Title: Hierarchy of Theories with Indefinite Causal Structures: A Second Look at the Causaloid Framework

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Collection: Quantizing Time

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Abstract: "The Causaloid framework [1] is useful to study Theories with Indefinite Causality; since Quantum Gravity is expected to marry the radical aspects of General Relativity (dynamic causality) and Quantum Theory (probabilistic-ness). To operationally study physical theories one finds the minimum set of quantities required to perform any calculation through physical compression. In this framework, there are three levels of compression: 1) Tomographic Compression, 2) Compositional Compression and 3) Meta Compression.

We present a diagrammatic representation of the Causaloid framework to facilitate exposition and study Meta compression. We show that there is a hierarchy of theories with respect to Meta compression and characterise its general form. Next, we populate the hierarchy. The theory of circuits forms the simplest case, which we express diagrammatically through Duotensors, following which we construct Triotensors using hyper3wires (hyperedges connecting three operations) for the next rung in the hierarchy. Finally, we discuss the implications for the field of Indefinite Causality.

[1] Journal of Physics A: Mathematical and Theoretical, 40(12), 3081"

Hierarchy of Theories with Indefinite Causal Structures: A Second Look at the Causaloid Framework

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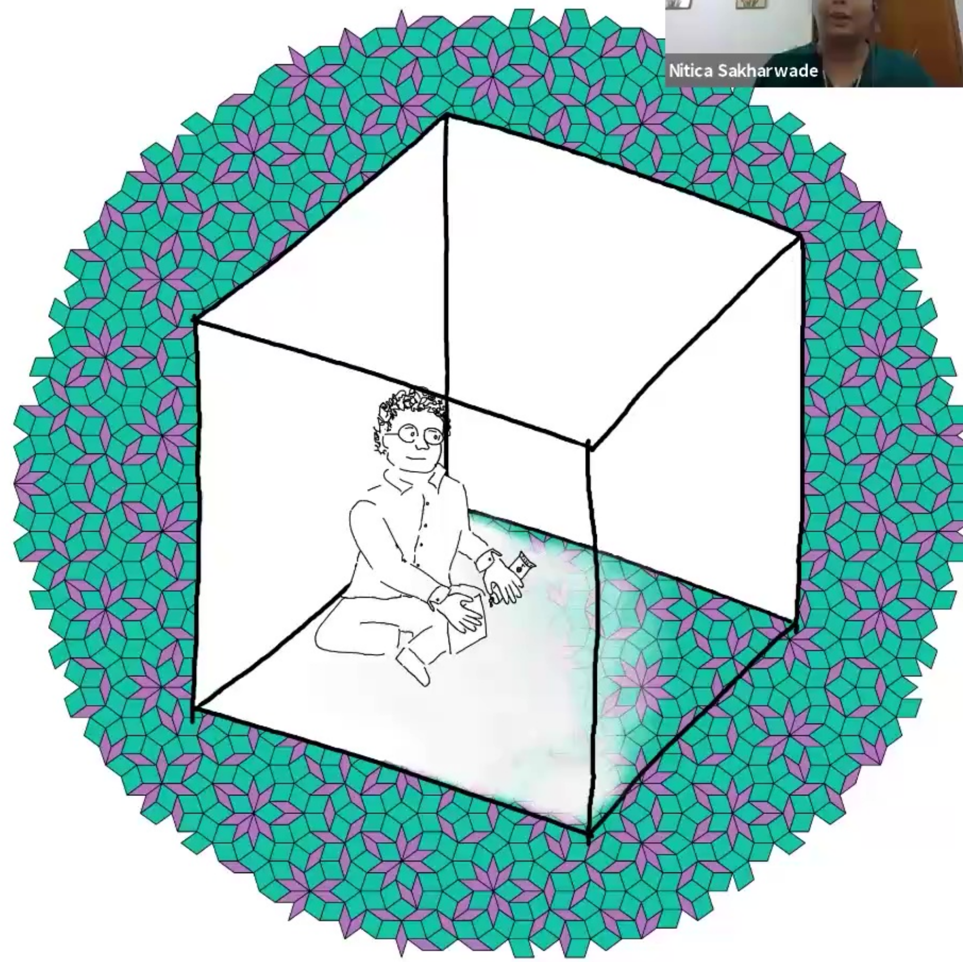
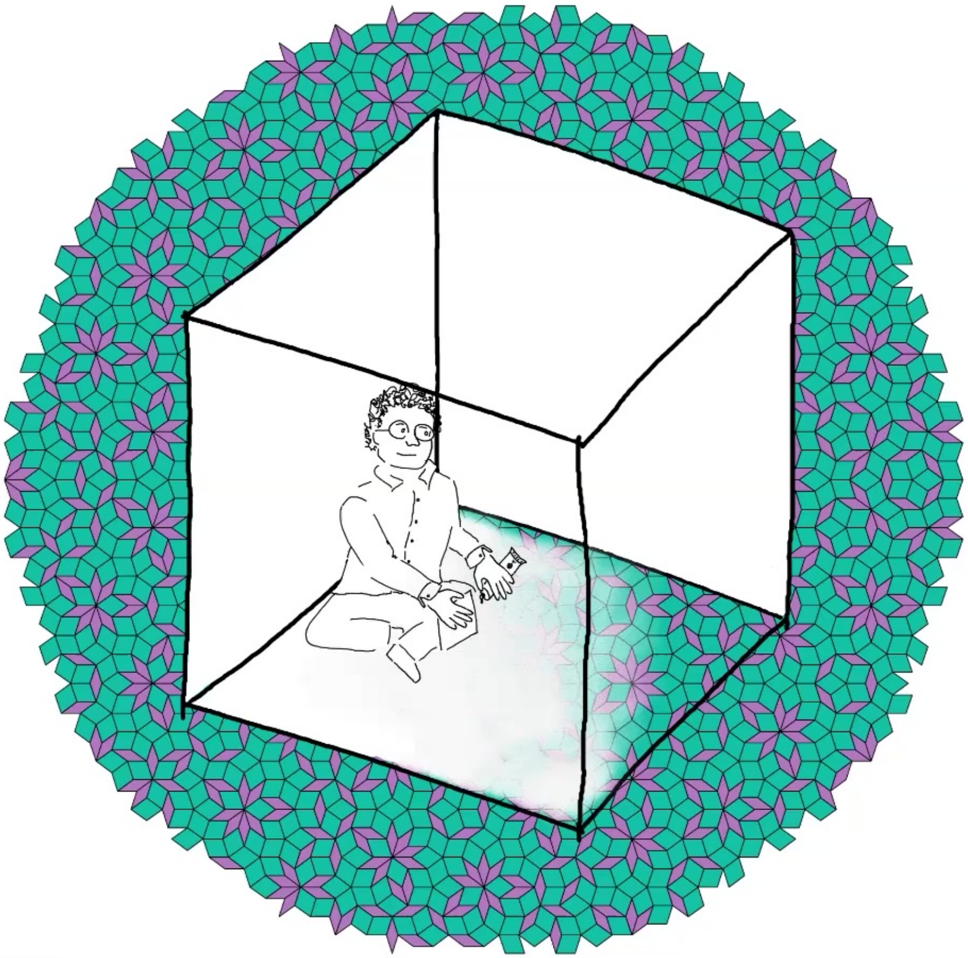
(Ongoing work with Lucien Hardy)

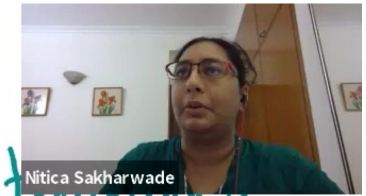
Quantizing Time (2021)



Prezi

Presenter view





Operational Methodology

Data:

- x: location
- a: actions
- o: observations

Sets:

- → R_i: Elementary Region
- → $F_{R_i} := F \cap R_i$: Procedure
- → $Y_{R_i} := Y \cap R_i$: Outcome

$$R_i \supseteq F_{R_i} \supseteq Y_{R_i}$$

"Thinking inside the box"

Pre-Compression (0^{th} level)

We want: $P(Y_{R_i}^{\alpha_{R_i}} \cup Y_{R-R_i} \mid F_{R_i}^{\alpha_{R_i}} \cup F_{R-R_i}) \equiv p_{\alpha_{R_i}} = R_{\alpha_{R_i}} \cdot P$

(Y_{R-R_i}, F_{R-R_i})

generalised prep for R_i

$(Y_{R_i}^{\alpha_{R_i}}, F_{R_i}^{\alpha_{R_i}})$

measurement for R_i

$$R = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad P = \begin{pmatrix} \vdots \\ p_{\alpha_{R_i}} \\ \vdots \end{pmatrix}$$

$\alpha_{R_i} \in \Gamma_{R_i}$

Pre-Compression (0^{th} level)

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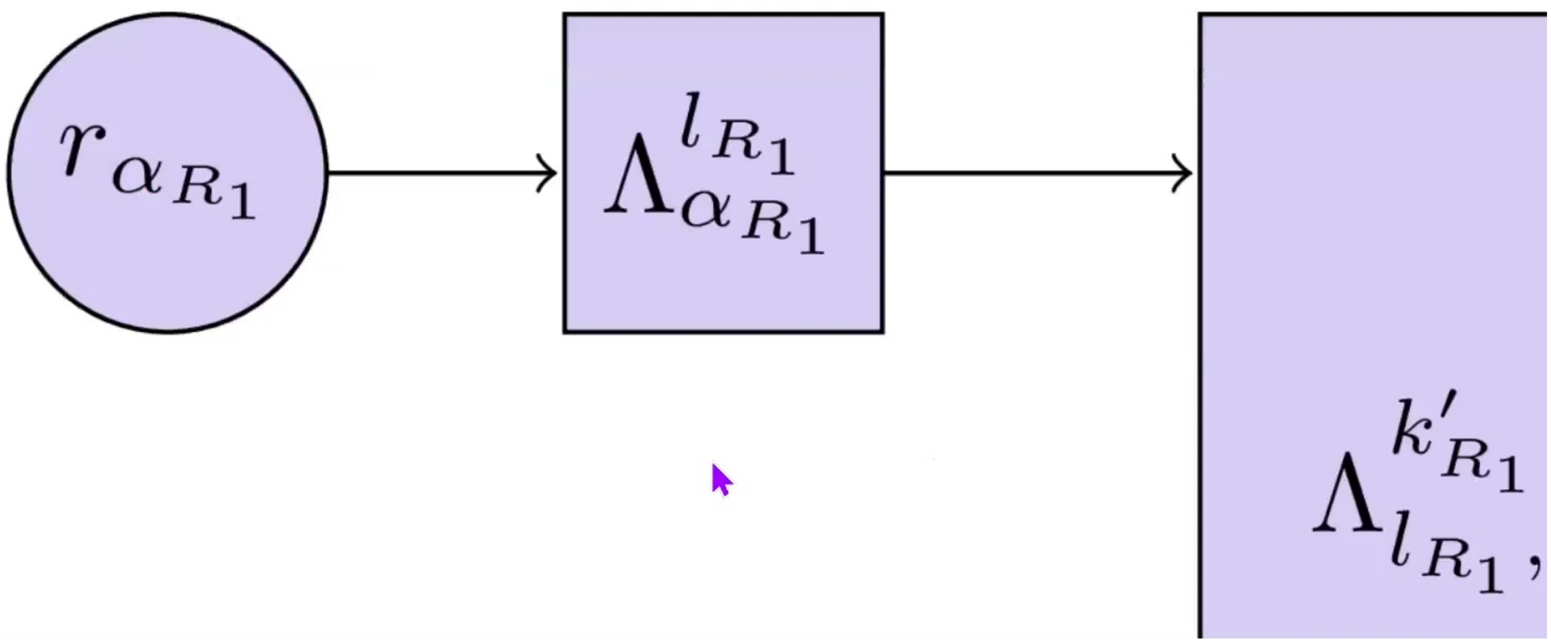
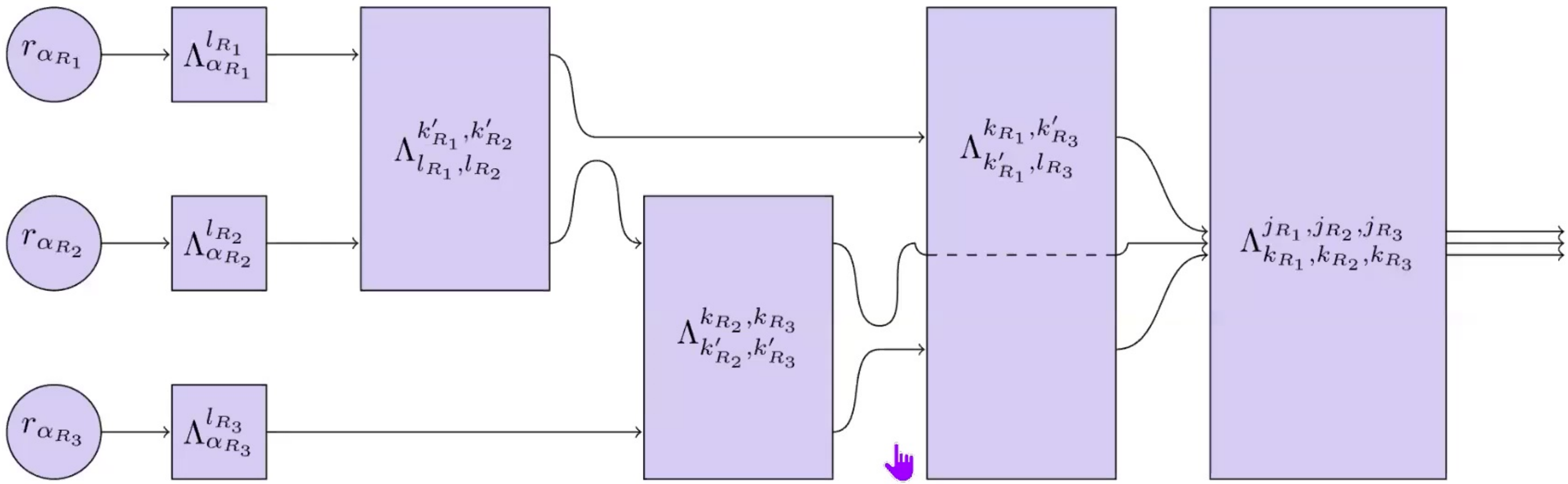
$\alpha_{R_i} \in \Gamma_{R_i}$

Consider



Consider three regions → $R = R_1 \cup R_2 \cup R_3$



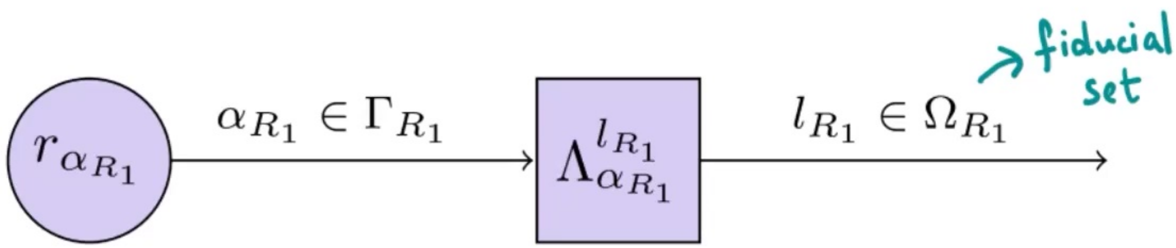
We will focus on this diagram to:

1) Explain the Causaloid framework

2) Introduce a Hierarchy for physical theories

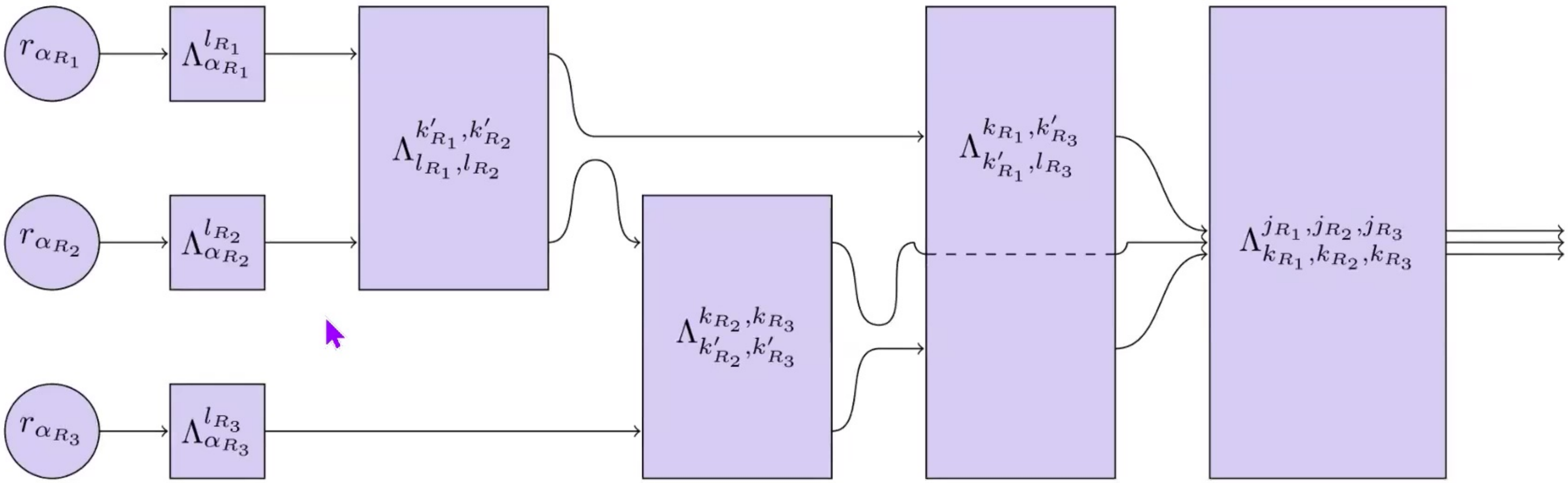
Tomographic Compression (1^{st} level)

We want: $P(Y^{\alpha_{R_i}}_{R_i} \cup Y_{R-R_i} \mid F^{\alpha_{R_i}}_{R_i} \cup F_{R-R_i}) \equiv P_{\alpha_{R_i}} = r_{\alpha_{R_i}} \cdot P$



$$\equiv \quad r_{\alpha_{R_1}} = \sum_{l_{R_1} \in \Omega_{R_1}} \lambda^{l_{R_1}}_{\alpha_{R_1}} r_{l_{R_1}}$$

Consider three regions → $R = R_1 \cup R_2 \cup R_3$



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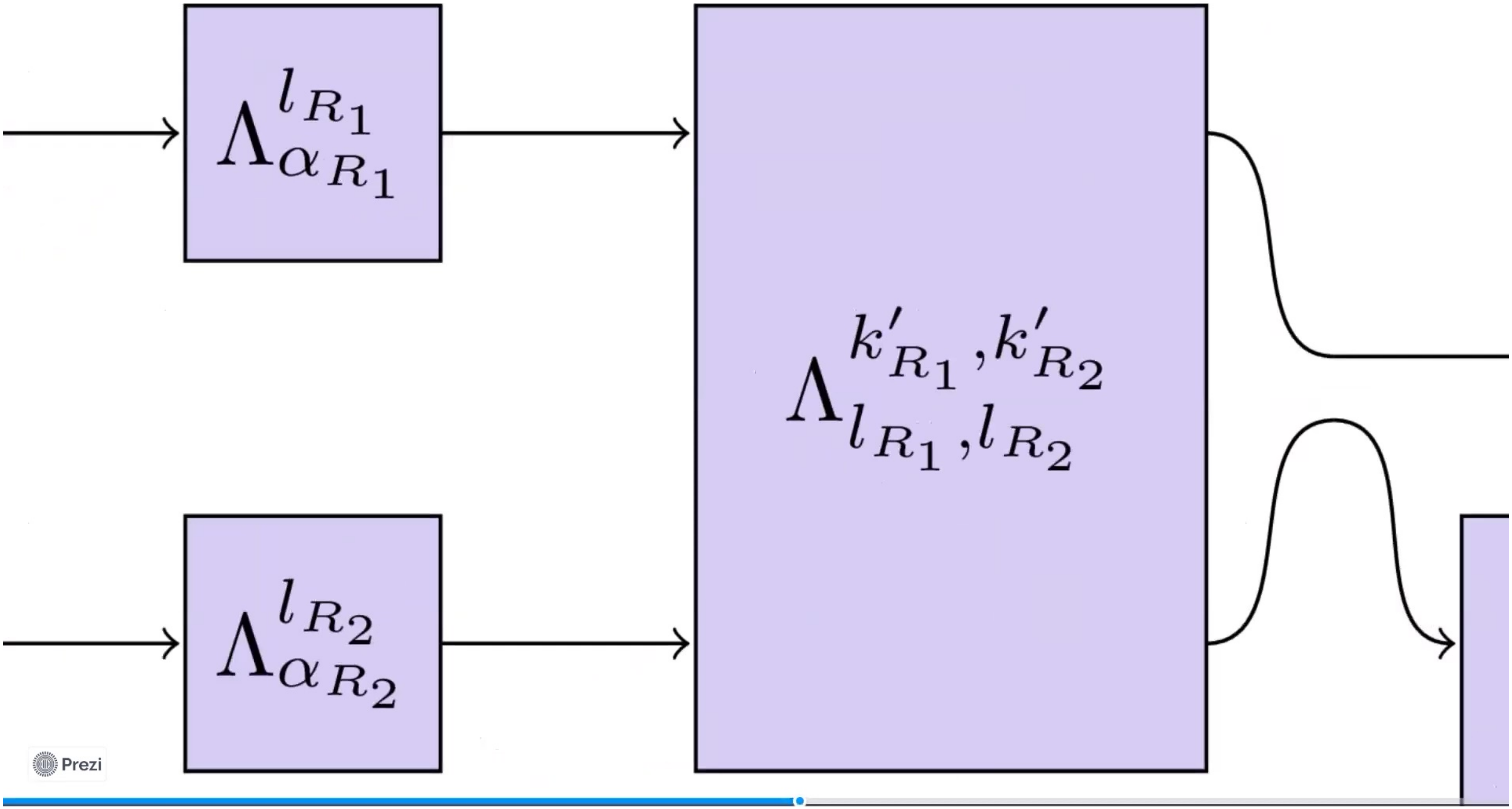
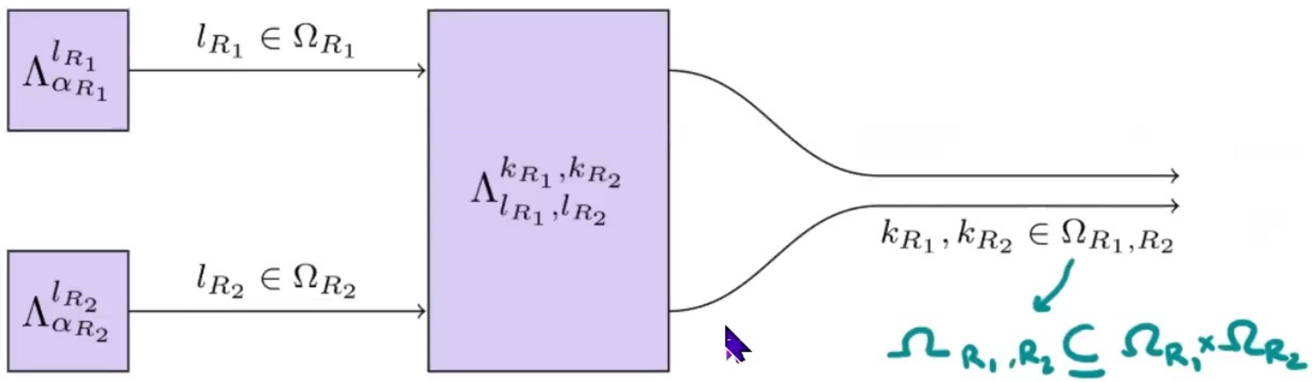


$\Lambda^{l_{R_1}}_{\alpha_{R_1}}$
$\Lambda^{l_{R_2}}_{\alpha_{R_2}}$
$\Lambda^{k'_{R_1},k'_{R_2}}_{l_{R_1},l_{R_2}}$
Prezi

Compositional Compression (2nd level)

We want:

$$P\left(Y_{R_i}^{\alpha_{R_i}} \cup Y_{R_j}^{\alpha_{R_j}} \cup Y_{R-R_i-R_j} \,\middle|\, F_{R_i}^{\alpha_{R_i}} \cup F_{R_j}^{\alpha_{R_j}} \cup F_{R-R_i-R_j}\right) \equiv P_{\alpha_{R_i}, \alpha_{R_j}} = r_{\alpha_{R_i}, \alpha_{R_j}} \cdot P$$

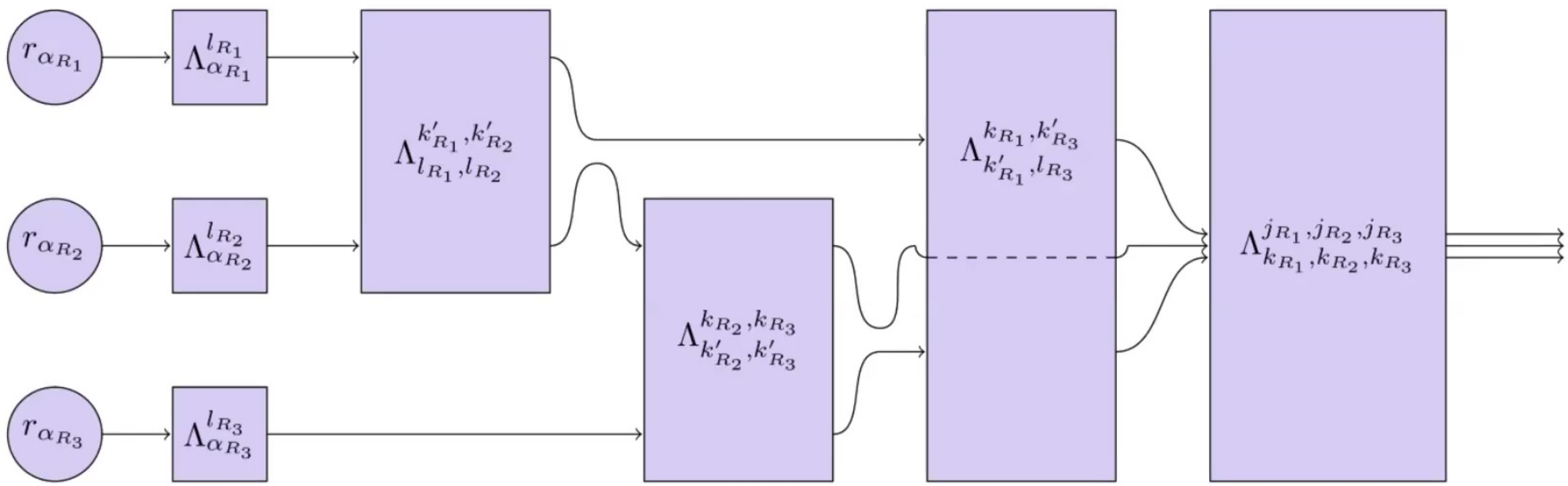


$$\equiv \; r_{l_{R_1}, l_{R_2}} = \sum_{k_{R_1}, k_{R_2} \in \Omega_{R_1,R_2}} \Lambda^{k_{R_1}, k_{R_2}}_{l_{R_1}, l_{R_2}} \, r_{k_{R_1}, k_{R_2}}$$

Meta Compression (3rd level) ...

We want :

$$\left\{ \{\Lambda^{l_{R_a}}_{\alpha_{R_a}}\}, \{\Lambda^{k_{R_a},k_{R_b}}_{l_{R_a},l_{R_b}}\}, \{\Lambda^{j_{R_a},j_{R_b},j_{R_c}}_{l_{R_a},l_{R_b},l_{R_c}}\}, \dots \right\} \to \Lambda \to \begin{array}{l} r_{\alpha_1,\alpha_2,\dots} \\ = r_{\alpha_1} \otimes^{\lambda} r_{\alpha_2} \otimes^{\lambda} \dots \end{array}$$



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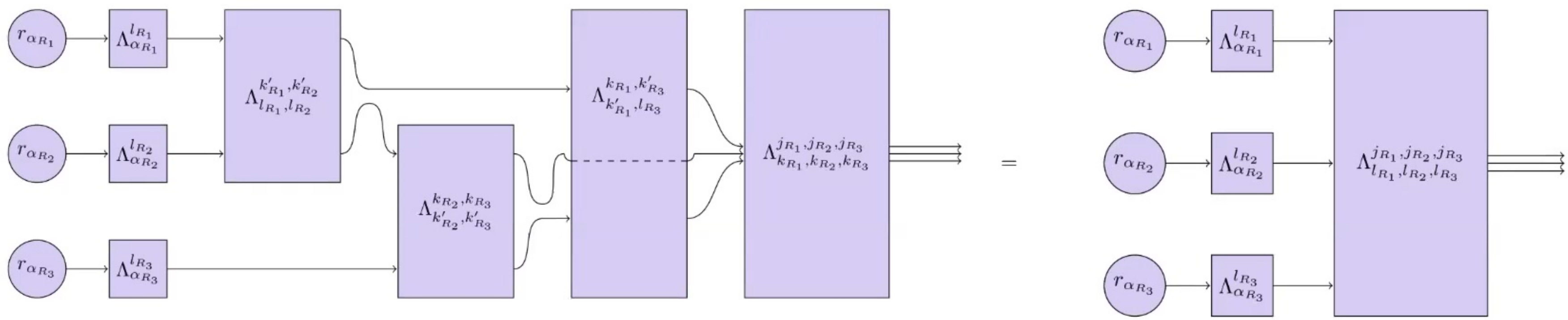
Meta Compression (3^{rd} level) ... and a Hierarchy

We want:

$$\left\{ \{\lambda^{l_{R_a}}_{\alpha_{R_a}}\}, \{\Lambda^{k_{R_a},k_{R_b}}_{l_{R_a},l_{R_b}}\}, \{\lambda^{j_{R_a},j_{R_b},j_{R_c}}_{l_{R_a},l_{R_b},l_{R_c}}\}, \dots \right\} \to \Lambda \to r_{\alpha_1,\alpha_2,\dots} = r_{\alpha_1} \otimes^{\lambda} r_{\alpha_2} \otimes^{\lambda} \dots \longrightarrow \left\{ \{\lambda^{l_{R_a}}_{\alpha_{R_a}}\}, \{\Lambda^{k_{R_a},k_{R_b}}_{l_{R_a},l_{R_b}}\} \right\} \to \Lambda$$



$$\equiv \quad r_{\alpha_{R_1},\alpha_{R_2},\alpha_{R_3}} \cdot P = \sum_{\substack{j_{R_1},j_{R_2},j_{R_3} \\ \in \Omega_{R_1,R_2,R_3}}} \sum_{\substack{k_{R_1},k_{R_3} \\ \in \Omega_{R_1,R_3}}} \sum_{\substack{k_{R_2},k'_{R_3} \\ \in \Omega_{R_2,R_3}}} \sum_{\substack{k'_{R_1},k'_{R_2} \\ \in \Omega_{R_1,R_2}}} \sum_{\substack{l_{R_1} \\ \in \Omega_{R_1}}} \sum_{\substack{l_{R_2} \\ \in \Omega_{R_2}}} \sum_{\substack{l_{R_3} \\ \in \Omega_{R_3}}} \lambda^{j_{R_1},j_{R_2},j_{R_3}}_{k_{R_1},k'_{R_2},k_{R_3}} \Lambda^{k_{R_1},k_{R_3}}_{k'_{R_1},k'_{R_3}} \Lambda^{k_{R_2},k'_{R_3}}_{k'_{R_2},l_{R_3}} \Lambda^{k'_{R_1},k'_{R_2}}_{l_{R_1},l_{R_2}} \Lambda^{l_{R_1}}_{\alpha_{R_1}} \Lambda^{l_{R_2}}_{\alpha_{R_2}} \Lambda^{l_{R_3}}_{\alpha_{R_3}} = \sum_{\substack{j_{R_1},j_{R_2},j_{R_3} \\ \in \Omega_{R_1,R_2,R_3}}} \sum_{\substack{l_{R_1} \\ \in \Omega_{R_1}}} \sum_{\substack{l_{R_2} \\ \in \Omega_{R_2}}} \sum_{\substack{l_{R_3} \\ \in \Omega_{R_3}}} \lambda^{j_{R_1},j_{R_2},j_{R_3}}_{l_{R_1},l_{R_2},l_{R_3}} \Lambda^{l_{R_1}}_{\alpha_{R_1}} \Lambda^{l_{R_2}}_{\alpha_{R_2}} \Lambda^{l_{R_3}}_{\alpha_{R_3}}$$

when $R_1 \to R_2 \to R_3$

$$\sum_{\substack{j_{R_1},j_{R_2},j_{R_3} \\ \in \Omega_{R_1,R_2,R_3}}} \lambda^{j_{R_1},j_{R_2},j_{R_3}}_{l_{R_1},l_{R_2},l_{R_3}} = \sum_{\substack{k_{R_2},k_{R_3} \\ \in \Omega_{R_2,R_3}}} \sum_{\substack{k_{R_1},k'_{R_2} \\ \in \Omega_{R_1,R_2}}} \Lambda^{k_{R_2},k_{R_3}}_{k'_{R_2},l_{R_3}} \Lambda^{k_{R_1},k'_{R_2}}_{l_{R_1},l_{R_2}}$$

such that $\Omega_{R_1,R_2,R_3} = (\Omega_{R_1,R_2} \times \Omega_{R_3}) \cap (\Omega_{R_1} \times \Omega_{R_2,R_3})$

$$\sum_{\substack{j_{R_1},j_{R_2},j_{R_3} \\ \in \Omega_{R_1,R_2,R_3}}} \lambda^{j_{R_1},j_{R_2},j_{R_3}}_{l_{R_1},l_{R_2},l_{R_3}} = \sum_{\substack{k_{R_2},k_{R_3} \\ \in \Omega_{R_2,R_3}}} \sum_{\substack{k_{R_1},k'_{R_2} \\ \in \Omega_{R_1,R_2}}} \lambda^{k_{R_2},k_{R_3}}_{k'_{R_2},l_{R_3}} \lambda^{k_{R_1},k'_{R_2}}_{l_{R_1},l_{R_2}}$$

Such that $\Omega_{R_1,R_2,R_3} = \left(\Omega_{R_1,R_2} \times \Omega_{R_3}\right) \cap \left(\Omega_{R_1} \times \Omega_{R_2,R_3}\right)$

Prezi

Hierarchy in $d<n$

$$\Omega_{R_1,R_2,\ldots,R_n} = \bigcap_{i\neq j\neq\ldots\neq d^{th}\text{index}}^{{}^nC_d} O\left(F_n\left(\Omega_{R_i,R_j,\ldots,R_{d^{th}\text{index}}}\right)\right)$$

O: ordering

F_n: Filling

$d=1$ No causal structure

$d=2$ Duotensors (Including Q Theory, C Prob Theory)

$d=3$ Triotensors (no Quantum analogue)

$d=4$ Quadrotensors (Quantum generalisation linked to Quarternions)

Hierarchy in $d<n$

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Thank you!

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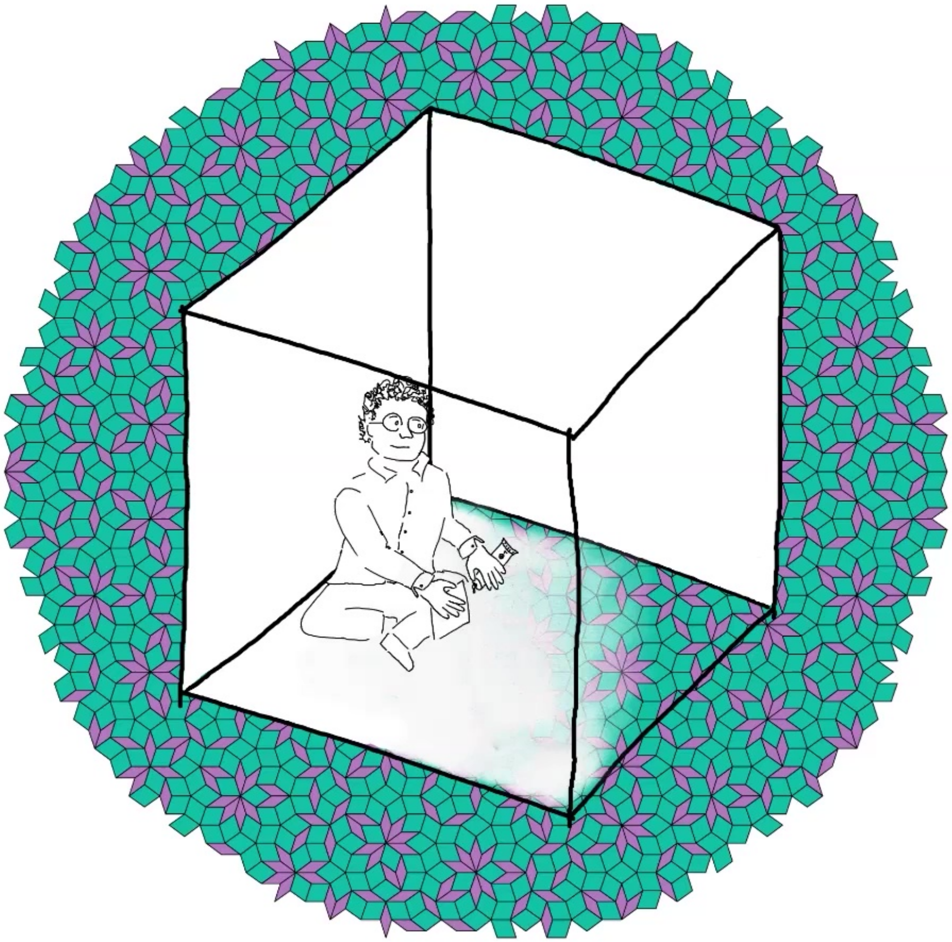

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Prezi

Presenter view