Title: Time and Noether's (first) theorem

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Abstract: "It is widely believed that the homogeneity of time is the symmetry related by Noether's (first) theorem to the conservation of energy, and indeed that it explains energy conservation. Both claims are questionable, and in particular seemingly hard to reconcile with the modern version of Noether's first theorem due independently to MartÃ-nes Alonso (1979) and Olver (1986).
The talk is based on: 'Do symmetries ""explain"" conservation laws? ...'
arXiv:2010.10909v1"

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time and Noether's (first) theorem

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Quantising Time, Perimeter Institute, June 2021

HB, `Do symmetries "explain" conservation laws? The modern converse Noether theorem vs pragmatism', arXiv:2010.10909v2

the arrow of time

reversal invariance

emergence

distant simultaneity

duration (connection with clocks)

action-reaction principle

homogeneity

slogans

- Time is what stops everything happening at once.
  John A. Wheeler/Albert Einstein
- Time is Nature’s way of getting round the law of non-contradiction.
- Time is money.
  Benjamin Franklin

the arrow of time

reversal invariance

emergence

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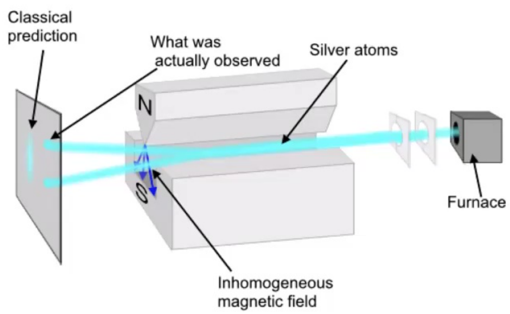
homogeneity



VS.

time is “weakly” measured

types of symmetries

variational *vs* dynamical

$$S = \int_R \mathcal{L}\left(\phi_i, \phi_{i,x}, x\right) \mathrm{d}x \xrightarrow[\textit{Principle}]{\textit{Hamilton's}} E_i \equiv \frac{\partial \mathcal{L}}{\partial \phi_i} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial \mathcal{L}}{\partial \phi_{i,x}}\right) = 0$$

the physics is here!*

*there are equations of motion/field equations that are not "variational".

Take the Fourier heat equation with constant diffusivity *D*:

$$\frac{\partial u}{\partial t} - D\nabla^2 u = 0$$

Time reversal non-symmetric

1768-1830

Consider the rescaling transformation $u \to cu$

This is a symmetry of the heat equation. Is there an associated Noether charge?

Problem: the heat equation does not have a Lagrangian density of type

$$\mathcal{L} = \mathcal{L}\left(u, \frac{\partial u}{\partial x_\mu}, x^\mu\right) \qquad \mu = 0, 1, 2, 3.$$

the trick leads to Noether

$$\mathcal{L} = D\nabla u \nabla v + \frac{1}{2}\left(v\frac{\partial u}{\partial t} - u\frac{\partial v}{\partial t}\right)$$

Hamilton's principle by varying v

$$\frac{\partial u}{\partial t} - D\nabla^2 u = 0$$

Heat equation with constant diffusivity
Time reversal non-symmetric

the transformation $u \to cu$ is a dynamical, but not a variational symmetry, except when $c = 1$.

Noether variation (quasi-symmetry)	$v \to v' = v + \epsilon$	$v \to v' = v + \mathbf{x}\cdot\epsilon$
Noether continuity equation	$\frac{\partial u}{\partial t} - D\nabla^2 u = 0$	$\frac{\partial u}{\partial t} - D\nabla^2 u = 0$
Noether charge	$\frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty} u\mathrm{d}\mathbf{x} = 0$ if the gradient of u falls off sufficiently fast	$\frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty} u x_i \mathrm{d}\mathbf{x} = 0$ if the current density falls off sufficiently fast

Ibragimov and Kolsrud (2004)

The quantum analogue

$$\mathcal{L}_{\text{free}} = -\frac{\hbar^2}{2m}\nabla\psi\nabla\psi^* + \frac{i\hbar}{2}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$

Hamilton's principle varying ψ^*

$$\frac{\partial\psi}{\partial t} - \frac{i\hbar}{2m}\nabla^2\psi = 0$$

Schrödinger equation for free particle
Time reversal invariant!

the transformation $\psi \to c\psi$ is a dynamical, but not a variational symmetry, except when $|c| = 1.$

Noether variation (quasi-symmetry)	$\psi \to \psi' = \psi + \varepsilon$	$\psi \to \psi' = \psi + \epsilon \cdot \mathbf{x}$
Noether continuity equation	$\frac{\partial\psi}{\partial t} - \frac{i\hbar}{2m}\nabla^2\psi = 0$	$\frac{\partial\psi}{\partial t} - \frac{i\hbar}{2m}\nabla^2\psi = 0$
Noether charge	$Q = \int_\Omega \psi d\mathbf{x}.$ if $\lvert\nabla\psi\rvert$ decreases faster than $\lvert\mathbf{x}\rvert^{-2}$	$Q_k = \int_\Omega \psi x_k \mathrm{d}\mathbf{x}$ if the current density falls off sufficiently fast

HRB & P. Holland *Am J Phys* (2004)

"There are some [quantities] whose constancy is of profound significance, deriving from the fundamental homogeneity and isotropy of space and time ..."

Landau and Lifshitz *Mechanics* 1976

"[When] I heard about Noether's insight … I was profoundly impressed. The revelation that these basic conservation laws follow from the assumption that physics is the same yesterday, today, and tomorrow; here, there, and everywhere; east, west, north, and south, was for me, as Einstein put it, essentially spiritual."

A. Zee, *Fearful Symmetry* 1986

Consider the free non-relativistic particle in 1 dimension

Hamilton's principle:

$$\mathcal{L} = \frac{1}{2}\dot{q}^2 \quad \Longrightarrow \quad \ddot{q} = 0 \quad \Longleftarrow \quad \mathcal{L}' = \frac{1}{2}\left[\dot{q}^2 - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q^2}{t}\right)\right]$$

$$q' = q + \epsilon t; \quad t' = t$$

"Boost"
quasi-symmetry of $\mathcal{L}$
strict symmetry of $\mathcal{L}'$ (Noether 1918)

$$q' = q + \left(t - \frac{2q}{\dot{q}}\right)\epsilon; \quad t' = t - \frac{2q}{\dot{q}^2}\epsilon$$

strict symmetry of $\mathcal{L}$ (Noether 1918)
quasi-symmetry of $\mathcal{L}'$

$$\frac{\mathrm{d}}{\mathrm{d}t}(q - \dot{q}t) = 0$$

Conservation of "centre of mass motion"

Which symmetry is "responsible" for the conservation law?

the free non-relativistic particle in 1 dimension c'td

Hamilton's principle:

$$\mathcal{L} = \frac{1}{2}\dot{q}^2 \quad \Longrightarrow \quad \ddot{q} = 0 \quad \Longleftarrow \quad \mathcal{L}' = \frac{1}{2}\left[\dot{q}^2 - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q^2}{t}\right)\right]$$

Noether

$$q' = q; \quad t' = t + \epsilon$$

symmetry of $\mathcal{L}$; quasi-symmetry of $\mathcal{L}'$

$$q' = q - \left(\frac{2q}{t}\right)\epsilon; \quad t' = t + \left(1 - \frac{2q}{\dot{q}t}\right)\epsilon$$

quasi-symmetry of $\mathcal{L}$; quasi-symmetry of $\mathcal{L}'$

$$q' = q - \dot{q}\epsilon; \quad t' = t$$

quasi-symmetry of $\mathcal{L}$; quasi-symmetry of $\mathcal{L}'$

$$q' = q - \dot{q}(1-T)\epsilon; \quad t' = t + T\epsilon; \quad T = \frac{1}{\mathcal{L}'}\left(\frac{q\dot{q}}{t} - \frac{1}{2}\dot{q}^2\right)$$

quasi-symmetry of $\mathcal{L}$; symmetry of $\mathcal{L}'$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\dot{q}^2\right) = 0$$

Conservation of energy

Which symmetry is "responsible" for the conservation law?

The Martínez Alonso-Olver (MAO) theorem

If $\mathcal{L}$ is a non-degenerate variational problem, there is a one-to-one correspondence between suitably defined equivalence classes of nontrivial conservation laws of the Euler-Lagrange equations and suitably defined equivalence classes of variational symmetries of the action $\int \mathcal{L} \mathrm{d}x$.

L. Martínez Alonso *Lett. Math. Phys.* (1979)

P. J. Olver *Applications of Lie groups to Differential Equations* (1986, 1993)

Why the widespread explanatory priority of symmetries?

- MAO theorem is little known?
- Properties of space, time often regarded as primordial
- Properties like mass and spin as Casimir invariants of the Poincaré group; they "are what they are because of the symmetries of the laws of nature". (Weinberg 1993)
- the 20th century heuristic paradigm in QED and particle physics:

symmetry ⟶ *action* ⟶ *experiments* (Zee 1993)

Lorentz invariance
Yang-Mills gauge symmetry

Conclusion

The widespread notion that symmetries “explain” conservation laws is probably due to pragmatic considerations.

Certainly not to modern Noetherian logic (MAO).

In particular doubts arise regarding the role of the homogeneity of time as the source of energy conservation.