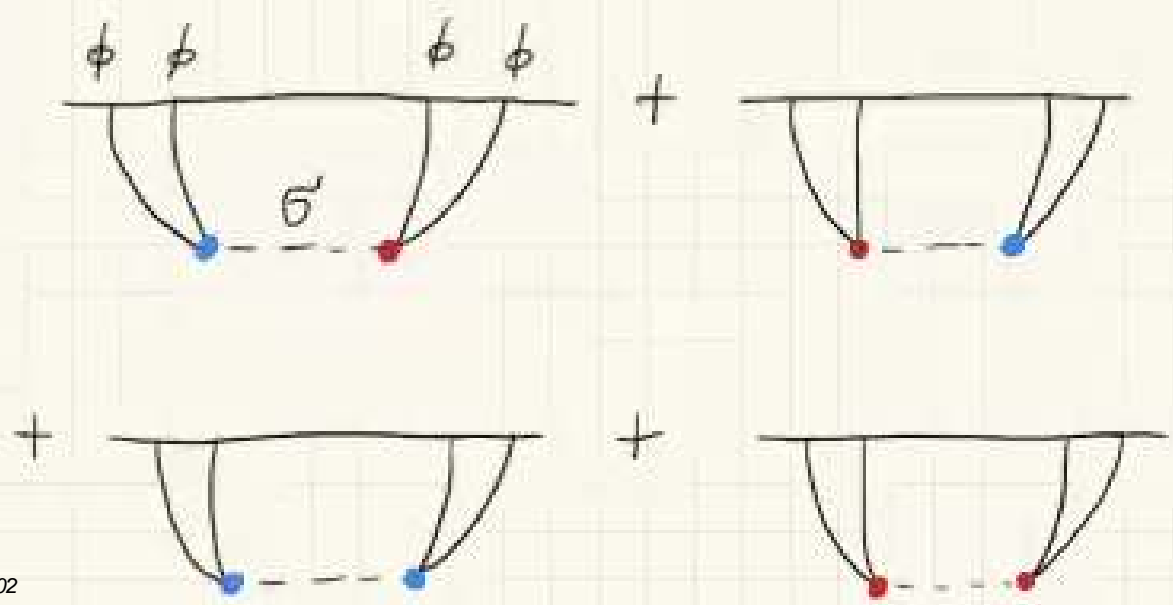
$$= \sum_J \int_0^\infty d\nu \langle \Omega | 12 | \nu, J \rangle \langle \nu, J | 34 | \Omega \rangle$$

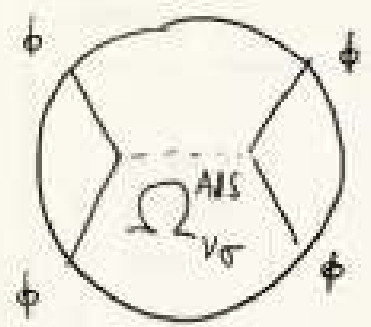
$$e_{-12}(\dots) = -m^2 + J(J+d-2)$$

Example

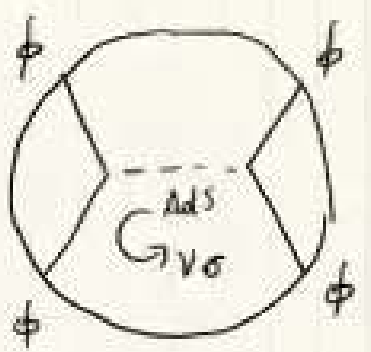
Exchange diagram in $g\phi^2\sigma$

light \leftarrow ϕ^2 \rightarrow heavy σ





$$\propto \int_{v_0}^{\infty}$$



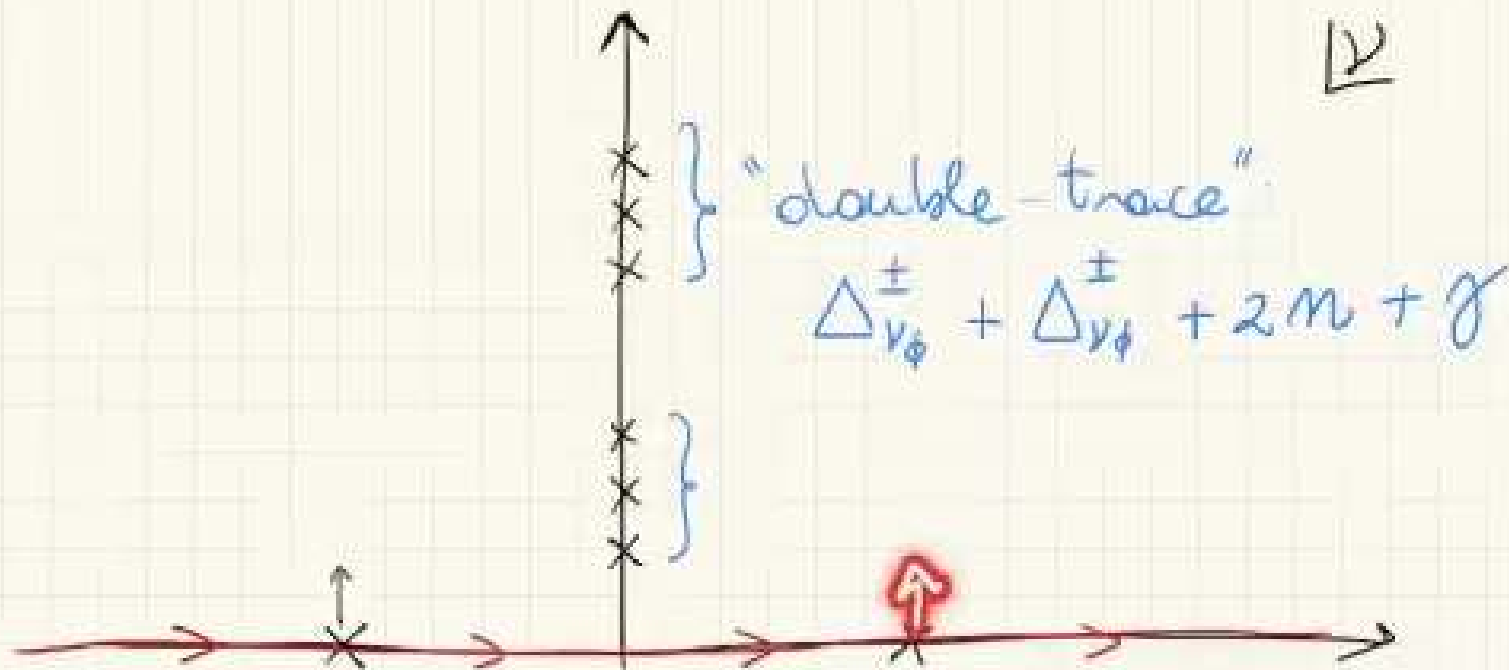
$$\propto \int_{-\infty}^{+\infty} dv \frac{1}{v^2 - v_0^2} \int_v$$

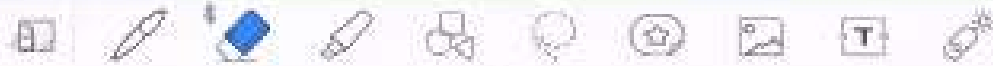
pole on the contour



Positivity of 2pt function $\langle \Omega | \phi^2 \phi^2 | \Omega \rangle$

$$\Rightarrow \text{Im} B_{\nu_\phi}(\nu) \geq 0$$

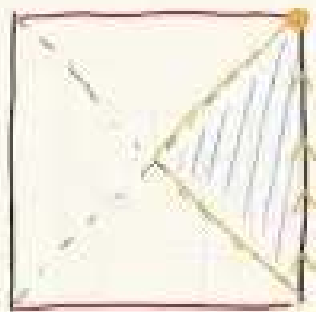




Comments on OPE and QNM

Physical origin of OPE?

Maybe static patch:



- * causal part of 1 point
- * discrete set of complex frequencies: QNM

→ they control the behavior at late time of bulk correlators



Future

* Beyond weak coupling:

$$O(N) \lambda \phi^4 @ \text{large } N$$

* Relation between CFT_ψ
and $CFT_{\text{correlators}}$

* Add $\dot{\phi}$: inflationary correlators