Title: Inequivalent clocks in quantum cosmology

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Abstract: Quantum cosmology faces the problem of time: the Universe has no background time, only interacting dynamical degrees of freedom within it. The relational view is to use one degree of freedom (which can be matter or geometry) as a clock for the others. In this talk we discuss a cosmological model of a flat FLRW universe filled with a massless scalar field and a perfect fluid. We study three quantum theories based on three different choices of (relational) clock and show that, if we require the dynamics to be unitary, all three make drastically different predictions regarding resolution of the classical (Big Bang) singularity or a possible quantum recollapse at large volume. The talk is based on [arXiv:2005.05357] and a second paper to appear on arXiv in May 2021. We plan to give two talks: one covering the foundations and general properties of the model, and one showing detailed results and physical interpretation. (We will merge these talks into one if the organisers decide to accept only one talk.)

# Inequivalent clocks in quantum cosmology

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joint work with Lucía Menéndez-Pidal
`arXiv:2005.05357` (Class. Quant. Grav. **37** (2020) 205018)
+ second paper on arXiv very soon

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## Relational clocks in classical GR

"What is observable in classical and quantum gravity?" [Rovelli 1991]

Due to diffeomorphism symmetry, there is no meaningful way to identify spacetime points by coordinates: the Ricci scalar $R(x_0)$ at a point identified by coordinates $x_0$ is **not** an observable quantity.

Similarly, in cosmology cannot ask "what was the spatial curvature of the Universe at $t = 0$?"

The (ADM) Hamiltonian in GR generates gauge transformations $\Rightarrow$ observable (gauge-invariant) quantities must be constants of motion (e.g., [Unruh/Wald 1989])

Way out: **material reference systems** which label spacetime points not by arbitrary coordinates but by the values taken by reference matter fields:
"Matter energy density when $\varphi = \varphi_0$" *is* observable (and a constant of motion!)

## The model

*Geometry:* homogeneous, isotropic, spatially flat universe with metric

$$\mathrm{d}s^2 = -N(\tau)^2\mathrm{d}\tau^2 + a(\tau)^2 h_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

where $h$ is a flat metric, $a(\tau)$ is the scale factor and $N(\tau)$ is the lapse function.

*Matter:* a free massless scalar $\phi(\tau)$ and perfect fluid with energy density $\rho(\tau)$ and equation of state parameter $w < 1$ (e.g., $w = 0$ for dust, $w = -1$ for dark energy) so that

$$m \equiv \rho(\tau)a(\tau)^{3(w+1)} = \text{const.}$$

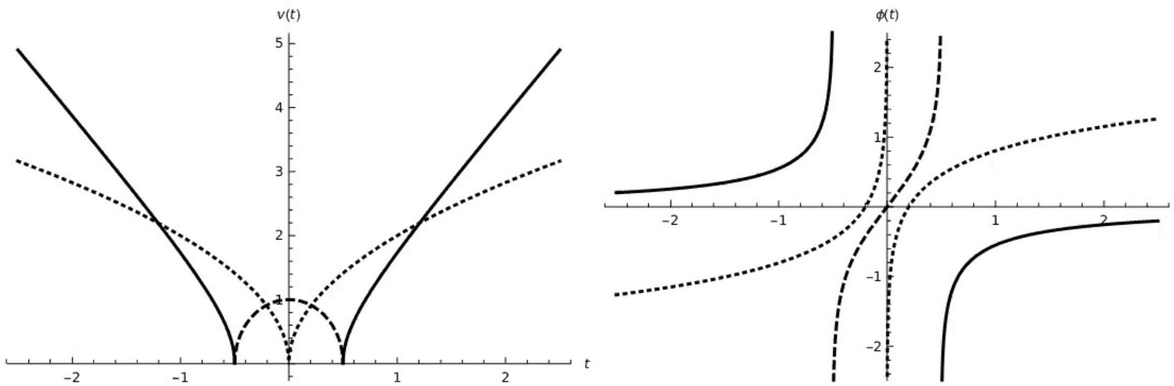
After suitable variable transformations, find Hamiltonian and canonical pairs

$$\mathcal{H} = \tilde{N}\left[-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda\right], \quad \{v, \pi_v\} = \{\varphi, \pi_\varphi\} = \{t, \lambda\} = 1$$

where $v \propto a^{\frac{3(1-w)}{2}}$, $\varphi \propto \phi$, $\lambda \propto m$, $\tilde{N} = Na^{-3w}$. $\tilde{N} = 1$ leads to simplest dynamics, with $\mathrm{d}t/\mathrm{d}\tau = 1$, so that $t$ is identified with "time".

## Solutions in $t$ time

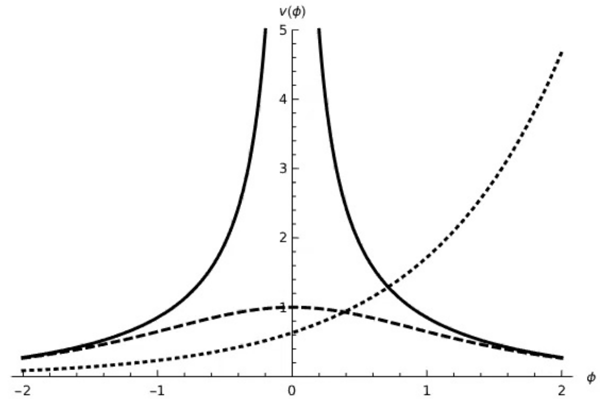
Classically, the variables $t$ and $\varphi$ evolve monotonically (if we exclude $\pi_\varphi = 0$) so are always good relational clocks. For $v$ this is true if $\lambda \not< 0$; for $\lambda < 0$ there is a turning point (recollapse of the Universe).

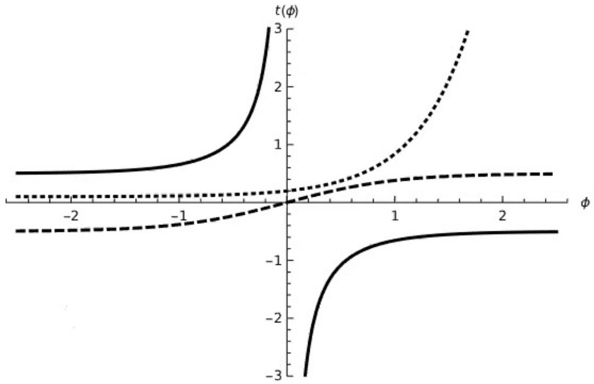



Classical solutions $v(t)$ and $\varphi(t)$ as functions of the clock $t$, with $\pi_\varphi = 1$ and $\lambda = 1$ (solid), $\lambda = -1$ (dashed) and $\lambda = 0$ (dotted).

**All** solutions have a (Big Bang/Big Crunch) singularity with $v \to 0$ and $\varphi \to \infty$.

## Solutions in $\varphi$ time




Parameters: $\pi_\varphi = 1$, $\lambda = 1$ (solid), $\lambda = -1$ (dashed) and $\lambda = 0$ (dotted).

When $\varphi$ is used as a clock, the Big Bang/Big Crunch singularity is pushed to $\varphi \to \pm\infty$. For $\lambda > 0$ there is a finite value of $\varphi$ where $v$ and $t$ diverge.

**The explicit form of cosmological solutions highly depends on the clock.**

## The role of unitarity

Classical solutions, when expressed in terms of one of the "natural" clock variables, can terminate at a finite time as measured by the clock.

In $t$ time this reflects the Big Bang/Big Crunch singularity of classical GR.

In $\varphi$ time and with $\lambda > 0$ it reflects the fact that $\varphi \to \varphi_0$ as the Universe expands and $\varphi$ becomes an "infinitely slow" clock asymptotically.

Classically, clocks are not defined beyond the point where the solution terminates. But what happens quantum mechanically? If we require quantum theory to be **unitary** any state must have a globally well-defined time evolutuion.
$\Rightarrow$ Evolution must extend beyond points where classical solution terminates!

*Conjecture* [Gotay/Demaret 1983]: *unitary slow-time quantum dynamics is always nonsingular, while unitary fast-time quantum dynamics inevitably leads to collapse.* We extend this conjecture to clocks reaching infinity in finite "time".

## The role of unitarity

Details of our quantum theory depend on additional choices:

- Operator ordering in the quantum version of the Hamiltonian;
- Inner product used to construct the physical Hilbert space.

We perform a reduced quantisation in which the clock is chosen before quantisation and other observables, classically *relational Dirac observables* for this clock, are represented as “time”-dependent expectation values.
For $t$ and $\varphi$ as clocks, which admit a splitting of system into *clock* + *rest*, seems equivalent to Dirac quantisation. [wip]

We find that **when unitarity is imposed** quantum solutions continue when classical solutions terminate. If $t$ is the clock, the singularity is resolved; if $\varphi$ is the clock, the Universe must recollapse at finite volume. If $v$ is the clock, unitarity is “trivial” and quantum states can follow classical solutions exactly.

## Relation to previous work

- The model was analysed by Gryb and Thébault in a series of papers (2018/19) using $t$ as clock; generic resolution of the singularity was found in the sense that $\langle v(t) \rangle \geq C_\psi > 0$ where $C_\psi$ is some state-dependent constant. We confirm and extend these results.
- Bojowald and Halnon (2018) studied the model using deparametrisation and an effective (semiclassical) approach, finding inequivalent results for different clocks since different factor orderings are needed.
- GR with a massless scalar field and fixed cosmological constant is similar to our model (for us, since $\Lambda$ is a conserved momentum, superpositions in $\Lambda$ are possible). This model was quantised by Pawłowski and Ashtekar (2012) using $\varphi$ as a clock. The authors found recollapse of the Universe at large volume, but no singularity resolution, consistent with our general framework.

# Thank you!

# Quantisation

- The Hamiltonian constraint becomes the Wheeler–DeWitt (WDW) equation

$$\left[\hbar^2 \frac{\partial^2}{\partial \varphi^2} - \hbar^2 \left(\frac{\partial}{\partial \log(v/v_0)}\right)^2 + i\hbar v^2 \frac{\partial}{\partial t}\right] \Psi(v, \varphi, t) = 0.$$

- The solutions of the WDW equation are Bessel functions of real or purely imaginary order.
- There is no notion of time evolution from the WDW equation $\Longrightarrow$ We are free to choose a dynamical variable to serve as clock. This is an example of the problem of time.
- Solution: choose a clock, build a Hilbert space on the remaining variables, analyse and repeat.

# The $v$-clock theory

The properties of the $v$-clock theory are:

- Inner product: $\langle \Psi | \Phi \rangle_{KG} = \mathrm{i} \int \mathrm{d}t \mathrm{d}\varphi \left[ \bar{\Psi} v \partial_v \Phi - \Phi v \partial_v \bar{\Psi} \right].$
- Unitarity: $\frac{\partial}{\partial v} \langle \Psi | \Phi \rangle = 0$ is trivial.

There is no boundary condition.

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# The $t$-clock theory

The properties of the $t$-clock theory are:

- Inner product $\langle \Psi | \Phi \rangle_{SCH} = \int \mathrm{d}\varphi \mathrm{d}v \; v \bar{\Psi} \Phi$.
- Unitarity: $\frac{\partial}{\partial t} \langle \Psi | \Phi \rangle_{SCH} = 0 \implies$ Boundary condition

$$\left[ v \bar{\Psi} \frac{\partial}{\partial v} \Phi - v \Phi \frac{\partial}{\partial v} \bar{\Psi} \right]_{v \to 0} = 0.$$

The boundary condition is at the classical singularity!

- The solutions are reflected from $v = 0$.

# The $\varphi$-clock theory

The properties of the $\varphi$-theory are:

- Inner product $\langle \Psi | \Phi \rangle_\varphi = i \int \mathrm{d}t \frac{\mathrm{d}v}{v} \left[ \bar{\Psi} \frac{\partial}{\partial \varphi} \Phi - \Phi \frac{\partial}{\partial \varphi} \bar{\Psi} \right]$.
- Unitarity: $\frac{\partial}{\partial \varphi} \langle \Psi | \Phi \rangle_\varphi = 0 \implies$ Boundary condition

$$\left[ v \bar{\Psi} \frac{\partial}{\partial v} \Phi - v \Phi \frac{\partial}{\partial v} \bar{\Psi} \right]_{v \to \infty} = 0.$$

The boundary condition is NOT at the classical singularity!

- Again, the wave functions are reflected but from $v = \infty$.

What happens to the singularity?

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# Results on singularity resolution

Method: build semiclassical states and compare expectations values versus classical solutions.

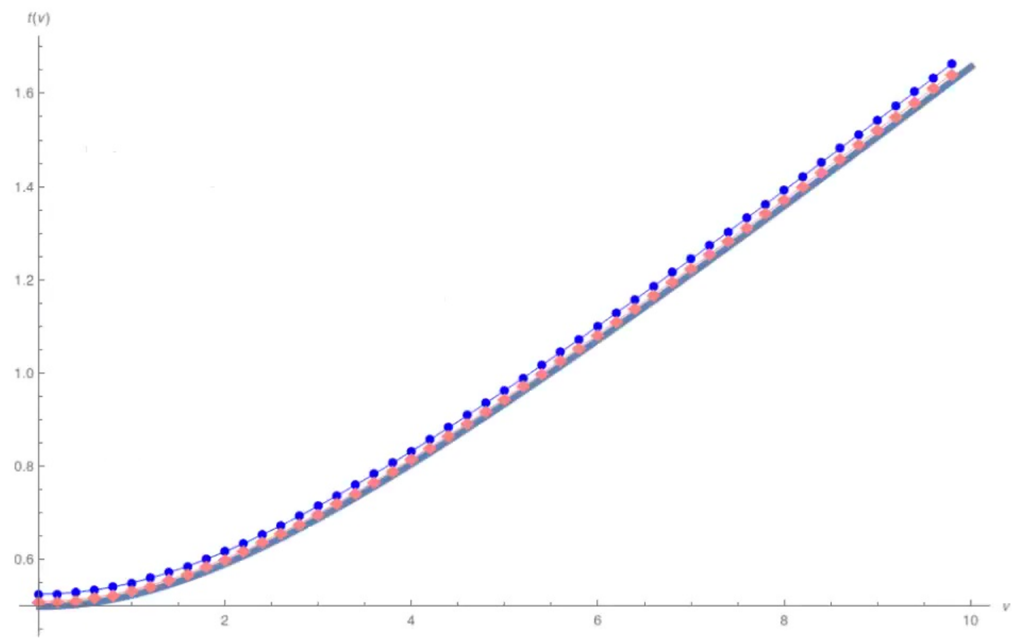


Figure: $\langle t(v) \rangle$ (dots) vs $t(v)$ (solid) ($\lambda = 10$)

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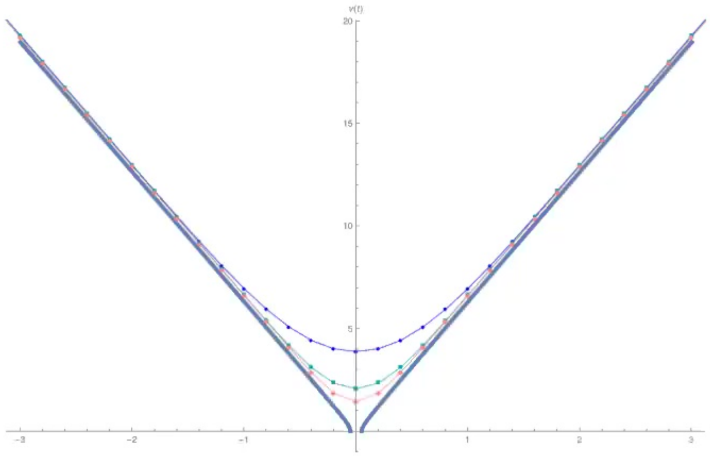
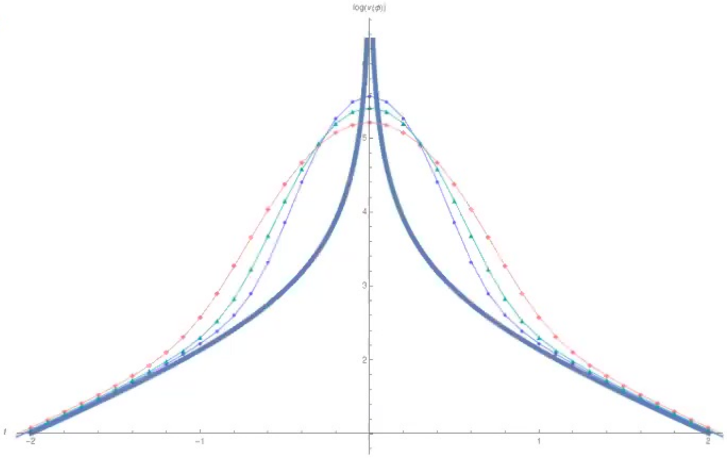


(a) $\langle v(t)\rangle$ (dots) vs $v(t)$ (solid) ($\lambda = 2$)



(b) $\log[\langle v(\varphi)\rangle]$ vs $\log[v(\varphi)]$ ($\lambda = 1$)

We see a smooth transition between the expanding and contracting phase of the universe. $\langle v(t)\rangle \geq C > 0$, and $\langle v(\varphi)\rangle \to 0$, only the Schrödinger theory resolves the singularity.

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# Summary and conclusion

| | $t$-clock | $\varphi$-clock | $v$-clock |
|---|---|---|---|
| Boundary condition | at $v = 0$ | at $v = \infty$ | No |
| Singularity resolution | Yes | No | No |
| Maximum volume | No | Yes | No |

Table: Differences between the 3 theories

In conclusion we found that:

- General covariance is lost.
- The choice of clock leads to boundary conditions that will influence the behaviour of the solutions.
- It is possible to guess the behaviour of the clocks: slow clocks resolve the singularity and fast clocks do not.

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