Title: The Issue of Time in Generally Covariant Theories

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Abstract: A possible solution of the problem of time in quantum gravitational systems is presented based on a relational description between the parameterized Dirac observables of the system under consideration and the clocks. The use of physical clocks required by a quantum gravitational description of time is shown to induce a loss of unitarity. The evolution is described by a Lindblad-type master equation unless it is possible to construct a perfect clock. I show that fundamental uncertainties in time measurements could arise due to quantum and gravitational effects, leading to the conclusion that there is always a loss of unitarity induced by the use of physical clocks. The extension of the analysis to physical reference frames in totally constrained systems is sketched.

The issue of time and reference frames in Quantum Generally Covariant Theories

Rodolfo Gambini

In collaboration with Jorge Pullin

INTRODUCTION

There is by now extensive literature addressing the problem of time in Classical and Quantum Gravity.

The heart of the problem lies in the fact that Einstein gravity is a fully constrained system whose *Hamiltonian* vanishes.
O*bservable* quantities are those that commute with the constraints, that is they are Dirac Observables, and therefore, they do not *evolve* and cannot represent time.

I will discuss here three approaches to this problem.

They have in common their relational character. In fact, one of the basic ingredients in the different proposals to describe evolution is the use of *relations* between different degrees of freedom in the theory .

The first and older approach is gauge fixing

The second approach is based on the concept of *relational or evolving Dirac observables.*

The third one is the conditional probabilities approach proposed by Page and Wootters.

Hoehn, Smith, and Lock e-Print: 1912.00033 [quant-ph] have recently shown that the three approaches are essentially equivalent, and they may be implemented at the quantum level.
At the quantum level all of them require the use of magnitudes only defined in the kinematical space before imposing the constraints and therefore it is not clear how to describe the evolution in terms of the observable time measured by a physical clock.

For us, the central problem of time in generally covariant systems is not to recover a Schroedinger or Heisenberg type of description in terms of an ideal external parameter t that is not a Dirac Observable . It is to find a description of the evolution where time is measured by clocks in a quantum generally covariant world.

We have noticed some time ago that a combination of the approaches discussed above provides us with such a description.

It is possible to define correlations between Dirac observables that describe both the system and the clock on equal footing and, in terms of them, define conditional probabilities.

As we shall see the procedure can be extended to physical frames of reference

The problem of time in totally constrained systems like Quantum Gravity:

I start by recalling how time is usually introduced in totally constrained systems.
In totally constrained systems, the Hamiltonian vanishes It is a linear combination of constraints:

$$H_T = \mu^\alpha \phi_\alpha(q,p)$$

The generator of the evolution also generates gauge transformations

Dirac observables are gauge invariant quantities

$$\{O(q,p),\phi_\beta(q,p)\} \approx 0 \qquad \{O(q,p),H_T(q,p)\} \approx 0$$

Therefore, they are constants of the motion. Only Dirac observables can be quantized in the space where the constraints are fulfilled: called the physical space. We live in a generally covariant Universe where all observable magnitudes are constants.

At the quantum level the observable magnitudes are operators acting on a Hilbert space H_{ph} defined by

$$|\psi>_{ph} \in H_{ph} \quad \hat{\phi}(\hat{q},\hat{p})|\psi>_{ph} = 0$$

If $\hat{Q}$ is a Dirac Observable, it satisfies $\hat{Q}|\psi>_{ph} \in H_{ph}$

because $[\hat{Q},\hat{\phi}]=0$ but a kinematical gauge dependent variable

$$\hat{q}^0 \ \text{such that} \ [\hat{q}^0,\hat{\phi}(q,p)] \neq 0$$

$$\hat{q}^0|\psi>_{ph} \notin H_{ph} \quad \hat{q}^0|\psi>_{kin} \in H_{kin}$$

The issue of time: If the physically relevant quantities in totally constrained systems as general relativity are constants of the motion, how can we describe the evolution?

1) Gauge fixing: One starts by identifying some dynamical variable of the classical kinematical space as a parameter that plays the role of time variable

$$\tau = f(q,p), \qquad \tau = q^0$$

2) Relational observables: Bergmann, DeWitt, Rovelli, Dittrich

$$\{Q_i(t), \phi_\alpha\} \approx 0 \qquad Q_i(t, q^a, p_a)\big|_{t=q^1} = q_i$$

For instance, for the relativistic particle.
One has two independent observables:

$$\phi = p_0^2 - p^2 - m^2$$

$$p, X \equiv q - \frac{p}{\sqrt{p^2+m^2}} q^0, \qquad Q(t, q^a, p_a) = X + \frac{p}{\sqrt{p^2+m^2}} t$$

$$Q(t = q^0, q^a, p_a) = q$$

Notice that one needs to assume that there are variables as q^0 that are physically observable, even though they are not Dirac observables.

The idea is that one promotes all variables to quantum operators and computes conditional probabilities among them. This idea appears simple, natural and attractive in a closed system.

Hoehn, Smith, and Lock arXiv:1912.00033v2 have recently shown how the Page-Wootters approach can be treated consistently but like the previously mentioned proposals it requires the use of quantities that are not Dirac observables: Positive operator-valued-measures defined in the kinematical space.

In any of these approaches one encounters the same issue: evolution can only be described in terms of a time variable that cannot be defined in the physical space of states.

Using real clocks: conditional probabilities in terms of evolving Dirac observables.

As we have seen, all the approaches require the use of variables which are not defined in the physical space. But then: how is it possible to describe the evolution in terms of physical clocks in general covariant systems like our Universe?
We have considered an approach where all reference to external parameters is abolished, and the evolution is defined in terms of correlations between Dirac observables.

First one chooses an evolving observable as the clock, let us call it $T(t)$.
Then one identifies the set of observables $O_1(t)\ldots O_N(t)$ that commute with T and describe the physical system whose evolution one wants to study. One computes

$$\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) = \lim_{\tau \to \infty} \frac{\int_{-\tau}^{\tau} dt \,\mathrm{Tr}\left(P_{O_0}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \,\mathrm{Tr}\left(P_{T_0}(t) \rho\right)}$$

t is the parameter used to define the evolving observables. This variable is treated as an ideal unobservable quantity that evolves at constant rate.

We have shown that this definition leads to the correct propagators plus quantum corrections.

Assuming that $\rho = \rho_{sys} \otimes \rho_{cl}$

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$$= \lim_{\tau \to \infty} \frac{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(U_{\mathrm{sys}}(t)^\dagger P_O(0) U_{\mathrm{sys}}(t) \rho_{\mathrm{sys}}\right) \mathrm{Tr}\left(U_{\mathrm{cl}}(t)^\dagger P_T(0) U_{\mathrm{cl}}(t) \rho_{\mathrm{cl}}\right)}{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_T(t)\rho_{\mathrm{cl}}\right) \mathrm{Tr}\left(\rho_{\mathrm{sys}}\right)}.$$

one can show that the system may be described in terms of an effective density matrix

$$\rho_{\mathrm{eff}}(T) := \int_{-\infty}^{\infty} dt \; U_{\mathrm{sys}}(t) \rho_{\mathrm{sys}} U_{\mathrm{sys}}(t)^\dagger \mathcal{P}_t(T) \qquad \mathcal{P}_t(T) := \frac{\mathrm{Tr}(P_{T_0}(0) U_{\mathrm{cl}}(t) \rho_{\mathrm{cl}} U_{\mathrm{cl}}(t)^\dagger)}{\int_{-\infty}^{\infty} dt \; \mathrm{Tr}(P_{T_0}(t)\rho_{\mathrm{cl}})}$$

$\int dt \mathcal{P}_t(T) = 1$ where $\mathcal{P}_t(T)$ is the probability that T is observed when the ideal time takes the value t

$$\mathcal{P}(O_0|T) := \frac{\mathrm{Tr}(P_{O_0}(0)\rho_{\mathrm{eff}}(T))}{\mathrm{Tr}(\rho_{\mathrm{eff}}(T))}$$

If the clock has certain dispersion the effective density operator will involve a superposition of unitary operators and the use of real clocks may lead to a loss of quantum coherence and therefore to corrections to the standard propagation

The underlying unitary evolution of the evolving Dirac observables in the ideal time *t* is crucial, yet unobservable. All we observe are the correlations in physical time, then it is not surprising that they present a fundamental level of decoherence due to the intrinsically quantum and gravitational limitations of real clocks.

If we assume the “real clock'' is behaving semi-classically the Schrödinger evolution is modified:

$$-i\hbar \frac{\partial \rho}{\partial T} = [\hat{H}, \rho] + \sigma(T)[\hat{H}, [\hat{H}, \rho]] + \ldots$$

Where *σ(T)* is the rate of spread of the wavefunction of the clock. The evolution is given by a master equation of the Lindblad type. ***Pure states evolve into mixed states.*** *The use of physical clocks required by a quantum gravitational description of time induces a fundamental loss of unitarity unless it is possible to define a perfect clock.* Perfect clocks allow to recover the Schroedinger evolution.

Of course, this loss of coherence is typical of imperfect clocks. It has been observed in Rabi oscillations.

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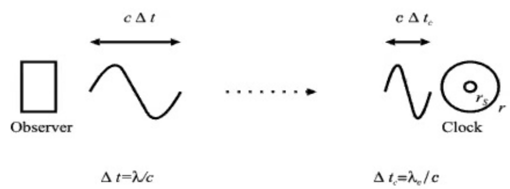
As we do not have a complete theory of quantum gravity this is a contentious issue. Phenomenological arguments have been given by Salecker-Wigner, Ng, Karolyhazy, Lloyd, and Frenkel leading to similar estimations based on two main effects: quantum fluctuations or black hole formation.

We have recently given a simple argument leading to a fundamental minimum uncertainty in the determination of time intervals consistent with the previous estimations. It only relies in the uncertainty principle and time dilation in a gravitational field.

$$\Delta E \Delta t_c > \hbar. \qquad t = \frac{t_c}{\sqrt{1 - \frac{r_S}{r}}},$$

One considers an electromagnetic source at the frequency that maximizes the probability that a particle transitions between levels. The source emits photons that interact with a microscopic system and induce transitions. This frequency is used as standard of time measurements. The precision grows with the frequency of the oscillator. But the higher the frequency of the oscillator the higher the uncertainty in the mass-energy of the microsystem and, due to time dilation, in the time interval measured by a macroscopic observer

Energy fluctuations induce uncertainties in the relation between t and t_c



$$\Delta t > t^{1/3} t_P{}^{2/3} \quad \Delta l > l^{1/3} l_P{}^{2/3}$$

R.G and J. Pullin J.Phys.Comm. 4 (2020) 6, 065008
e-Print: 2006.08730

If the best accuracy one can get with a clock is the given above, the master equation will induce a decay of the out of diagonal terms of the density matrix.

$$\rho(T)_{nm} = \rho_{nm}(0) e^{-i\omega_{nm}T} e^{-\omega_{nm}^2 T_{\text{Planck}}^{4/3} T^{2/3}}.$$

Pure states evolve into statistical mixtures, the out of diagonal terms of the density matrix in the energy basis asymptotically vanish, and the system would present a fundamental loss of coherence due to these effects.

The computation of conditional probabilities in terms of evolving Dirac observables may be extended to include physical reference frames.

Quantum Reference Frames

This loss of coherence is also present when one uses a relational description of positions with respect to physical reference frames. Systems that can be described by pure states in an external frame must be described using mixed states in terms of relative coordinates.
S.D. Bartlett, T. Rudolph, R. Spekkens and P.S. Turner, New J. Phys. 11, 063013 (2009)

The example of the sl(2R) Model.

This is a model with two Hamiltonian constraints and a diffeomorphism constraint.
This allows to address the issue of "different foliations" in time.
The canonical pairs are (u_i, p_i) and (v_i, π_i) $i = 1 \dots N$ The constraints are,

$$H_1 = \frac{1}{2}\sum_{i=1}^{N} p_i^2 - \frac{1}{2}\sum_{i=1}^{N} v_i^2$$

$$H_2 = \frac{1}{2}\sum_{i=1}^{N} \pi_i^2 - \frac{1}{2}\sum_{i=1}^{N} u_i^2$$

$$D = u.p - v.\pi$$

And their algebra is,

$$\begin{aligned} \{H_1, H_2\} &= D, \\ \{H_1, D\} &= 2H_1, \\ \{H_2, D\} &= -2H_2. \end{aligned}$$

M. Montesinos, C. Rovelli and T. Thiemann, Phys. Rev. D 60 (1999)

The observables of the model are: $O_{ij} = x_i^1 x_j^2 - x_i^2 x_j^1.$

Where:
$$\vec{x}^1 = (u_1, \dots u_N, \pi_1, \dots \pi_N),$$
$$\vec{x}^2 = (p_1, \dots p_N, v_1, \dots v_N)$$

There exist 2 (2N-3) independent observables

For instance, for N=2 we have two independent Dirac observables and two parameterized Dirac observables depending on three parameters *x,y,z* corresponding to three kinematical variables.

$$U^1 = \frac{x(z\cos\phi + \epsilon_2\epsilon_1 y\sin\phi) + r}{\epsilon_2 y\cos\phi - z\epsilon_1\sin\phi}$$

$$P_1 = \epsilon_2 y\sin\phi + \epsilon_1 z\cos\phi$$

$$U_1(x = u_2, y = v_1, z = v2, o_1, o_2) = u_1$$

One can introduce a time structure and an inner product such that the parameterized Dirac observables turn out to be, at the quantum level, self-adjoint operators on the corresponding Hilbert space that evolve unitarily. However, for N=2 it is not possible to introduce physical clocks and frames, because we do not have enough commuting parameterized observables that can play the role of reference frames.

In order to implement a physical reference frame, we need to consider the N=4 model
There exist ten independent observables, which we will call $\phi_1 \ldots \phi_9, \text{and } \rho$

One can show that the ten independent observables can be promoted to self adjoint operators and identify self adjoint parameterized Dirac observables that depend on a suitable triad of kinematical parameters.

Conditional probabilities may be introduced by identifying three independent commuting Dirac observables $X_1(x,y,z)\ X_2(x,y,z)\ X_3(x,y,z)$ that play the role of physical clocks and rods.

We know how to identify *simple* clock and frame variables in the kinematical space by extending the Hohn, Smith, and Lock technique.
This allows identifying an integration measure that correspond to a given choice of frame variables for the computation of conditional probabilities in the SL(2R) model.

$$P(O^0|{X_1}^0,{X_2}^0,{X_3}^0)=\frac{\int dx\int dy\int dz\,\mu(x,y,z)\mathrm{Tr}(P_{O^0}P_{{X_1}^0}P_{{X_2}^0}P_{{X_3}^0}\rho P_{{X_1}^0}P_{{X_2}^0}P_{{X_3}^0})}{\int dx\int dy\int dz\,\mathrm{Tr}(P_{{X_1}^0}P_{{X_2}^0}P_{{X_3}^0}\rho)\,\mu(x,y,z)}$$

Deviations from covariance induced by physical reference frames?

In the general relativistic case, instead of a finite number of parameters one should parameterize the system with functions that describe for instance the foliation used.

Even if the quantum theory includes the Dirac observables that are present in classical G.R., the metric like any frame dependent object could show deviations from covariance, due to Planck scale limitations in the definition of physical reference systems in terms of conditional probabilities.

To know when the measurement is carried out, we need to observe a gauge dependent quantity. There is not any physical clock able to measure an elapsed time in terms of the kinematical variable q^0. We shall call the quantity that plays the role of q^0 ideal time.

The requirement that the relational observables be self-adjoint is very restrictive in any totally constrained system and imposes strong limitations on the type of ideal time that can be used at the quantum level. Generically, the problem appears when the "equal time" surfaces $T = T_0$ are not transversal to the orbits. In non-relativistic quantum mechanics only the Newtonian time is transversal to the orbits.
In the relativistic case any spacelike surface is transversal to the orbits of the particles. We will further restrict the kind of ideal times that we will consider to "simple" clocks in the sense of Hoehn Smith and Lock. Clocks that change at a constant rate along the dynamical trajectories

$\{T, H_C\} = u(H_C)$ constant of the motion and such that $C_H = H_C + H_S \approx 0,$

it is assumed that clock and system do not interact.

From now on we will restrict the notion of relational Dirac observables to the self adjoint operators that evolve unitarily in the ideal time.

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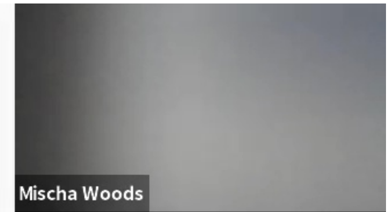

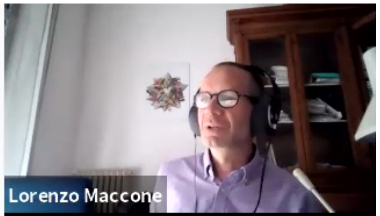

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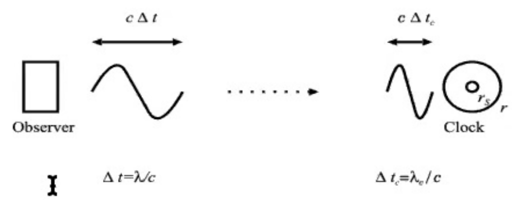

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