Title: Time in Physics and Intuitionistic Mathematics

Speakers: Nicolas Gisin

Collection: Quantizing Time

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Abstract: "Physics is formulated in terms of timeless axiomatic mathematics. However, time is essential in all our stories, in particular in physics. For example, to think of an event is to think of something in time. A formulation of physics based of intuitionism, a constructive form of mathematics built on time-evolving processes, would offer a perspective that is closer to our experience of physical reality and may help bridging the gap between static relativity and quantum indeterminacy.

Historically, intuitionistic mathematics was introduced by L.E.J. Brouwer with a very subjectivist view where an idealized mathematician continually produces new information by solving conjectures. Here, in contrast, Iâ€™ll introduce intuitionism as an objective mathematics that incorporates a dynamical/creative time and an open future. Standard (classical) mathematics appears as the view from the â€œend of timeâ€• and the usual real numbers appear as the hidden variables of classical physics. Similarly, determinism appears as indeterminism seen from the â€œend of timeâ€•.

Relativity is often presented as incompatible with indeterminism. Hence, at the end of this presentation Iâ€™ll argue that these incompatibility arguments are based on unjustified assumptions and present the â€œrelativity of indeterminacyâ€•.

References:

C. Posy, Mathematical Intuitionism, Cambridge Univ. Press, 2020.

N. Gisin, Indeterminism in Physics, Classical Chaos and Bohmian Mechanics. Are Real Numbers Really Real?, Erkenntnis (2019), https://doi.org/10.1007/s10670-019-00165-8

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Flavio Del Santo and N. Gisin, Physics without determinism: Alternative interpretations of classical physics, Physical Review A 100.6 (2019).

N. Gisin, Mathematical languages shape our understanding of time in physics, Nature Physics 16, 114-119 (2020).

N. Gisin Indeterminism in Physics and Intuitionistic Mathematics, arXiv:2011.02348

Flavio Del Santo and N. Gisin, The Relativity of Indeterminacy, arXiv:2101.04134"

Time in Physics and Intuitionist Mathematics

Nicolas Gisin

Group of Applied Physics, University of Geneva

Schaffhausen Institute of Technology, SIT.org-Geneva

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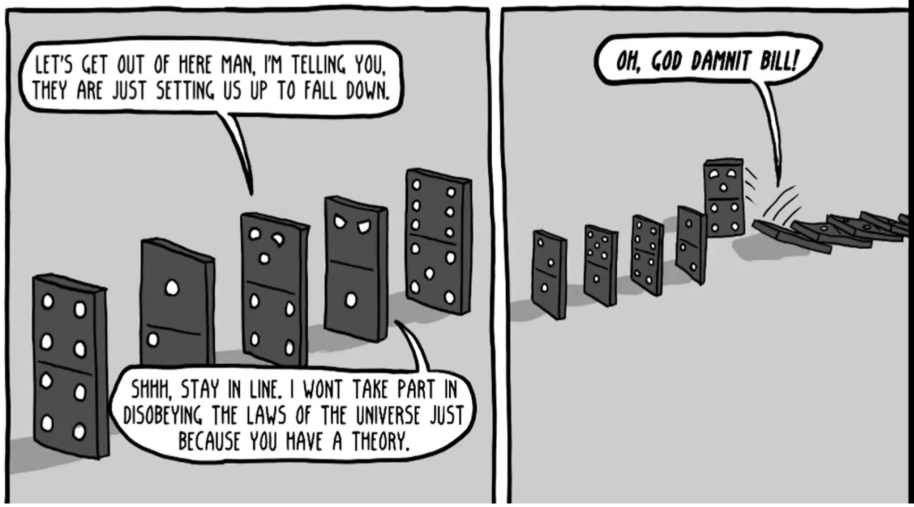
- Motivations
- Classical chaos
- **Real numbers are not really real**
- Mathematical language<u>**S**</u>
- Intuitionistic maths
- Intuitionistic logic, the continuum and arithmetic
- The relativity of indeterminacy



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What is Time?

If nobody asks me, I know; but if I were desirous to explain it to one that should ask me, plainly I do not know." **St. Augustine**

Time is (not only) what ideal clocks measure.
Parmenides-geometric Time: when what matters is being.

An essential aspect of time is that there is a time before and a time after an event; e.g. times before and after a random bit is produced, or before and after I made a decision.

Motivations. 1: Stories

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- Physicists produce models of reality.
- The models should be as faithful as possible:

⇒ correct empirical predictions, and

⇒ **allow humans to tell stories about How Nature Does It.**

⇒No way to tell stories without the passage of time: **"To think of an event is to think of something in time", Yuval Dolev.**

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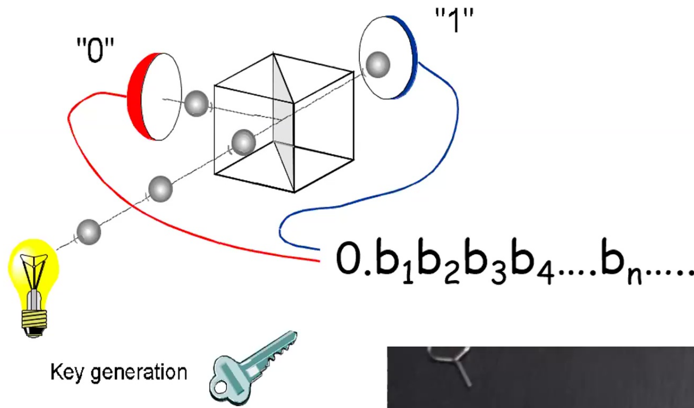
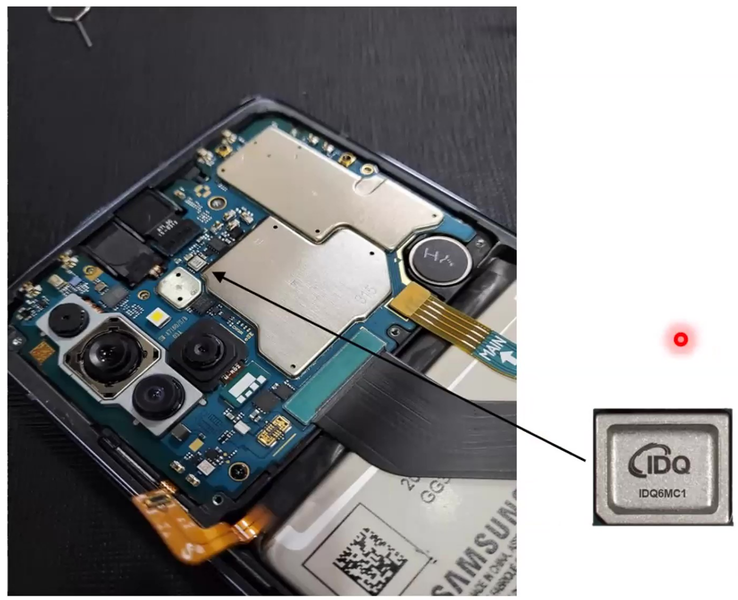
Motivations. 2: Quantum Randomness



New information gets created as time passes



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Quantum Indeterminacy

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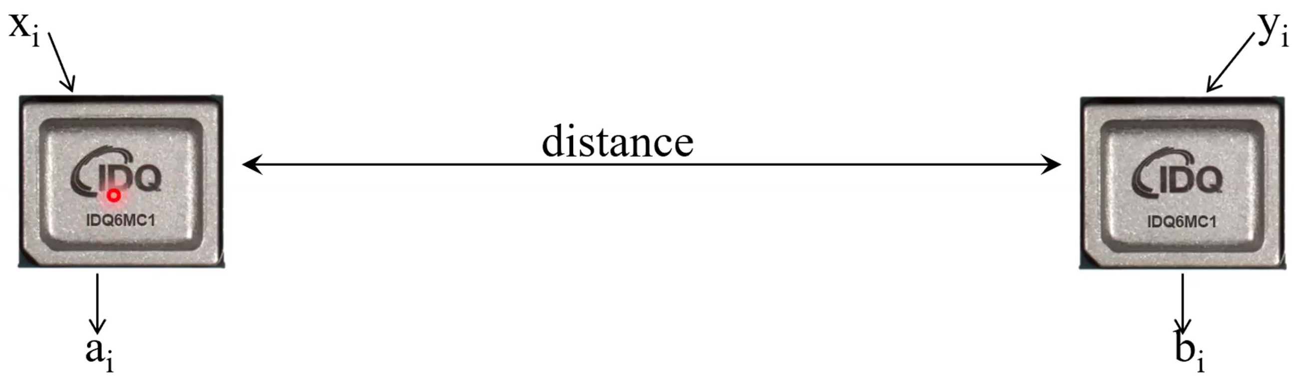

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Quantum Indeterminacy

- Assume no instantaneous communication at a distance.
- Assume no conspiracy: there are independent variables.

Experimental fact:
Prob(a⊕b=x·y) > 3/4

x_i

a_i

distance

Theorem:
⇒ The outcomes are truly random

y_i

b_i

N. Gisin
Quantum Chance
Springer 2014.
N. Brunner et al.,
RMP 86, 419
(2014).

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nature

nature International weekly journal of science

LETTERS

Ran

S. Pironio ... schenk[6],
D. Hayes

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→ Our world is not deterministic.
The Universe is open: Nature is able of acts of pure creation

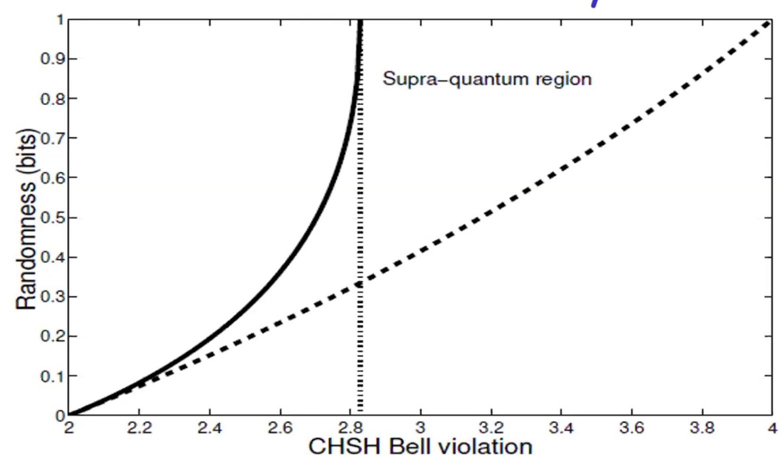
$$\text{Prob}(a \oplus b = x \cdot y) > 3/4$$

x_i →  → a_i

distance

y_i →  → b_i

Theorem:
⇒ The outcomes are truly random



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An essential aspect of time is that there is a time before and a time after an event; e.g. times before and after a random bit is produced.

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Time is (also) the accumulation of little events.
Heraclitus-creative Time:
when what matters is change.

Deterministic creation is not real creation because whatever novelty arises is really just an unfolding of what came before.

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Intuitionist maths: a first encounter

F. Del Santo & NG, *PRA* 100.6 (2019).

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- Numbers are there to count:
  How can it be that most of the so-called real numbers are not computable?

Let's define some integers:

1. n1=0 if every even integer bwt 4 and 10^4 is the sum of 2 primes, and n1=1 otherwise.
2. n2=0 if every even integer bwt 4 and 10^{100} is the sum of 2 primes, and n2=1 otherwise.
3. n3=0 if every even integer larger than 4 is the sum of 2 primes, and n3=1 otherwise (Goldbach conjecture).

- There is no known finite method to compute n3.
- Nevertheless, every student knows that in order not to fail an exam he has to claim that n3 has a determined value.
- Except if it is an exam in intuitionist mathematics. Then the student should answer that the value of n3 is indeterminate and that the law of the excluded middle is not valid.

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- Except if it is an exam in intuitionist mathematics. Then the student should answer that the value of n3 is indeterminate and that the law of the excluded middle is not valid.
- And the value of n3 may evolve over time ...

… from indeterminate to determinate.

Intuitionist stand-point

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- Carl Posy: “We humans have finite memories, finite attention spans and finite lives. So we can fully grasp only finitely many finite sized pieces of a compound thing. There’s no infinite helicopter allowing us to survey the whole terrain or to tell how things will look at the end of time.”
- Erret Bishop: “The classicist wishes to describe God's mathematics; the constructivist, to describe the mathematics of finite beings, man's mathematics for short … Constructive mathematics does not postulate a pre-existent universe, with objects lying around waiting to be collected and grouped into sets, like shells on a beach.”
- Brouwer: **“Nature simply has not yet fully determined all objects”**. This can be compared to the “uncertainty” principle used in quantum mechanics.
- That’s the essence of intuitionism. But I see that you get worried: how could there be things, including mathematical objects, not fully determined?
- To calm a bit your worries, let me state that with intuitionist mathematics you can compute and prove theorems, though not always the same theorems as in classical mathematics or not following the same proofs. For sure, everything one can do on a (classical) computer can be done with intuitionist mathematics. Hence, all of physics can be done.
- Climate physics uses truncated numbers and stochastic remainders.

Palmer, T. N. *Nat. Rev. Phys.* **1**, 463–471 (2019).

The mathematical language we speak has a huge influence on the world-view that physics presents to us.

- The dependence of intuitionism on time is essential: statements can become provable in the course of time and therefore might become intuitionistically valid while not having been so before.
  Standford Encyclopedia of Philosophy.
- Brouwer, the father of intuitionism, introduced into his mathematics the concept of an ideal mathematician – the creating subject – who continually produces new information by solving mathematical conjectures.
- I will present intuitionism without any ideal mathematician and motivate it by the physical concept of indeterminism. Brouwer would not have liked my presentation. I am a physicist, hence a (naive?) realist.

- My main claim is that intuitionist mathematics is the natural mathematical tool to describe the passage of time and indeterminism
  in physics, like derivative is the natural tool to describe velocity.

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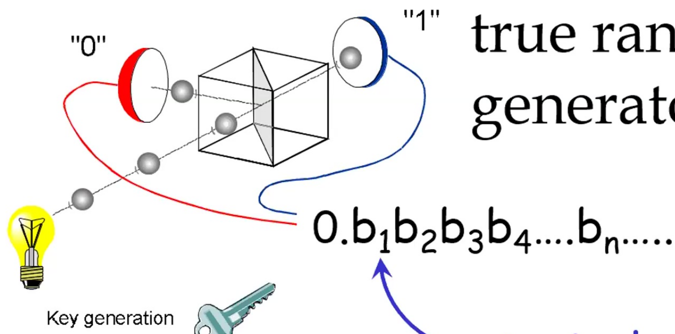
Typical real numbers

- Sinc … names and algo … ain an infi … tion.
- One … s to all que … uman lang …

When we say:
Let $x_0 \in \Re$
what we effectively say is:
let x_0 denote an infinite amount of information.

- The only good way of thinking of a typical real number is the unlimited string of outcomes of a true random number generator.



typical real number

G. Chaitin, The Labyrinth of the Continuum, in Meta Math!, Vintage 2008

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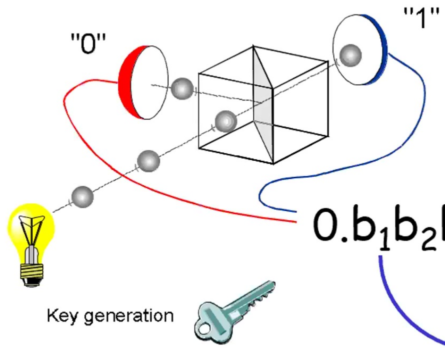
Typical real numbers

- All real numbers one encounters are exceptional:
  they have a name and are defined by a (finite) algorithm: $\sqrt{2}$, π, 77/125, etc.
- The bits of typical real numbers have no structure: the bits are random, as random as quantum measurement outcomes:



$0.b_1b_2b_3b_4....b_n.....$

typical real number

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Whether Newtonian classical mechanics is deterministic or not, is not a scientific question; it depends on the physical significance one associates with mathematical real numbers

- Intuitionism brings classical closer to quantum.

The Classical and Intuitionistic continuum

D. Hilbert

L.E.J. Brouwer

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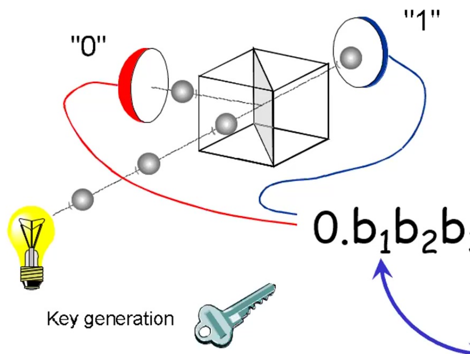

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Mathematical real numbers are not Physically real

and

Mathematical real numbers are Physically random

And these **random numbers should be at the basis of scientific determinism?**

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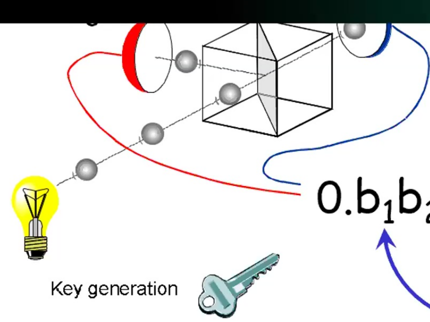

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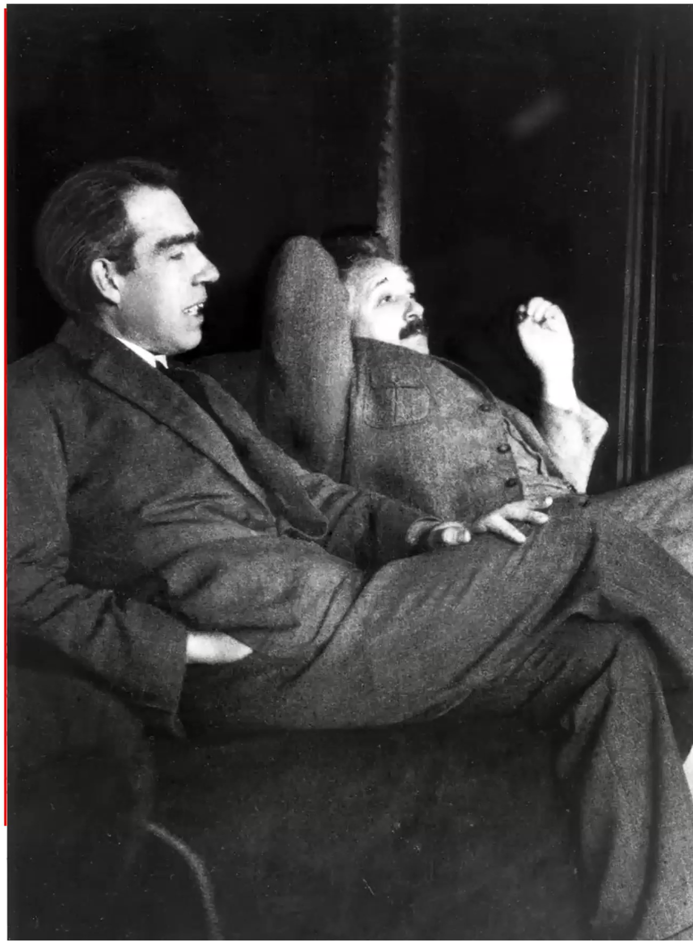
Einstein – Bohr



Einstein – Bergson (Paris 1922)

The Classical and Intuitionistic continuum

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D. Hilbert

L.E.J. Brouwer

<u>Classical mathematics</u>:
Every real number is an individual <u>completed</u> entity.
All digits are given at once.
The continuum is a collection of individual points.

Real numbers exists outside of time in some ideal Platonistic world.

<u>Intuitionistic mathematics</u>:
Real numbers are processes that develop in time.
Digits are not all given at once (except computable numbers).
The continuum is a "viscous collection" of processes.

Time is essential to intuitionism: at any instant only finite information exists.

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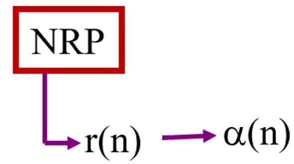
Intuitionist Mathematics

Choice sequences $\alpha(n)$, choices made by an idealized mathematician

Nature has the power to produce true randomness, i.e. to produce new information
(what else?)



At each time step (instant) n a Natural Random Process (NRP) outputs some random number r(n).

$\alpha(n)=fct(\alpha(n-1),n,r(1),...,r(n)) \in$ computable number,

where fct is a computable function and the series $\alpha(n)$ converges:

e.g. $|\alpha(n)-\alpha(n-1)| \leq 2^{-n}$

My definition is close to what Brouwer called «projections of choice sequences» or «Intentional sequences». The function fct plays the role of Brouwer's spreads & fans.

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Examples of intuitionist numbers

1. Totally random numbers

$\alpha(n)=\alpha(n-1)+r(n)\cdot 2^{-n} = 0.b_1b_2b_3b_4....b_n$

2. Computable numbers

$\alpha(n)=fct(\alpha(n-1),n,r(1)..r(k)) \Rightarrow \alpha(n)$, $n>>k$, is a pseudo-random series, where r(1)..r(k) is the seed.

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Examples of intuitionist numbers

4. Mortal numbers

$$\alpha(n) = \frac{1}{2} + (-1)^{r(n)} \cdot 10^{-n}$$

$r(n)=0 \Rightarrow \alpha(n)>1/2$
$r(n)=1 \Rightarrow \alpha(n)<1/2$

until, by chance, the last n/2 random bits r(j) all happen to have the same value, and $n \geq 4$ is even, then the series terminates - dies:

$$\alpha(n)=\alpha(n-1) \text{ for all future n.}$$

Because the probability of termination decreases exponentially, there is an a priori probability that the sequence goes on for ever.

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Intuitionist Logic

- Time is essential in intuitionist mathematics,
  Stanford Encyclopaedia of Philosophy, *Intuitionism in the Philosophy of Mathematics.*
- The statement r(n)=0 is indeterminate before the n^{th} time step, the n^{th} instant.
- The proposition $\alpha < ½$ is indeterminate as long as the mortal numbers didn't die.
- The proposition «It will rain in exactly one year from now at Piccadilly Circus» is, at present, indeterminate.

⇒ **The law of the excluded middle is not valid.**

⇒ No non-constructive existence proofs.

Surprising !

… but makes plenty of sense in an indeterministic world, i.e. in a world in which the future is open.

- Brouwer: excluded middle fails because the world is not-determinate, so truths about it are indeterminate.

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The intuitionist continuum

- Elements of the continuum (intuitionist «real numbers») are evolving sequences of computable numbers (Brouwer's choice sequences).
- For some sequences it is never determined (at any finite time) whether they are larger or smaller than ½.
- Hence, one can't cut the continuum into larger and smaller than ½.
- Brouwer's theorem:

  All total functions are continuous (no step fct).

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Theorem

If f is a continuous function such $f(a) < 0$ and $f(b) > 0$ (where $a < b$), then for any $\varepsilon > 0$ one can construct a value of x between a and b, for which $|f(x)| < \varepsilon$.

- The standard proof of Gleason's theorem does not hold intuitionistically. However, F. Richman proved that Gleason's original formulation is in fact constructively provable.

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The mathematical language we use when speaking physics has a huge influence on the world-view that physics presents to us.

Indeterministic Physics	Intuitionist Mathematics
Past, present and future are **not** all given at once	Digits of real numbers are **not** all given at once
Time passes	Numbers are processes
Indeterminism	Numbers contain finite information
The present is thick	The continuum is viscous
The future is open	No law of the excluded middle
Becoming	Choice sequences
Experiencing	Intuitionism

The Relativity of Indeterminacy

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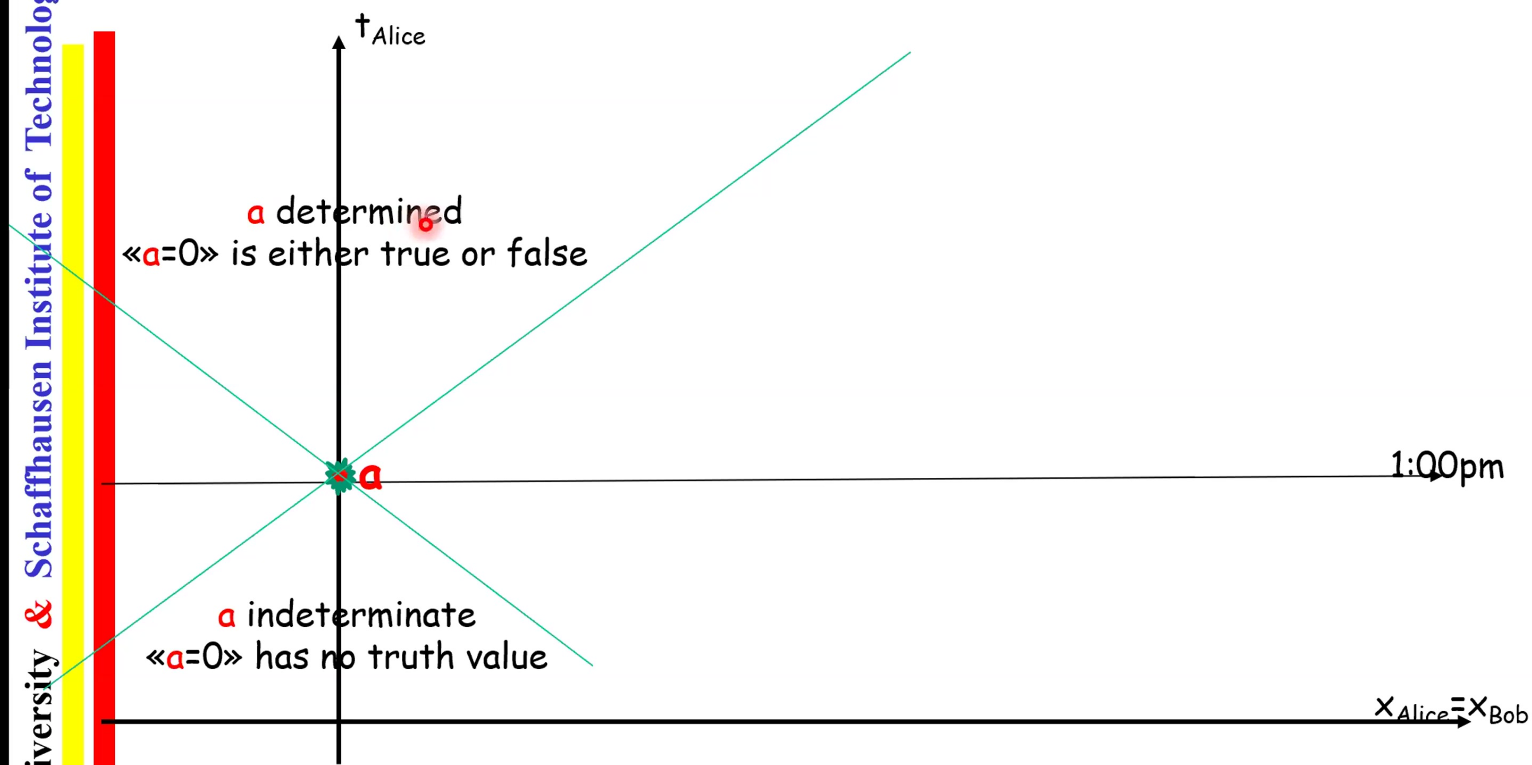

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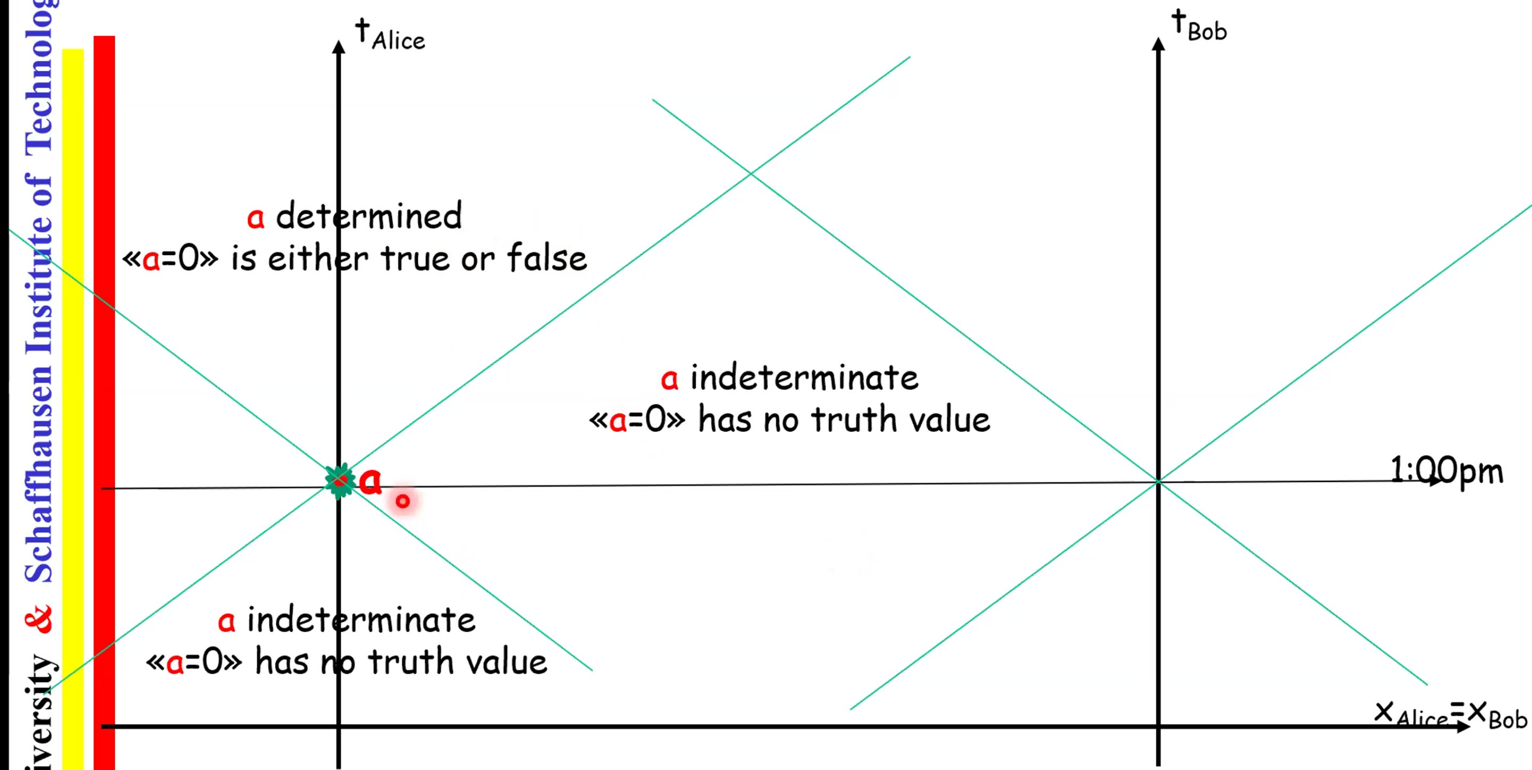

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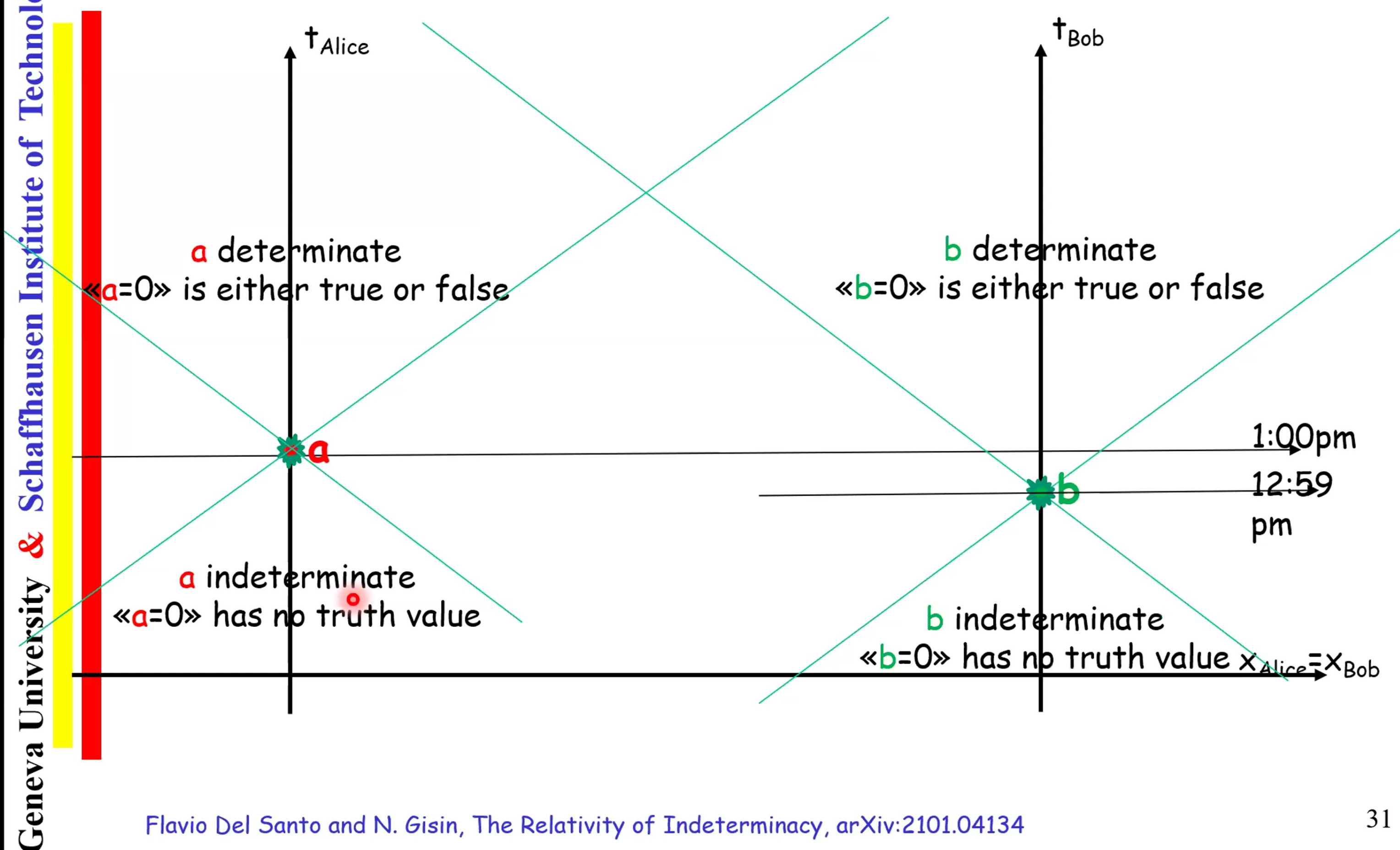

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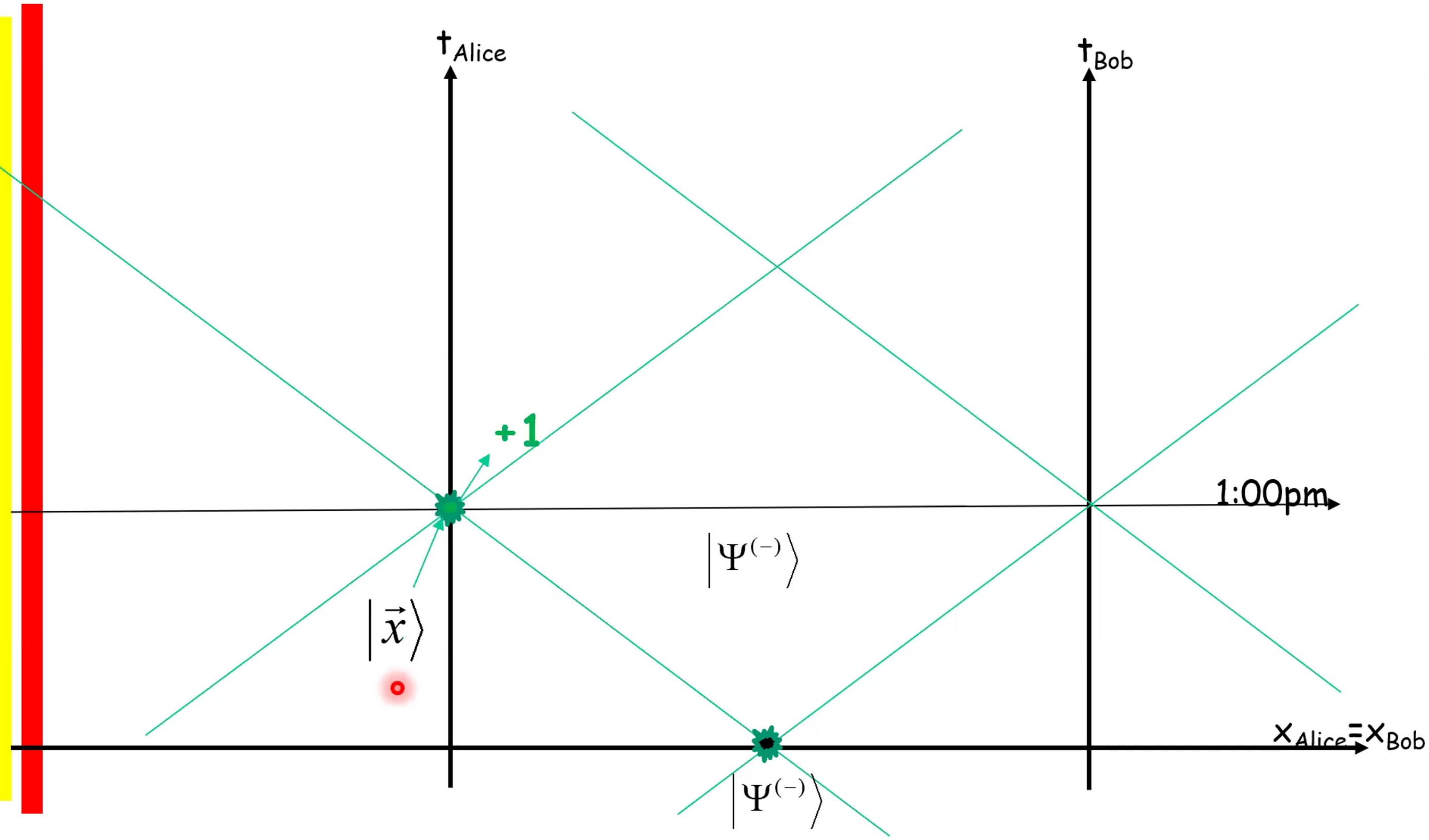

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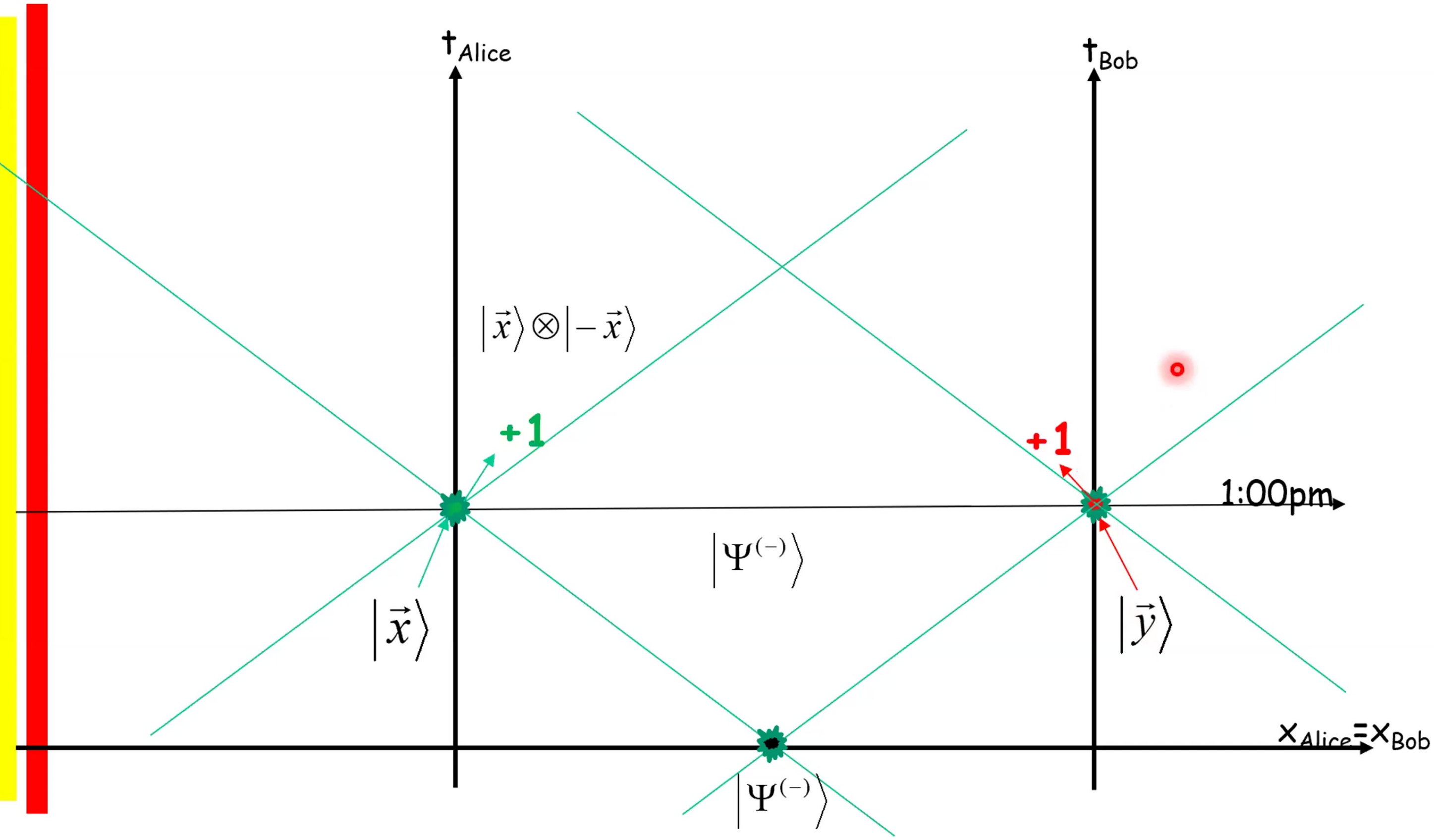
The Relativity of Indeterminacy



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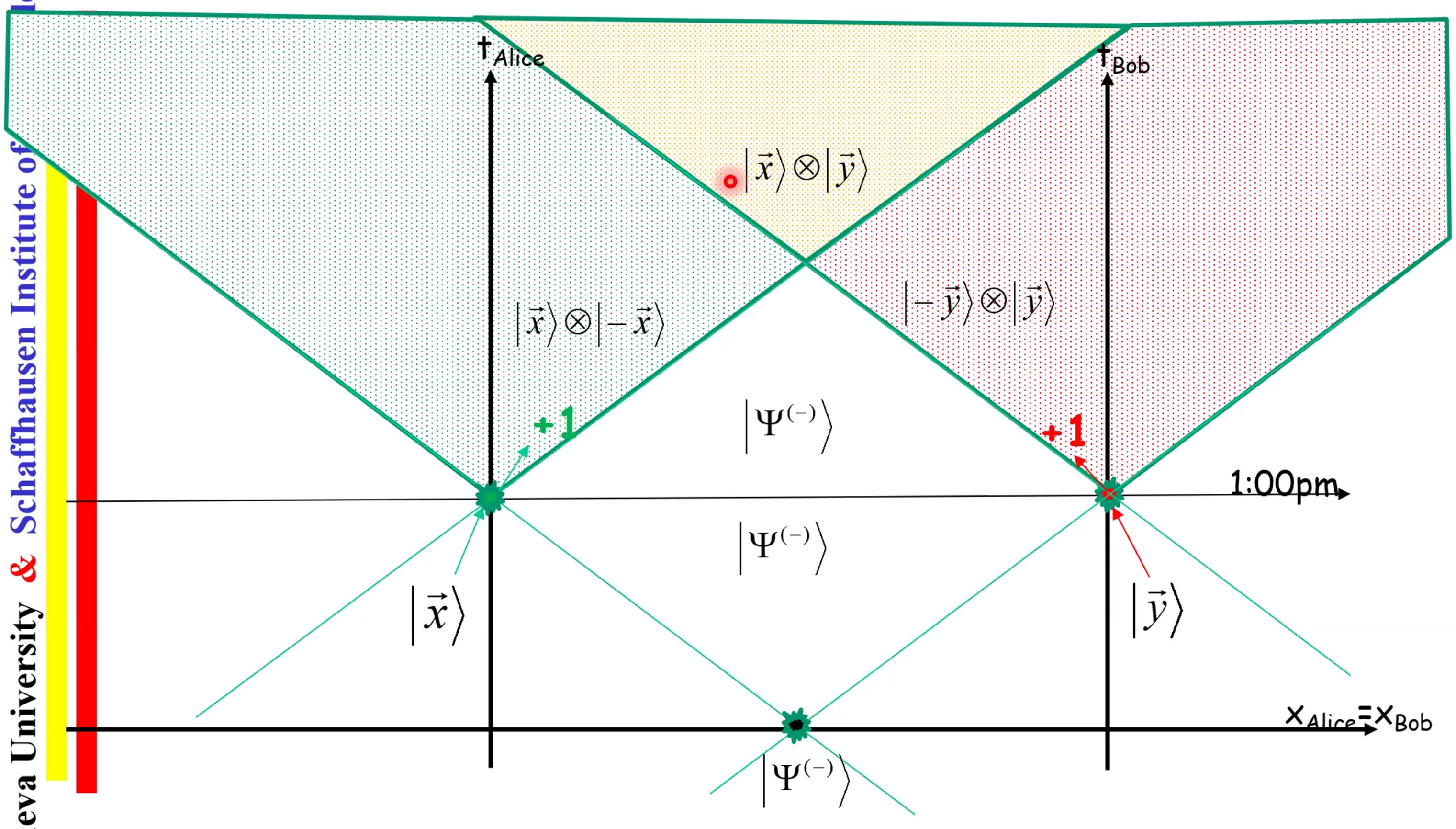

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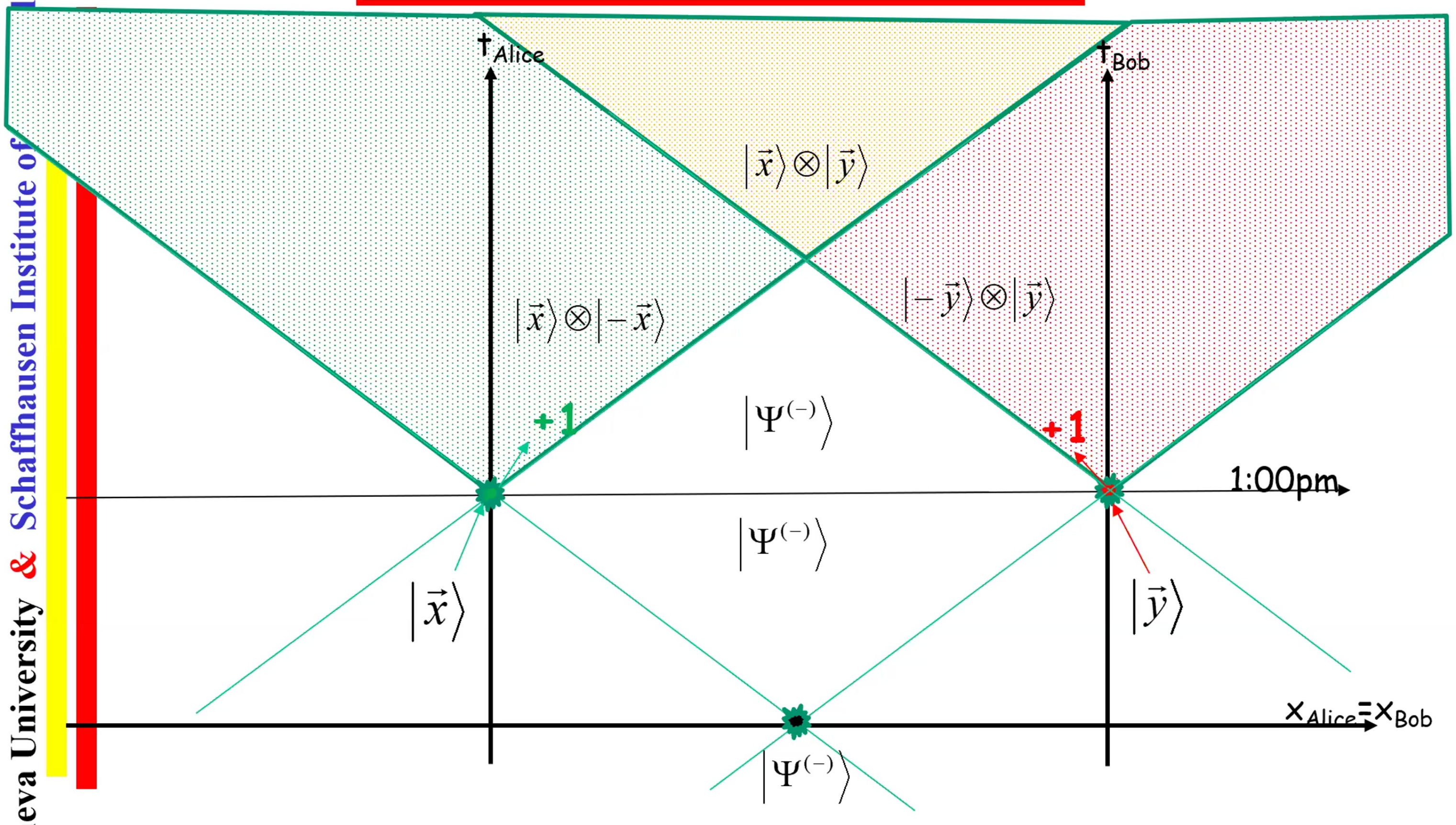
The Relativity of Indeterminacy

No wavefunction of the Universe



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Conclusion

- *The mathematical language we speak when "talking physics" impacts on our world-view:*

  some conclusions one is tempted to infer from physics – like determinism and the illusion of time – are, actually, inspired by the language, not by the facts.

  - Classical real numbers are the hidden variables of classical mechanics.

  - Classical mathematics assumes a view from the end of time.

  - The law of the excluded middle holds only if one assumes a look from the "end of time", that is, a God's eye view.

- *Intuitionism brings classical closer to quantum.*
- *Indeterminacy is relative.*

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- N. Gisin, *Indeterminism in Physics, Classical Chaos and Bohmian Mechanics. Are Real Numbers Really Real?*, Erkenntnis (2019).
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