Title: Non-teleology and motion of a tidally perturbed Schwarzschild black hole

Speakers: Zeyd Sam

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 11, 2021 - 12:00 PM

URL: http://pirsa.org/21070000

Abstract: The prospect of gravitational wave astronomy with EMRIs has motivated increasingly accurate perturbative studies of binary black hole dynamics. Studying the apparent and event horizon of a perturbed Schwarzschild black hole, we find that the two horizons are identical at linear order regardless of the source of perturbation. This implies that the seemingly teleological behaviour of the linearly perturbed event horizon, previously observed in the literature, cannot be truly teleological in origin. The two horizons do generically differ at second order in some ways, but their Hawking masses remain identical. In the context of tidal distortion by a small companion, we also show how the perturbed event horizon in a small-mass-ratio binary is effectively localized in time, and we numerically visualize unexpected behaviour in the black holeâ€™s motion around the binaryâ€™s center of mass.

capra_talk_2.pdf - Adobe Acrobat Reader DC (32-bit)

Non-teleology and motion of a tidally perturbed Schwarzschild black hole

Zeyd Sam*

in collab. with Riccardo Bonetto* and Adam Pound*

Mathematical Sciences, University of Southampton*
Bernoulli Institute, University of Groningen*

24th Capra, Perimeter Institute, June 11 2021

Motivation - Black hole horizons

- Spacetime metric

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \mathcal{O}(\epsilon^3)$$

- Horizon's radial profile

$$r_{\mathcal{H}}(v) = 2M + \epsilon r^{(1)}(v) + \epsilon^2 r^{(2)}(v) + \mathcal{O}(\epsilon^3)$$

- **Event horizon (EH)**: defining feature of BH - but intrinsically teleological - location determined by entire future history of the Universe
- **Apparent horizon (AH)**: Alternative characterization of surface using locally identifiable criteria

$$\vartheta = 0$$

  - Always coincides with EH or lies entirely inside the BH - converse not true. Differences less studied in pert. theory, AH previously not studied for inspirals

First order EH

- Evolution equation

$$\partial_v r^{(1)} - \kappa_0 r^{(1)} = -2M\kappa_0 h^{(1)}_{vv}(v, \theta^A)$$

where $\kappa_0 := 1/(4M)$

- Teleological solution

$$r^{(1)}_{lm}(v) = 2M\kappa_0 \int_v^\infty e^{-\kappa_0(v'-v)} h^{(1lm)}_{vv}(v')dv'$$

- Temporal localization; slowly varying amplitudes, rapidly oscillating phases $h_{vv} = \sum_k h^{(1,k)}_{vv}(\tilde{v})e^{i\varphi_k}$

$$r^{(1)}_{\boldsymbol{k}} = \frac{2Mh^{(1,\boldsymbol{k})}_{vv}}{1 + 4iM\Omega_{\boldsymbol{k}}}$$

First order AH

- Expand the expansion scalar

$$\vartheta = \epsilon \vartheta^{(1)}(v, \theta^A) + \epsilon^2 \vartheta^{(2)}(v, \theta^A) + O(\epsilon^3)$$

- To solve $\vartheta^{(1)} = 0$, decompose into harmonic modes to find

$$r^{(1)}_{lm}(v) = \frac{2M h^{(1lm)}_{vv} - \lambda_1^2 h^{(1lm)}_{v+} - \partial_v h^{(1lm)}_{\circ}}{1 + \lambda_1^2}$$

where $\lambda_1^2 = l(l+1)$

EH vs AH

- Well known that $\vartheta^{(1)} = 0$ for an EH perturbed to first order [Poisson 04] (at times $\geq t_{rr}$ before plunge)
- EH and AH identical at first order!
- Why is this interesting?
  - Location of EH is not truly teleological
  - But "teleological" effects have been observed at first order...

Visualizing the horizon

- Embedding, parabolic encounter, EMRI scenarios [Hartle 73-74], [Vega et al. 11], [O'Sullivan, Hughes, and Penna 14-17]
- Construct a closed 2-surface in Euclidean space with the same intrinsic curvature as the horizon. Radial profile

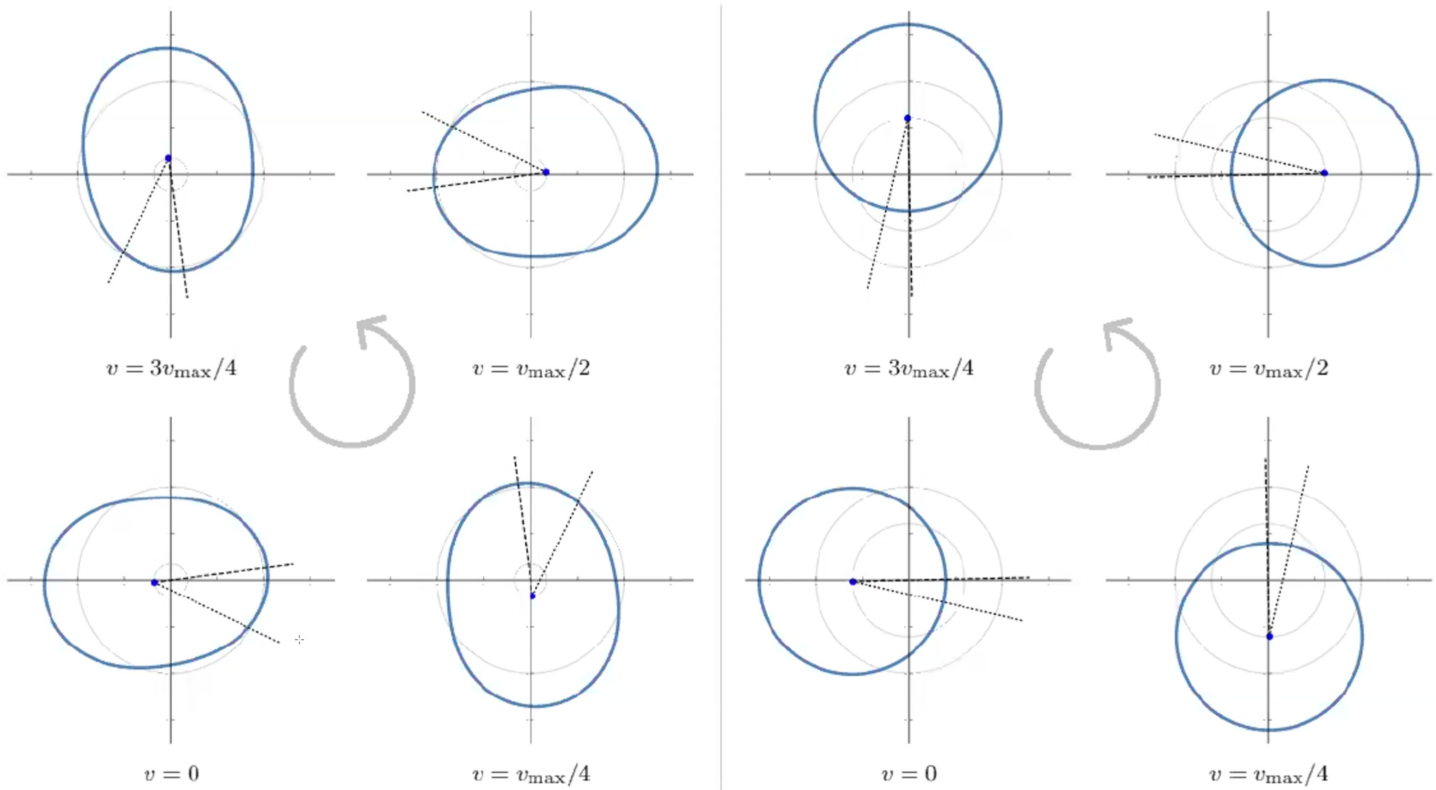
$$r_{\mathrm{E}} = 2M\left[1 + 2\epsilon M^2 \sum_{lm} \frac{\mathcal{R}^{(1)}_{lm} Y_{lm}(\theta^A)}{(l+2)(l-1)} e^{-i\Omega_m v}\right]$$

  - where $\mathcal{R}^{(1)}$ 1st order Ricci scalar in 2D, Ω_m orbital frequency
- Modify the embedding

$$x^i_{\mathrm{E}}(\theta^A; v) = x^i_{\mathrm{BH}}(v) + r_{\mathrm{E}}(\theta^A; v)\Omega^i(\theta^A)$$

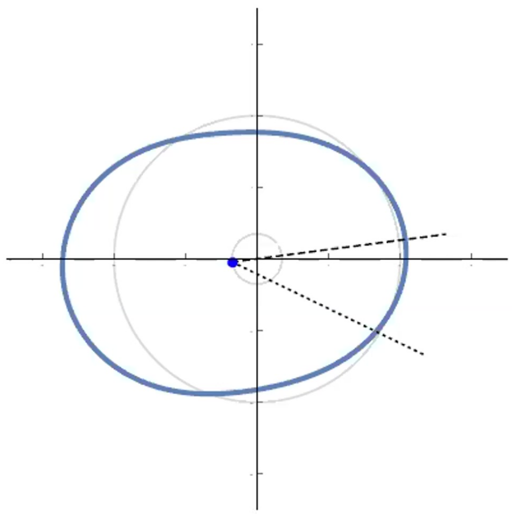
  - At each v, $r_{\mathrm{E}}(\theta^A; v)$ drawn around the BH's "center" $x^i_{\mathrm{BH}}(v)$
  - Uniform translation $x^i_{\mathrm{BH}}(v)$

Plots in 2D, $r_0 = 7M$, $r_0 = 25M$



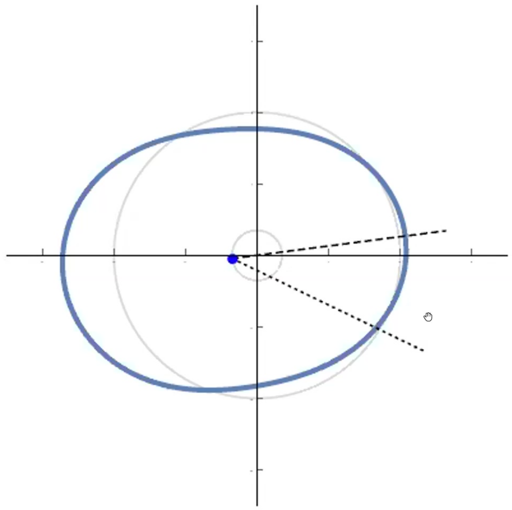
Tidal bulge and Teleological arguments

- Lead angle of tidal bulge discussed many times in the past e.g. [Hartle 73], [Fang, Lovelace 05], [Vega, Poisson, Massey 11], [O'Sullivan, Hughes 14]
- Literature: tidal lead due to teleological nature of EH, that it effectively anticipates the companion's future location
- Deformation leads the metric perturbation by a time of order $1/\kappa_0$
- Explanation cannot be wholly correct...



Non-teleological arguments

- At linear order, EH indistinguishable from AH
  - radial profiles $r^{(1)}_{\mathrm{EH}\ lm}(v) = r^{(1)}_{\mathrm{AH}\ lm}(v)$
  - thus, EH location on a slice of constant v is completely determined by information on that slice
  - ~~truly teleological explanation~~...
- AH is not pulled toward the companion; it is repelled
  - So, tidal bulge of horizon cannot be interpreted in the same way as the tidal bulge of a fluid body



Zeno's paradox of the arrow

- Definition of AH on a slice requires knowledge of the null vectors on that slice
- Know the velocities (tangent vectors) at a given v, so we have some weak info. about the future - we know where null curves are heading
- In previous literature (e.g. Hartle),

$$\partial_v r^{(1)} - \kappa_0 r^{(1)} = -2M\kappa_0 h^{(1)}_{vv}$$

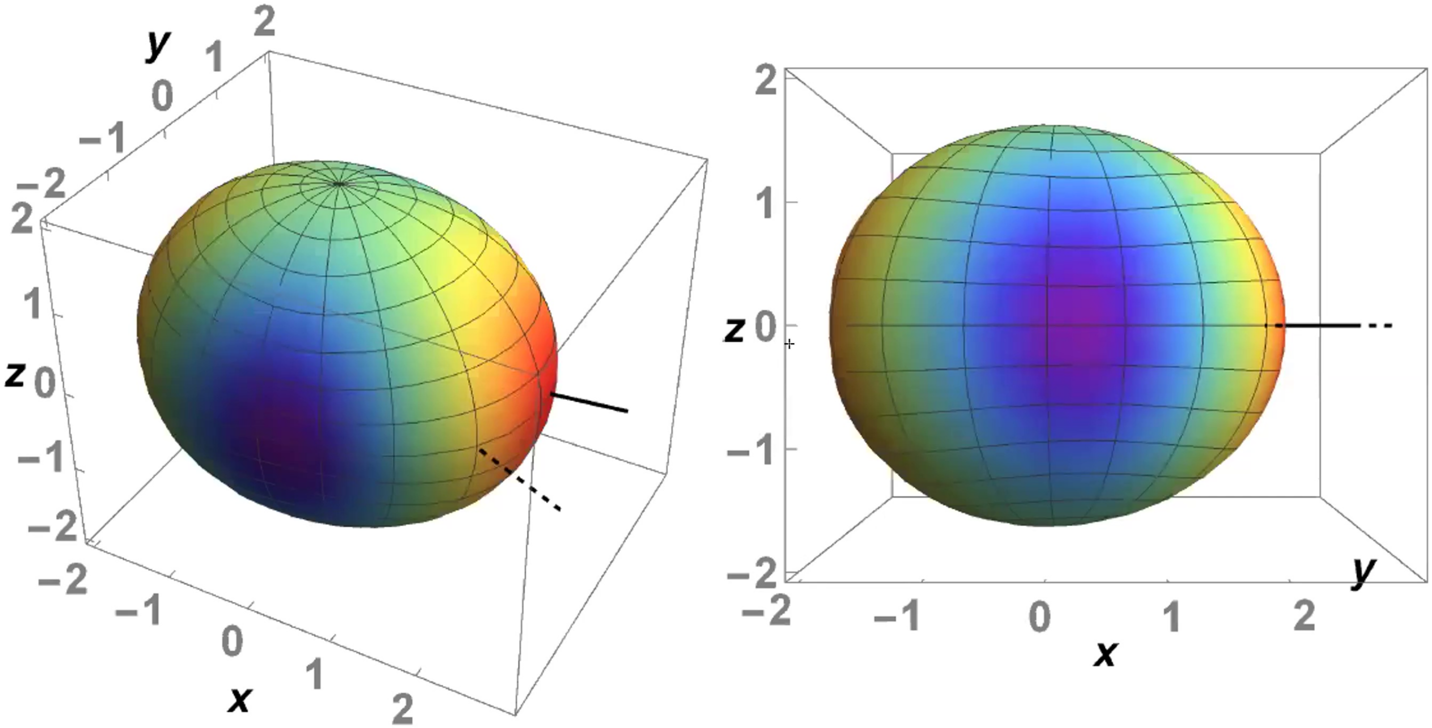
  interpreted as encoding a teleological "lead time" $1/\kappa_0$, but that term arises purely from considering a null vector at a given instant
- The "paradox" is resolved the same way Zeno's paradox is resolved: tangent vectors are properties of a system at a given instant

Summary and Outlook

- Extended literature by including BH motion
- Provided non-teleological argument for tidal lead
- Details on 2nd order calculations...
- Next, extend visualizations to 2nd order?

Thank you for listening

Plots 3D



Thank you for listening