

Title: Matter Unification at the TeV scale: Flavour anomalies and muon (g-2)

Speakers: Clara Murgui

Series: Particle Physics

Date: July 13, 2021 - 1:00 PM

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Abstract: Several anomalies have been recently reported by different laboratory experiments: the flavor anomalies involving B meson semileptonic and leptonic decays by the LHCb and B-factories, as well as the anomalous muon (g-2) by the Fermilab (g-2) collaboration. These deviations, if not coming from underestimated experimental or theoretical uncertainties, are pointing to new degrees of freedom around the few TeV scale. Enlarging the field content of the Standard Model may lead to baryon number violation, whose aggressive experimental constraints can rule out a wide range of attractive candidates. Motivated by its safeness under unacceptable baryon number violation and the possibility for having TeV scale physics, I will introduce the simplest theory for matter (leptons-quarks) unification based on the Pati-Salam symmetry and show how this theory can address both the flavor anomalies and the muon (g-2) with the scalar leptoquarks that it predicts.

Zoom Link: <https://pitp.zoom.us/j/95090784229?pwd=Q21oQUVkeXFiSmt5S3hKcGJ3SlEyZz09>



# Matter Unification at the TeV scale: Flavour anomalies and muon (g-2)

Clara Murgui

In collaboration with Pavel Fileviez Pérez (CWRU), Alexis Plascencia (CWRU)  
and Mark B. Wise (Caltech)

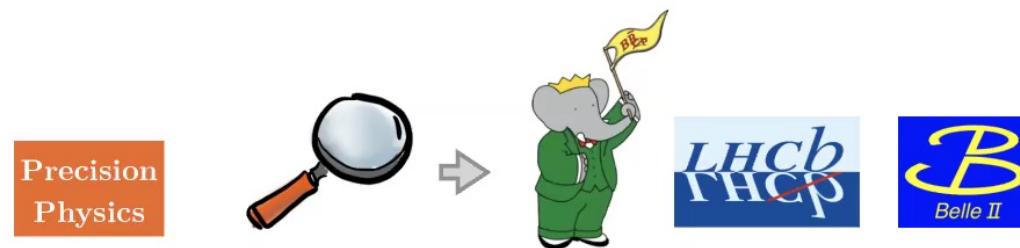
July 13th 2021  
Perimeter Institute



# Accessing High Energies

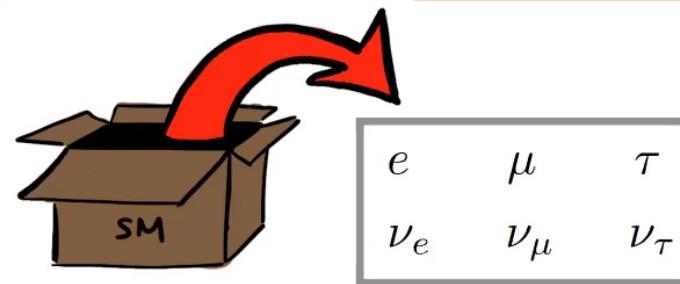


$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{O} \left( \frac{\text{Energy}}{\Lambda_{\text{NP}}} \right)^n$$



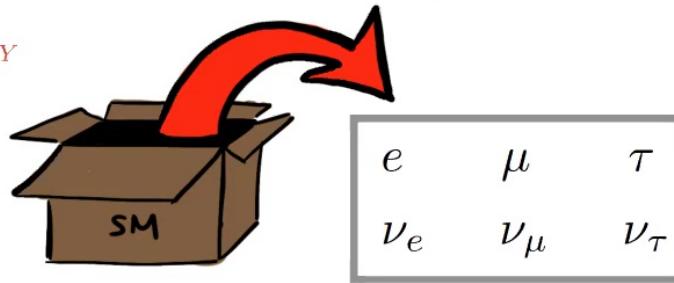


# Lepton Flavour Universality (Violation)



# Lepton Flavour Universality (Violation)

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$



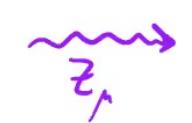
$e$        $\mu$        $\tau$



$e_L$        $\mu_L$        $\tau_L$        $(\nu_e)_L$        $(\nu_\mu)_L$        $(\nu_\tau)_L$

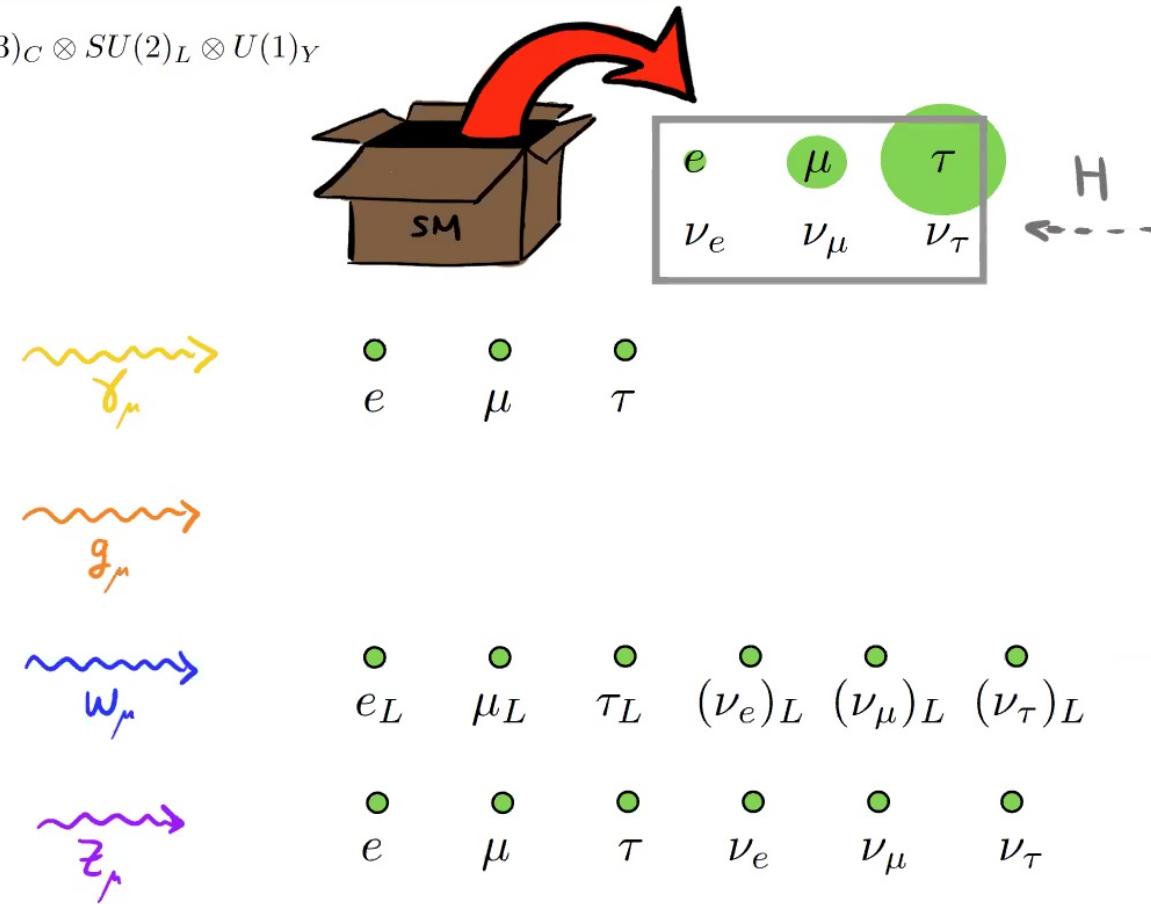


$e$        $\mu$        $\tau$        $\nu_e$        $\nu_\mu$        $\nu_\tau$

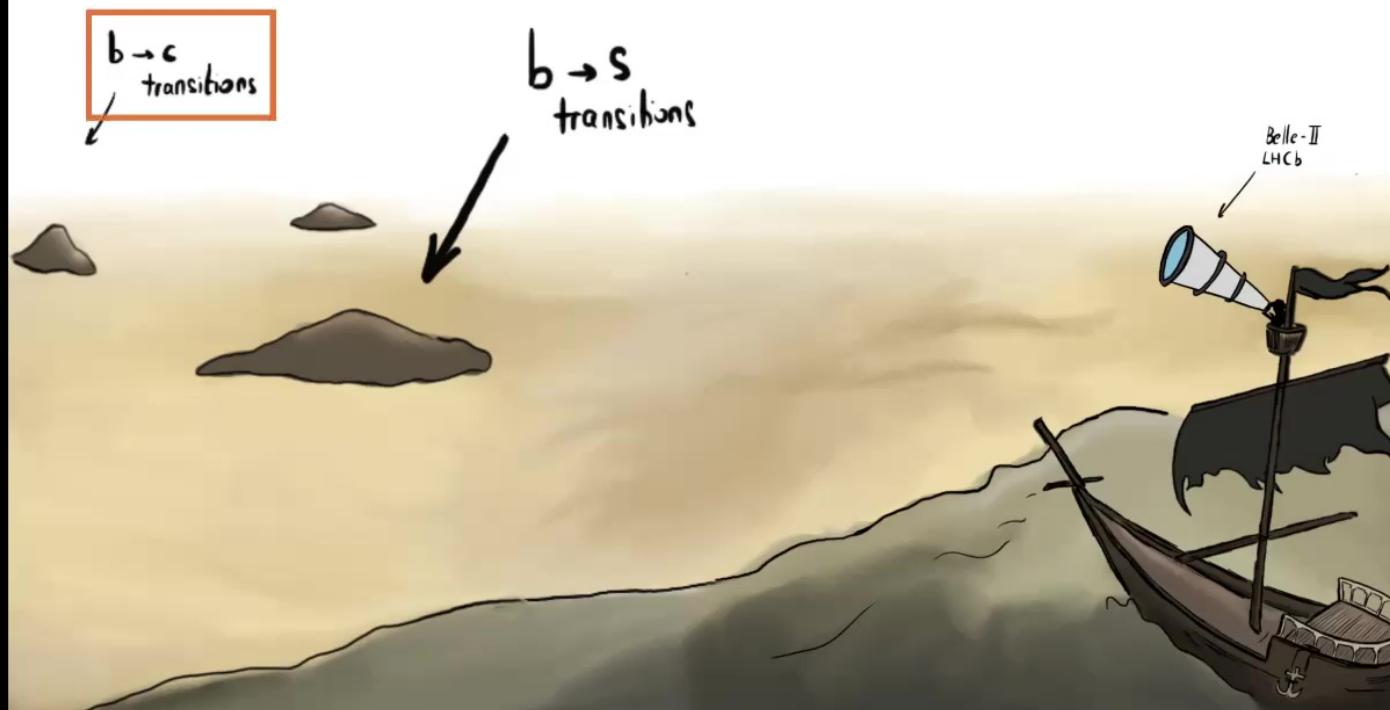


# Lepton Flavour Universality (Violation)

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

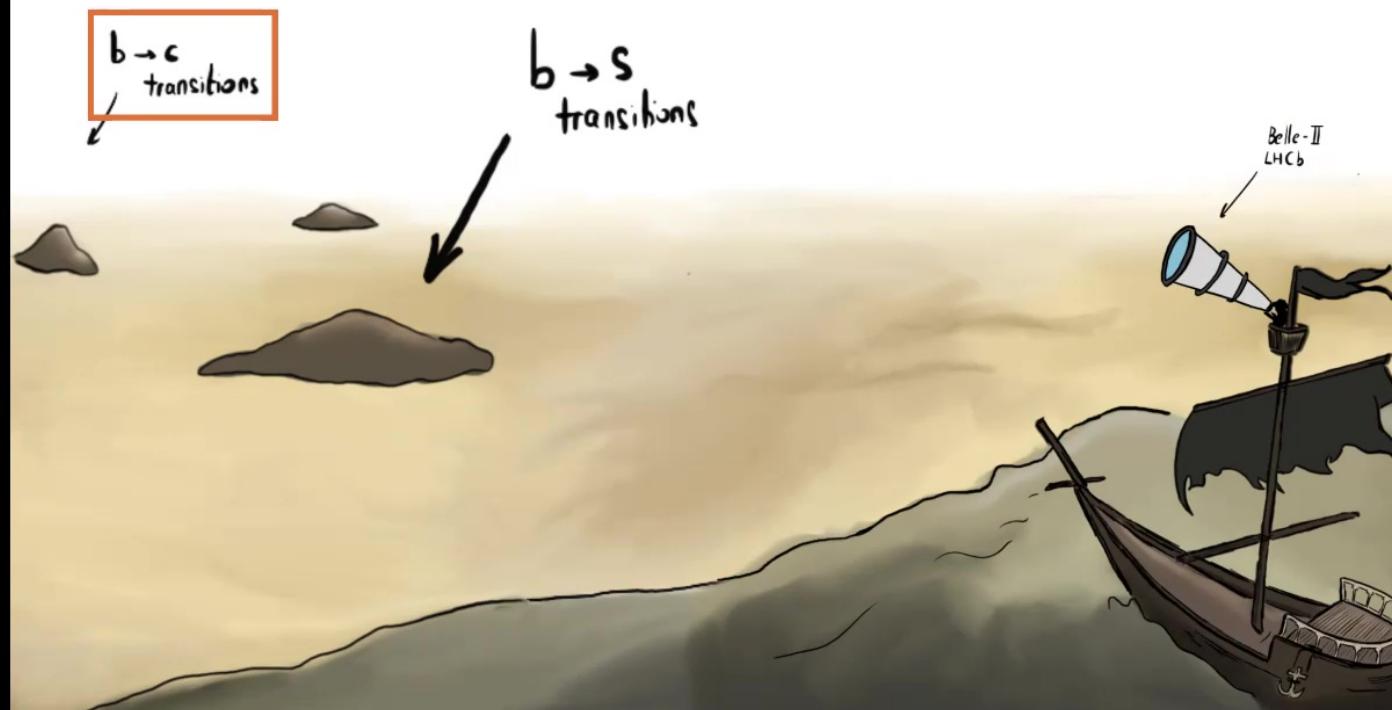


# Anomalies in $b \rightarrow c$ transitions

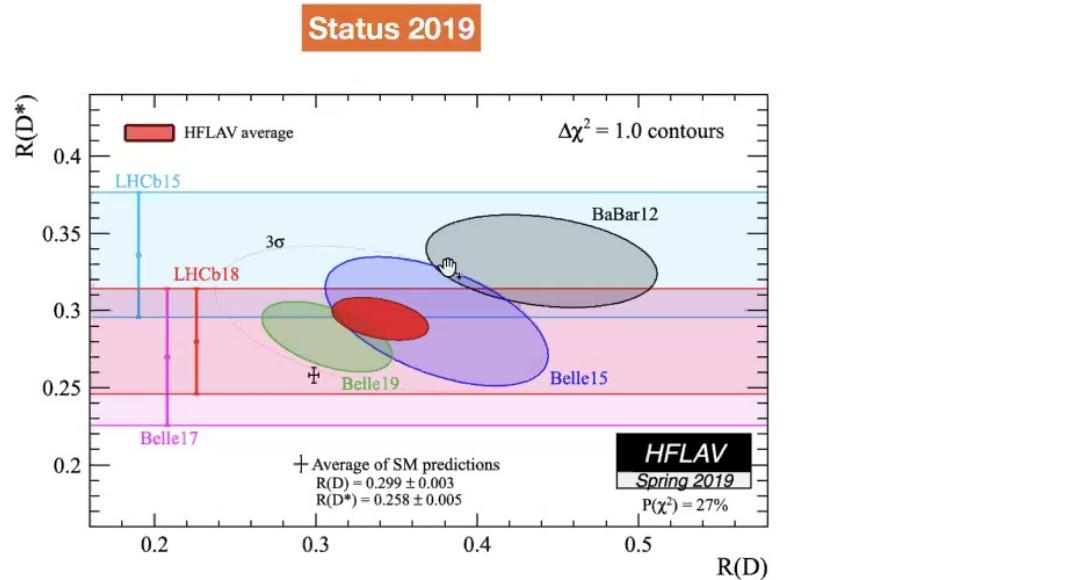


# Anomalies in $b \rightarrow c$ transitions

[Based on Refs. [1904.09311](#) and [2004.06726](#), in collaboration with Martin Jung, Rusa Mandal, Ana Peñuelas and Antonio Pich.]



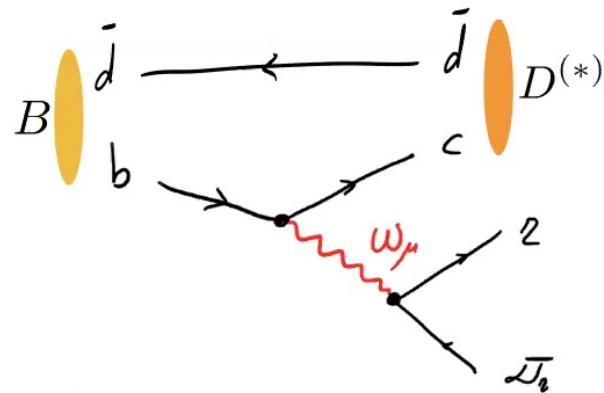
# Anomalies in $b \rightarrow c$ transitions



$3.9\sigma$	$4.0\sigma$	$3.9\sigma$	$3.6\sigma$	$3.1\sigma$	[LHCb, 1506.08614, 1708.08856, 1711.02505]
2015	2016	2017	2018	2019	[Belle, 1507.03233, 1607.07923, 1612.00529, 1709.00129, 1904.02440]
					[BaBar, 1205.5442, 1303.0571]

# Anomalies in $b \rightarrow c$ transitions

$$\Rightarrow \mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\bar{\nu}_\ell)}$$

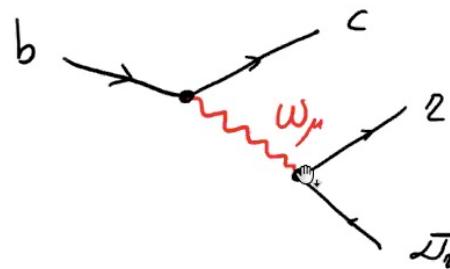


$$\mathcal{R}_{D^{(*)}} = \frac{|V_{cb}|^2 \langle \text{hadronic current} | \boxed{b} \boxed{c} \rangle \langle \text{leptonic current} | \boxed{2} \boxed{1} \rangle}{|V_{cb}|^2 \langle \text{hadronic current} | \boxed{b} \boxed{c} \rangle \langle \text{leptonic current} | \boxed{e, \mu} \boxed{1} \rangle}$$

# Anomalies in $b \rightarrow c$ transitions



⇒  $\mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\bar{\nu}_\ell)}$  **3.1  $\sigma$**   
HFLAV, up to date



Tree level process!!

# Anomalies in $b \rightarrow c$ transitions

Pattern of deviations in B-meson decays involving b to c transitions  
pointing to “the same direction”

$$\Rightarrow \mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\bar{\nu}_\ell)} \quad 3.1 \sigma$$

HFLAV, up to date

$$\Rightarrow \mathcal{R}_{J/\Psi} \equiv \frac{\mathcal{B}(B_c \rightarrow J/\Psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \rightarrow J/\Psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$$

LHCb, 2017

$$R_{J/\Psi SM} \sim 0.25 - 0.28$$

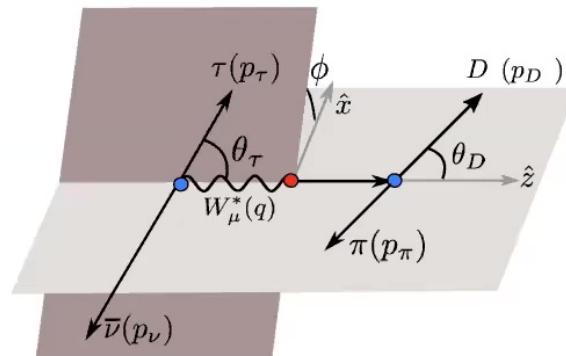
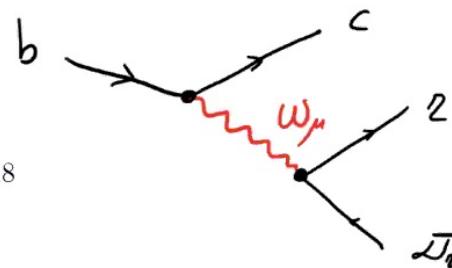
$$\Rightarrow \bar{\mathcal{P}}_\tau^{D^*} = -0.38 \pm 0.51^{+0.21}_{-0.16}$$

Belle, 2016

$$\mathcal{P}_\tau(D^*)_{SM} = -0.499 \pm 0.003$$

$$\Rightarrow \bar{F}_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \quad 1.6 \sigma$$

Belle, 2019

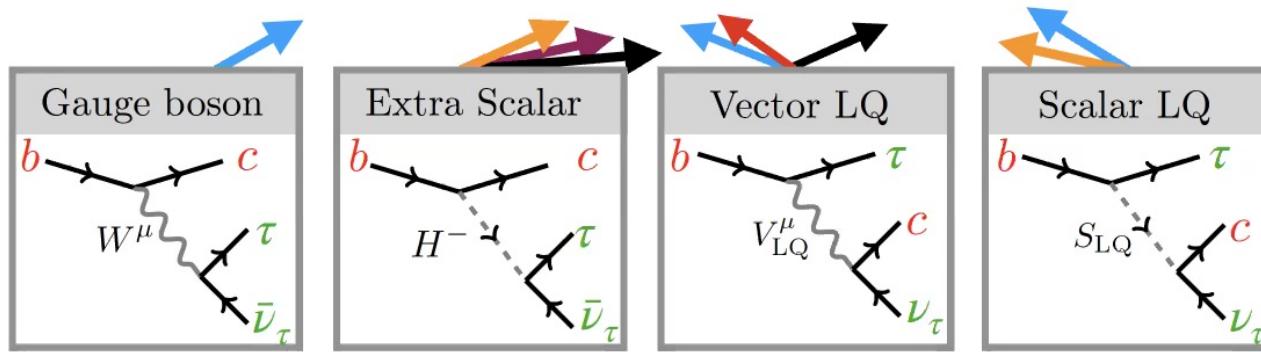


## Bottom-up approach



- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \textcolor{red}{C}_{V_R} \mathcal{O}_{V_R} + \textcolor{orange}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{red}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$



$$\boxed{\mathcal{O}_{V_L}} = (\bar{c} \gamma^\mu P_L b)(\bar{\ell} \gamma_\mu P_L \nu_\ell),$$

$$\boxed{\mathcal{O}_{S_R}} = (\bar{c} P_R b)(\bar{\ell} P_L \nu_\ell),$$

$$\boxed{\mathcal{O}_T} = (\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\ell} \sigma_{\mu\nu} P_L \nu_\ell),$$

$$\boxed{\mathcal{O}_{V_R}} = (\bar{c} \gamma^\mu P_R b)(\bar{\ell} \gamma_\mu P_L \nu_\ell),$$

$$\boxed{\mathcal{O}_{S_L}} = (\bar{c} P_L b)(\bar{\ell} P_L \nu_\ell),$$

# Bottom-up approach

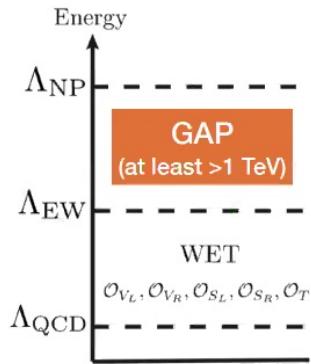


- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \textcolor{red}{C}_{V_R} \mathcal{O}_{V_R} + \textcolor{orange}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{purple}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$

- Assumptions:

→ EFT 



$$\begin{aligned}\mathcal{O}_{V_L} &= (\bar{c} \gamma^\mu P_L b)(\bar{\ell} \gamma_\mu P_L \nu_\ell), & \mathcal{O}_{V_R} &= (\bar{c} \gamma^\mu P_R b)(\bar{\ell} \gamma_\mu P_L \nu_\ell), \\ \mathcal{O}_{S_R} &= (\bar{c} P_R b)(\bar{\ell} P_L \nu_\ell), & \mathcal{O}_{S_L} &= (\bar{c} P_L b)(\bar{\ell} P_L \nu_\ell), \\ \mathcal{O}_T &= (\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\ell} \sigma_{\mu\nu} P_L \nu_\ell),\end{aligned}$$

# Bottom-up approach



- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \textcolor{red}{C}_{V_R} \mathcal{O}_{V_R} + \textcolor{brown}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{violet}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$

- Assumptions:

→ EFT

→ New physics only in the **third generation**

NP effects negligible in  $b \rightarrow c(e, \mu) \bar{\nu}_{(e, \mu)}$  analysis [Jung, Straub, 1801.01112]

$$\begin{aligned}\boxed{\mathcal{O}_{V_L}} &= (\bar{c} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu P_L \nu_{\textcolor{brown}{\tau}}), & \boxed{\mathcal{O}_{V_R}} &= (\bar{c} \gamma^\mu P_R b)(\bar{\tau} \gamma_\mu P_L \nu_{\textcolor{brown}{\tau}}), \\ \boxed{\mathcal{O}_{S_R}} &= (\bar{c} P_R b)(\bar{\tau} P_L \nu_{\textcolor{brown}{\tau}}), & \boxed{\mathcal{O}_{S_L}} &= (\bar{c} P_L b)(\bar{\tau} P_L \nu_{\textcolor{brown}{\tau}}). \\ \boxed{\mathcal{O}_T} &= (\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\tau} \sigma_{\mu\nu} P_L \nu_{\textcolor{brown}{\tau}}),\end{aligned}$$

# Bottom-up approach



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$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \cancel{\textcolor{red}{C}_{V_R}} \mathcal{O}_{V_R} + \textcolor{orange}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{red}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$

- Assumptions:

→ EFT ✓

→ New physics only in the **third generation**,

→  $C_{V_R}$  lepton flavour universal  $\Rightarrow C_{V_R}^\tau \underset{\oplus}{\sim} 0$   
 $C_{V_R}^\ell \equiv C_{V_R} + \mathcal{O}\left(\frac{v^4}{\Lambda_{\text{NP}}^4}\right)$

Assuming SMEFT and no significant effect from NP in  $b \rightarrow c(e, \mu)\bar{\nu}_{(e, \mu)}$  [Jung, Straub, 1801.01112]

$$\begin{aligned} \boxed{\mathcal{O}_{V_L}} &= (\bar{c} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu P_L \nu_{\tau}), & \boxed{\mathcal{O}_{V_R}} &= (\bar{c} \gamma^\mu P_R b)(\bar{\tau} \gamma_\mu P_L \nu_{\tau}), \\ \boxed{\mathcal{O}_{S_R}} &= (\bar{c} P_R b)(\bar{\tau} P_L \nu_{\tau}), & \boxed{\mathcal{O}_{S_L}} &= (\bar{c} P_L b)(\bar{\tau} P_L \nu_{\tau}). \\ \boxed{\mathcal{O}_T} &= (\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\tau} \sigma_{\mu\nu} P_L \nu_{\tau}), \end{aligned}$$

# Bottom-up approach



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- Assumptions:

- ➔ EFT ✓
- ➔ New physics only in the **third generation**,
- ➔  $C_{V_R}$  lepton flavour universal  $\Rightarrow C_{V_R}^\tau \sim 0$
- ➔ CP conserving W.C.

Fitted complex W.C. without significant improvement

$$\begin{aligned}\boxed{\mathcal{O}_{V_L}} &= (\bar{c} \gamma^\mu P_L b)(\bar{\tau} \gamma_\mu P_L \nu_{\tau}), & \boxed{\mathcal{O}_{V_R}} &= (\bar{c} \gamma^\mu P_R b)(\bar{\tau} \gamma_\mu P_L \nu_{\tau}), \\ \boxed{\mathcal{O}_{S_R}} &= (\bar{c} P_R b)(\bar{\tau} P_L \nu_{\tau}), & \boxed{\mathcal{O}_{S_L}} &= (\bar{c} P_L b)(\bar{\tau} P_L \nu_{\tau}). \\ \boxed{\mathcal{O}_T} &= (\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\tau} \sigma_{\mu\nu} P_L \nu_{\tau}),\end{aligned}$$

# Bottom-up approach

- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \textcolor{orange}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{red}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$

- Inputs:

$\Rightarrow \mathcal{R}_D$

$\Rightarrow \mathcal{R}_{D^*}$

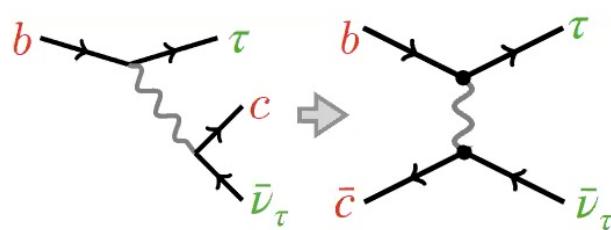
$\Rightarrow \Gamma(B \rightarrow D^{(*)} \tau \bar{\nu}_\tau)$

$\Rightarrow B_c \rightarrow \tau \bar{\nu}_\tau$

$\Rightarrow F_{\mathfrak{Q}}^{D^*}$

$B \rightarrow D^{(*)} \tau \bar{\nu}$

$B_c \rightarrow \tau \bar{\nu}$



$$\mathcal{B}(B_c \rightarrow \tau \bar{\nu}_\tau) = \#|V_{cb}|^2 \times \left| 1 + \textcolor{blue}{C}_{V_L} - \textcolor{red}{C}_{V_R} + \frac{m_{B_c}^2}{m_\tau(m_b + m_c)} (\textcolor{orange}{C}_{S_R} - \textcolor{red}{C}_{S_L}) \right|^2$$



# Bottom-up approach

- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \textcolor{orange}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{red}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$



- Inputs:

- ➔  $\mathcal{R}_D$
- ➔  $\mathcal{R}_{D^*}$
- ➔  $\Gamma(B \rightarrow D^{(*)} \tau \bar{\nu}_\tau)$
- ➔  $B_c \rightarrow \tau \bar{\nu}_\tau$
- ➔  $F_L^{D^*}$

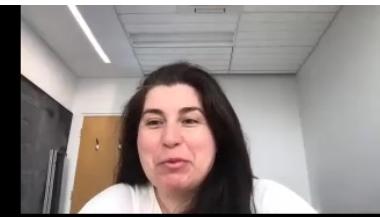
- Bc lifetime:

$\Rightarrow \text{Br}(B_c \rightarrow \tau \bar{\nu}_\tau) \leq 30 - 40\%$   
[Alonso et al., 2016]

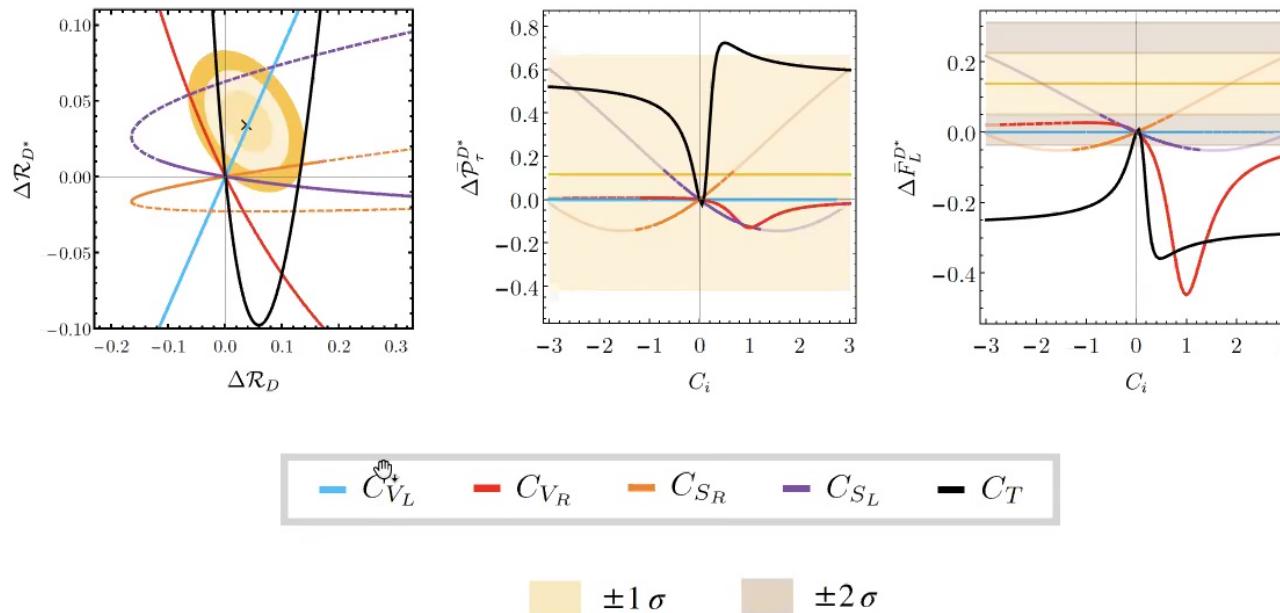
- Bound LEP Z peak:

[Akeroyd et al., 2017]  
 $\Rightarrow \text{Br}(B_c \rightarrow \tau \bar{\nu}_\tau) \leq 10\%$

$$\mathcal{B}(B_c \rightarrow \tau \bar{\nu}_\tau) = \#|V_{cb}|^2 \times \left| 1 + \textcolor{blue}{C}_{V_L} - \textcolor{red}{C}_{V_R} + \frac{m_{B_c}^2}{m_\tau(m_b + m_c)} (\textcolor{orange}{C}_{S_R} - \textcolor{red}{C}_{S_L}) \right|^2$$



# Fit independent analysis



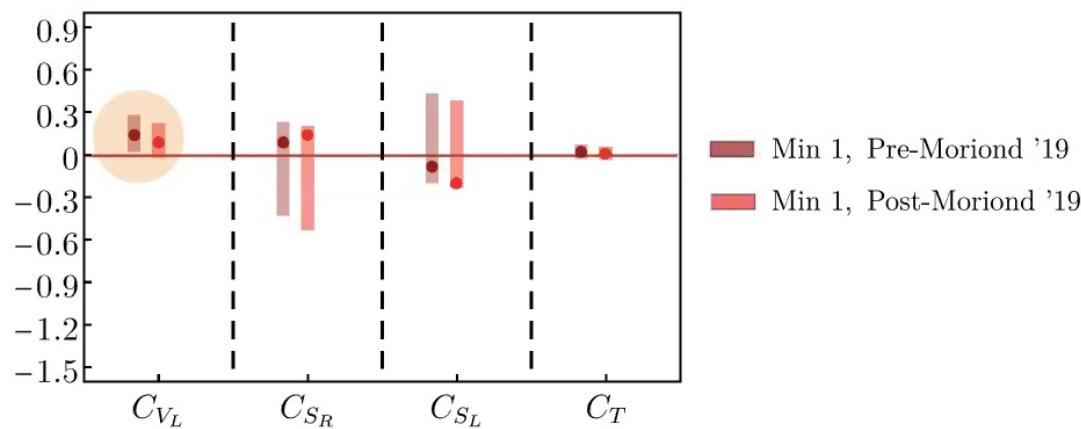
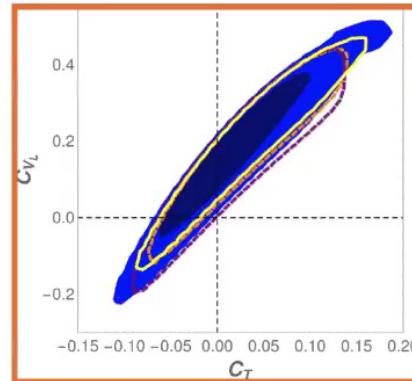
# Global Fit

- SM:

$$\chi^2_{SM} = 65.5 / 57 \text{ d.o.f.}$$

- New Physics:

$$\chi^2_{min1b} = 37.4 / 54 \text{ d.o.f.}$$



# Bottom-up approach



- Most general effective dim 6 Hamiltonian:

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4 G_F}{\sqrt{2}} V_{cb} [(1 + \textcolor{blue}{C}_{V_L}) \mathcal{O}_{V_L} + \cancel{\textcolor{red}{C}_{V_R}} \mathcal{O}_{V_R} + \textcolor{brown}{C}_{S_R} \mathcal{O}_{S_R} + \textcolor{brown}{C}_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T] + \text{h.c.}$$

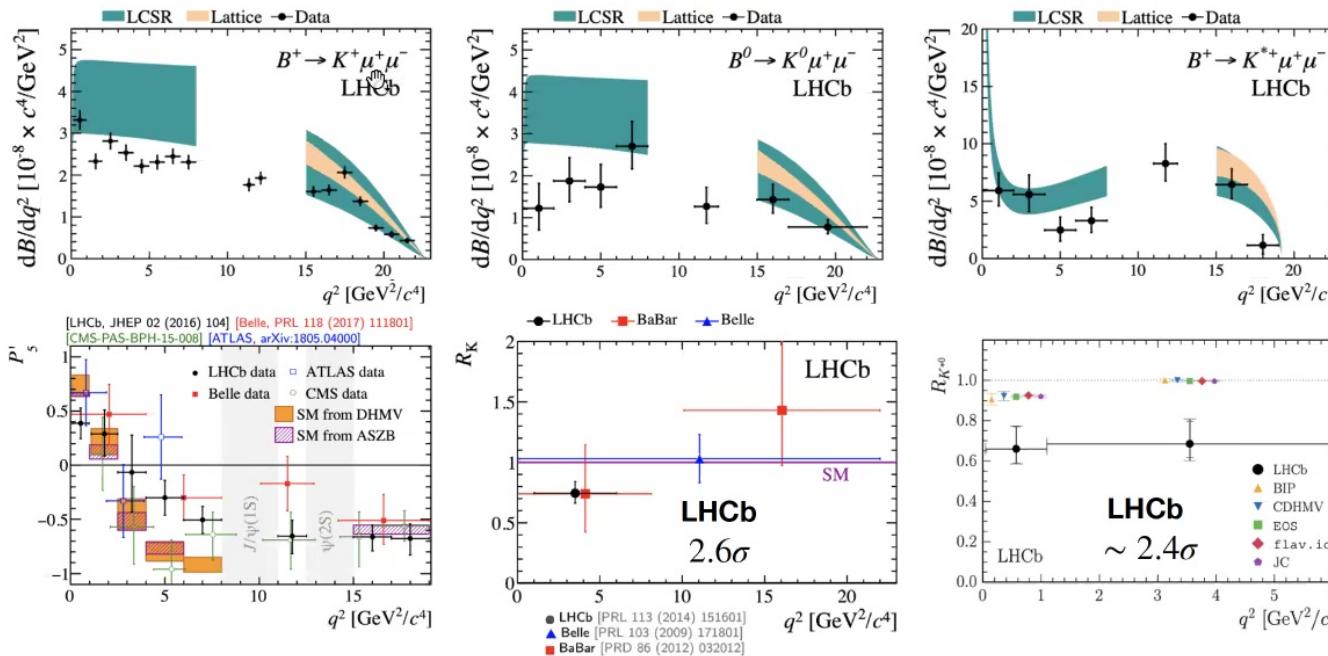
- Theoretical assumptions:

- ➔ EFT ✓ [C. Bobeth et al., two months ago]
- ➔ New physics only in the **third generation**,
- ➔  $C_{V_R}$  lepton flavour universal  $\Rightarrow C_{V_R}^\tau \sim 0$
- ➔ CP conserving W.C.

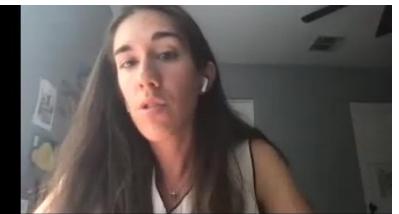
- Experimental measurements

An unidentified or underestimated systematic uncertainty...

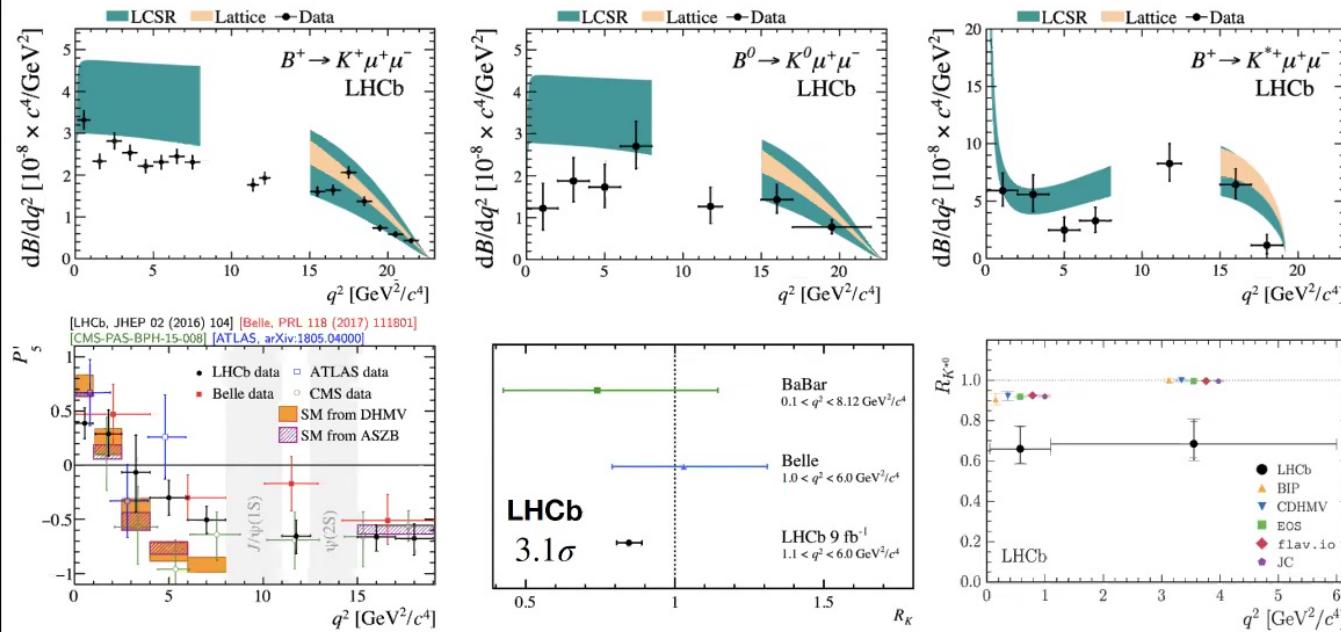
# Anomalies in $b \rightarrow s$ transitions



Status 2017

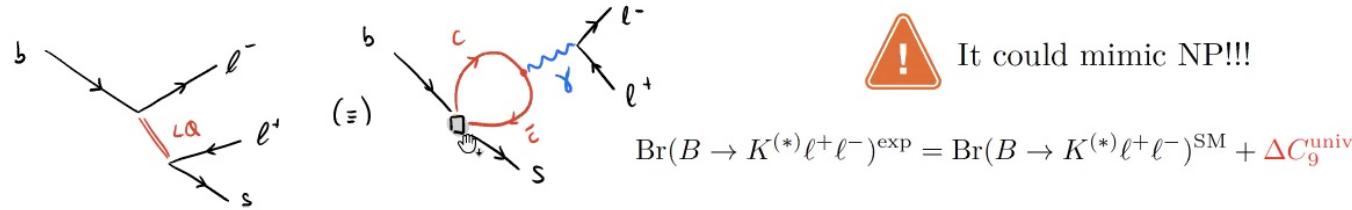
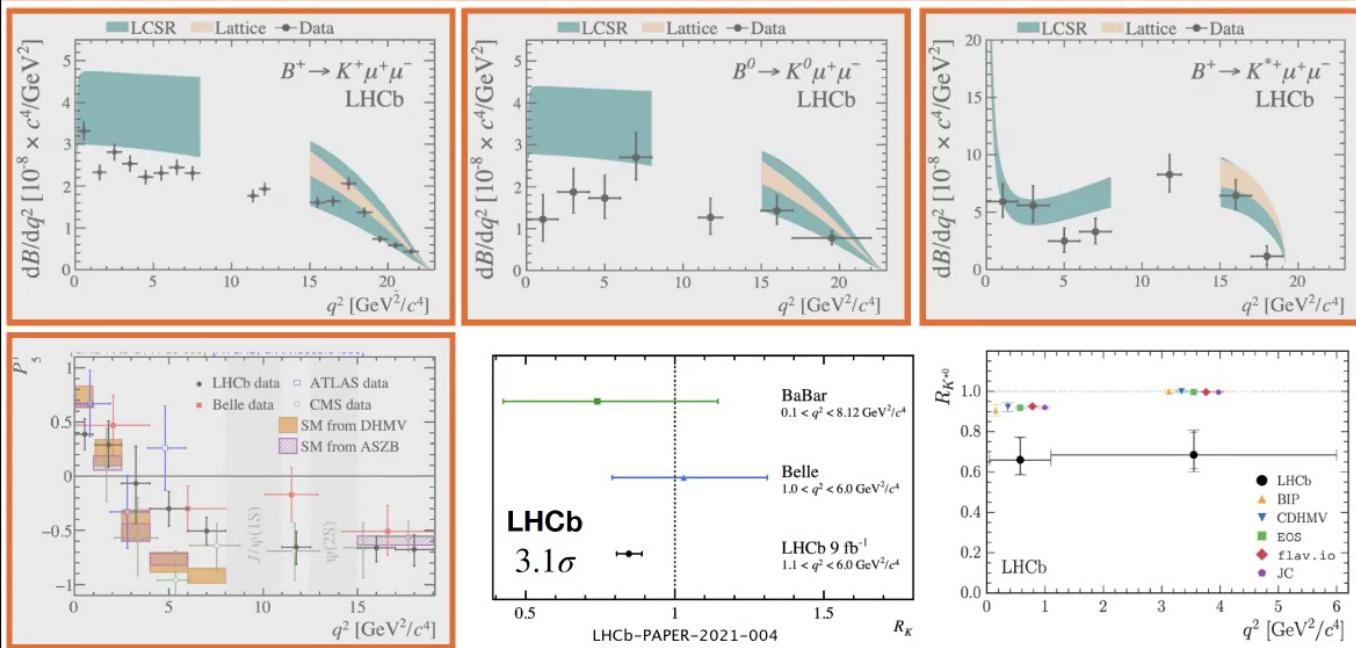


# Anomalies in $b \rightarrow s$ transitions



Status NOW

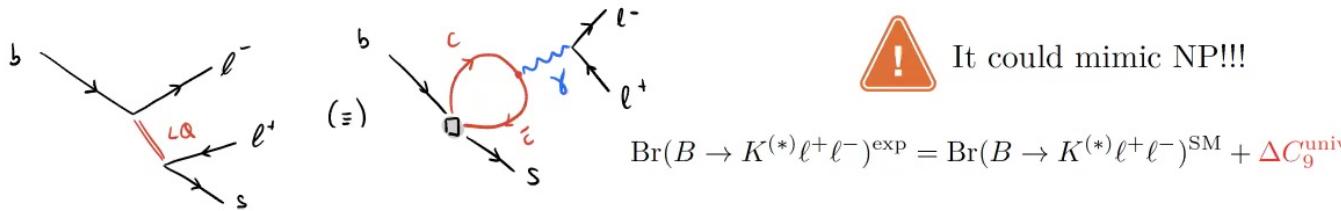
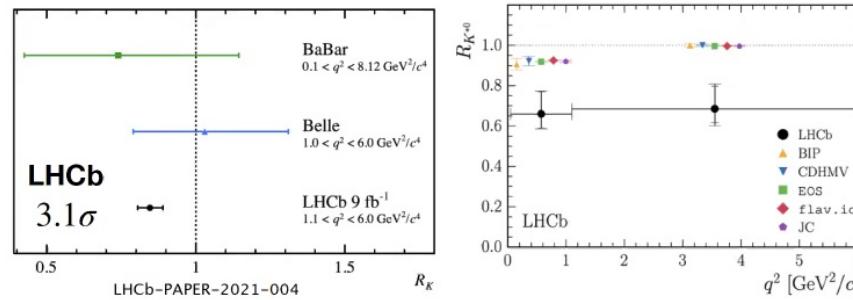
# Anomalies in $b \rightarrow s$ transitions



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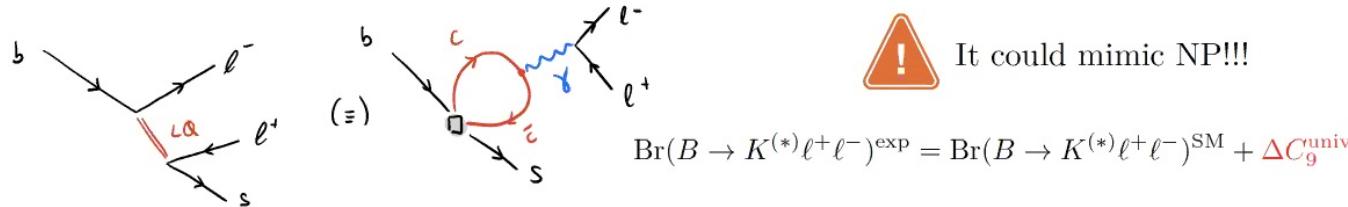
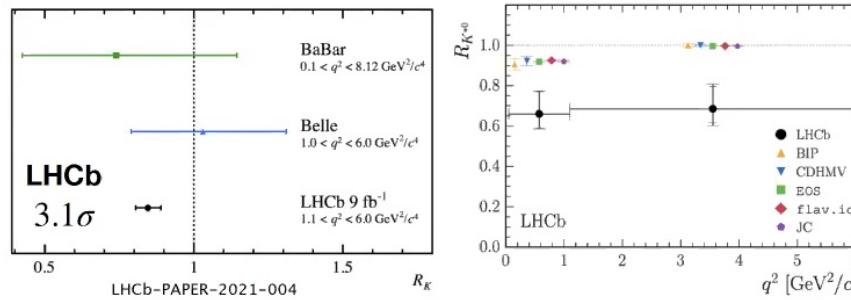
$$\mathcal{R}_K = \frac{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K \mu^+ \mu^-)}{dq^2} dq^2}{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K e^+ e^-)}{dq^2} dq^2} \stackrel{\text{SM}}{=} 1 \pm \mathcal{O}(10^{-2}) \text{ EM correction}$$



# Anomalies in $b \rightarrow s$ transitions



$$\mathcal{R}_K = \frac{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K \mu^+ \mu^-)}{dq^2} dq^2}{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K e^+ e^-)}{dq^2} dq^2} \stackrel{\text{SM}}{\simeq} \frac{\Delta_{\text{SM}} + \Delta C_9^{\text{univ}}}{\Delta_{\text{SM}} + \Delta C_9^{\text{univ}}} \simeq 1$$

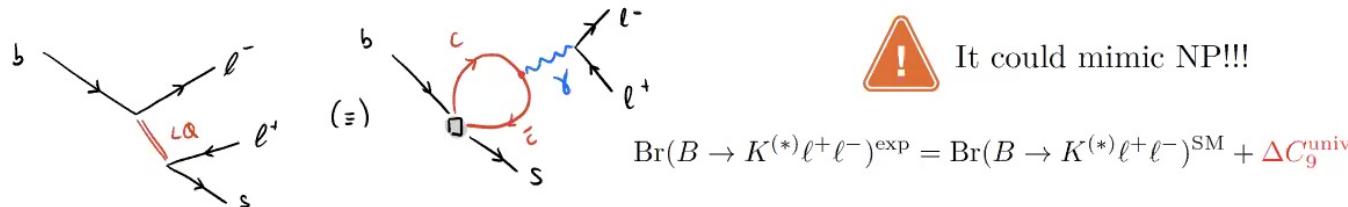
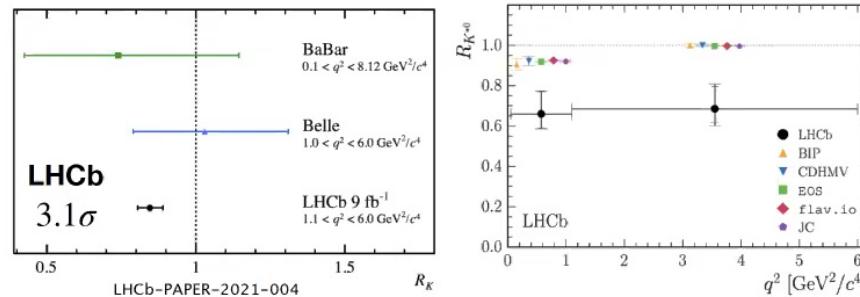


# Anomalies in $b \rightarrow s$ transitions



$$\mathcal{R}_K = \frac{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K \mu^+ \mu^-)}{dq^2} dq^2}{\int_{1.1 \text{ GeV}^2}^{6.0 \text{ GeV}^2} \frac{d\text{Br}(B \rightarrow K e^+ e^-)}{dq^2} dq^2} \stackrel{\text{SM}}{\simeq} \frac{\Delta_{\text{SM}} + \Delta C_9^{\text{univ}}}{\Delta_{\text{SM}} + \Delta C_9^{\text{univ}}} \simeq 1$$

→ Clean Observables!



# Anomalies in $b \rightarrow s$ transitions

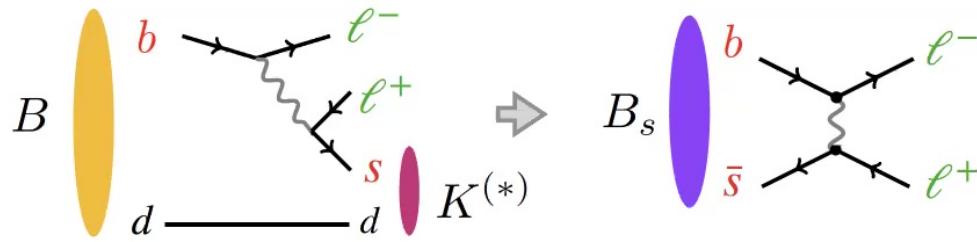


- Clean observables:

$3.1\sigma$   $\mathcal{R}_K^{\text{exp}}(1.1 < q^2 < 6.0 \text{ GeV}^2/c^4) = 0.846^{+0.042}_{-0.039} {}^{+0.013}_{-0.012}$

$2.1\text{-}2.5\sigma$   $\mathcal{R}_{K^*}^{\text{exp}} = \begin{cases} 0.66^{+0.11}_{-0.07} \text{ (stat)} \pm 0.03 \text{ (syst)} & \text{for } 0.045 < q^2 < 1.1 \text{ GeV}^2/c^4, \\ 0.69^{+0.11}_{-0.07} \text{ (stat)} \pm 0.05 \text{ (syst)} & \text{for } 1.1 < q^2 < 6.0 \text{ GeV}^2/c^4. \end{cases}$

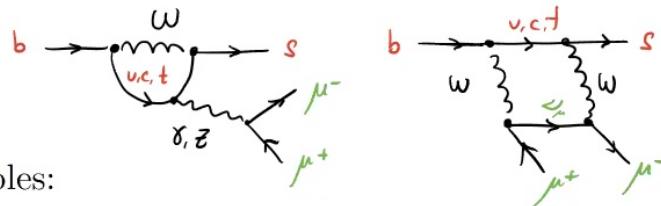
$2.1\sigma$   $\text{Br}(B_s \rightarrow \mu^+ \mu^-)^{\text{exp}} = 2.69^{+0.37}_{-0.35} \times 10^{-9}$



# Anomalies in $b \rightarrow s$ transitions



- New Physics competes with the SM at the loop level!

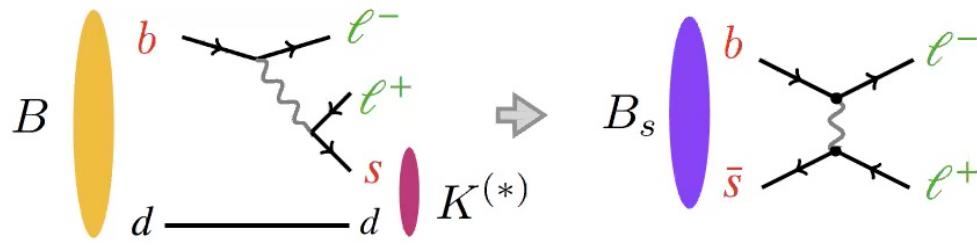


- Clean observables:

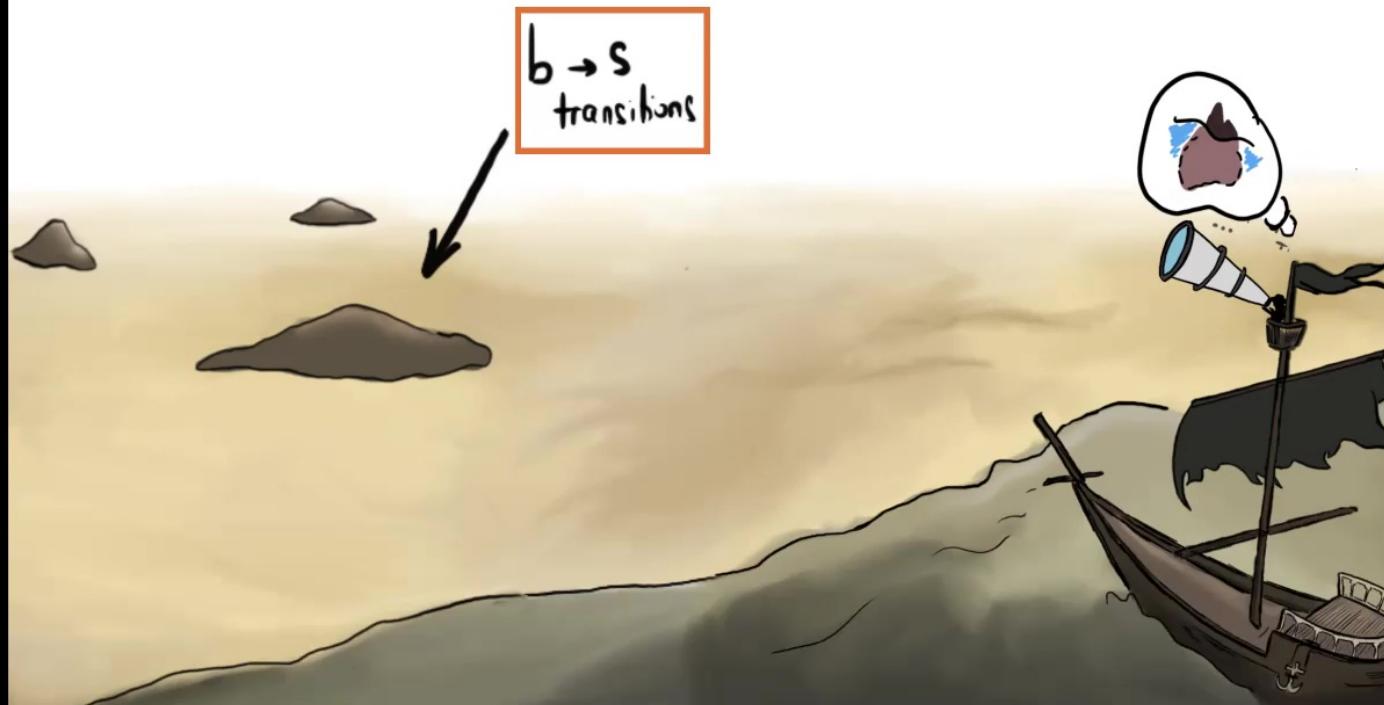
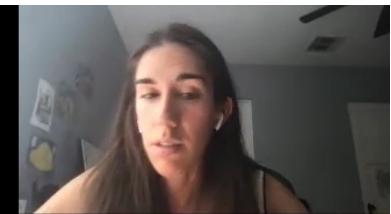
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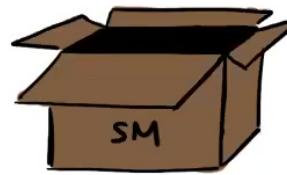
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## Anomalies in $b \rightarrow s$ transitions

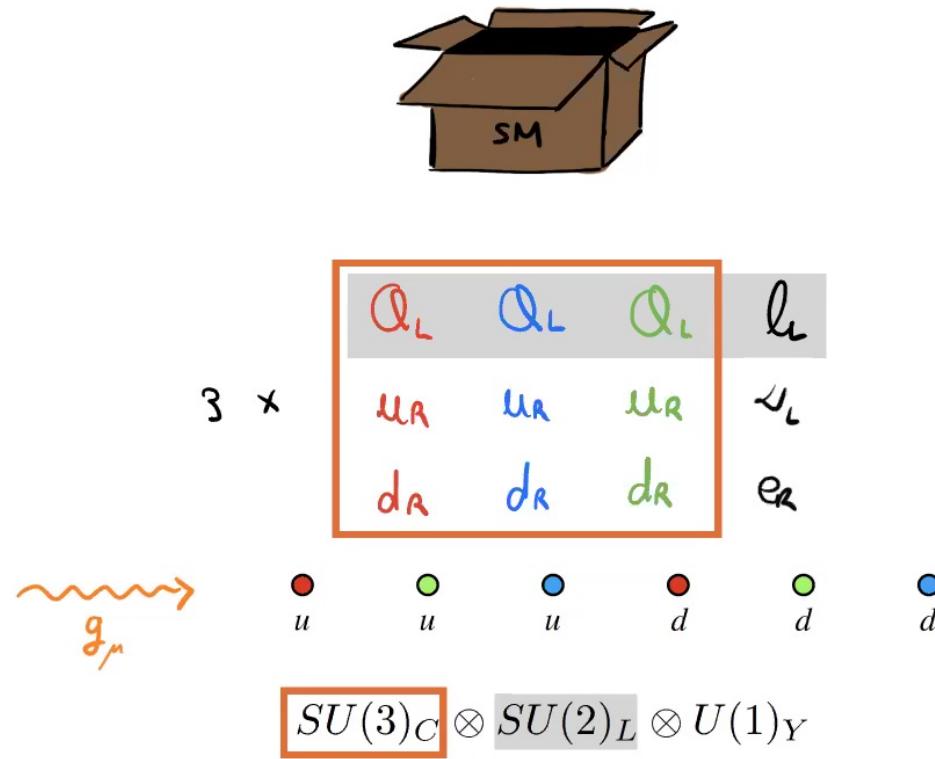


# Unification of Matter

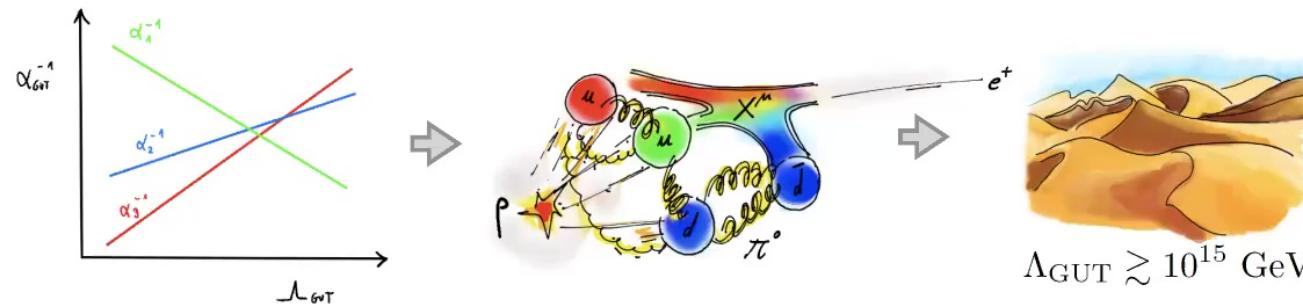


$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

# Unification of Matter



# Unification of Matter



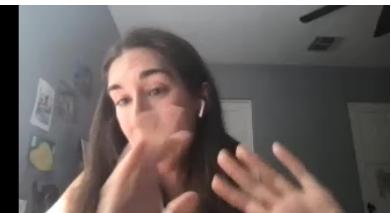
3 ×

$Q_L$	$Q_L$	$Q_L$	$l_e$
$u_R$	$u_R$	$u_R$	$\nu_e$
$d_R$	$d_R$	$d_R$	$e_R$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$



# Unification of Matter: Pati-Salam



$$3 \times \left\{ \begin{array}{c|c} Q_L & Q_L \\ u_R & u_R \\ d_R & d_R \end{array} \mid \begin{array}{c} l_e \\ \nu_e \\ e_R \end{array} \right\}$$

↑ May leptons be  
the 4<sup>th</sup> color?

$$\text{PS} \supset SU(4)_C \otimes SU(2)_L \otimes U(1)_R$$

[J. Pati and A. Salam 1974] [P. Fileviez Perez and M. B. Wise 2013]

# Unification of Matter: Pati-Salam

$$F_{QL} \sim (4, 2, 0) = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_L$$

$$F_u = (u^c_{\text{Q}_L} \nu^c)_L \sim (\bar{4}, 1, -1/2)$$

$$F_d = (d^c \ e^c)_L \sim (\bar{4}, 1, 1/2)$$



$$SU(4)_C \otimes SU(2)_L \otimes U(1)_R$$

[P. Fileviez Perez and M. B. Wise 2013]



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$$\chi = (\chi_{\textcolor{red}{u}}, \chi_{\textcolor{green}{u}}, \chi_{\textcolor{blue}{u}}, \langle \chi_R^0 \rangle)$$

$$\cancel{SU(4)_c \otimes SU(2)_L \otimes U(1)_R} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$SU(4)_C \otimes SU(2)_L \otimes U(1)_R$$



# Vector LQ $U_1^\mu \sim (3, 1, 2/3)$

$$F_{QL} \sim (4, 2, 0) = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_L$$

$$\begin{aligned} F_u &= (u^c \quad \nu^c)_L \sim (\bar{4}, 1, -1/2) \\ F_d &= (d^c \quad e^c)_L \sim (\bar{4}, 1, 1/2) \end{aligned}$$

$$V_{15}^\mu \sim (15, 1, 0) = \underbrace{\begin{pmatrix} G^\mu & U_1^\mu / \sqrt{2} \\ (U_1^\mu)^* / \sqrt{2} & 0 \end{pmatrix}}_{SU(4)} + T_4 B'^\mu$$

$$\chi = (\chi_{\textcolor{red}{u}}, \chi_{\textcolor{green}{u}}, \chi_{\textcolor{blue}{u}}, \langle \chi_R^0 \rangle) \quad \Rightarrow \quad M_{U_1} \sim g_4 v_\chi \quad ?$$

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$$\mathcal{L}_K \supset \frac{g_4}{\sqrt{2}} U_4^\mu (\bar{Q}_L \gamma_\mu \ell_L + \bar{u}_R \gamma_\mu \nu_R + \bar{d}_R \gamma_\mu e_R) + \text{h.c.}$$

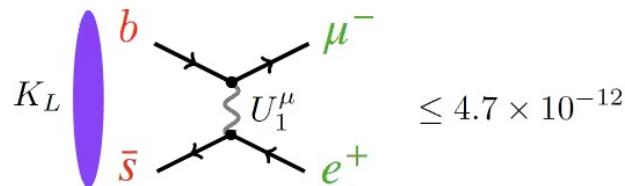
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$$\cancel{SU(4)_c} \otimes SU(2)_L \otimes \cancel{U(1)_R} \Rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

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# Vector LQ $U_1^\mu \sim (3, 1, 2/3)$



$$\leq 4.7 \times 10^{-12}$$

$$V_{15}^\mu \sim (15, 1, 0) = \underbrace{\begin{pmatrix} G^\mu \\ (U_1^\mu)^*/\sqrt{2} & \begin{matrix} U_1^\mu/\sqrt{2} \\ 0 \end{matrix} \end{pmatrix}}_{SU(4)} + T_4 B'^\mu$$

$$\mathcal{L}_K \supset \frac{g_4}{\sqrt{2}} U_1^\mu (\bar{Q}_L \gamma_\mu \ell_L + \bar{u}_R \gamma_\mu \nu_R + \bar{d}_R \gamma_\mu e_R) + \text{h.c.}$$

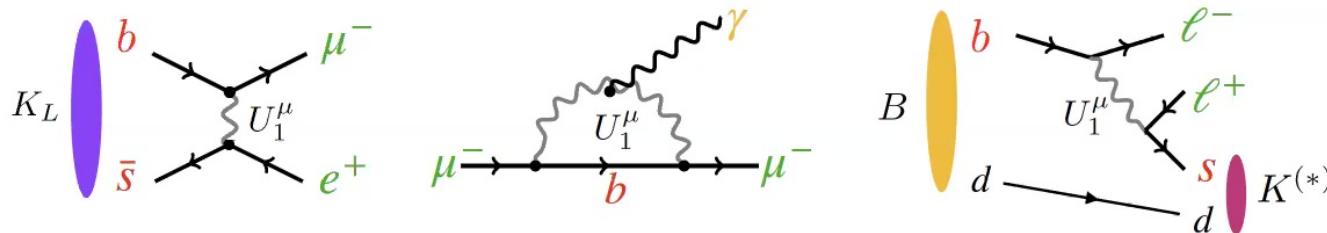
$$\chi = (\chi_u, \chi_e, \chi_{\mu}, \langle \chi_R^0 \rangle) \quad \Rightarrow \quad M_{U_1} \sim g_4 v_\chi \gtrsim 10^3 \text{ TeV}$$

$$\cancel{SU(4)_c} \otimes SU(2)_L \otimes \cancel{U(1)_R} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

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$$\mathcal{L}_K \supset \frac{g_4}{\sqrt{2}} U_1^\mu \left( \cdots + \bar{d}_R U_R^\dagger \not{\partial}^\mu E_R e_R \right) + \text{h.c.}$$

Naive bound!

$$\chi = (\chi_u, \chi_d, \chi_s, \langle \chi_R^0 \rangle) \quad \Rightarrow \quad M_{U_1} \sim g_4 v_\chi \gtrsim 10^3 \text{ TeV}$$

~~$SU(4)_c \otimes SU(2)_L \otimes U(1)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$~~

$$SU(4)_C \otimes SU(2)_L \otimes U(1)_R$$



# Unification of Matter

$$F_{QL} \sim (4, 2, 0) = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_L$$

$$\begin{aligned} F_u &= (u^c \quad \nu^c)_L \sim (\bar{4}, 1, -1/2) \\ F_d &= (d^c \quad e^c)_L \sim (\bar{4}, 1, 1/2) \end{aligned}$$

$$\mathcal{L}_Y = Y_1 F_{QL} F_u H + Y_3 H^\dagger F_{QL} F_d$$

$$M_u = Y_1 \frac{v_1}{\sqrt{2}}$$

$$M_d = Y_3 \frac{v_1}{\sqrt{2}}$$

$$M_\nu^D = Y_1 \frac{v_1}{\sqrt{2}}$$

⊕

$$M_e = Y_3 \frac{v_1}{\sqrt{2}}$$

$$H \sim (1, 2, 1/2)_{\text{SM}}$$

$$SU(4)_c \otimes SU(2)_L \otimes U(1)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q$$

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$$\Phi \sim (15, 2, 1/2) = \begin{pmatrix} \Phi_{\text{MW}} & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2, \quad H \sim (1, 2, 1/2)_{\text{SM}}$$

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# Inverse seesaw mechanism



- Add a fermion singlet  $S \sim (1, 1, 0)$

$$-\mathcal{L}_{QL}^\nu = Y_5 F_u \chi S + \frac{1}{2} \mu S S + \text{h.c.}$$
$$\xrightarrow{\langle \chi \rangle} M_\chi^D = Y_5 v_\chi / \sqrt{2}$$

- Mass matrix for neutral fermions:

$$(\nu \nu^c S) \begin{pmatrix} 0 & \text{EW} & 0 \\ \text{EW} & 0 & \text{LQ} \\ 0 & \text{LQ} & \mu \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ S \end{pmatrix}$$

$$M_\chi^D \gg M_\nu^D \gg \mu \quad \xrightarrow{\text{diag}} \quad m_\nu \approx \mu \text{EW} / \text{LQ}$$

# Inverse seesaw mechanism



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$\xrightarrow{\langle \chi \rangle} M_\chi^D = Y_5 v_\chi / \sqrt{2}$

- Mass matrix for neutral fermions:

$$(\nu \nu^c S) \begin{pmatrix} 0 & \text{EW} & 0 \\ \text{EW} & 0 & \text{LQ} \\ 0 & \text{LQ} & \mu \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ S \end{pmatrix}$$

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$$\xrightarrow{\langle \chi \rangle} M_\chi^D = Y_5 v_\chi / \sqrt{2}$$

- Mass matrix for neutral fermions:

$$(\nu \nu^c S) \begin{pmatrix} 0 & M_\nu^D & 0 \\ (M_\nu^D)^T & 0 & M_\chi^D \\ 0 & (M_\chi^D)^T & \mu \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ S \end{pmatrix}$$

$$M_\chi^D \gg M_\nu^D \gg \mu \quad \xrightarrow{\qquad} \quad m_\nu \approx \mu (M_\nu^D)^2 / (M_\chi^D)^2,$$

No need for  $\langle \chi \rangle$  to be large!!

[P. Fileviez Perez and M. B. Wise 2013]

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$$\mathcal{L}_Y = Y_1 F_{QL} F_u H + Y_3 H^\dagger F_{QL} F_d + Y_2 F_{QL} F_u \Phi + Y_4 \Phi^\dagger F_{QL} F_d + \text{h.c.}$$

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$$SU(4)_c \otimes SU(2)_L \otimes U(1)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q$$

# Unification of Matter



- The theory predicts scalar LQs:

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{\text{SM}} \quad \Phi_4 \sim (3, 2, 7/6)_{\text{SM}}$$

$$-\mathcal{L}_Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + Y_2 \ell_L \Phi_4 (u^c)_L + Y_4 Q_L \Phi_4^\dagger (e^c)_L + Y_4 \ell_L \Phi_3^\dagger (d^c)_L + \text{h.c.}$$

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$$Q_B(\Phi_3) = -1/3,$$

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$$\text{e.g.} \quad \frac{1}{\Lambda} u_R^\alpha d_R^\beta (\Phi_3^\dagger)^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}, \quad \frac{1}{\Lambda} d_R^\alpha d_R^\beta \Phi_4^\gamma H^\dagger \epsilon_{\alpha\beta\gamma} \quad [\text{Arnold, Fornal, Wise, 2013}]$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{O} \left( \frac{\text{Energy}}{\Lambda_{\text{NP}}} \right)^n \quad \uparrow$$

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

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$$Q_B(\Phi_3) = -1/3, \quad Q_L(\Phi_3) = 1, \quad Q_B(\Phi_4) = 1/3, \quad Q_L(\Phi_4) = -1$$

[C.M, M. B. Wise, 2105.14029]

$$\frac{1}{\Lambda_{\text{PS}}^3} F_d^A F_u^B (\Phi^\dagger)_D^C \chi^D \chi^E H^\dagger \epsilon_{ABCD} \xrightarrow{\langle \rangle} \frac{v_\chi^2}{\Lambda_{\text{PS}}^3} d_R^\alpha u_R^\beta (\Phi_3^\dagger)^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}$$

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e.g.  $\frac{1}{\Lambda} u_R^\alpha d_R^\beta (\Phi_3^\dagger)^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}$ ,  $\frac{1}{\Lambda} d_R^\alpha d_R^\beta \Phi_4^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}$  [Arnold, Fornal, Wise, 2013]



$$SU(3)_c \otimes \cancel{SU(2)_L} \otimes \cancel{U(1)_Y} \Rightarrow SU(3)_c \otimes U(1)_Q$$

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$$\frac{1}{\Lambda_{\text{PS}}^3} F_d^A F_u^B (\Phi^\dagger)_D^C \chi^D \chi^E H^\dagger \epsilon_{ABCD} \xrightarrow{\langle \rangle} \frac{v_\chi^2}{\Lambda_{\text{PS}}^3} d_R^\alpha u_R^\beta (\Phi_3^\dagger)^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}$$

$$\Phi \sim (15, 2, 1/2) = \begin{pmatrix} \Phi_{\text{MW}} & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2, \quad H \sim (1, 2, 1/2)_{\text{SM}}$$

$$SU(4)_c \otimes SU(2)_L \otimes U(1)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$



# Unification of Matter



- The theory predicts scalar LQs:

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{\text{SM}} \quad \Phi_4 \sim (3, 2, 7/6)_{\text{SM}}$$

$$-\mathcal{L}_Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + Y_2 \ell_L \Phi_4 (u^c)_L + Y_4 Q_L \Phi_4^\dagger (e^c)_L + Y_4 \ell_L \Phi_3^\dagger (d^c)_L + \text{h.c.}$$

$$Q_B(\Phi_3) = -1/3, \quad Q_L(\Phi_3) = 1, \quad Q_B(\Phi_4) = 1/3, \quad Q_L(\Phi_4) = -1$$

[C.M. M. B. Wise, 2105.14029]

$$\frac{1}{\Lambda_{\text{PS}}^3} F_d^A F_u^B (\Phi^\dagger)_D^C \chi^D \chi^E H^\dagger \epsilon_{ABCD} \xrightarrow{\langle \rangle} \frac{v_\chi^2}{\Lambda_{\text{PS}}^3} d_R^\alpha u_R^\beta (\Phi_3^\dagger)^\gamma H^\dagger \epsilon_{\alpha\beta\gamma}$$

$v_\chi \sim \mathcal{O}(10^3)$  TeV, and  $\Lambda_{PS} > 10^{15}$  GeV  $\Rightarrow$   $\Phi_3$  and  $\Phi_4$  are safe if embedded in Pati-Salam!

$$\Phi \sim (15, 2, 1/2) = \begin{pmatrix} \Phi_{\text{MW}} & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2, \quad H \sim (1, 2, 1/2)_{\text{SM}}$$

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# Pati-Salam shielding

- The theory predicts scalar LQs:

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{\text{SM}}$$

$$\Phi_4 \sim (3, 2, 7/6)_{\text{SM}}$$

$$-\mathcal{L}_Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + Y_3 \ell_L \Phi_4 (u^c)_L + Y_4 \ell_L \Phi_3^\dagger (d^c)_L + \text{h.c.}$$

$$Q_B(\Phi_3) = -1/6$$

$$Q_L(\Phi_4) = -1$$

$$\frac{1}{\Lambda_{PS}^3} F_u^A F_d^B (\Phi^\dagger \chi)^C \chi^D + \frac{v_\chi}{\Lambda_{PS}^3} d_R^\alpha u_R^\beta (\Phi_3^\dagger)^\gamma \epsilon H^\dagger \epsilon_{\alpha\beta\gamma},$$

$$v_\chi \sim \mathcal{O}(10) \text{ TeV, and } \Lambda$$

$\Phi_3$  and  $\Phi_4$  are safe if embedded  
in Pati-Salam!

$$\Phi \sim \begin{pmatrix} \Phi_{MW} & \Phi_3 \\ \Phi_4 & \end{pmatrix} \oplus T_1 H, \quad H \sim (1, 2, 1/2)_{\text{SM}}$$

$$SU(4)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

# Unification of Matter



- The theory predicts scalar LQs:

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{\text{SM}} \quad \Phi_4 \sim (3, 2, 7/6)_{\text{SM}}$$

$$-\mathcal{L}_Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + Y_2 \ell_L \Phi_4 (u^c)_L + \textcolor{red}{Y_4} Q_L \Phi_4^\dagger (e^c)_L + \textcolor{red}{Y_4} \ell_L \Phi_3^\dagger (d^c) + \text{h.c.}$$

$$M_u = Y_1 \frac{v_1}{\sqrt{2}} + \frac{1}{2\sqrt{6}} Y_2 \frac{v_2}{\sqrt{2}} \quad M_d = Y_3 \frac{v_1}{\sqrt{2}} + \frac{1}{2\sqrt{6}} \textcolor{red}{Y_4} \frac{v_2}{\sqrt{2}},$$

$$M_\nu^D = Y_1 \frac{v_1}{\sqrt{2}} - \frac{3}{2\sqrt{6}} Y_2 \frac{v_2}{\sqrt{2}} \quad M_e = Y_3 \frac{v_1}{\sqrt{2}} - \frac{3}{2\sqrt{6}} \textcolor{red}{Y_4} \frac{v_2}{\sqrt{2}}.$$

$$\Phi \sim (15, 2, 1/2) = \begin{pmatrix} \Phi_{\text{MW}} & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2, \quad H \sim (1, 2, 1/2)_{\text{SM}}$$

$$SU(4)_c \otimes SU(2)_L \otimes U(1)_R \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q$$

# Pati-Salam shielding

- The theory predicts scalar LQs:

$$\Phi_3 \sim (\bar{3}, 2, -1/6)_{\text{SM}}$$

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[P. Fileviez, C. I and A. D. Plascencia, 2104.11229]



## Scalar LQ: $\Phi_3 \sim (\bar{3}, 2, -1/6)$

$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R^b (\phi_3^{1/3})^* \nu_L^a + \boxed{Y_4^{ab} \bar{d}_R^b (\phi_3^{-2/3})^* e_L^a} + \text{h.c.}$$

- $\phi_3^{-2/3}$  contributes to  $b \rightarrow s$  transitions!

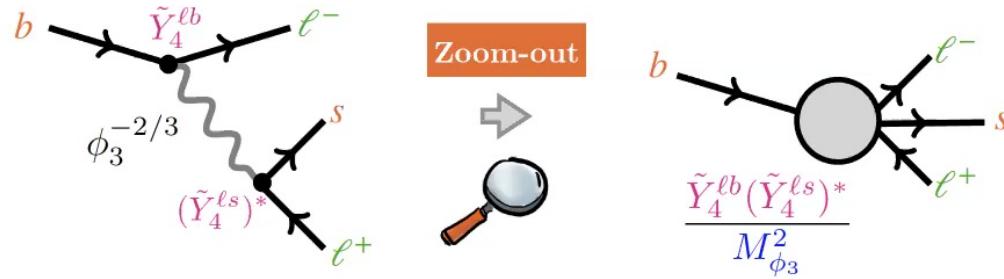


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- $\phi_3^{-2/3}$  contributes to  $b \rightarrow s$  transitions!



$$\mathcal{L}_{\text{eff}}^{\phi_3^{-2/3}} \supset \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} [C'_{9\ell\ell} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell) + C'_{10\ell\ell} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma^5 \ell)]$$

$$\Rightarrow C'_{10\ell\ell} - C'_{9\ell\ell} = \left( \frac{\sqrt{2}\pi}{G_F V_{tb} V_{ts}^* \alpha} \right) \frac{\tilde{Y}_4^{\ell 3} (\tilde{Y}_4^{\ell 2})^*}{4M_{\phi_3^{-2/3}}^2}$$

# Scalar LQ: $\Phi_3 \sim (\bar{3}, 2, -1/6)$

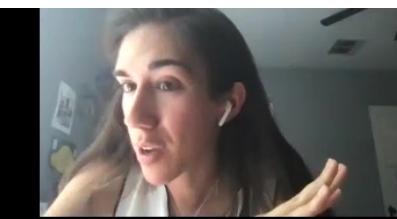
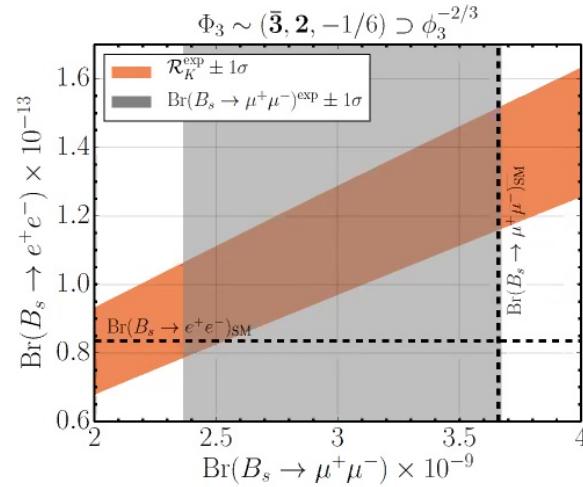
$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R^{b*} (\phi_3^{1/3})^* \nu_L^a + Y_4^{ab} \bar{d}_R^b (\phi_3^{-2/3})^* e_L^a + \text{h.c.}$$

$C'_{10\ell\ell} = -C'_{9\ell\ell}$

- $\phi_3^{-2/3}$  contributes to  $b \rightarrow s$  transitions!

$$\text{Br}(B_s \rightarrow \ell^+ \ell^-) = f_2(C'_{10\ell\ell})$$

$$\mathcal{R}_{K^{(*)}} = \frac{f_2(C'_{10\mu\mu})}{f_2(C'_{10ee})}$$



# Scalar LQ: $\Phi_3 \sim (\bar{3}, 2, -1/6)$



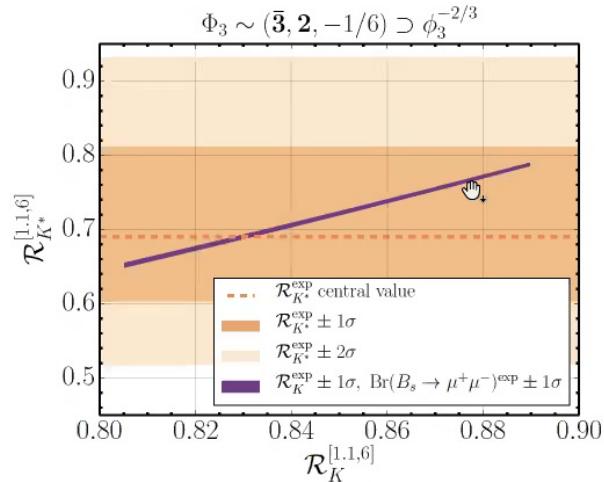
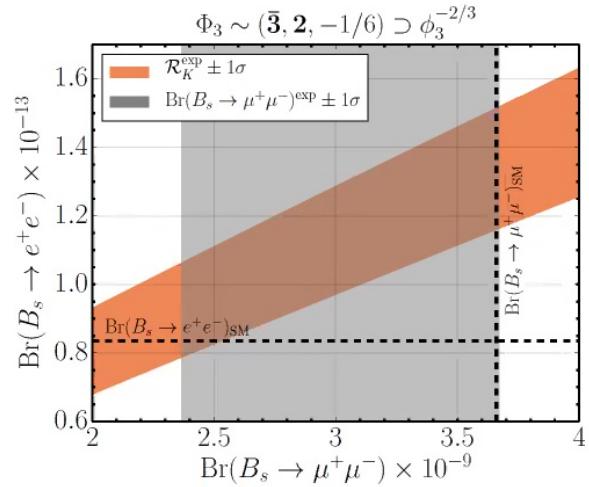
$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R (\phi_3^{1/3})^* \nu_L^a + Y_4^{ab} \bar{d}_R (\phi_3^{-2/3})^* e_L^a + \text{h.c.}$$

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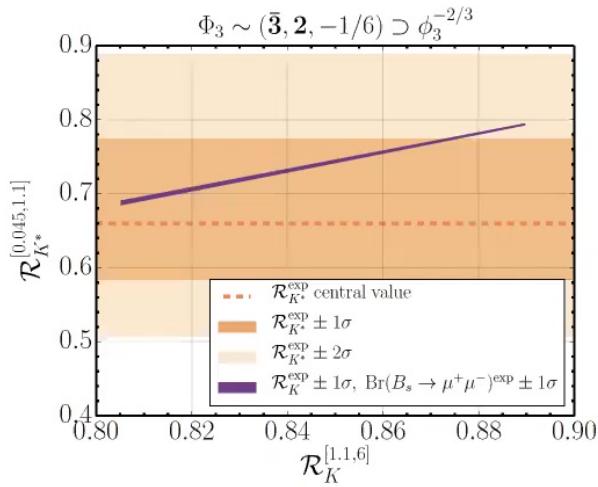
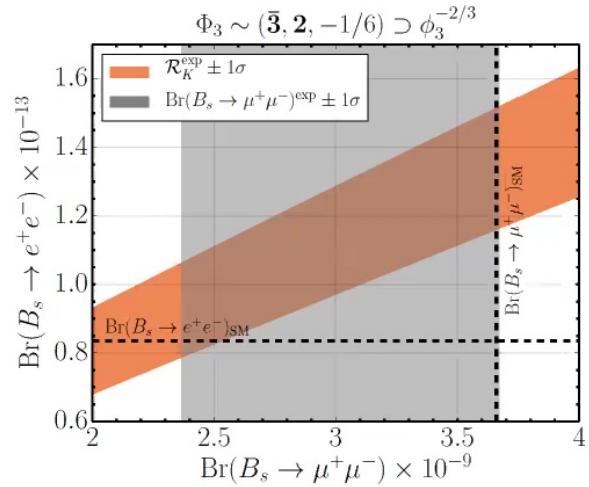
$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R (\phi_3^{1/3})^* \nu_L^a + Y_4^{ab} \bar{d}_R (\phi_3^{-2/3})^* e_L^a + \text{h.c.}$$

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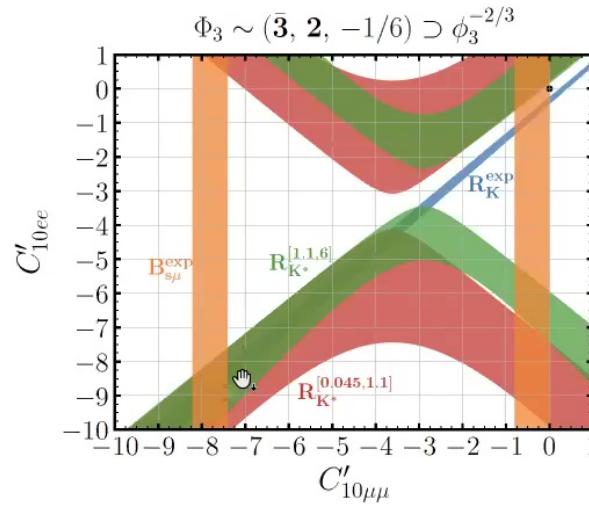
$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R^{b*} (\phi_3^{1/3})^* \nu_L^a + Y_4^{ab} \bar{d}_R^{b*} (\phi_3^{-2/3})^* e_L^a + \text{h.c.}$$

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$C'_{10\ell\ell} = -C'_{9\ell\ell}$

- $\phi_3^{-2/3}$  contributes to  $b \rightarrow s$  transitions! (also to other processes...)

Feynman diagram showing the decay  $K_L \rightarrow \mu^+ \mu^-$  through a loop involving the scalar field  $\Phi_3$ . The loop consists of two crossed yellow lines representing the field  $\Phi_3$ , with vertices labeled  $Y_{\phi_3}^{ed}$  and  $Y_{\phi_3}^{\mu d}$ . The external lines are labeled  $K_L$  and  $\mu^+ \mu^-$ .

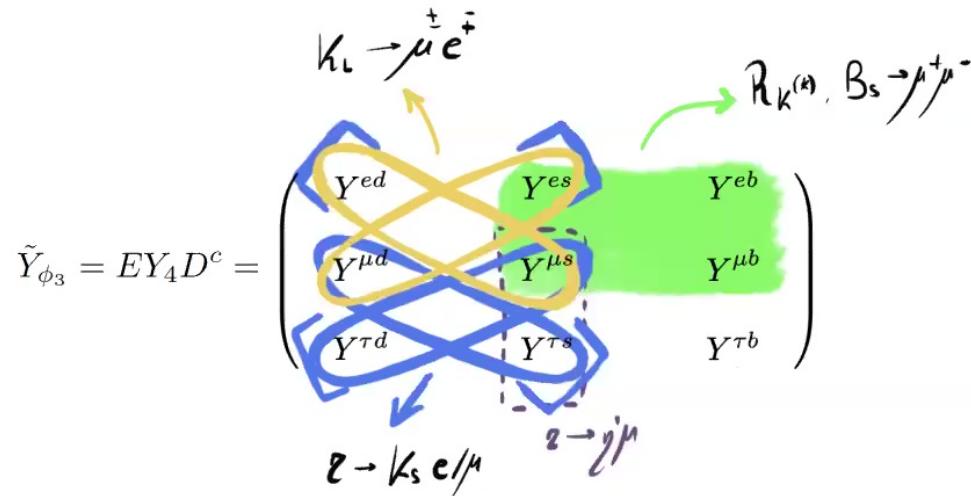
$$\tilde{Y}_{\phi_3} = E Y_4 D^c = \left( \begin{array}{ccc} Y_{\phi_3}^{ed} & Y_{\phi_3}^{es} & Y_{\phi_3}^{eb} \\ Y_{\phi_3}^{\mu d} & Y_{\phi_3}^{\mu s} & Y_{\phi_3}^{\mu b} \\ Y_{\tau d} & Y_{\tau s} & Y_{\tau b} \end{array} \right)$$

## Scalar LQ: $\Phi_3 \sim (\bar{3}, 2, -1/6)$

$$\Phi_3 = \begin{pmatrix} \phi_3^{1/3} \\ \phi_3^{-2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_3} = Y_4^{ab} \bar{d}_R (\phi_3^{1/3})^* \nu_L^a + Y_4^{ab} \bar{d}_R (\phi_3^{-2/3})^* e_L^a + \text{h.c.}$$

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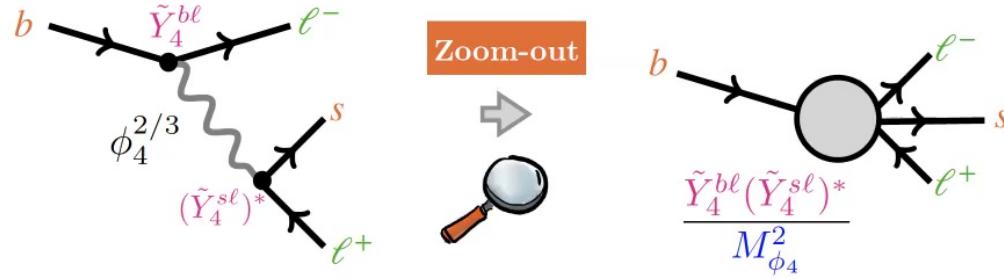
$$\tilde{Y}_{\phi_3} = E Y_4 D^c = \begin{pmatrix} \cdot & \bullet & \bullet \\ \cdot & \bullet & \bullet \\ \cdot & \cdot & ? \end{pmatrix}$$



## Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$

$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_4} \supset Y_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + \boxed{Y_4^{ab} \bar{e}_R^b (\phi_4^{2/3})^* d_L^a} + \text{h.c.}$$

- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions!



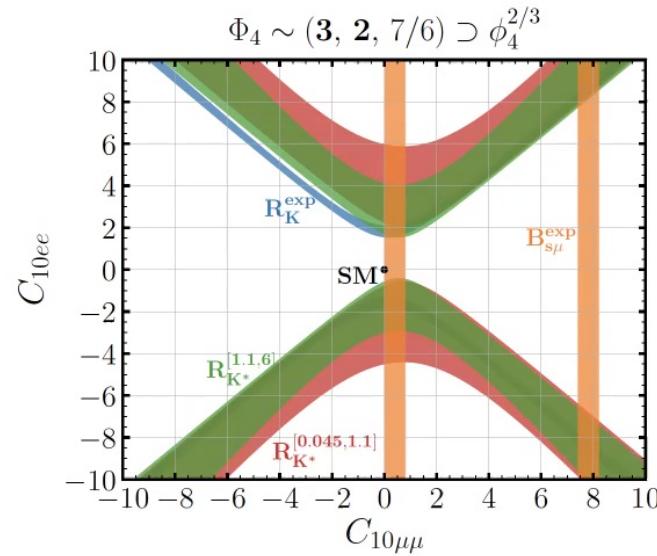
$$\mathcal{L}_{\text{eff}}^{\phi_4^{2/3}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} [C_{9\ell\ell} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) + C_{10\ell\ell} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)]$$

$$\Rightarrow C_{10\ell\ell} = C_{9\ell\ell} = - \left( \frac{\pi\sqrt{2}}{G_F V_{tb} V_{ts}^* \alpha} \right) \frac{\tilde{Y}_4^{3\ell}(\tilde{Y}_4^{2\ell})^*}{4M_{\phi_4^{2/3}}^2}$$

## Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$

$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_4} \supset Y_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + Y_4^{ab} \bar{e}_R^b (\phi_4^{2/3})^* d_L^a + \text{h.c.}$$

$\bullet \phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions!



# Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$

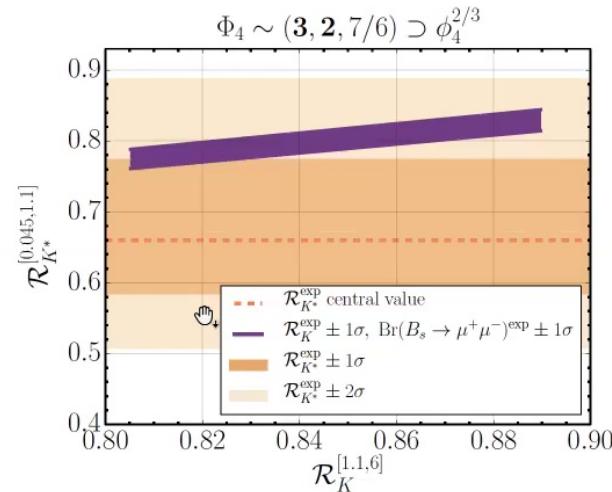
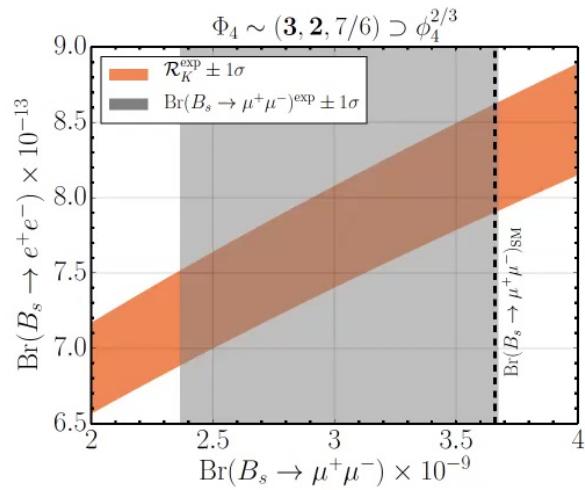
$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix}$$

$$-\mathcal{L}_Y^{\Phi_4} \supset Y_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + Y_4^{ab} \bar{e}_R^b (\phi_4^{2/3})^* d_L^a + \text{h.c.}$$

- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions!

$$\text{Br}(B_s \rightarrow \ell^+ \ell^-) = f_2(\mathcal{C}_{10\ell\ell})$$

$$\mathcal{R}_{K^{(*)}} = \frac{f_2(\mathcal{C}_{10\mu\mu})}{f_2(\mathcal{C}_{10ee})}$$

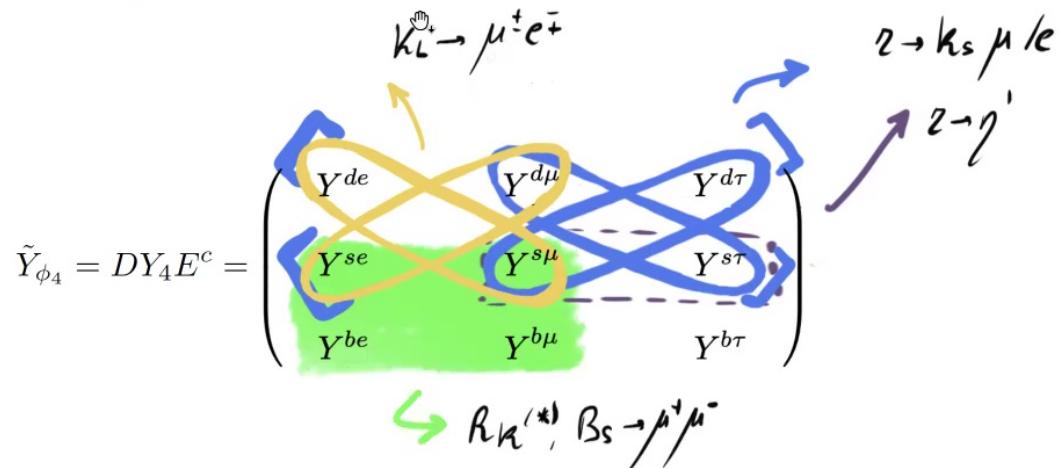


## Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$



$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_4} \supset Y_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + \tilde{Y}_{\phi_4}^{ab} \bar{e}_R^b (\phi_4^{2/3})^* d_L^a + \text{h.c.}$$

- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions! (and also to other processes...)



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- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions! (and also to other processes...)

$$\tilde{Y}_{\phi_4} = DY_4E^c = \begin{pmatrix} \cdot & \cdot & \cdot \\ \bullet & \bullet & \cdot \\ \bullet & \bullet & ? \end{pmatrix}$$



## Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$

$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_4} \supset \tilde{Y}_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + \tilde{Y}_{\phi_4}^{ab} \bar{e}_{\phi_4}^b (\phi_4^{2/3})^* d_L^a + \text{h.c.}$$

- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions! (and also to other processes...)

$$\tilde{\tilde{Y}}_4 = K_2 V_{\text{CKM}} K_1 \tilde{Y}_{\phi_4}$$

$$\tilde{\tilde{Y}}_4 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \bullet & \bullet & \cdot \\ \bullet & \bullet & ? \end{pmatrix}$$



## Scalar LQ: $\Phi_4 \sim (3, 2, 7/6)$



$$\Phi_4 = \begin{pmatrix} \phi_4^{5/3} \\ \phi_4^{2/3} \end{pmatrix} \quad -\mathcal{L}_Y^{\Phi_4} \supset \tilde{Y}_4^{ab} \bar{e}_R^b (\phi_4^{5/3})^* u_L^a + \tilde{Y}_{\phi_4}^{ab} \bar{e}_R^b (\phi_4^{2/3})^* d_L^a + \text{h.c.}$$

- $\phi_4^{2/3}$  contributes to  $b \rightarrow s$  transitions! (and also to other processes...)

$$\tilde{Y}_4 \simeq \tilde{Y}_{\phi_4}$$

$$Y_4 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ Y^{ce} & Y^{c\mu} & \cdot & ? \\ Y^{te} & Y^{t\mu} & ? & ? \end{pmatrix}$$

⇒  $\text{Br}(t \rightarrow c\mu^+ \mu^-) \sim 2 \times 10^{-7}$

$\oplus_4$

## Bonus: $(g - 2)_\mu$

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$

Fermilab Muon g-2, 2021

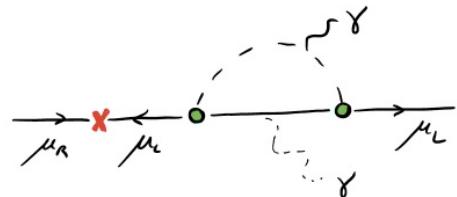


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Fermilab Muon g-2, 2021

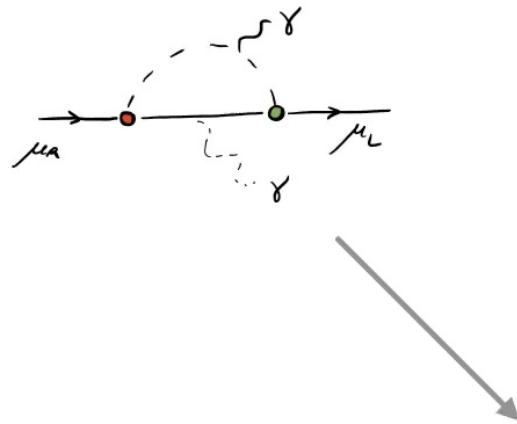


$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} \right]$$

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$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$

Fermilab Muon g-2, 2021



$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} + \frac{m_q}{m_\mu} \text{Re}[\lambda_{\alpha L}^{2j} (\lambda_{\alpha R}^{2j})^*] f'_{\text{loop}} \right]$$

## Bonus: $(g - 2)_\mu$



$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$

Fermilab Muon g-2, 2021

$$-\mathcal{L}_Y \supset Y_4 Q_L \Phi_4^\dagger(e^c)_L + \boxed{Y_4 \ell_L \Phi_3^\dagger(d^c)_L} + \text{h.c.}$$

$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} + \frac{m_q}{m_\mu} \text{Re}[\lambda_{\alpha L}^{2j} (\lambda_{\alpha R}^{2j})^*] f'_{\text{loop}} \right]$$

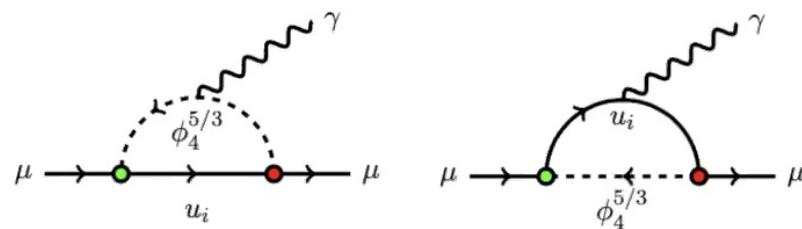
## Bonus: $(g - 2)_\mu$



$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$

Fermilab Muon g-2, 2021

$$-\mathcal{L}_Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + [Y_2 \ell_L \Phi_4 (u^c)_L \pm Y_4 Q_L \Phi_4^\dagger (e^c)_L] + Y_4 \ell_L \Phi_3^\dagger (d^c)_L + \text{h.c.}$$



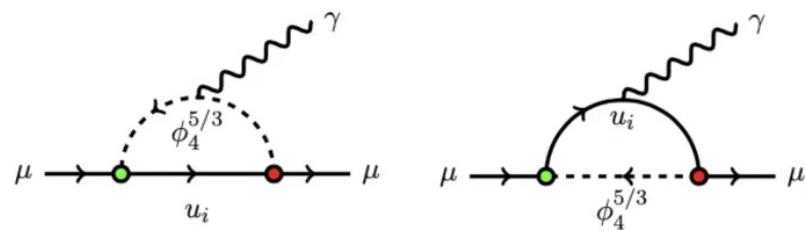
Chiral enhancement!

$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} + \boxed{\frac{m_q}{m_\mu}} \text{Re}[\lambda_{\alpha L}^{2j} (\lambda_{\alpha R}^{2j})^*] f'_{\text{loop}} \right]$$

## Bonus: $(g - 2)_\mu$

$$-\mathcal{L} \supset \tilde{Y}_{\phi L} \phi_4^{5/3} (u^c)_L + \boxed{\tilde{Y}_{\phi_4}} \left( u_L (\phi_4^{5/3})^* (e^c)_L + d_L (\phi_4^{2/3})^* (e^c)_L \right) + \tilde{Y}_{\phi_3} e_L (\phi_3^{-2/3})^* (d^c)_L + \text{h.c.}$$

$$-\mathcal{L} \supset \bar{e}_i \left( \boxed{\lambda_R^{ij}} P_L + \boxed{\lambda_L^{ij}} P_R \right) u^j \left( \phi_4^{5/3} \right)^* + \text{h.c.}$$



$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} + \frac{m_q}{m_\mu} \text{Re}[\lambda_{\alpha L}^{2j} (\lambda_{\alpha R}^{2j})^*] f'_{\text{loop}} \right]$$

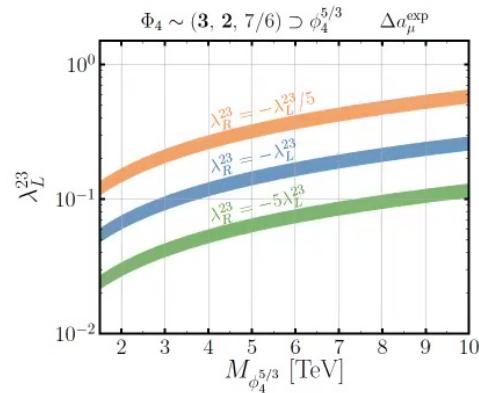


# Bonus: $(g - 2)_\mu$



$$-\mathcal{L} \supset \tilde{Y}_2 e_L \phi_4^{5/3} (u^c)_L + \boxed{\tilde{Y}_{\phi_4}} \left( u_L (\phi_4^{5/3})^* (e^c)_L + d_L (\phi_4^{2/3})^* (e^c)_L \right) + \tilde{Y}_{\phi_3} e_L (\phi_3^{-2/3})^* (d^c)_L + \text{h.c.}$$

$$-\mathcal{L} \supset \bar{e}_i \left( \lambda_R^{ij} P_L + \lambda_L^{ij} P_R \right) u^j \left( \phi_4^{5/3} \right)^* + \text{h.c.}$$



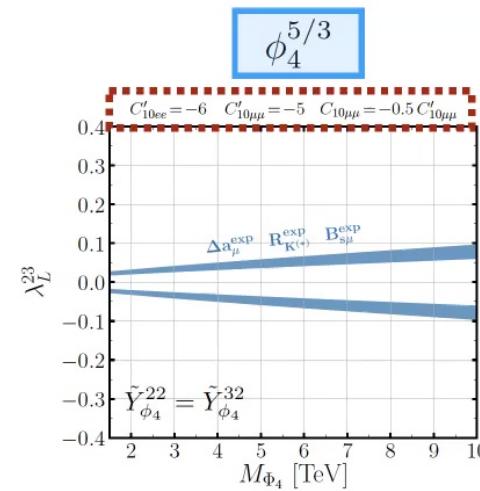
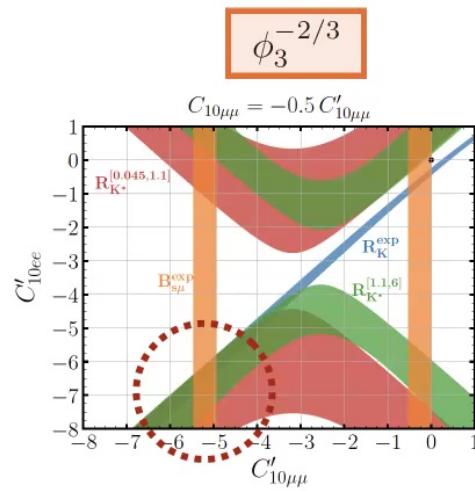
$$\Delta a_\mu^\alpha = \frac{-3}{16\pi^2} \frac{m_\mu^2}{M_{\Phi_\alpha}^2} \sum_j \left[ \left( |\lambda_{\alpha L}^{2j}|^2 + |\lambda_{\alpha R}^{2j}|^2 \right) \times f_{\text{loop}} + \frac{m_q}{m_\mu} \text{Re}[\lambda_{\alpha L}^{2j} (\lambda_{\alpha R}^{2j})^*] f'_{\text{loop}} \right]$$

Bonus:  $(g - 2)_\mu$

$$-\mathcal{L} \supset \tilde{Y}_2 e_L \phi_4^{5/3} (u^c)_L + \tilde{Y}_{\phi_4} \left( u_L (\phi_4^{5/3})^* (e^c)_L + d_L (\phi_4^{2/3})^* (e^c)_L \right) + \tilde{Y}_{\phi_3} e_L (\phi_3^{-2/3})^* (d^c)_L + \text{h.c.}$$

$$\tilde{Y}_{\phi_3} = \begin{pmatrix} \cdot & \text{\fbox{}} & \text{\fbox{}} \\ \cdot & \text{\fbox{}} & \text{\fbox{}} \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\tilde{Y}_{\phi_4} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \text{O} & \cdot \\ \cdot & \text{O} & \cdot \end{pmatrix}$$

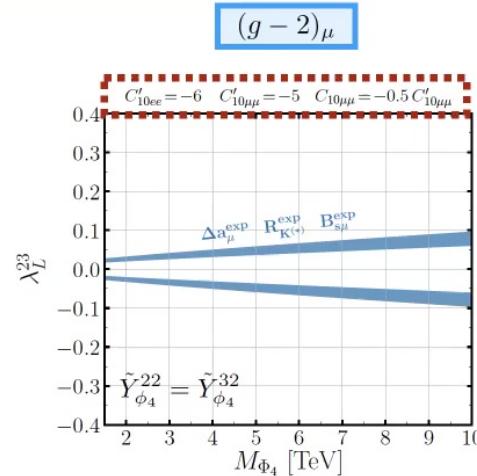
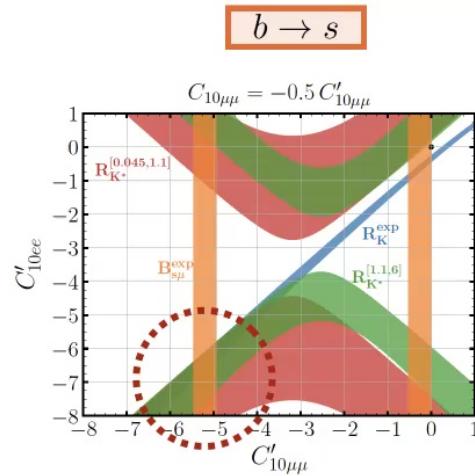


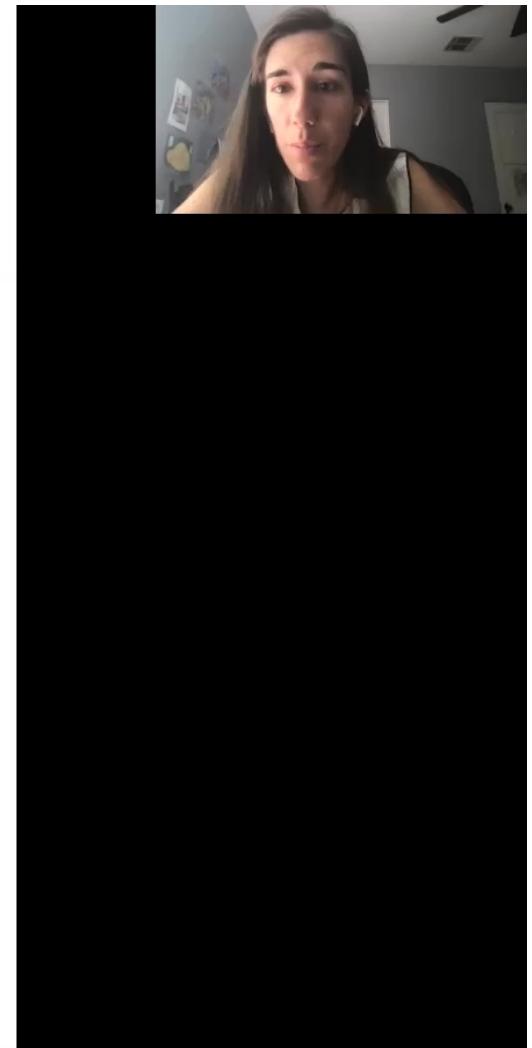
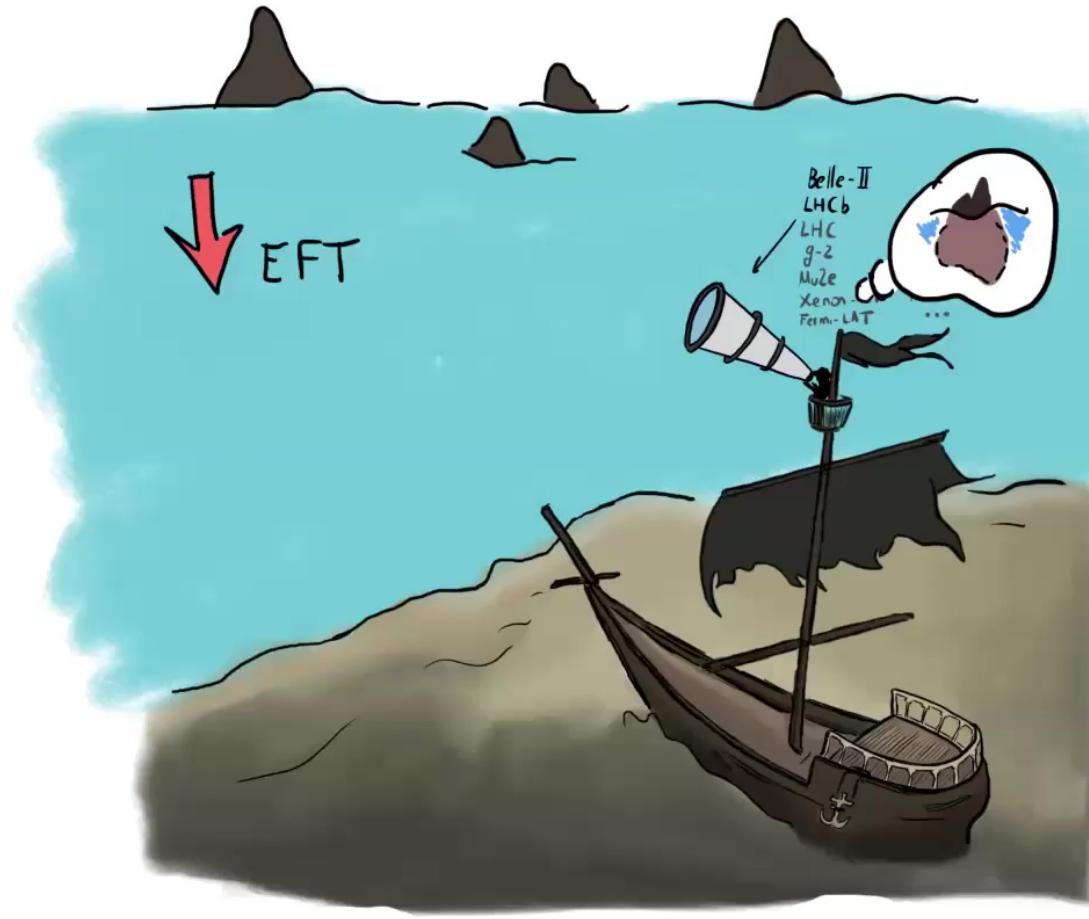
## Bonus: $(g - 2)_\mu$

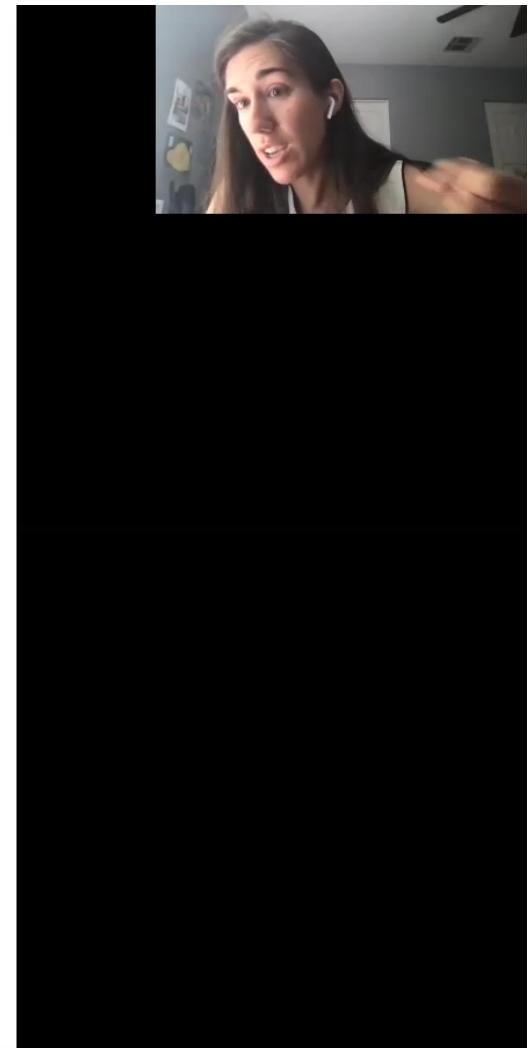
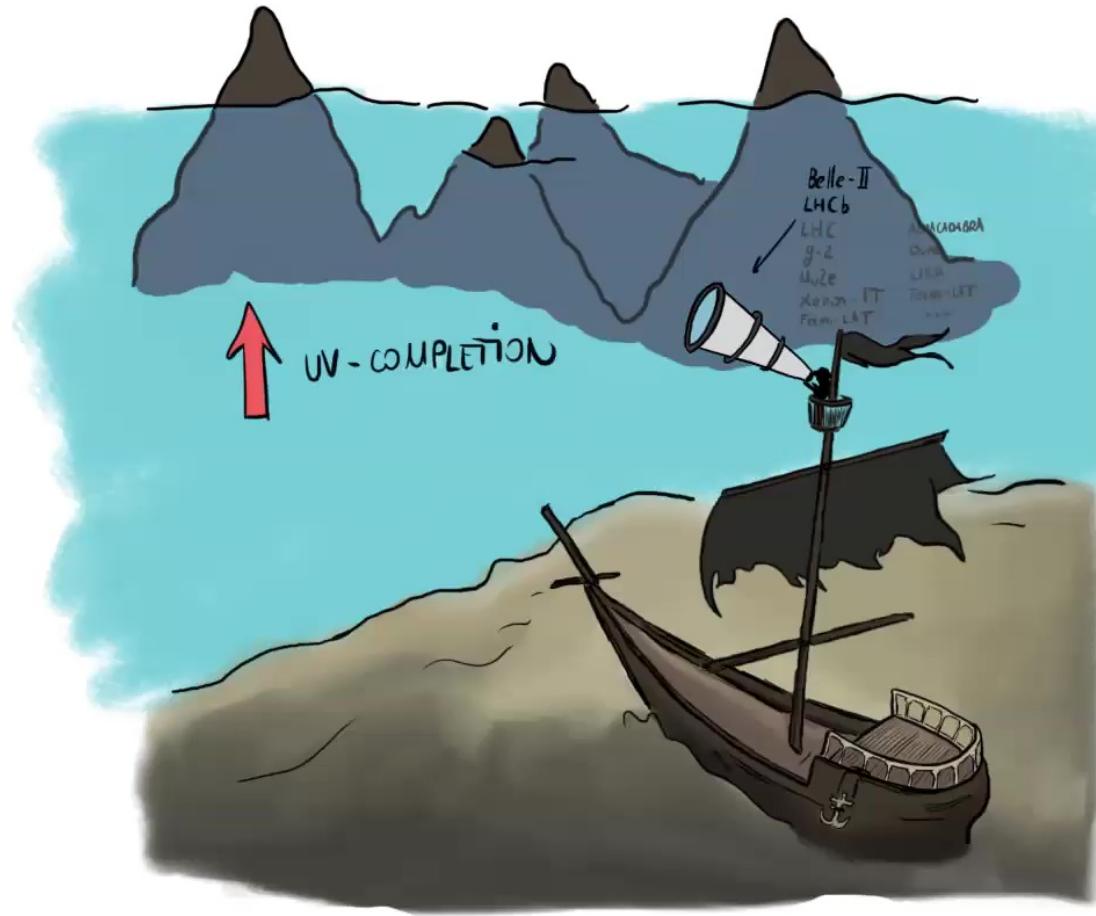
$$-\mathcal{L} \supset \tilde{Y}_2 \mu_L \phi_4^{5/3} (u^c)_L + \tilde{Y}_{\phi_4} \left( u_L (\phi_4^{5/3})^* (\mu^c)_L + d_L (\phi_4^{2/3})^* (\mu^c)_L \right) + \tilde{Y}_{\phi_3} e_L^i (\phi_3^{-2/3})^* (d^c)_L + \text{h.c.}$$

$$\tilde{Y}_{\phi_3} = \begin{pmatrix} \cdot & \begin{matrix} \square & \end{matrix} & \begin{matrix} \square & \end{matrix} \\ \cdot & \ddots & \\ \cdot & & \cdot \end{pmatrix}$$

$$\tilde{Y}_{\phi_4} = \begin{pmatrix} \cdot & & & \\ \cdot & \begin{matrix} \cdot & \end{matrix} & & \\ \cdot & \begin{matrix} \square & \end{matrix} & \ddots & \\ & & & \cdot \end{pmatrix}$$









Thank you!