Title: Interaction and Evolution in Classical and Quantum Physics, and Indefinite Causal Structure

Speakers: Erik Curiel

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Abstract: In classical mechanics, the representations of dynamical evolutions of a system and those of interactions the system can have with its environment are different vector fields on the space of states: evolutions and interactions are conceptually, physically and mathematically different in classical physics, and those differences arise from the generic structure of the very dynamics of classical systems ("Newton's Second Law"). Correlatively, there is a clean separation of the system's degrees of freedom from those of its environment, in a sense one can make precise. I present a theorem showing that these features allow one to reconstruct the entire flat affine 4-dimensional geometry of Newtonian spacetime---the dynamics is inextricably tied to the underlying spacetime structure. In quantum theory (QT), contrarily, the representations of possible evolutions and interactions with the environment are exactly the same vector fields on the space of states ("add another self-adjoint operator to the Hamiltonian and exponentiate"): there is no difference between "evolution" and "interaction" in QT, at least none imposed by the structure of the dynamics itself. Correlatively, in a sense one can make precise, there is no clean separation of the system's degrees of freedom from those of the environment. Finally, there is no intrinsic connection between the dynamics and the underlying spacetime structure: one has to reach in and attach the dynamics to the spacetime geometry by hand, a la Wigner (e.g.). How we distinguish interaction from evolution in QT and how we attach the dynamics to a fixed underlying spacetime structure come from imposing classical concepts foreign to the theory. Trying to hold on to such a distinction is based on classical preconceptions, which we must jettison if we are to finally come to a satisfying understanding of QT. These observatons offer a way to motivate and make sense of, inter alia, the idea of indefinite causal structures.

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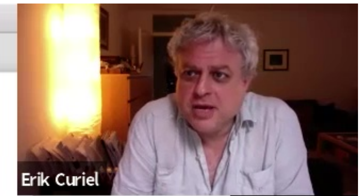



# Interaction, Evolution, Space and Time in Classical Mechanics and in Quantum Mechanics

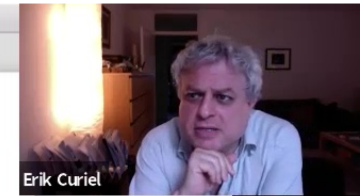
## Quantizing Time, PI, 18. Jun 2021


Erik Curiel

Munich Center For Mathematical Philosophy
Ludwig-Maximilians-Universität

Black Hole Initiative
Harvard University

**email**: `erik@strangebeautiful.com`




I will draw out and make precise some differences in the mathematical representations of “interaction” and “evolution” in classical mechanics and quantum mechanics, and begin to draw some conceptual lessons from that analysis.

I will *not* propose a “solution” to the measurement problem, rather a different way of looking at it, trying to approach it—an attempt to reconceptualize what a good answer to it might look like.

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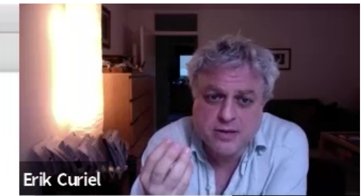


in sum, classical (Newtonian and Lagrangian) mechanics:

1. characterizes "interaction" and "evolution" as naturally distinguished mathematically, physically and conceptually by the dynamics alone (and so intrinsic to it)
2. $\Rightarrow$ natural distinction between configuration and momentum
3. $\Rightarrow$ natural characterization of free evolution
4. $\Rightarrow$ construction of (*a priori* relation to) spacetime geometry
5. clean separation of system's degrees of freedom from environment's

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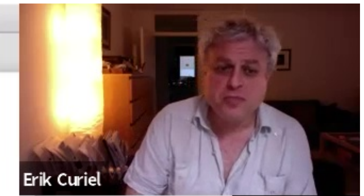


in sum, in quantum mechanics:

1. NO natural distinction between interaction and evolution intrinsic to dynamics
2. $\Rightarrow$ NO natural distinction between configuration and momentum
3. $\Rightarrow$ NO natural characterization of free evolution
4. $\Rightarrow$ NO construction of (*a priori* relation to) spacetime geometry
5. NO clean separation of system's degrees of freedom from environment's

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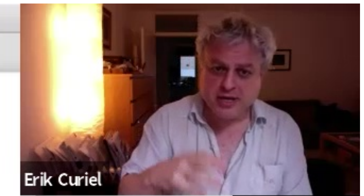


## classical system

1. space of states $\mathcal{S}$ is even-dimensional manifold
2. evolution governed by Newton's Second Law (family of dynamical vector fields $\mathfrak{D}$)
3. "interaction": applying an "external force" to the system ("intervention")
4. has distinguished "free" evolution ("isolation" = no external force)

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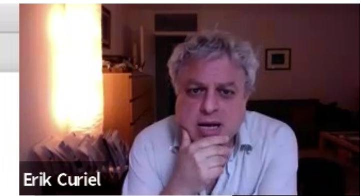


# Newton's Second Law

**free particle**

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{0} \end{aligned} \tag{1}$$

dynamical vector field has components: $(\mathbf{v}, \mathbf{0})$

**with interaction turned on ("hit it with a stick")**

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{F}_{\text{stick}} \end{aligned} \tag{2}$$

dynamical vector field: $(\mathbf{v}, \mathbf{F}_{\text{stick}})$
interaction vector field: $(\mathbf{0}, \mathbf{F}_{\text{stick}})$

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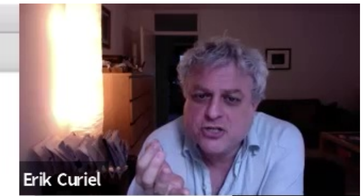


## Definition

*A* classical system *is fully characterized by:*

1. *space of states is even-dimensional manifold;*
2. *the family of interaction vector fields $\mathfrak{I}$ has the structure of a vector space;*
3. *the family of dynamical vector fields $\mathfrak{D}$ has the structure of an affine space modeled on $\mathfrak{I}$.*

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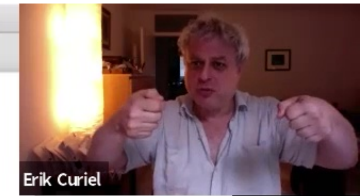


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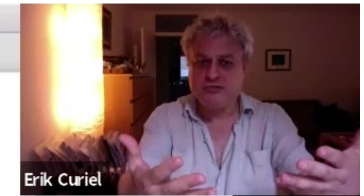


conceptually, the family of possible interactions allows one to distinguish "configuration" from "velocity" quantities in a principled way:

- interactions directly modify the system's evolutions only in "velocital directions", *i.e.*, as generalized "accelerations"
- correlatively, the physical meaning of "configuration" is that those quantities encode the possible interactions the system can have with its environment
- "velocity" quantities are always dynamical derivatives of "configuration" ones (in sense of affine structure on family of dynamical vector fields)
- then: free dynamical vector field distinguished by fact that only configurative quantities change

(one can make this all precise and rigorous, for finite-dimensional systems AND fields with infinite-dimensional spaces of state)

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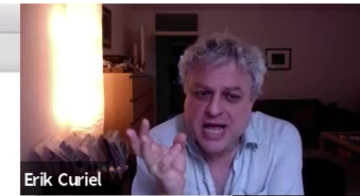
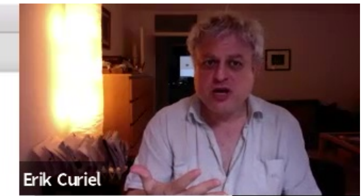



**this is what I mean by "classical system":**

1. mathematical and conceptual distinction between "configuration" and "velocity" encoded in the dynamics
2. mathematical and conceptual distinction between "evolution" and "interaction" encoded in the difference between configuration and velocity
3. and so a naturally distinguished "free" evolution

**always a clean separation between a system's degrees of freedom and those of its environment**—no matter the interaction, I can determine the system's evolution without knowing any details about the environment's degrees of freedom or its evolution: I treat the environment like a black box, everything relevant encoded in interaction vector field




**Theorem (Curiel 2019)**

*The Euclidean geometry of ordinary space, the metrical structure of time, and the flat affine geometry of 4-dimensional Newtonian spacetime is entirely determined by the dynamical structure of a classical system.*

in classical mechanics, evolution characterizes time and its geometry, while interaction (including "isolation") characterizes space and its geometry; both jointly determine the full 4-dimensional flat affine geometry

the distinction between time and space is built in to the structure of the dynamics, as is the geometry of spacetime itself

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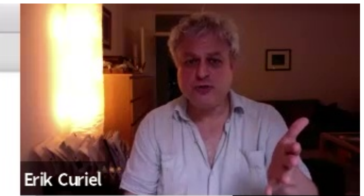
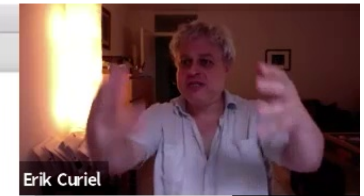



# Lagrangian mechanics

**Theorem (Curiel 2014)**

*Given classical system with space of states $\mathcal{S}$ and families of vector fields $\mathfrak{D}$ and $\mathfrak{I}$:*

1. *one can reconstruct the system's configuration space $\mathcal{C}$ from $\mathcal{S}$, based on the affine structure of $\mathfrak{D}$;*
2. *$\mathcal{S}$ is canonically isomorphic to $T\mathcal{C}$ (using the distinguished free dynamical vector field to define the isomorphism);*
3. *$\mathfrak{D}$ is then isomorphic to the family of vector fields on $T\mathcal{C}$ representing all solutions to the Euler-Lagrange equation ("second-order vector fields", i.e., lifts of vector fields from configuration space to the tangent bundle);*
4. *$\mathfrak{I}$ is isomorphic to the family of vector fields on $T\mathcal{C}$ representing generalized forces ("vertical vector fields", pointing straight up and down the fibers);*
5. *in particular, the vertical vector fields have the structure of a vector space, and the second-order vector fields the structure of an affine space modeled on it.*




if I know only the space of states as an abstract manifold and how to solve the Euler-Lagrange equation, I can reconstruct **everything else**

$\Longrightarrow$ *the dynamics by itself automatically defines each of and encodes the difference between "configuration" and "velocity" and, correlatively, between "evolution" and "interaction", and uniquely determines "isolation" ("free evolution")*

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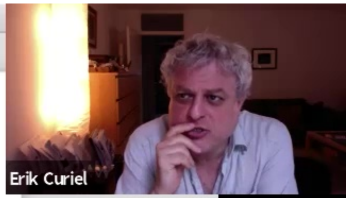
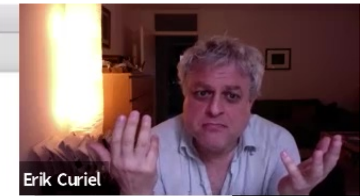


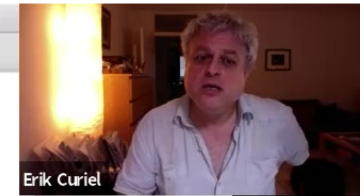

1. dynamical vector fields ("evolutions") are unitary flows on Hilbert space
2. unitary flows are exponentiated Hamiltonians (self-adjoint operators)
3. "interaction" is just adding another Hamiltonian
4. only fixed relation between "configuration" and "momentum" is canonical commutation relation

$$[Q,\, P] = i\hbar I$$




if I give you a Hilbert space and its family of self-adjoint operators, *i.e.*, the dynamics:

1. you can't distinguish interactions from evolutions ("external interventions" such as measurements)
2. you can't tell me what's configuration and what's momentum
3. you can't tell what is free evolution ("isolation")
4. indeed, even if I give you a "standard" Hamiltonian in some funky basis, and ask you to decompose it into its "free" part and its "interaction" part, you won't be able to do it




I need an explicit representation both of degrees of freedom of the environment, and their inter-twining with those of the system, in order to appropriately and adequately treat the system "during an interaction". More specifically, I need:

1. *to change the space of states I use to treat the system*, now the tensor product of its "isolated" Hilbert space with that of the environment;
2. to change its set of physical quantities, now the algebra of observables on the tensor-product Hilbert space;
3. to change the structure of the quantities encoding evolution, now Hamiltonians on the tensor-product.

That most quantum of quantum mechanical phenomena has now made its entrance, *entanglement*.

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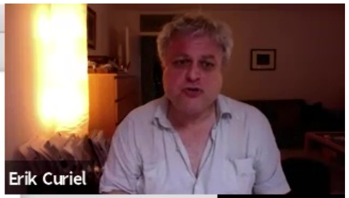
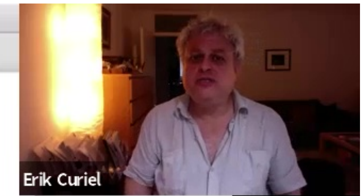



**$\Longrightarrow$ no clean separation in quantum mechanics between a system's degrees of freedom and those of its environment**

if I want to understand how a system evolves under interaction with the environment, I have to know how to model the environment's degrees of freedom and dynamics

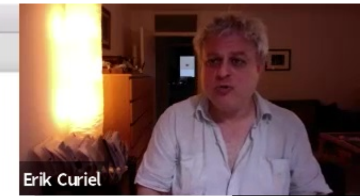



How do we distinguish the $Q$s and the $P$s in quantum mechanics, and so define a "free" evolution? Two ways:

1. by fiat
2. introduce representation of Poincaré or Galileian group *à la* Wigner, and define $Q$ as the generator of momentum translations, *etc.*

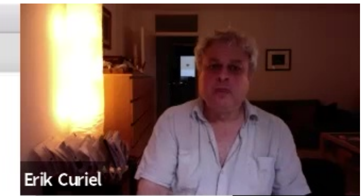
In both cases, we must explicitly and by hand hook the dynamics up to the background spacetime structure to get a principled distinction between configuration and momentum, and so define "free" and "interacting"

we don't get it from the dynamics alone
as in classical mechanics




# possible responses

1. modify quantum mechanics/look for interpretation that makes difference between "evolution" and "interaction" clear, as in classical mechanics—this is, implicitly, most common approach in most popular "interpretations" (many worlds with preferred basis, Bohm, GRW, . . . —but information-theoretic approaches?)
2. reconceptualize "interaction" in quantum mechanics so as to accord with these facts
3. get rid of the distinction between evolution and interaction altogether, and reconceptualize "dynamics"




connection to indefinite causal orders *à la* Hardy's causaloid framework and Brukner *et al.*'s process matrix formalism?

conjectures:

1. configuration and momentum are not naturally and univocally distinguished by the dynamics, because different causal orders require different momentum operators, to respect the different null cones
2. one way to interpret a process matrix: as a fixing of the $P$s and $Q$s in the output Hilbert space

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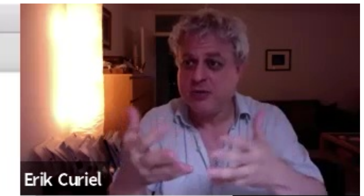


similarities, points of contact with Rovelli's relational quantum mechanics:

1. "properties of systems" refer to *interactions*, not to systems *simpliciter*
2. "all evolution is *relational*", *i.e.*, "evolution" is the *same thing* as "interaction"

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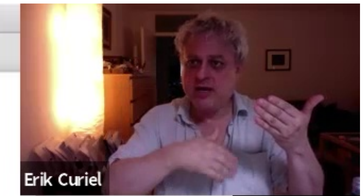


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