Title: "TIME IN QUANTUM GRAVITY - From the fundamental level to the classical limit"

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Collection: Quantizing Time

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Abstract: Time cannot be both absolute (as in quantum mechanics) and dynamical (as in general relativity). I present general arguments for the absence of time at the most fundamental level of quantum gravity. I discuss possible concepts that could replace it and present the recovery of standard time as an approximate concept. My discussion is restricted to quantum geometrodynamics, but I argue for the validity of my conclusions beyond that scheme.

TIME IN QUANTUM GRAVITY

From the fundamental level to the classical limit

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Quantizing time?

To quantize is a verb – suggests to turn classical time t into an operator $\hat{t}$; but does this make sense?

Wolfgang Pauli (1933):

We thus conclude that one must completely go without the introduction of an operator t and that the time t in wave mechanics must necessarily be considered as an ordinary number ('c-number').

Unruh and Wald (1989):

...in ordinary Schrödinger quantum mechanics for a system with a Hamiltonian bounded from below, no dynamical variable can correlate monotonically with the Schrödinger time t, and thus the role of t in the interpretation of Schrödinger quantum mechanics cannot be replaced by that of a dynamical variable.

Contents

Time in quantum mechanics

- Time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi;$$

- can be obtained from the classical constraint

$$p_t + H \approx 0.$$

What is more fundamental: the time-dependent or the time-independent Schrödinger equation?

Recovery of the time-dependent Schrödinger equation

Neville Mott (1931):

...we shall confine ourselves ... to the somewhat idealised case of the collision between an α particle and an atom consisting of a single electron bound in the field of an infinitely heavy nucleus. We have to show that the same probabilities of excitation result whether we consider the problem as a one-body problem ... or as a two-body problem ...

Making the ansatz

$$\Psi(\mathbf{r}, \mathbf{R}) = f(\mathbf{r}, \mathbf{R})\mathrm{e}^{\mathrm{i}kZ},$$

where $\mathbf{r}$ and $\mathbf{R} \equiv (X, Y, Z)$ are the positions of the electron and α-particle, respectively, Mott derives the time-dependent Schrödinger equation (with t related to Z) from the (more fundamental) time-independent Schrödinger equation $H\Psi = 0$; he explicitly refers to the Born–Oppenheimer approximation (1927).

Approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String (M-) theory
- ...

See e.g. C.K. *Quantum Gravity*, 3rd ed. (Oxford 2012)

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[1]

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[1] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt.* Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die 'Hindernisse' oder 'Öffnungen' nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine 'undulatorische Mechanik' zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Quantum geometrodynamics

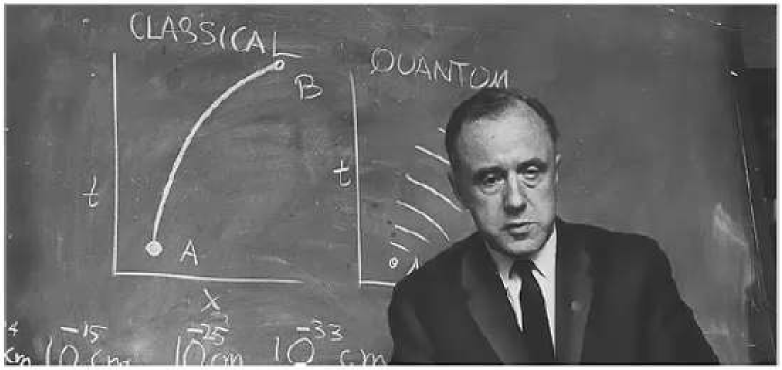


Figure: John Archibald Wheeler at Princeton University 1967

Application of Schrödinger's procedure to general relativity leads to

$$\hat{\mathcal{H}}_\perp \Psi \equiv \left(-16\pi G \hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (16\pi G)^{-1} \sqrt{h} \left({}^{(3)}R - 2\Lambda \right) + \sqrt{h} \hat{\rho} \right) \Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}^a \Psi \equiv -2\nabla_b \frac{\hbar}{\mathrm{i}} \frac{\delta \Psi}{\delta h_{ab}} + \sqrt{h} \hat{j}^a \Psi = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- Classical version: Absence of non-dynamical spacetime ("background independence")

  As Albert Einstein wrote: "Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann."

  ("It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.")

- Quantum version: Absence of spacetime (corresponds to absence of classical trajectory in quantum mechanics); only three-geometry remains.
- The (locally) hyperbolic nature of the Wheeler–DeWitt equation may allow the introduction of an intrinsic time ("ephemeris time").
- Open question: What happens to the probability interpretation? Possible answer in the context of relational observables and conditional probabilities.

  (E.g. L. Chataignier, *Phys. Rev. D* **103**, 026013 (2021), and his talk on Friday, June 18.)

Quantum Cosmology

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

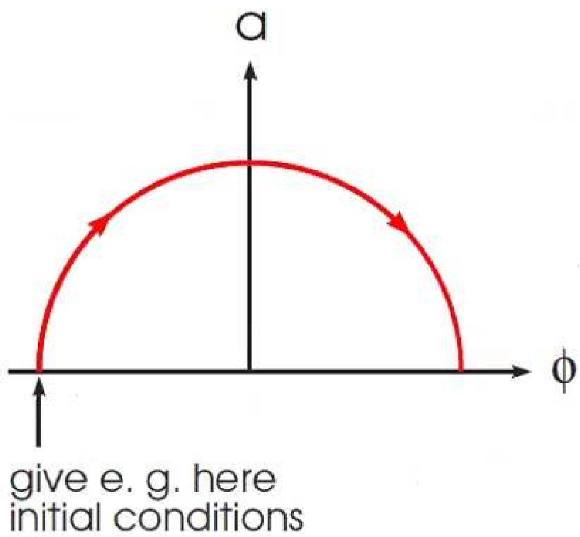
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

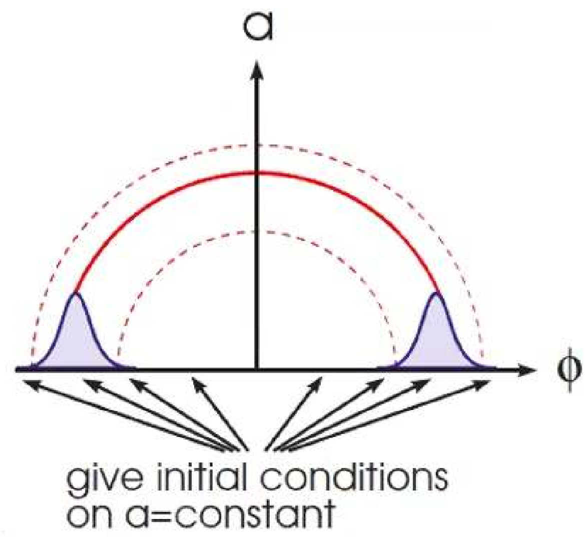
Factor ordering chosen in order to achieve covariance in minisuperspace

Determinism in classical and quantum theory

Classical theory



Quantum theory



Recollapsing part is deterministic successor of expanding part

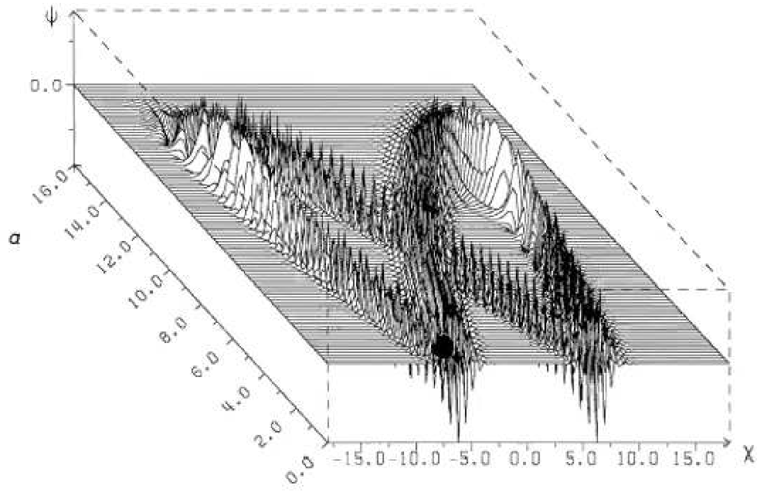
'Recollapsing' wave packet must be present 'initially'

No intrinsic difference between 'big bang' and 'big crunch'!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K. (1990)

Emergence of (semiclassical) time

Quantum field theory in curved spacetime can be obtained from canonical quantum general relativity by a Born-Oppenheimer type of expansion with respect to the Planck-mass squared, $m_{\rm P}^2 = \hbar/G$.

$$\Psi[h_{ab}, \phi] \equiv \exp\left(\frac{\mathrm{i}}{\hbar} S[h_{ab}, \phi]\right)$$

Expansion of S:

$$S[h_{ab}, \phi] = m_{\rm P}^2 S_0 + S_1 + m_{\rm P}^{-2} S_2 + \ldots$$

Insert this into the Wheeler-DeWitt equation and compare different orders of $m_{\rm P}^2$.

(*Lapchinsky and Rubakov* 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, ...)

- m_{P}^4: S_0 is independent of ϕ
- m_{P}^2: Hamilton-Jacobi equation for S_0
- m_{P}^0: Equation for S_1 that can be simplified by introducing

$$f \equiv D[h_{ij}] \exp\left(\frac{\mathrm{i}}{\hbar} S_1\right)$$

  and demanding the "WKB prefactor equation" for D.
- Introduce a local "bubble" (Tomonaga-Schwinger) time functional by

$$\frac{\delta}{\delta\tau(\mathbf{x})} := G_{abcd} \frac{\delta S_0}{\delta h_{ab}} \frac{\delta}{\delta h_{cd}}$$

Note the analogy with Mott's work from 1931!

$$i\hbar \frac{\delta f}{\delta \tau} = \hat{\mathcal{H}}_{\perp}^{\mathrm{m}} f$$

- τ is not a scalar function.
- This equation can be integrated to yield a (functional) Schrödinger equation.
- It describes the limit of quantum field theory in curved spacetime.
- Next order (m_{P}^{-2}): quantum-gravitational corrections (modify the power spectrum of the CMB anisotropies)

Quantum-gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

(*C.K. and Singh* (1991); *Barvinsky and C.K.* (1998))

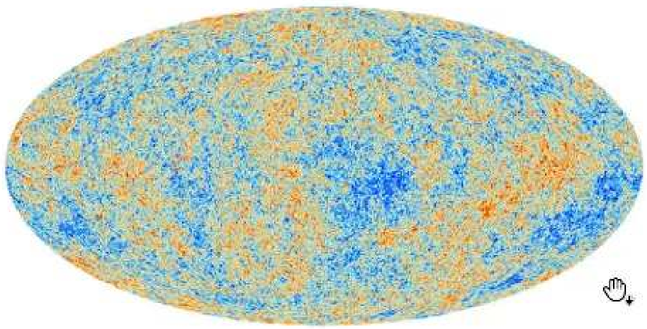


Figure credit: ESA/PLANCK Collaboration

C.K. and M. Krämer, *Phys. Rev. Lett.*, **108**, 021301 (2012); D. Bini, G. Esposito, C.K., M. Krämer, and F. Pessina, *Phys. Rev. D*, **87**, 104008 (2013); A. Y. Kamenshchik, A. Tronconi, and G. Venturi, *Phys. Lett. B* **734**, 72 (2014); D. Brizuela, C.K., M. Krämer, *Phys. Rev. D* **93**, 104035 (2016); *ibid.* **94**, 123527 (2016); D. Brizuela, C.K., M. Krämer, S. Robles-Pérez, *ibid.* **99**, 104007 (2019); L. Chataignier and M. Krämer, *Phys. Rev. D*, **103**, 066005 (2021); …

What happens to quantum superpositions?

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe? Answer: by decoherence

Decoherence in quantum cosmology

- 'System': global degrees of freedom (scale factor, inflaton field, ...)
- 'Environment': small density fluctuations, gravitational waves, ...

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) ('system') experiences decoherence by gravitons ('environment') according to

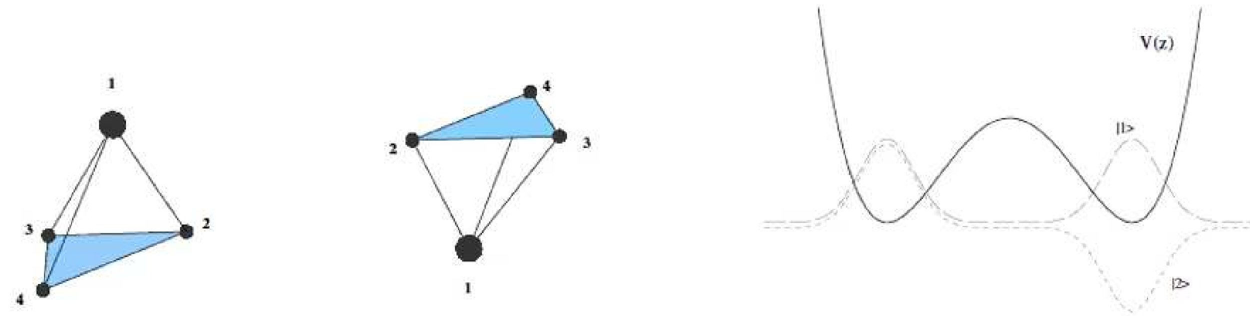
$$\rho_0(a,a') \to \rho_0(a,a') \exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right) \ , \ C > 0$$

The Universe assumes classical properties at the beginning of inflation

(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality



dynamical origin: decoherence through scattering by light or air molecules

Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

Example for decoherence factor: $\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Arrow of time from timeless quantum cosmology

Fundamental asymmetry with respect to "intrinsic time":

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha \to -\infty} \right] \right) \Psi = 0$$

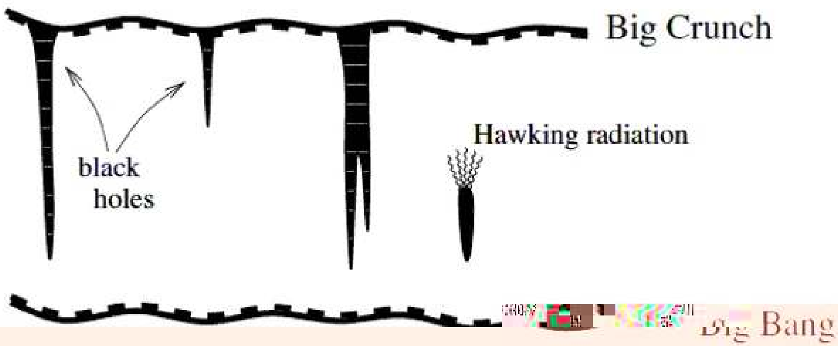
Is compatible with simple boundary condition:

$$\Psi \overset{\alpha \to -\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

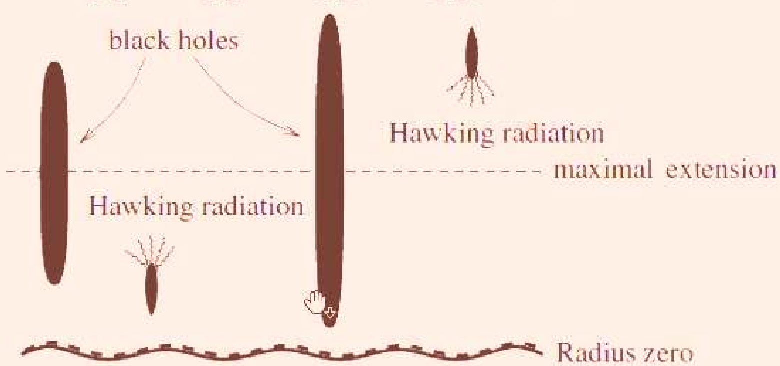
Entropy increases with increasing α, since entanglement with other degrees of freedom increases;
this defines in the semiclassical limit the direction of semiclassical time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

Two questions

- Does the probability interpretation of quantum theory make sense at the most fundamental level of quantum gravity?
- Is there a sensible notion of time beyond the semiclassical level?

- m_{P}^4: S_0 is independent of ϕ
- m_{P}^2: Hamilton-Jacobi equation for S_0
- m_{P}^0: Equation for S_1 that can be simplified by introducing

$$f \equiv D[h_{ij}] \exp\left(\frac{\mathrm{i}}{\hbar} S_1\right)$$

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