Title: Relational dynamics: interacting clocks and systems and quantum time dilation

Speakers: Mehdi Ahmadi

Collection: Quantizing Time

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Abstract: Time is absolute in quantum mechanics, whereas it is dynamical in general relativity. This is considered as one of the main obstacles towards unifying quantum theory and gravity. Relational quantum dynamics offers a possible solution by treating clocks as internal quantum systems, which promotes time to a dynamical quantity. This talk begins with a quick overview of time in relational quantum dynamics. We then explain that the inclusion of an interaction term coupling the clock and system causes the system dynamics to be governed by a time-nonlocal SchrÃ¶dinger equation. Moreover, we demonstrate a quantum time dilation phenomena wherein we analyze the effect of non-classical states of quantum clocks on relativistic time dilation.

Relational dynamics: interacting clocks and systems and quantum time dilation

Mehdi Ahmadi
Santa Clara University

In collaboration with Alexander R. H. Smith

Outline

- Relational quantum dynamics (Page & Wootters)
- Interacting clocks and systems

Quantizing time: Interacting clocks and systems

Alexander R. H. Smith[1] and Mehdi Ahmadi[2] Quantum 3, 160 (2019).

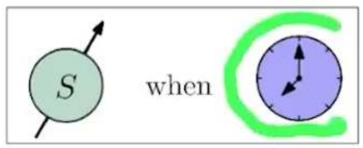
- Quantum time dilation

Quantum clocks observe classical and quantum time dilation

Alexander R. H. Smith & Mehdi Ahmadi

Nature Communications **11**, Article number: 5360 (2020) | Cite this article

- Relational quantum dynamics (Page & Wootters)



- Interacting clocks and systems
- Quantum time dilation

- **Relational quantum dynamics** (Page & Wootters)
  - For an asymptotically-flat universe, the long-range gravitational field provides a **superselection rule for the total energy**, which then causes t to be inaccessible.
  - Communication in the absence of synchronized clocks
    - Physical states: $\mathscr{G}[\rho] = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} dt U(t) \rho U^{\dagger}(t) = \sum_i \Pi_{E_i} \rho \Pi_{E_i}$

Quantum parameter estimation with imperfect reference frames

Dominik Šafránek[1], Mehdi Ahmadi[2,1] and Ivette Fuentes[1]

Published 3 March 2015 • © 2015 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 17, March 2015

Citation Dominik Šafránek *et al* 2015 *New J. Phys.* **17** 033012

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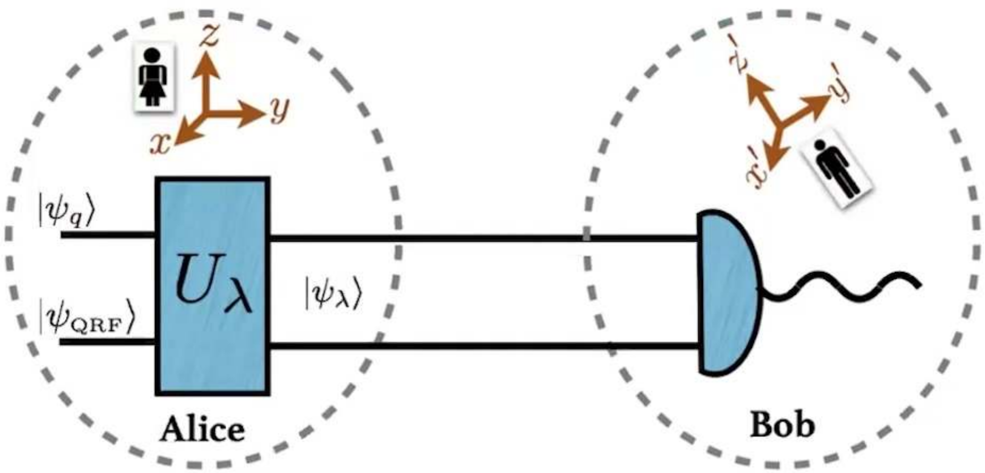
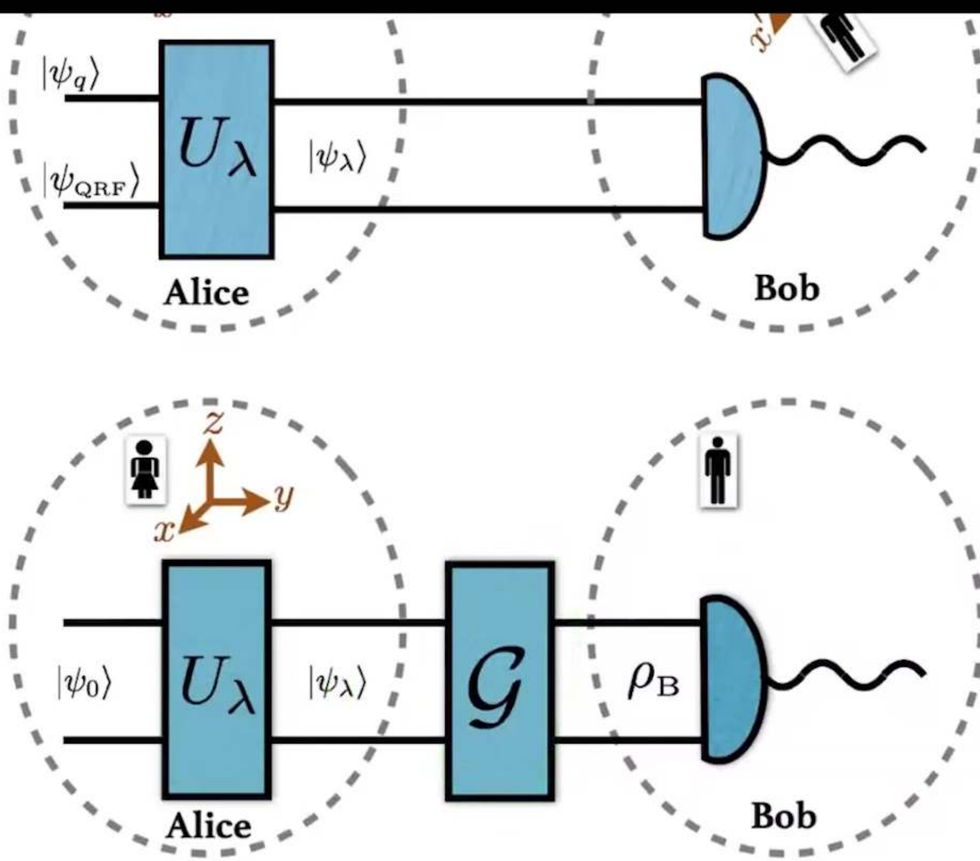
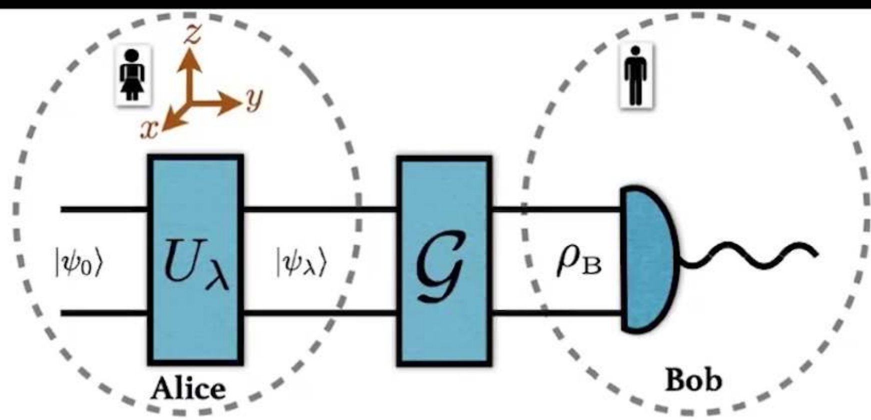



$|\psi_q\rangle$
$|\psi_{\mathrm{QRF}}\rangle$
U_λ
$|\psi_\lambda\rangle$
Alice
Bob
z
y
x
$|\psi_0\rangle$
U_λ
$|\psi_\lambda\rangle$
$\mathcal{G}$
ρ_{B}
Alice
Bob


Mehdi Ahmadi



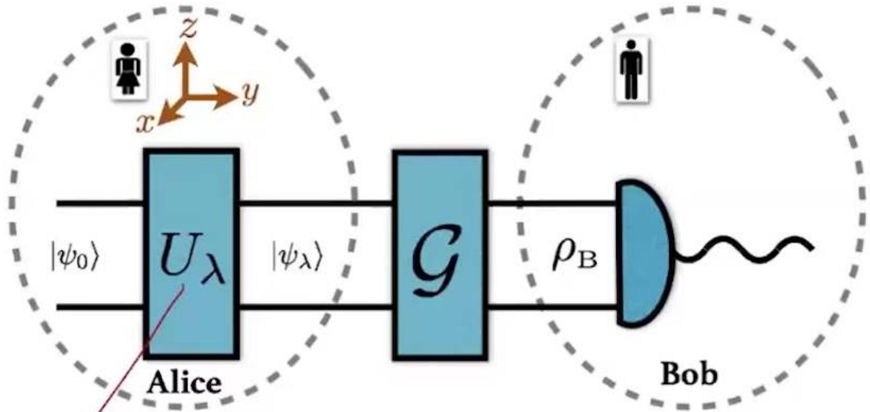
Quantum Fisher information loss:

ρ_λ $\rho_{\lambda+\varepsilon}$

$$l(\rho_\lambda, \hat{G}) = H(\rho_\lambda) - H(\mathcal{G}[\rho_\lambda])$$

$$l(\rho, \hat{G}) = 4\sum_i \frac{(\mathrm{Cov}_\rho(\hat{P}_i, \hat{K}))^2}{p_i} \quad \text{where} \quad \hat{G} = \sum_i g_i \hat{P}_i$$

$$\mathrm{Cov}_\rho(\hat{A}, \hat{B}) = \frac{1}{2}\langle\{\hat{A} - \langle\hat{A}\rangle, \hat{B} - \langle\hat{B}\rangle\}\rangle_\rho = \frac{1}{2}\langle\{\hat{A}, \hat{B}\}\rangle_\rho - \langle\hat{A}\rangle_\rho\langle\hat{B}\rangle_\rho$$



Quantum Fisher information loss:

$U_\lambda = e^{-i\hat{K}\lambda}$

$\rho_\lambda \quad \rho_{\lambda+\varepsilon}$

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relates Alice's to Bob's

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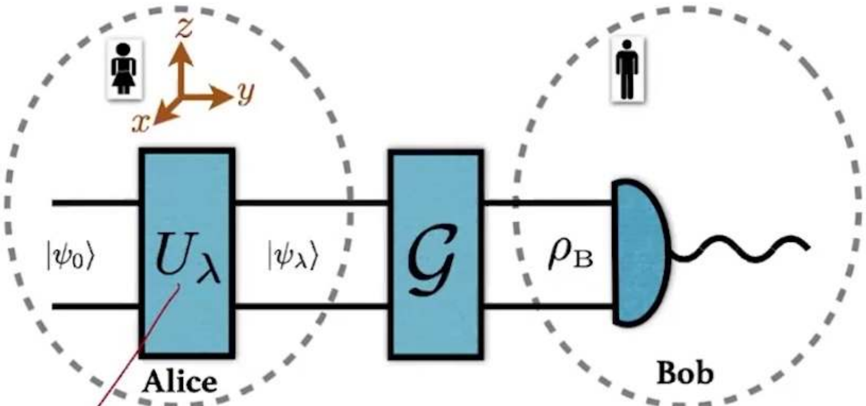

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■ **Relational quantum dynamics** (Page & Wootters)

- For an asymptotically-flat universe, the long-range gravitational field provides a **superselection rule for the total energy**, which then causes t to be inaccessible.
  - Physical states: $\mathscr{G}[\rho] = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} dt U(t) \rho U^{\dagger}(t) = \sum_i \Pi_{E_i} \rho \Pi_{E_i}$
- The **Wheeler-DeWitt equation** implies that the quantum state of the universe is independent of t.
  - Physical states: $H|\Psi\rangle\rangle = 0$
  - $|\Psi\rangle\rangle = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} dt U(t) |\Psi\rangle \Longrightarrow \Pi_{E_0} \rho \Pi_{E_0}$

■ **Examples of quantum clocks**

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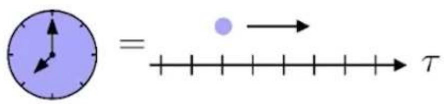
The ideal clock

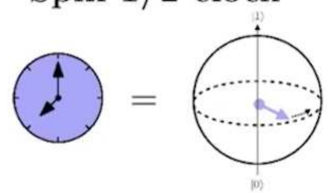


Clock Hilbert space
$\mathcal{H}_C \simeq L^2(\mathbb{R})$

Clock Hamiltonian
$H_C = P_C$

Clock states
$|\tau\rangle = e^{-iP_C\tau}|\tau_0\rangle$

Spin-1/2 clock



Clock Hilbert space
$\mathcal{H}_C \simeq \mathbb{C}^2$

Clock Hamiltonian
$H_C = \Omega\sigma_z$

Clock states
$|\tau\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i\Omega\tau}|1\rangle\right)$

■ Examples of quantum clocks

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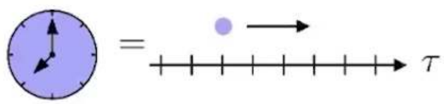

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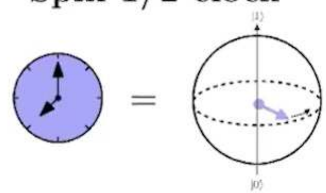

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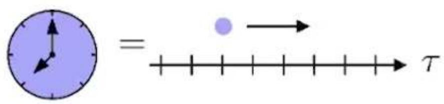

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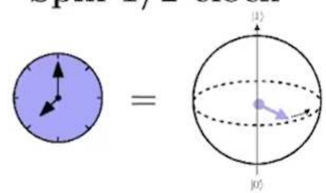

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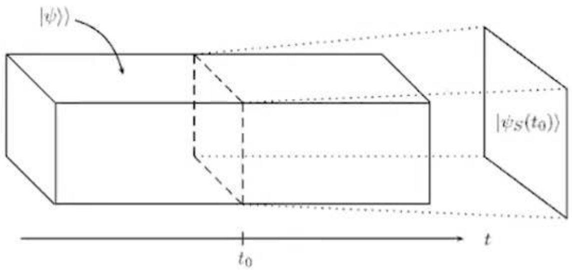
$\langle\tau|\tau'\rangle \neq 0$

■ Recovering Schrödinger's equation

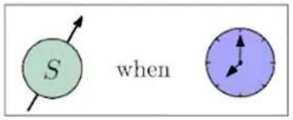


V. Giovannetti, S. Lloyd, and L. Maccone, *Quantum time*, Phys. Rev. D 92, 045022 (2015)

- The physical state: $H|\Psi\rangle\rangle = (H_S + H_{clock})|\Psi\rangle\rangle = 0$

$$|\Psi\rangle\rangle = \left(\int dt\ |t\rangle\langle t| \otimes I_S\right)|\Psi\rangle\rangle = \int dt\ |t\rangle\,|\psi_S(t)\rangle$$

- State of the system at time *t*: $|\psi_S(t)\rangle := (\langle t| \otimes I_S)\,|\Psi\rangle\rangle$



- Take the derivative wrt *t* and impose the Hamiltonian constraint: $i\frac{d}{dt}|\psi_S(t)\rangle = H_S\,|\psi_S(t)\rangle$

- Relational quantum dynamics (Page & Wootters)
- Interacting clocks and systems $H = H_C \otimes I_S + I_C \otimes H_S + H_{int}$
- Quantum time dilation

■ Clock-system interaction

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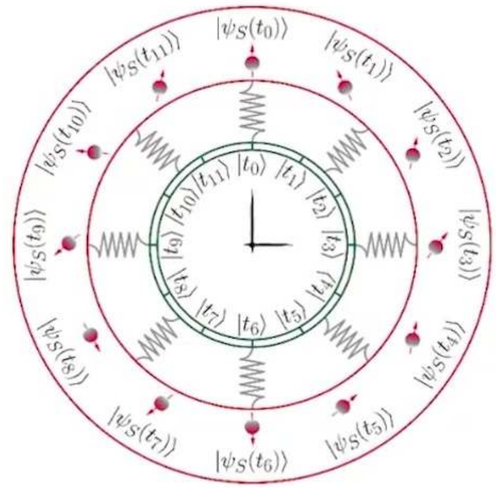

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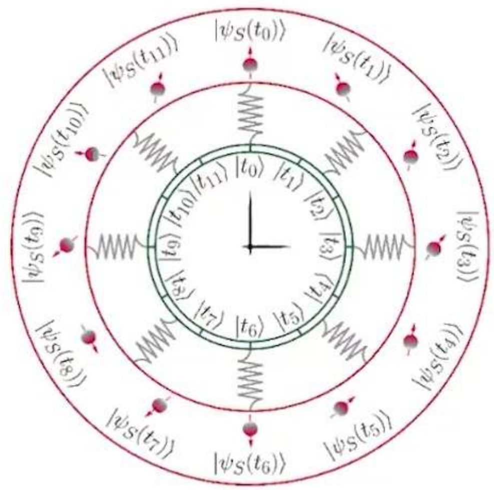
Integral operator adjoint



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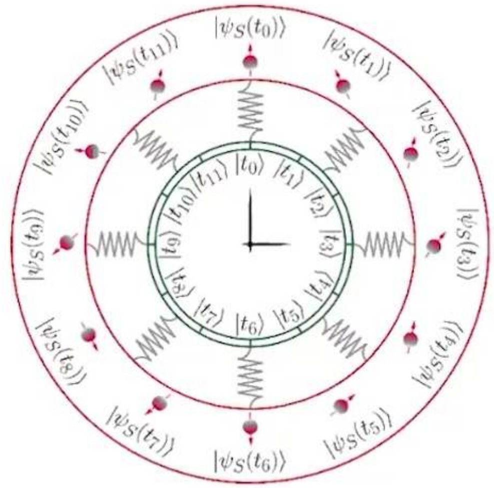

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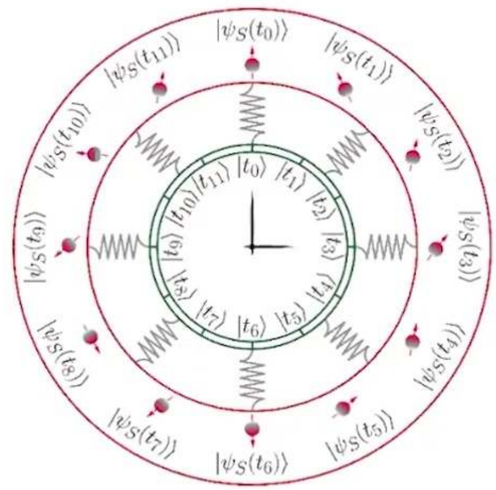

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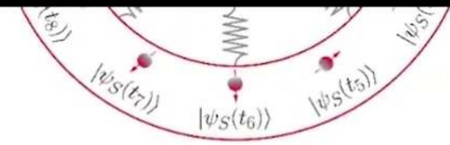
Int... operator adjoint



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$$V(t,t_0) = U(t,t_0)\left[I_S - i\lambda \int_{t_0}^{t} ds\, U(s,t_0)^\dagger \int du\, \bar{K}(s,u)U(u,t_0) + \mathcal{O}\big(\lambda^2\big)\right]$$

↓ unitary

- **Example: Gravitationally interacting clock and system**

- **Solving the modified Schrödinger equation:** $i\frac{d}{dt}\left|\psi_S(t)\right\rangle = \left(H_S + H_K\right)\left|\psi_S(t)\right\rangle$

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- **Example: Gravitationally interacting clock and system**

$$H_{int} = -\frac{Gm_C M_S}{d} = -\frac{G}{c^4 d}H_C \otimes H_S$$

$$K(t,t') := \langle t|\, H_{int}\, |t'\rangle = -\frac{G}{c^4 d} i\delta'(t'-t)H_S$$

$$i\frac{d}{dt}\left|\psi_S(t)\right\rangle = \left[H_S + \frac{G}{c^4 d}H_S^2 + \mathcal{O}\left(\frac{G^2}{c^8 d^2}\right)\right]\left|\psi_S(t)\right\rangle$$

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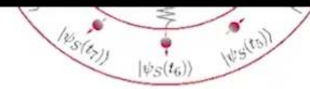
adjoint



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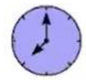
10. Wiseman, H. M. & Milburn, G. J. *Quantum Measurement and Control.* (Cambridge University Press, Cambridge, 2010).

Quantum clock

- A quantum clock is defined by a quadruple:  $= \{\mathscr{H}_C, H_{clock}, \rho, T_C\}$
- The **elapsed time** τ is unitarily encoded: $\rho(\tau) = e^{-iH_{clock}\tau}\rho\, e^{iH_{clock}\tau}$
- Our clock observable T_C should satisfy the following conditions:

  1. On average, it should estimate the elapsed time τ:

  $$\langle T_C \rangle_{\rho(\tau)} = \tau$$

  2. The variance of T_C should be independent of the elapsed time τ:

  $$\langle \left(\Delta T_C\right)^2 \rangle_{\rho(\tau)} = \langle \left(\Delta T_C\right)^2 \rangle_{\rho}$$

- **Covariant POVMs** satisfy these two conditions.
  - **Covariance:** $e^{iH_{clock}\tau}\hat{E}_C(\xi)e^{-iH_{clock}\tau} = \hat{E}_C(\xi - \tau)$

■ Constructing optimal clocks using covariant quantum estimation

5. Holevo, A. S. *Probabilistic and Statistical Aspects of Quantum Theory*, Vol. 1 of *Statistics and Probability* (North-Holland, Amsterdam, 1982).

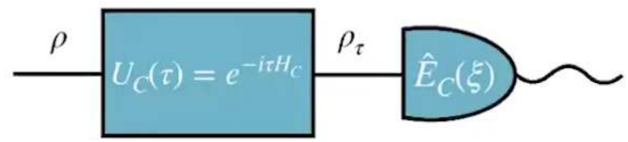
$\rho = |\psi_{clock}\rangle\langle\psi_{clock}|$ $\qquad \rho_\tau = U_C(\tau)\rho U_C^\dagger(\tau)$

$\hat{E}_C(\xi) = \mu\,|\,\xi\rangle\langle\xi\,|$

M. J. W. Hall, in "Quantum Communications and Measurement" (R. Hudson, V. P. Belavkin, and O. Hirota, Eds.), Plenum, New York, 1995.

$$p(\xi\,|\,\tau) = \mathrm{tr}\left[\hat{E}_C(\xi)\rho_\tau\right]$$

$$F(\tau;\rho_\tau) = \int d\xi\, p(\xi\,|\,\tau)\left(\frac{\mathrm{d}\ln p(\xi\,|\,\tau)}{d\tau}\right)^2$$



- **Covariance:** $e^{iH_{clock}\tau}\hat{E}_C(\xi)e^{-iH_{clock}\tau} = \hat{E}_C(\xi - \tau)$
  - Proper time and clock energy uncertainty relation:

$$\langle(\Delta T_C)^2\rangle_\rho \geq \frac{1}{F(\tau;\rho_\tau)} \geq \frac{1}{4\langle(\Delta H_{clock})^2\rangle_\rho}$$

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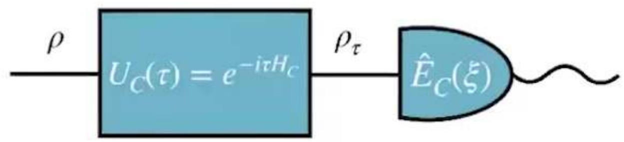

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$|\psi_{clock}\rangle$ gaussian $\qquad$ Covariance

■ A relativistic free particle

- Lorentz-invariant action: $\mathcal{S} = \int d\tau \left[-mc^2 + P_q \frac{dq}{d\tau} - H^{\text{clock}} \right]$

- The corresponding Hamiltonian constraint:

$$C_H := \eta_{\mu\nu} P^\mu P^\nu + M^2 c^4 \approx 0$$

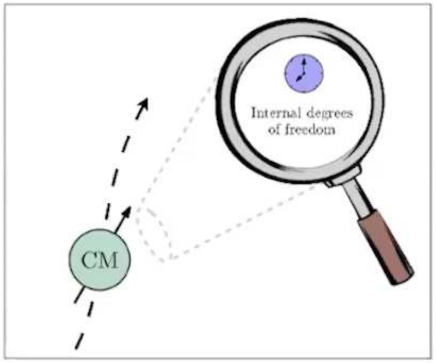
$$C_H = C_+ C_- \qquad C_\pm := P^0 c \pm \sqrt{\eta_{ij} P^i P^j c^2 + M^2 c^4}$$

  - Restricting to the positive energy sector: $C_+ := P^0 c + \sqrt{\eta_{ij} P^i P^j c^2 + M^2 c^4}$

- The quantum constraint:

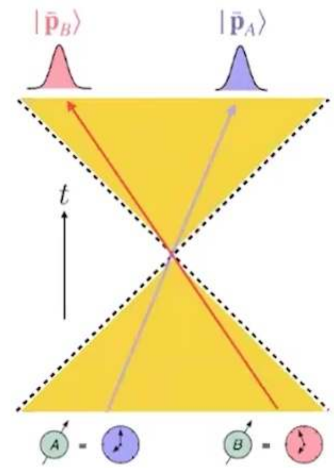
$$C_+ |\Psi\rangle\rangle = \left(P^0 c + H_{clock} + H_{cm} + H_{int} \right) |\Psi\rangle\rangle = 0$$

$$H_{int} = -\frac{1}{mc^2} \left(H_{cm} \otimes H_{clock} + \frac{1}{2} H_{cm}^2 \right) + \mathcal{O}\left(\frac{1}{m^2 c^4} \right)$$



■ **Time dilation of clock A relative to clock B**

$$\tau_A = \frac{\gamma_B}{\gamma_A}\tau_B = \left[1 - \frac{\bar{\mathbf{p}}_A^2 - \bar{\mathbf{p}}_B^2}{2m^2c^2}\right]\tau_B + \cdots$$

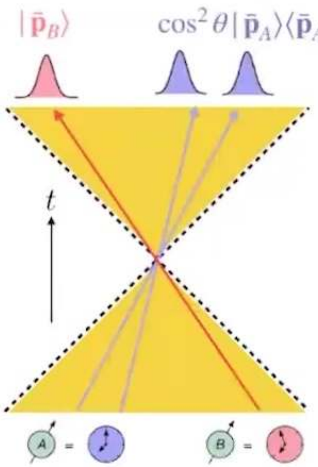


$$\text{Prob}\left[T_A = \tau_A \text{ when } T_B = \tau_B\right] = \frac{\text{Prob}\left[T_A = \tau_A \;\&\; T_B = \tau_B\right]}{\text{Prob}\left[T_B = \tau_B\right]}$$
$$= \frac{\langle\langle\Psi\,|\,E_A(\tau_A)E_B(\tau_B)\,|\,\Psi\rangle\rangle}{\langle\langle\Psi\,|\,E_B(\tau_B)\,|\,\Psi\rangle\rangle}$$

$$\langle T_A\rangle = \left[1 - \frac{\bar{\mathbf{p}}_A^2 - \bar{\mathbf{p}}_B^2}{2m^2c^2}\right]\tau_B + \cdots$$

■ **Quantum contribution**

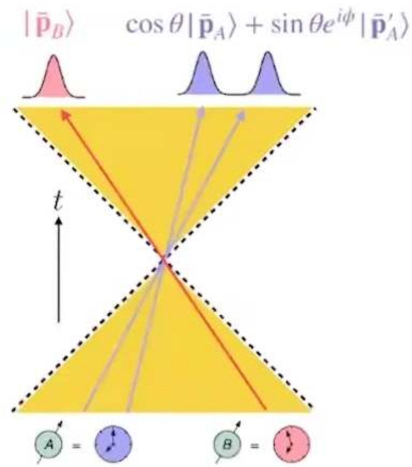
■ Quantum contribution



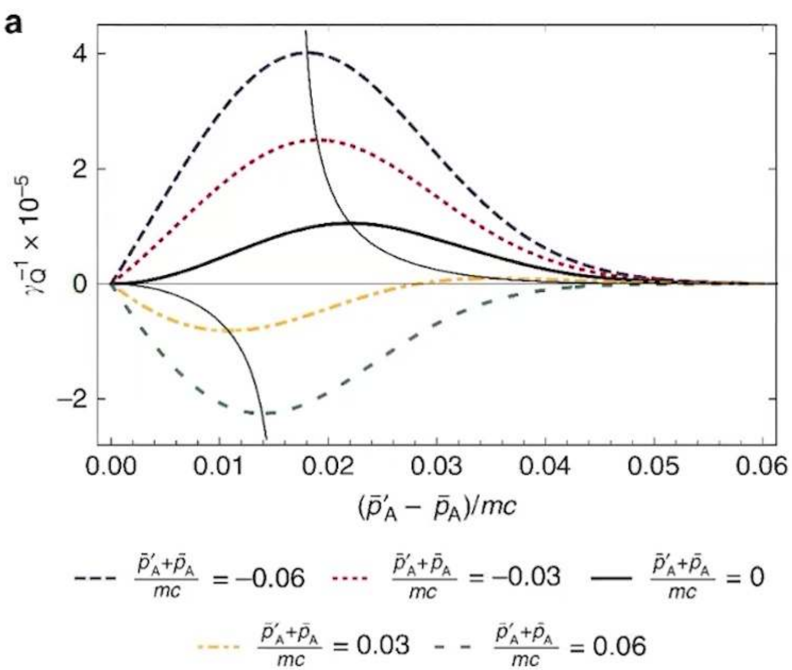
$$\langle T_A\rangle = \left(\gamma_{\mathrm{C}}^{-1} + \gamma_{\mathrm{Q}}^{-1}\right)\tau_B$$

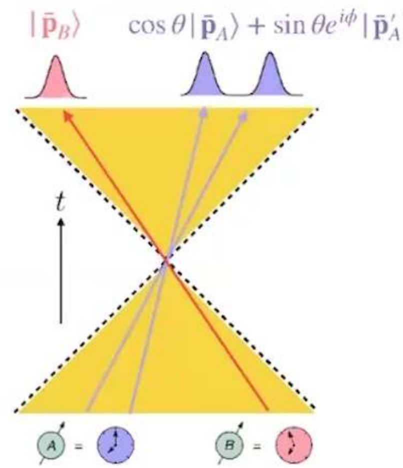
$$\gamma_{\mathrm{C}}^{-1} := \frac{\bar{\mathbf{p}}_A^2\cos^2\theta + \bar{\mathbf{p}}_A'^2\sin^2\theta}{2m^2c^2}$$

$$\gamma_{\mathrm{Q}}^{-1} := \frac{\cos\phi\sin 2\theta\left[\left(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A\right)^2 - 2\left(\bar{\mathbf{p}}_A'^2 - \bar{\mathbf{p}}_A^2\right)\cos 2\theta\right]}{8m^2c^2\left[\cos\phi\sin 2\theta + e^{(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2/4\Delta^2}\right]}$$

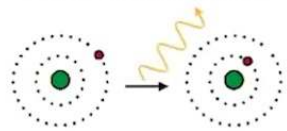


$$\gamma_{\mathrm{Q}}^{-1} := \frac{\cos\phi \sin 2\theta \left[\left(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A\right)^2 - 2\left(\bar{\mathbf{p}}'^2_A - \bar{\mathbf{p}}^2_A\right)\cos 2\theta \right]}{8m^2c^2 \left[\cos\phi \sin 2\theta + e^{(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2/4\Delta^2}\right]}$$





Quantum time dilation in atomic spectra

Atomic center of mass in a momentum superposition		Spontaneous emission from excited atom
	$\otimes$	
$\cos\theta\,\lvert\bar{\mathbf{p}}\rangle + \sin\theta\, e^{i\phi}\,\lvert\bar{\mathbf{p}}'\rangle$		$\lvert e\rangle \to \lvert g\rangle$

$$\hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{field}} + \hat{H}_{\text{af}}$$

$$\hat{H}_{\text{atom}} = \frac{\hat{\mathbf{p}}^2}{2m} - \frac{1}{8}\frac{\hat{\mathbf{p}}^4}{m^3c^2} + \hbar\Omega\left(1 - \frac{1}{2}\frac{\hat{\mathbf{p}}^2}{m^2c^2}\right)\lvert e\rangle\langle e\rvert$$

$$\hat{H}_{\text{af}} = -\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}^{\perp} - \frac{1}{2m}\left[\hat{\mathbf{p}}\cdot\left(\hat{\mathbf{B}}\times\hat{\mathbf{d}}\right) + \left(\hat{\mathbf{B}}\times\hat{\mathbf{d}}\right)\cdot\hat{\mathbf{p}}\right]$$

P. T. Grochowski, A. R. H. Smith, A. Dragan, K. Dębski, *Quantum time dilation in atomic spectra*, Phys. Rev. Research 3, 023053(2021)
M. Sonnleitner and S. M. Barnett, *Mass-energy and anomalous friction in quantum optics*, Phys. Rev. A 98, 042106 (2018)

Piotr T. Grochowski[1,*] Alexander R. H. Smith[2,3,†] Andrzej Dragan[4,5,‡] and Kacper Dębski[4,§]

Spontaneous emission rate experiences quantum time dilation:

$$\frac{\Gamma_{\text{sup}} - \Gamma_{\text{cl}}}{\Gamma_0} = \gamma_{\text{Q}}^{-1}$$

$$\gamma_{\text{Q}}^{-1} := \frac{\cos\phi \sin 2\theta \left[\left(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A \right)^2 - 2 \left(\bar{\mathbf{p}}'^2_A - \bar{\mathbf{p}}^2_A \right) \cos 2\theta \right]}{8m^2c^2 \left[\cos\phi \sin 2\theta + e^{(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2 / 4\Delta^2} \right]}$$

which suggests **quantum time dilation is universal!**

Effects of relativistic momentum coherence on atomic transitions

Kacper Dębski[1,*] Piotr T. Grochowski[2,3,†]
Alexander R. H. Smith[4,5,‡] Mehdi Ahmadi[6,§] and Andrzej Dragan[1,7,¶]

PHYSICAL REVIEW RESEARCH 3, 023055 (2021)

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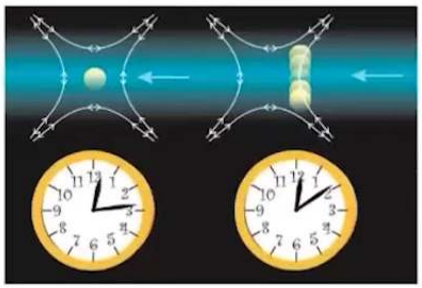
Is quantum time dilation observable?

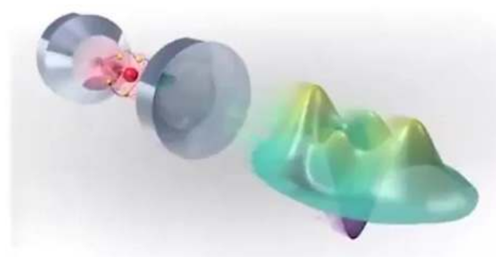
Time dilation observed for clocks moving several meters per second

C. W. Chou, D. B. Hume, T. Rosenband, D. J. Wineland, *Optical Clocks and Relativity*, Science 329, 1630 (2010)



Atomic superpositions separated by several meters per second have been created.

P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben, Phys. Rev. Lett.102, 240402 (2009)



An estimate of the quantum contribution is $\gamma_Q^{-1} \approx 10^{-15}$

This quantum contribution is within the resolution of current atomic clock provided the coherence time of the superposition is longer than the atom's lifetime.


Thanks for your attention!
Mischa Woods