Title: The effective action of superrotation modes

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Series: Quantum Gravity

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Abstract: Asymptotically flat spacetimes are invariant under an infinite-dimensional symmetry group comprised of superrotations and supertranslations. These symmetries are spontaneously broken, leading to an infinite degeneracy of gravitational vacua in asymptotically flat spacetimes. Starting from an analysis of four-dimensional asymptotically flat gravity in first order formulation, I will describe how superrotation parametrization modes labelling distinct superrotation vacua are governed by an Alekseevâ€“Shatashvili action on the celestial sphere. I will also comment on a recent construction of a two-dimensional effective action for the Goldstone modes of broken supertranslation invariance.

The effective action of superrotation modes

Jakob Salzer (Harvard University)

Quantum Gravity seminar, Perimeter June 10th

Motivation (i)

- The infrared structure of gravity in asymptotically flat spacetimes is remarkably rich.

- Known since work of Bondi, van der Burg, Metzner, Sachs 1962 but many new insights/perspectives uncovered recently

- BMS: defined boundary conditions for asymptotically flat spacetimes at null infinity

What is the set of asymptotic symmetries preserving these boundary conditions?

Motivation (ii)

BMS: infinite-dimensional group! $SO(3,1) \ltimes \mathcal{T}$

$\mathcal{T}$: supertranslations parametrized by function on sphere

•) vacuum (Minkowski space) only invariant under Poincaré
infinite vacuum degeneracy $\longrightarrow$ spontaneous symmetry breaking [Geroch '77, Ashtekar '81]

•) gravitational memory effect can be regarded as vacuum transition [Strominger, Zhiboedov '14]

•) Requiring gravitational S-matrix to be BMS invariant
$\Longrightarrow$ Weinberg's soft graviton theorem [He, Lysov, Mitra, Strominger '14]

Motivation (iii)

BMS: infinite-dimensional group! $\quad \underbrace{SO(3,1)}_{\text{Lorentz group}} \ltimes \mathcal{T}$

Lorentz group = global conformal trafo of 2-sphere

allow local trafo's

$\downarrow$

[Barnich, Troessaert '11]

extended BMS: $\hookrightarrow \underbrace{(Vir \otimes Vir)}_{\text{local conformal transformations}} \ltimes \mathcal{T}$

local conformal transformations : superrotations

more recently: •) $Diff(S^2) \ltimes \mathcal{T}$ generalized BMS [Campiglia, Laddha]

•) include Weyl rescalings [Barnich, Lambert ; Freidel, Oliveri, Pranzetti, Speziale]

$\Longrightarrow$ Vacuum is highly degenerate under these symmetries

Motivation (iv)

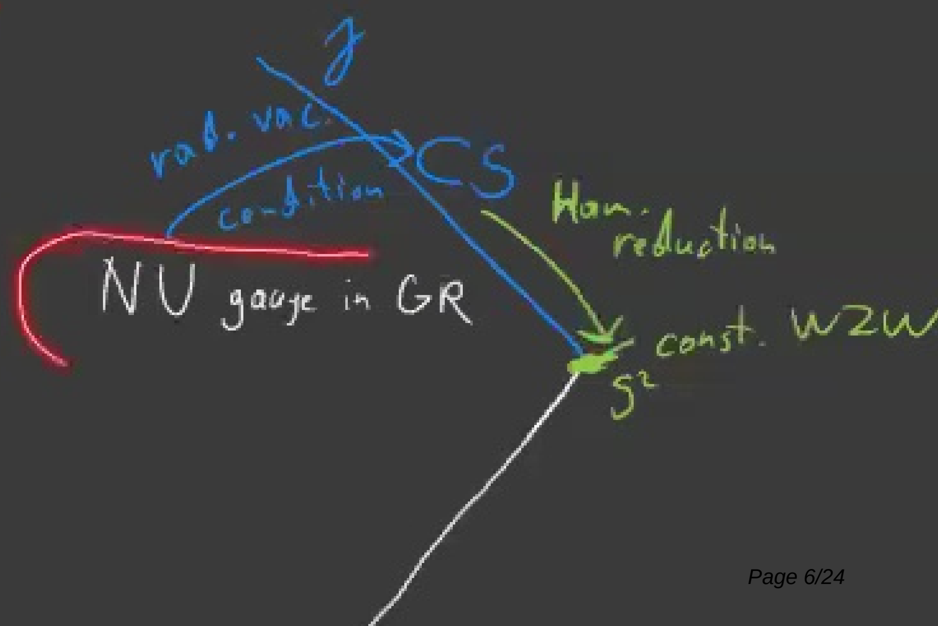
This talk: Identify a theory of radiative vacua at $\mathscr{I}$ as subsector of GR $\Longrightarrow$ reduces to theory on sphere

Outline

1) Asymptotically flat gravity in Newman – Unti gauge

2) The vacuum conditions & Chern-Simons theory on $\mathscr{I}$

3) The effective action of superrotation modes



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1) Asymptotically flat gravity in Newman–Unti gauge

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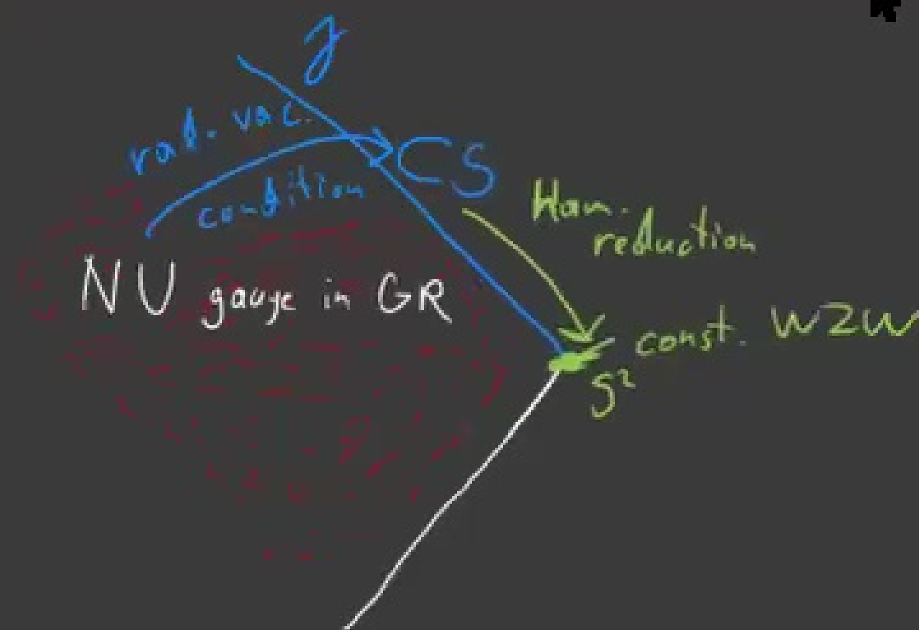
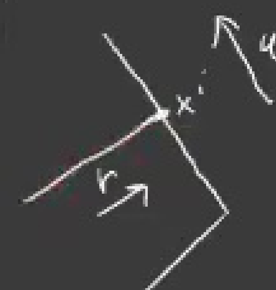
4) Comments & Conclusion



Asymptotically flat gravity in Newman–Unti gauge

$$ds^2 = g_{uu}\, du^2 - 2\, du\, dr + g_{ui}\, du\, dx^i + g_{ij}\, dx^i dx^j$$

$$g_{uu} = O(r) \qquad g_{ui} = O(1) \qquad g_{ij} = r^2 \gamma_{ij} + r C_{ij} + \cdots$$



work in first-order formulation

$$ds^2 = \eta_{IJ}\, E^I \otimes E^J$$

$$\eta_{IJ} = \begin{array}{c} \begin{array}{ccc} \hat{u} & \hat{r} & \overbrace{\quad}^{a,b} \end{array} \\ \begin{pmatrix} 0 & -1 & \\ -1 & 0 & \\ & & \delta^a{}_b \end{pmatrix}_{IJ} \end{array}$$

$$E^{\hat{u}} = du$$

$$E^{\hat{r}} = dr + E^r{}_\mu\, dx^\mu$$

AF spacetimes in NU gauge (ii)

tetrad expansion

$$E^{\hat{u}} = du$$

$$E^{\hat{r}} = dr + (r\, h_\mu + h^{(0)}_\mu + \dots)\, dx^\mu$$

$$E^a = (r\, e^a_\mu + e^{a(0)}_\mu + \dots)\, dx^\mu$$

spin connection expansion

$$\Omega_\mu{}^{ab} = \omega_\mu \varepsilon^a{}_b + O(r^{-1})$$

$$\Omega_\mu{}^{\hat{r}a} = \sigma^a_\mu + O(r^{-1})$$

$$\Omega_r{}^{IJ} = 0$$

$$e^a_u = 0\,, \quad e^a_i = \Theta(u,x^i)\,\bar{e}^a_i(x^i)\,, \quad h_u = \partial_u \log \Theta$$

$$\Downarrow$$

$$\gamma_{ij} \equiv e^a_i e^b_j \delta_{ab} = \Theta^2(u,x)\,\bar{\gamma}_{ij}$$

fixed determinant

Einstein's equation: •) $T^I = dE^I + \Omega^I{}_J \wedge E^J$ (torsion constraint)

Asymptotic Symmetries

$$\delta E^I = \mathcal{L}_\xi E^I + \Lambda^I{}_J E^J = 0 \quad + \text{subleading in } r$$

Local Lorentz

λ^{ab} .... rotation of spatial vierbein : $e^a \longrightarrow \lambda \varepsilon^a{}_b e^b$

λ^a .... null rotation around $E^{\hat{u}}$: $h_n \longrightarrow \lambda^a e^a$; $e^a \longrightarrow -\lambda^a h_n$

Diffeomorphisms

$\xi^u = \ominus (T(x^i) + \ldots)$ supertranslations

$\xi^i = Y^i(x^i)$ diff(S^2) superrotations

$\xi^r = \omega(u, x^i) \cdot r + \ldots$ Weyl trafo's

$$T_{12} = Y_1^i \partial_i T_2 + \frac{1}{2} T_1 D_i Y_2^i - (1 \leftrightarrow 2) \, ,$$

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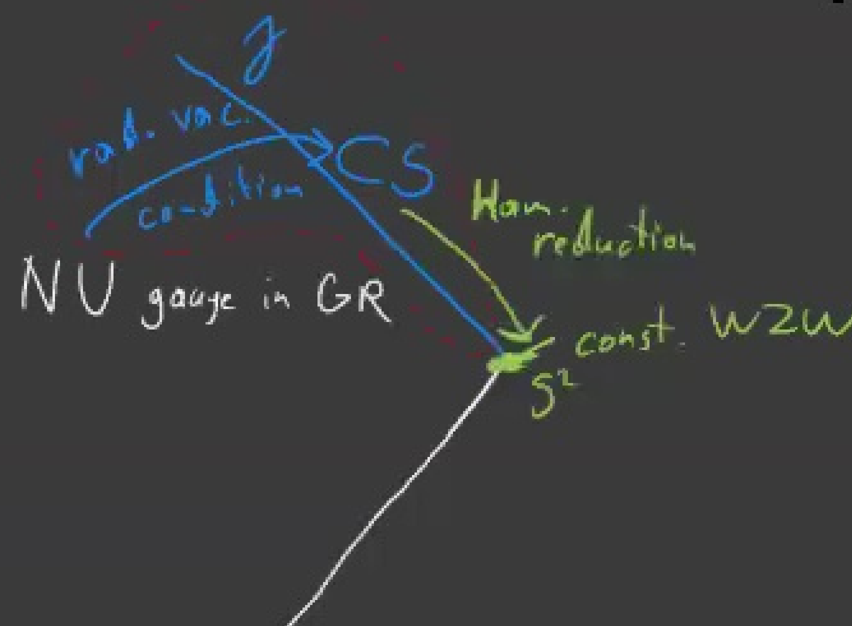

Radiative vacua of AF spacetimes

Space of radiative vacua on $\mathscr{J}$: depend on u, x^i only

•) torsion constraint holds to leading order

$$dh - e^a \wedge b_a = 0 \;;\; de^a + \omega\, \varepsilon^a{}_b \wedge e^b - h \wedge e^a = 0$$

•) scalar curvature vanishes like $\sim r^{-2}$ (implied by Einstein for reasonable matter fields)

$$d\omega - \varepsilon_{ab}\, e^a \wedge b^b = 0$$

•) no radiation @ $\mathscr{J}^+$: leading magnetic Weyl tensor$\big|_{\mathscr{J}} = 0$

$$db^a + h \wedge b^a + \omega\, \varepsilon^a{}_b \wedge b = 0$$

Chern-Simons theory for radiative vacum

Consider the following algebra: $\{H, P_a, B_a, J\}$

$$[J, B_a] = \epsilon_a{}^b B_b \qquad [H, B_a] = B_a \qquad [B_a, P_b] = H\delta_{ab} - J\epsilon_{ab}$$

$$[J, P_a] = \epsilon_a{}^b P_b \qquad [H, P_a] = -P_a$$

This is $so(3,1)$;

Define

$$A = h H + e^a P_a + b^a B_a + \omega J \qquad so(3,1)\text{-valued gauge field}$$

[subalgebra generated by $\{J, B_a\}$ is $iso(2)$

$SO(3,1)/$ ... 3d light-cone (think $\mathrm{Mink}_4 \simeq ISO(3,1)/SO(3,1)$)]

Chern-Simons theory for radiative vacuum

$A = h\,H + e^a P_a + b^a B_a + \omega J$ field strength: $F(A) = dA + A \wedge A$

$$\boxed{F(A) = 0 \iff \text{rad. vacuum conditions}}$$

we can write down an action on $\mathcal{J}$ for this equation

$$S_{CS} = \frac{k}{2\pi}\int_{\mathcal{J}} tr\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right) \quad \text{Chern-Simons}$$

invariant under gauge trafo's $\delta_\lambda A = d\Lambda + [A, \Lambda]$ $\Lambda \in so(3,1)$

reproduces trafo's under asymptotic symmetries $\{\lambda, \lambda^a, \omega, \Upsilon\}$

Radiative vacua of AF spacetimes

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Chern-Simons theory for radiative vacuum

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$$S_{cs} = \frac{k}{2\pi} \int_{\mathscr{I}} \mathrm{tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$ Chern-Simons

invariant under gauge trafo's $\delta_\lambda A = d\Lambda + [A, \Lambda]$ $\quad \Lambda \in so(3,1)$

reproduces trafo's under asymptotic symmetries $\{\lambda, \lambda^a, \omega, Y\}$

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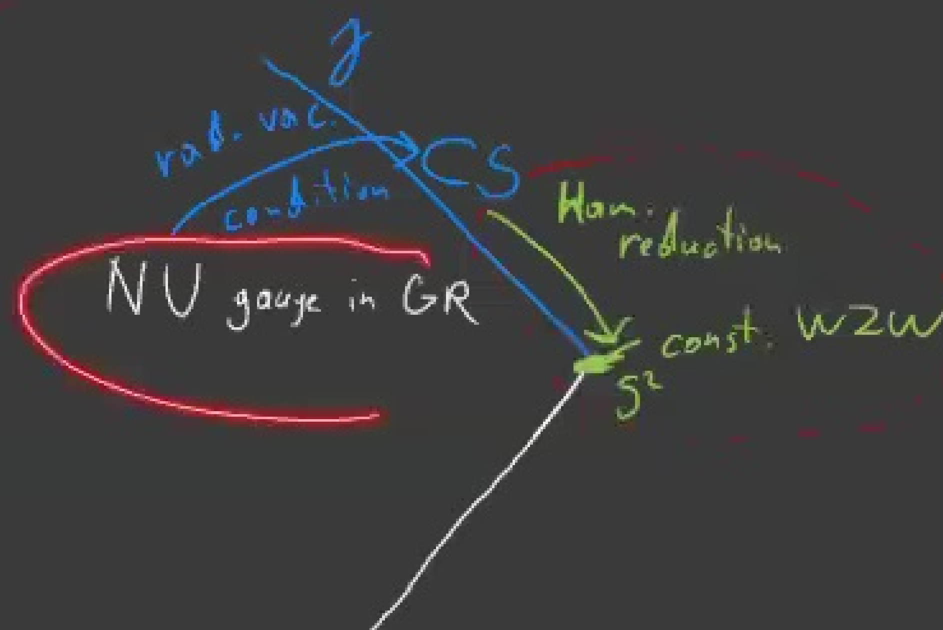


Hamiltonian reduction of CS theory

•) Chern - Simons theories are topological $\Longrightarrow$ non-trivial excitations reside on boundary

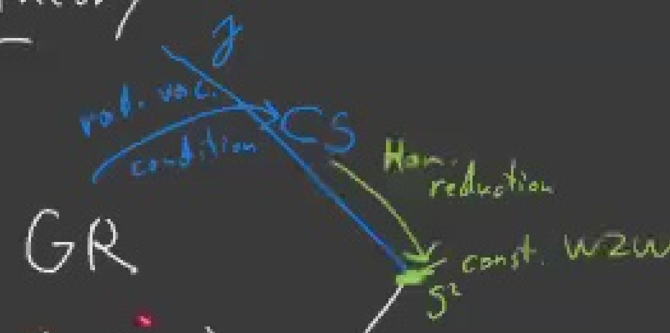
•) reduce CS theory $\longrightarrow$ 2d CFT: Wess-Zumino-Witten model on bdy

•) boundary conditions on fields of CS theory $\Longrightarrow$ constraints on WZW model

•) well-known procedure from 3d gravity in (A)dS/ flat space

•) Caveat: global properties of superrotation vacua not sufficiently well-understood $\Longleftrightarrow$ allow topology changes of $\mathcal{J}$?

Hamiltonian reduction of CS theory



CS theory in Hamiltonian form:

$$S[A] = \int_{\mathscr{I}^+} d^3x \, \mathrm{tr}\left(A_u \partial_{\bar z} A_z - A_z \partial_{\bar z} A_u + 2 A_{\bar z} F_{zu} \right)$$

Lagrange multiplier

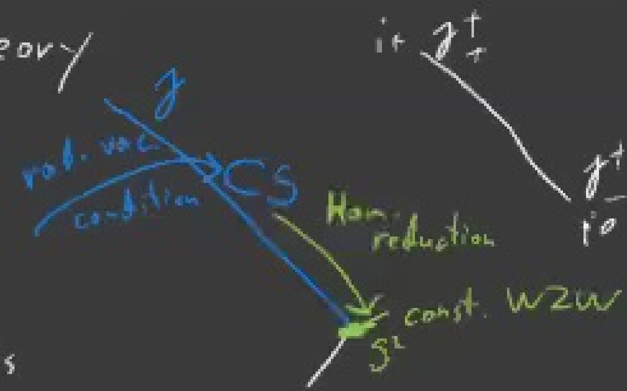
constraint

solve constraint: $F_{uz} = 0 \implies A_u = G^{-1}\partial_u G \; ; \; A_z = G^{-1}\partial_z G$

$$G \in SO(3,1)$$

$$S[G] = \int dz\, d\bar z \; \mathrm{tr}\left(G^{-1}\partial_z G \, G^{-1}\partial_{\bar z} G \right) \quad \Big\} \; SO(3,1) \text{ WZW theory}$$

Hamiltonian reduction of CS theory



impose boundary conditions @ $\mathcal{J}^+_-$

$$\left.\begin{aligned} \bullet)\quad & e^a_u\big|_{\mathcal{J}^+_-} = 0 \\ & h_u\big|_{\mathcal{J}^+_-} = \partial_u \log \Theta \end{aligned}\right\} \text{ for compatibility w/ 4d gravity analysis}$$

$$e^a e_a\big|_{\mathcal{J}^+_-} = \Theta^2 \gamma_{z\bar{z}}\, dz\, d\bar{z}$$

metric on sphere

arbitrary but fixed

we had $\quad A_u = G^{-1}\partial_u G \qquad G = e^{\Pi^a P_a}\, e^{\Lambda H + \tilde{\Lambda} J}\, e^{\beta^a B_a}$

boundary conditions impose constraints e.g.: $\; e^a_u\big|_{\mathcal{J}^+_-} = 0 \Longrightarrow \partial_u \Pi^a = 0$

$A = e^a P_a$

constraints can be solved in terms of $\Pi^a(z,\bar{z})$ only

The effective action of superrotation modes

$$\boxed{S[\Pi, \bar{\Pi}] = \frac{k}{2\pi} \int_{S^2} dz\, d\bar{z} \left[\frac{\partial_z \partial_{\bar{z}} \Pi\, \partial_z \Pi}{(\partial_z \Pi)^2} + \frac{\partial_z \partial_{\bar{z}} \bar{\Pi}\, \partial_{\bar{z}} \bar{\Pi}}{(\partial_{\bar{z}} \bar{\Pi})^2} \right].}$$

Alekseev-Shatashvili action of superrotation modes

•) equation of motion: $\partial_{\bar{z}} T = 0 \implies \partial_{\bar{z}} \Pi = 0 \Rightarrow \Pi = \Pi(z)$

$$T = -\frac{1}{2}\{\Pi, z\} = \frac{\partial^3 \Pi}{\partial \Pi} - \frac{3}{2}\left(\frac{\partial^2 \Pi}{\partial \Pi}\right)^2 \quad \text{Schwarzian derivative}$$

•) symmetries: $\delta_\varepsilon \Pi = \varepsilon \partial \Pi \implies$ charge: $Q[\varepsilon] = \int dz\, \varepsilon(z)\, T(z)$

•) gauge symmetry: $PSL(2,\mathbb{C}) \simeq SO(3,1) \quad \Pi \longmapsto \frac{a\Pi + b}{c\Pi + d}$

The effective action of superrotation modes

$$\boxed{S[\Pi,\bar{\Pi}] = \frac{k}{2\pi}\int_{S^2} dz\, d\bar{z} \left[\frac{\partial_z\partial_{\bar{z}}\Pi\, \partial_z\Pi}{(\partial_z\Pi)^2} + \frac{\partial_{\bar{z}}\partial_z\bar{\Pi}\, \partial_{\bar{z}}\bar{\Pi}}{(\partial_{\bar{z}}\bar{\Pi})^2} \right].}$$

Translate results back to 4d gravity formulation:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\gamma_{z\bar{z}}\,dz\,d\bar{z} + r\left(C_{zz}\,dz^2 + C_{\bar{z}\bar{z}}\,d\bar{z}^2\right) + \cdots$$

$$C_{zz} = 2u\,T + C^{T}_{zz}$$

determines supertranslation vacuum

$T = -\frac{1}{2}\{\Pi(z), z\}$ determines superrotation vacuum

The effective action of superrotation modes

$$\boxed{S[\Pi, \bar{\Pi}] = \frac{k}{2\pi}\int_{S^2} dz\, d\bar{z} \left[\frac{\partial \partial_{\bar{z}} \Pi\, \partial_z \Pi}{(\partial_z \Pi)^2} + \frac{\partial_{\bar{z}} \partial_z \bar{\Pi}\, \partial_{\bar{z}} \bar{\Pi}}{(\partial_{\bar{z}} \bar{\Pi})^2} \right]}\,.$$

Translate results back to 4d gravity formulation:

$$ds^2 = -du^2 - 2du\,dr + r^2 \gamma_{z\bar{z}}\, dz\, d\bar{z} + r\left(C_{zz}\, dz^2 + C_{\bar{z}\bar{z}}\, d\bar{z}^2 \right) + \dots$$

$$C_{zz} = 2u\,T + C^T_{zz}$$

C^T_{zz} ← determines supertranslation vacuum

$T = -\frac{1}{2}\{\Pi(u), z\}$ determines superrotation vacuum

Comments & Conclusions

- Leading order fields in NU gauge obey equations of $SO(3,1)$ CS theory when restricted to radiative vacua
- $SO(3,1)$ CS theory for vacuum sector leads to Alekseev-Shatashvili theory on boundary of $\mathcal{I}$
- effective 2d description of superrotation vacuum modes
- relation to ongoing celestial CFT program?
- better understanding of superrotation vacua needed!

Thank You!