

Title: Relational dynamics: interacting clocks and systems and quantum time dilation

Speakers: Mehdi Ahmadi

Collection: Quantizing Time

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Abstract: Time is absolute in quantum mechanics, whereas it is dynamical in general relativity. This is considered as one of the main obstacles towards unifying quantum theory and gravity. Relational quantum dynamics offers a possible solution by treating clocks as internal quantum systems, which promotes time to a dynamical quantity. This talk begins with a quick overview of time in relational quantum dynamics. We then explain that the inclusion of an interaction term coupling the clock and system causes the system dynamics to be governed by a time-nonlocal Schrödinger equation. Moreover, we demonstrate a quantum time dilation phenomena wherein we analyze the effect of non-classical states of quantum clocks on relativistic time dilation.

Relational dynamics: interacting clocks and systems and quantum time dilation

**Mehdi Ahmadi
Santa Clara University**



In collaboration with Alexander R. H. Smith



Outline

- Relational quantum dynamics (Page & Wootters)

- Interacting clocks and systems

Quantizing time: Interacting clocks and systems

Alexander R. H. Smith¹ and Mehdi Ahmadi²

Quantum 3, 160 (2019).

- Quantum time dilation

Quantum clocks observe classical and quantum time dilation

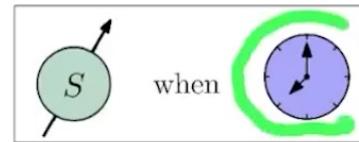
Alexander R. H. Smith  & Mehdi Ahmadi 

Nature Communications 11, Article number: 5360 (2020) | [Cite this article](#)





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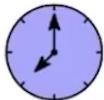


Mehdi Ahmadi

■ Relational quantum dynamics (Page & Wootters)

- For an asymptotically-flat universe, the long-range gravitational field provides a **superselection rule for the total energy**, which then causes t to be inaccessible.
- Communication in the absence of synchronized clocks

► Physical states: $\mathcal{G}[\rho] = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} dt U(t) \rho U^\dagger(t) = \sum_i \Pi_{E_i} \rho \Pi_{E_i}$



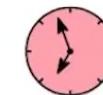
Quantum parameter estimation with imperfect reference frames

Dominik Šafránek¹, Mehdi Ahmadi^{2,1} and Ivette Fuentes¹

Published 3 March 2015 • © 2015 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

[New Journal of Physics](#), [Volume 17](#), [March 2015](#)

Citation Dominik Šafránek et al 2015 *New J. Phys.* **17** 033012



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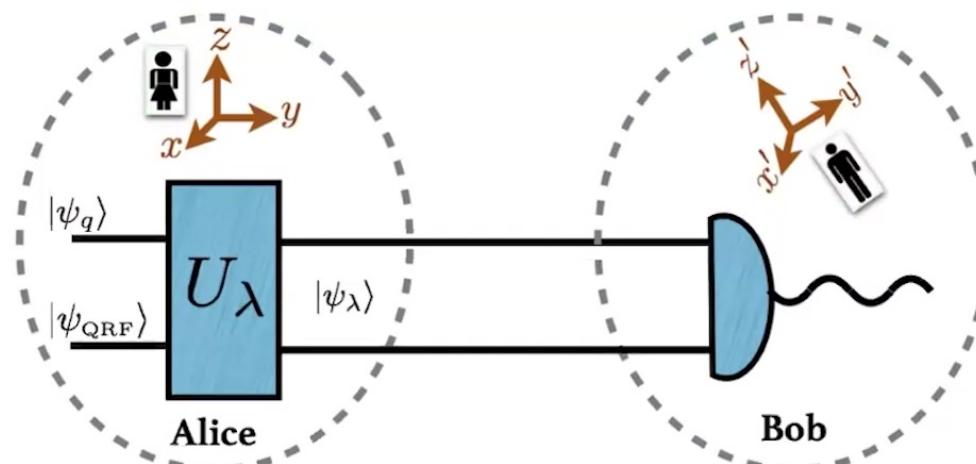
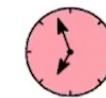
Quantum parameter estimation with imperfect reference frames

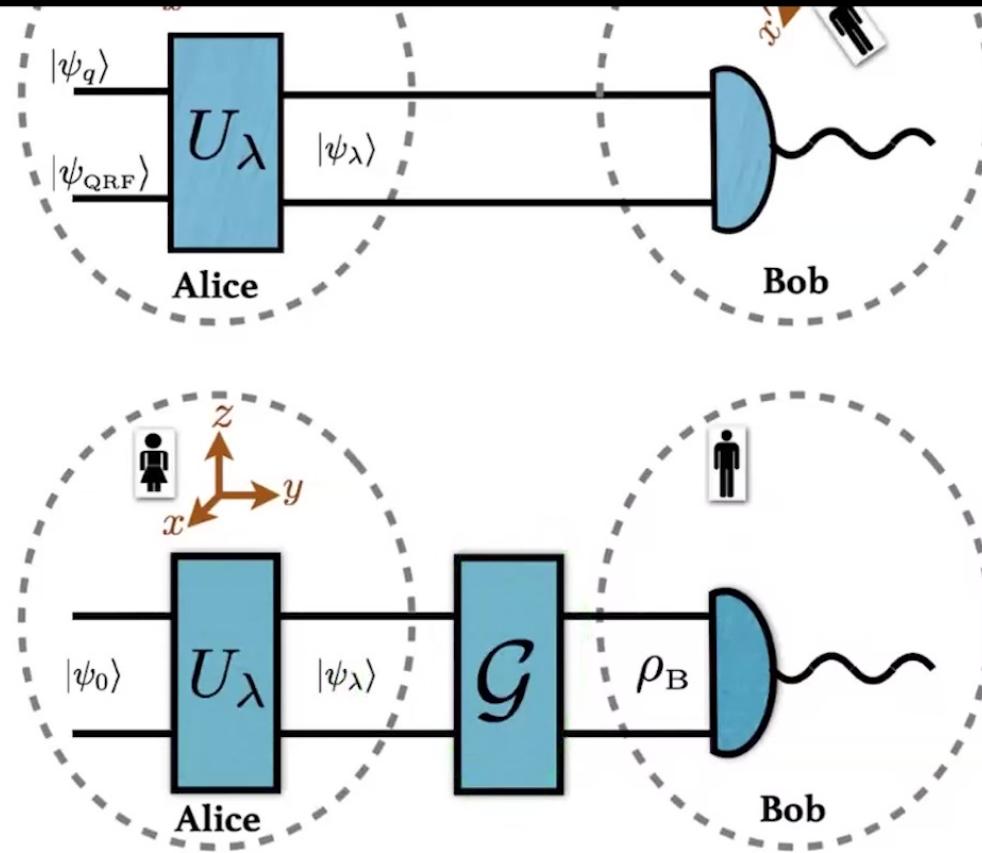
Dominik Šafránek¹, Mehdi Ahmadi^{2,1} and Ivette Fuentes¹

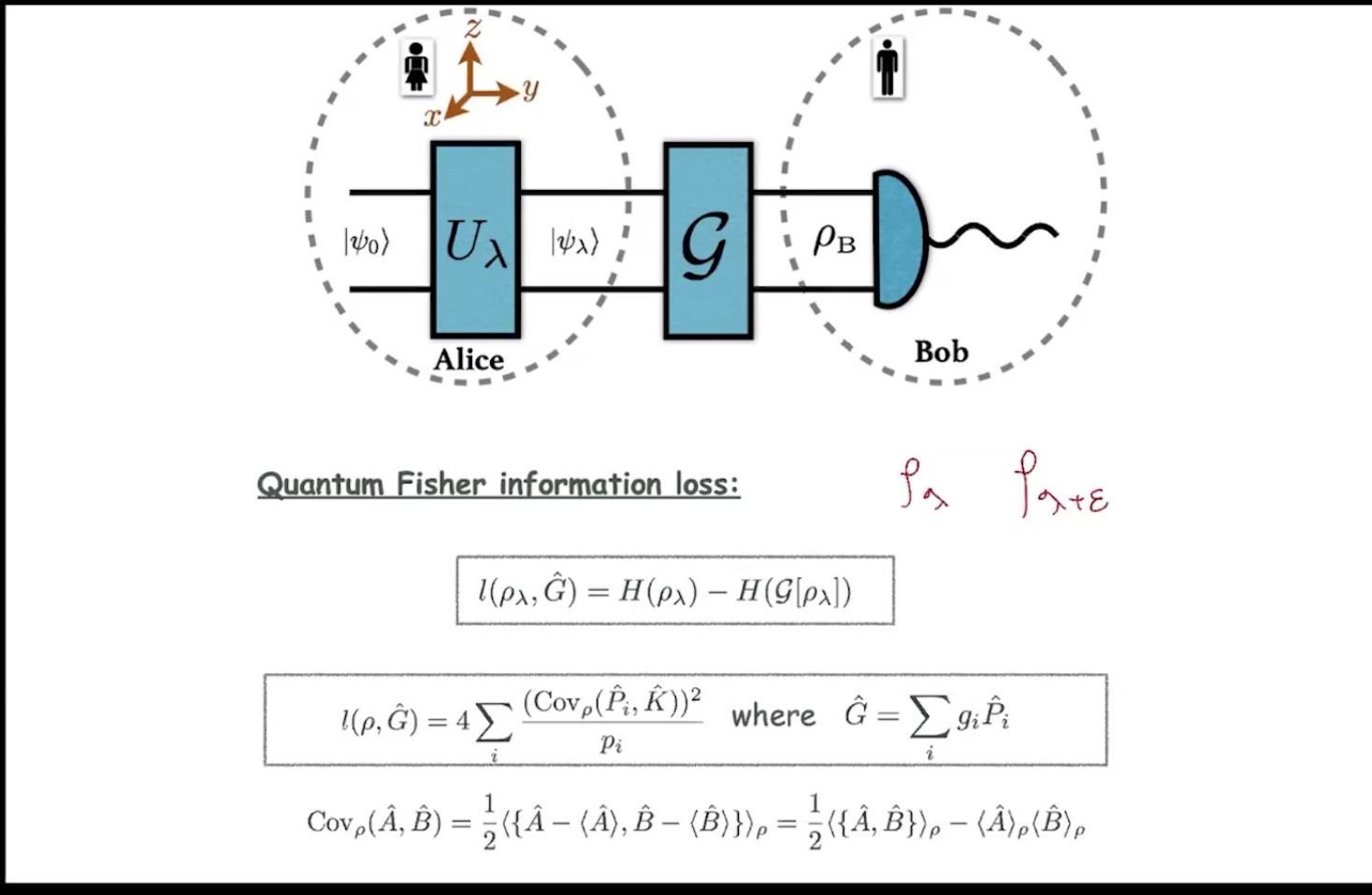
Published 3 March 2015 • © 2015 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

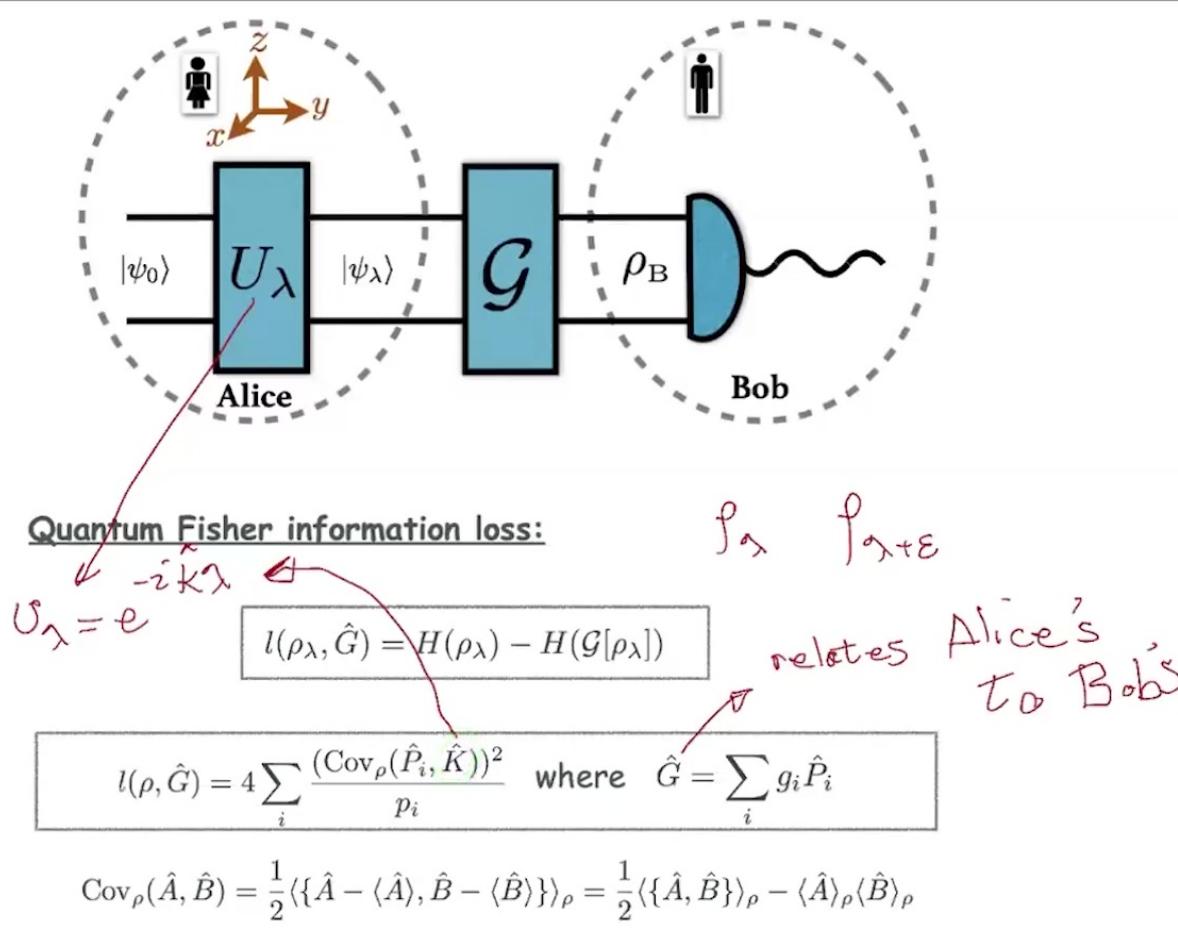
[New Journal of Physics, Volume 17, March 2015](#)

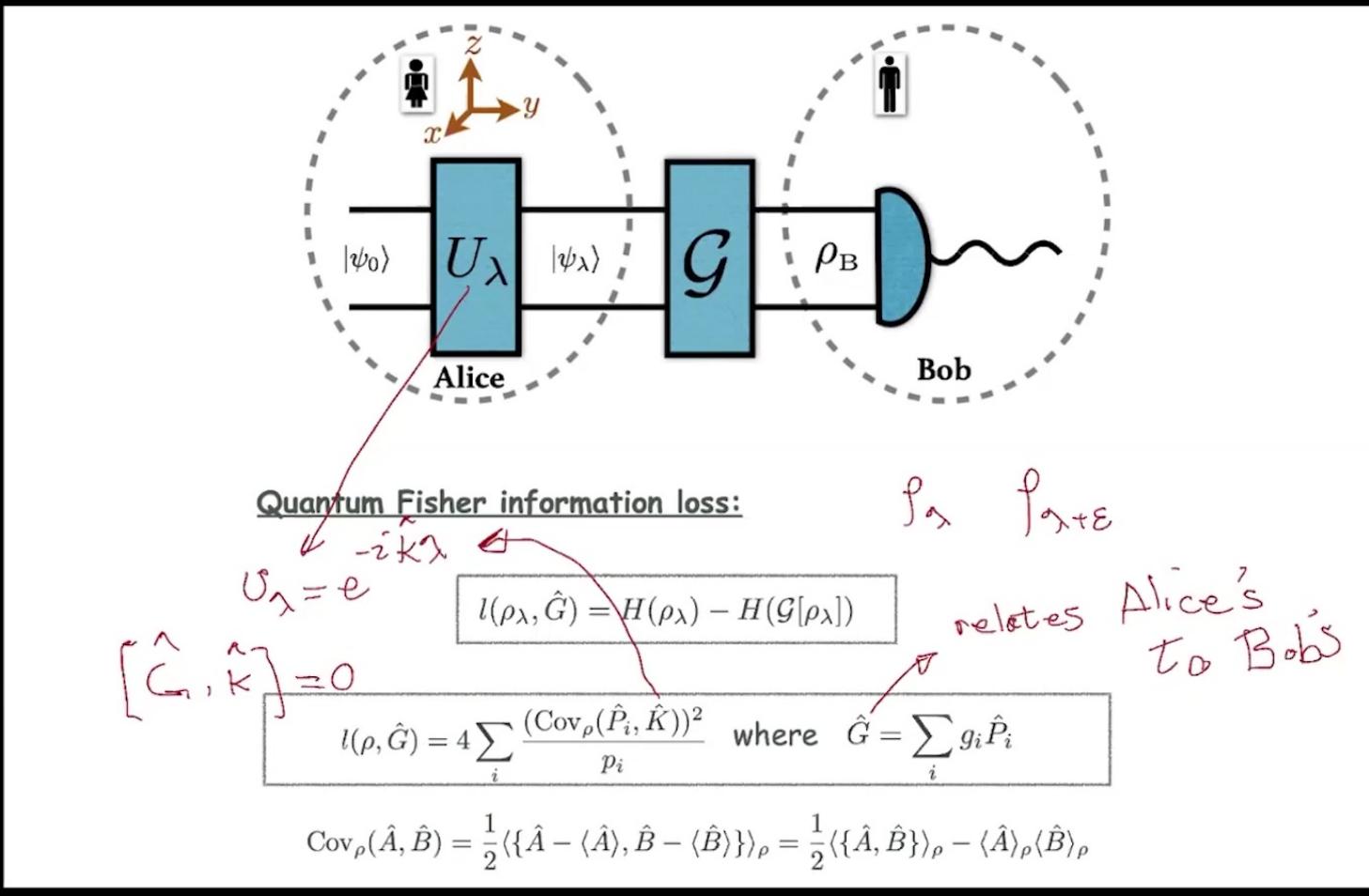
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- For an asymptotically-flat universe, the long-range gravitational field provides a **superselection rule for the total energy**, which then causes t to be inaccessible.

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- The **Wheeler-DeWitt equation** implies that the quantum state of the universe is independent of t .

► Physical states: $H |\Psi\rangle\rangle = 0$

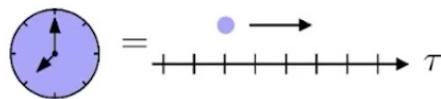
► $|\Psi\rangle\rangle = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} dt U(t) |\Psi\rangle \implies \Pi_{E_0} \rho \Pi_{E_0}$

■ Examples of quantum clocks



■ Examples of quantum clocks

The ideal clock



Clock Hilbert space

$$\mathcal{H}_C \simeq L^2(\mathbb{R})$$

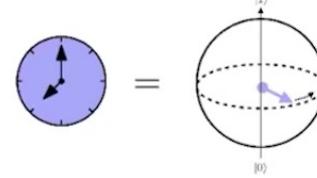
Clock Hamiltonian

$$H_C = P_C$$

Clock states

$$|\tau\rangle = e^{-iP_C\tau} |\tau_0\rangle$$

Spin-1/2 clock



Clock Hilbert space

$$\mathcal{H}_C \simeq \mathbb{C}^2$$

Clock Hamiltonian

$$H_C = \Omega \sigma_z$$

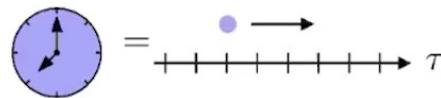
Clock states

$$|\tau\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{-i\Omega\tau} |1\rangle)$$



■ Examples of quantum clocks

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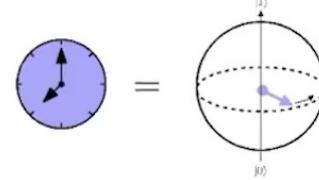
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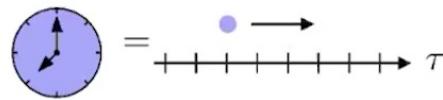
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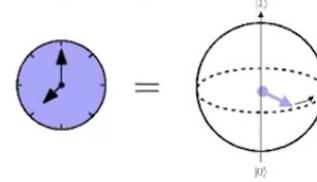
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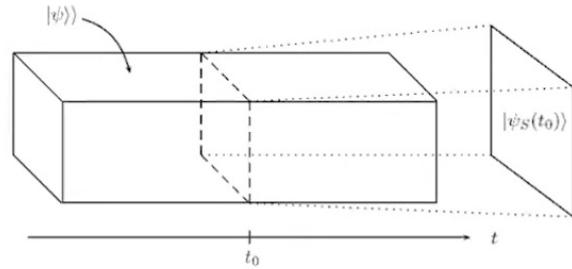
$$|\tau\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{-i\Omega\tau} |1\rangle)$$

$$\langle \tau | \tau' \rangle \neq 0$$



Mehdi Ahmadi

■ Recovering Schrödinger's equation



V. Giovannetti, S. Lloyd, and L. Maccone, *Quantum time*, Phys. Rev. D 92, 045022 (2015)

- The physical state: $H|\Psi\rangle = (H_S + H_{clock})|\Psi\rangle = 0$

$$|\Psi\rangle = \left(\int dt |t\rangle \langle t| \otimes I_S \right) |\Psi\rangle = \int dt |t\rangle |\psi_S(t)\rangle$$

- State of the system at time t : $|\psi_S(t)\rangle := (\langle t| \otimes I_S) |\Psi\rangle$



- Take the derivative wrt t and impose the Hamiltonian constraint: $i \frac{d}{dt} |\psi_S(t)\rangle = H_S |\psi_S(t)\rangle$



- Relational quantum dynamics (Page & Wootters)
- Interacting clocks and systems $H = H_C \otimes I_S + I_C \otimes H_S + H_{int}$
- Quantum time dilation



■ Clock-system interaction

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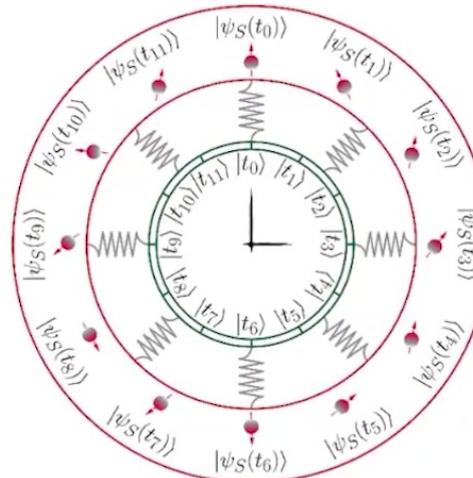
$$i \frac{d}{dt} |\psi_S(t)\rangle = H_S |\psi_S(t)\rangle + \langle t| H_{int} |\Psi\rangle$$

$$i \frac{d}{dt} |\psi_S(t)\rangle = H_S |\psi_S(t)\rangle + \int dt' K(t, t') |\psi_S(t')\rangle$$

$$K(t, t') := \langle t| H_{int} |t'\rangle$$

- The modified Schrödinger equation: $i \frac{d}{dt} |\psi_S(t)\rangle = (H_S + H_K) |\psi_S(t)\rangle$

- Solving the modified Schrödinger equation: $|\psi_S(t)\rangle = V(t, t_0) |\psi_S(t_0)\rangle = \left[\sum_n \lambda^n V_n(t, t_0) \right] |\psi_S(t_0)\rangle$





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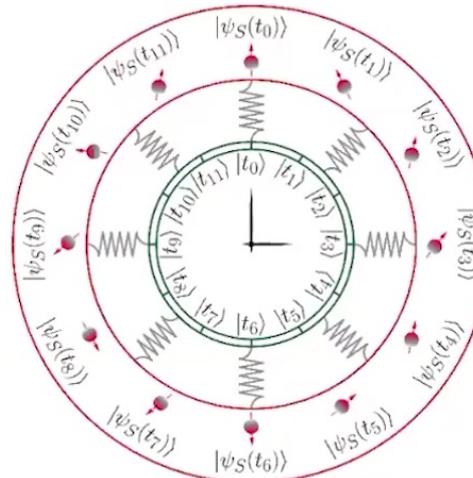
Integral
operator
adjoint

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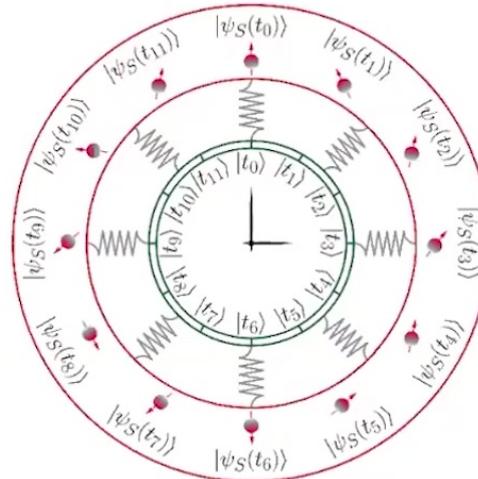
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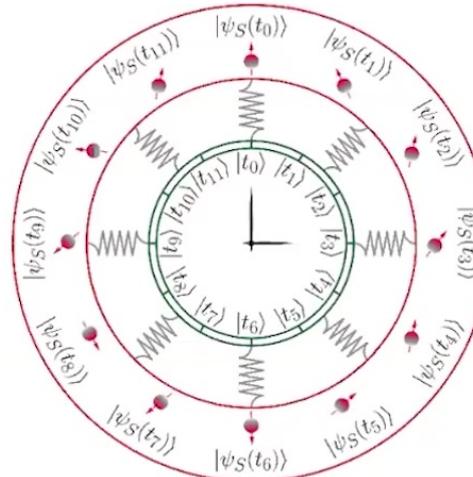
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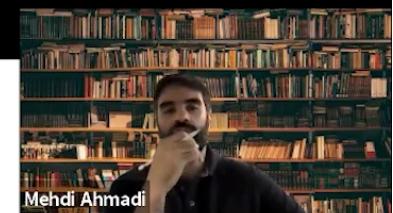
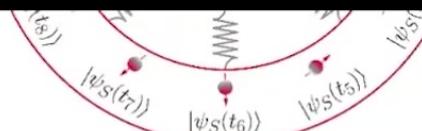


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Invert operators
adjoint



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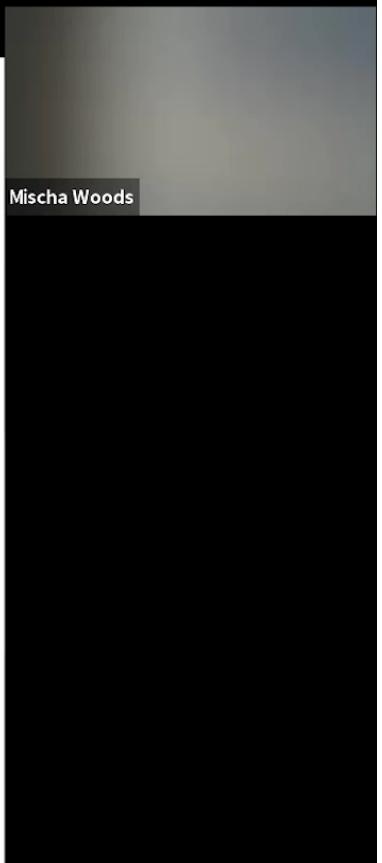
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$$V(t, t_0) = U(t, t_0) \left[I_S - i\lambda \int_{t_0}^t ds U(s, t_0)^\dagger \int du \bar{K}(s, u) U(u, t_0) + \mathcal{O}(\lambda^2) \right]$$

↓ unitary

- Example: Gravitationally interacting clock and system**



- Solving the modified Schrödinger equation: $i\frac{d}{dt}|\psi_S(t)\rangle = (H_S + H_K)|\psi_S(t)\rangle$

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- Example: Gravitationally interacting clock and system

$$H_{int} = -\frac{Gm_C M_S}{d} = -\frac{G}{c^4 d} H_C \otimes H_S$$

$$K(t, t') := \langle t | H_{int} | t' \rangle = -\frac{G}{c^4 d} i \delta'(t' - t) H_S$$

$$i\frac{d}{dt}|\psi_S(t)\rangle = \left[H_S + \frac{G}{c^4 d} H_S^2 + \mathcal{O}\left(\frac{G^2}{c^8 d^2}\right) \right] |\psi_S(t)\rangle$$

$$i \frac{d}{dt} |\psi_S(t)\rangle = H_S |\psi_S(t)\rangle + \int dt' K(t, t') |\psi_S(t')\rangle$$

open adjoint

$$K(t, t') := \langle t | H_{int} | t' \rangle$$

$$H_{int} = \propto \bar{H}_{int}$$

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$$V(t, t_0) = U(t, t_0) \left[I_S - i\lambda \int_{t_0}^t ds U(s, t_0)^\dagger \int du \bar{K}(s, u) U(u, t_0) + \mathcal{O}(\lambda^2) \right]$$

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■ Quantum clock

- A quantum clock is defined by a quadruple:  $\{ \mathcal{H}_C, H_{clock}, \rho, T_C \}$
- The **elapsed time** τ is unitarily encoded: $\rho(\tau) = e^{-iH_{clock}\tau} \rho e^{iH_{clock}\tau}$
- Our clock observable T_C should satisfy the following conditions:

1. On average, it should estimate the elapsed time τ :

$$\langle T_C \rangle_{\rho(\tau)} = \tau$$

2. The variance of T_C should be independent of the elapsed time τ :

$$\langle (\Delta T_C)^2 \rangle_{\rho(\tau)} = \langle (\Delta T_C)^2 \rangle_{\rho}$$

- **Covariant POVMs** satisfy these two conditions.

► **Covariance:** $e^{iH_{clock}\tau} \hat{E}_C(\xi) e^{-iH_{clock}\tau} = \hat{E}_C(\xi - \tau)$

10. Wiseman, H. M. & Milburn, G. J. *Quantum Measurement and Control*. (Cambridge University Press, Cambridge, 2010).



■ Constructing optimal clocks using covariant quantum estimation

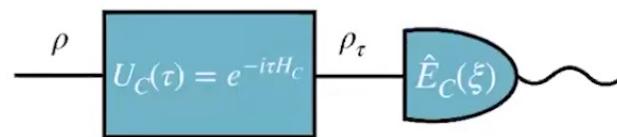
$$\rho = |\psi_{clock}\rangle\langle\psi_{clock}| \quad \rho_\tau = U_C(\tau)\rho U_C^\dagger(\tau)$$

$$\hat{E}_C(\xi) = \mu |\xi\rangle\langle\xi|$$

M. J. W. Hall, in "Quantum Communications and Measurement" (R. Hudson, V. P. Belavkin, and O. Hirota, Eds.), Plenum, New York, 1995.

$$p(\xi|\tau) = \text{tr} [\hat{E}_C(\xi)\rho_\tau]$$

$$F(\tau; \rho_\tau) = \int d\xi p(\xi|\tau) \left(\frac{d \ln p(\xi|\tau)}{d\tau} \right)^2$$



- **Covariance:** $e^{iH_{clock}\tau}\hat{E}_C(\xi)e^{-iH_{clock}\tau} = \hat{E}_C(\xi - \tau)$

- ▷ Proper time and clock energy uncertainty relation:

$$\langle (\Delta T_C)^2 \rangle_\rho \geq \frac{1}{F(\tau; \rho_\tau)} \geq \frac{1}{4 \langle (\Delta H_{clock})^2 \rangle_\rho}$$

5. Holevo, A. S. *Probabilistic and Statistical Aspects of Quantum Theory*, Vol. 1 of *Statistics and Probability* (North-Holland, Amsterdam, 1982).





■ Constructing optimal clocks using covariant quantum estimation

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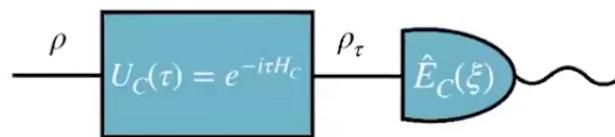
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- **Covariance:** $e^{iH_{clock}\tau}\hat{E}_C(\xi)e^{-iH_{clock}\tau} = \hat{E}_C(\xi - \tau)$

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(Handwritten notes: "Clock" is written above the first term, "gaussian" is written below the first term, and "Covariance" is written below the second term, with arrows pointing from the handwritten text to the corresponding terms in the equation.)

■ A relativistic free particle

- Lorentz-invariant action: $\mathcal{S} = \int d\tau \left[-mc^2 + P_q \frac{dq}{d\tau} - H^{\text{clock}} \right]$

- The corresponding Hamiltonian constraint:

$$C_H := \eta_{\mu\nu} P^\mu P^\nu + M^2 c^4 \approx 0$$

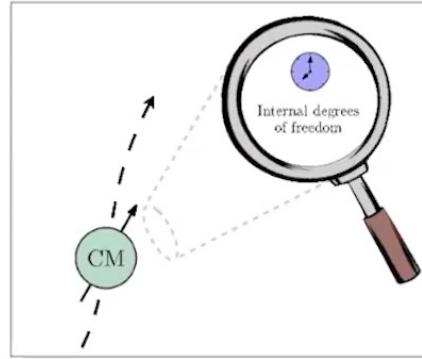
$$C_H = C_+ C_- \quad C_\pm := P^0 c \pm \sqrt{\eta_{ij} P^i P^j c^2 + M^2 c^4}$$

► Restricting to the positive energy sector: $C_+ := P^0 c + \sqrt{\eta_{ij} P^i P^j c^2 + M^2 c^4}$

- The quantum constraint:

$$C_+ |\Psi\rangle = (P^0 c + H_{\text{clock}} + H_{cm} + H_{int}) |\Psi\rangle = 0$$

$$H_{int} = -\frac{1}{mc^2} \left(H_{cm} \otimes H_{clock} + \frac{1}{2} H_{cm}^2 \right) + \mathcal{O}\left(\frac{1}{m^2 c^4}\right)$$

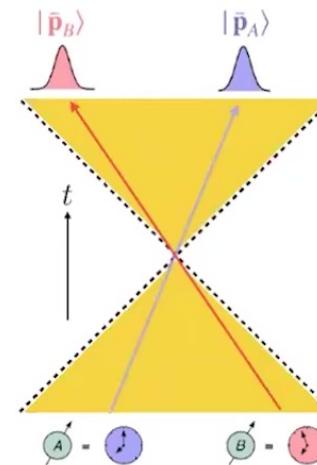


■ Time dilation of clock A relative to clock B

$$\tau_A = \frac{\gamma_B}{\gamma_A} \tau_B = \left[1 - \frac{\bar{p}_A^2 - \bar{p}_B^2}{2m^2c^2} \right] \tau_B + \dots$$

$$\begin{aligned} \text{Prob}[T_A = \tau_A \text{ when } T_B = \tau_B] &= \frac{\text{Prob}[T_A = \tau_A \text{ & } T_B = \tau_B]}{\text{Prob}[T_B = \tau_B]} \\ &= \frac{\langle \langle \Psi | E_A(\tau_A) E_B(\tau_B) | \Psi \rangle \rangle}{\langle \langle \Psi | E_B(\tau_B) | \Psi \rangle \rangle} \end{aligned}$$

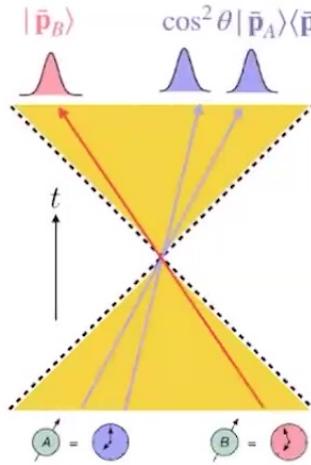
$$\langle T_A \rangle = \left[1 - \frac{\bar{p}_A^2 - \bar{p}_B^2}{2m^2c^2} \right] \tau_B + \dots$$



■ Quantum contribution



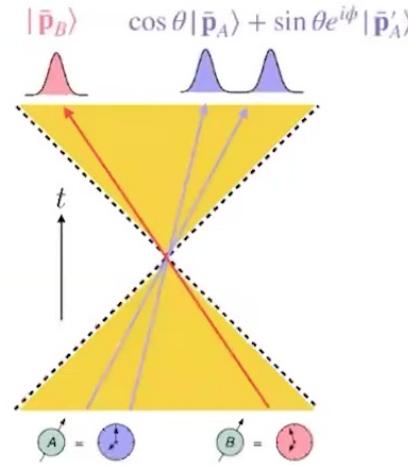
■ Quantum contribution

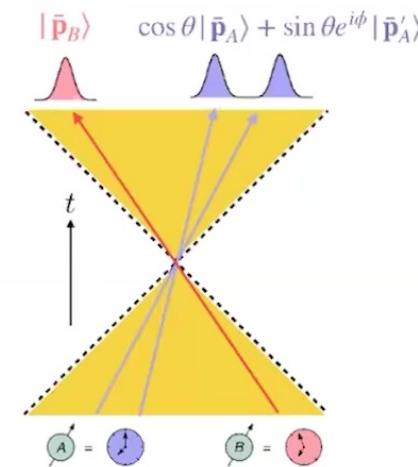
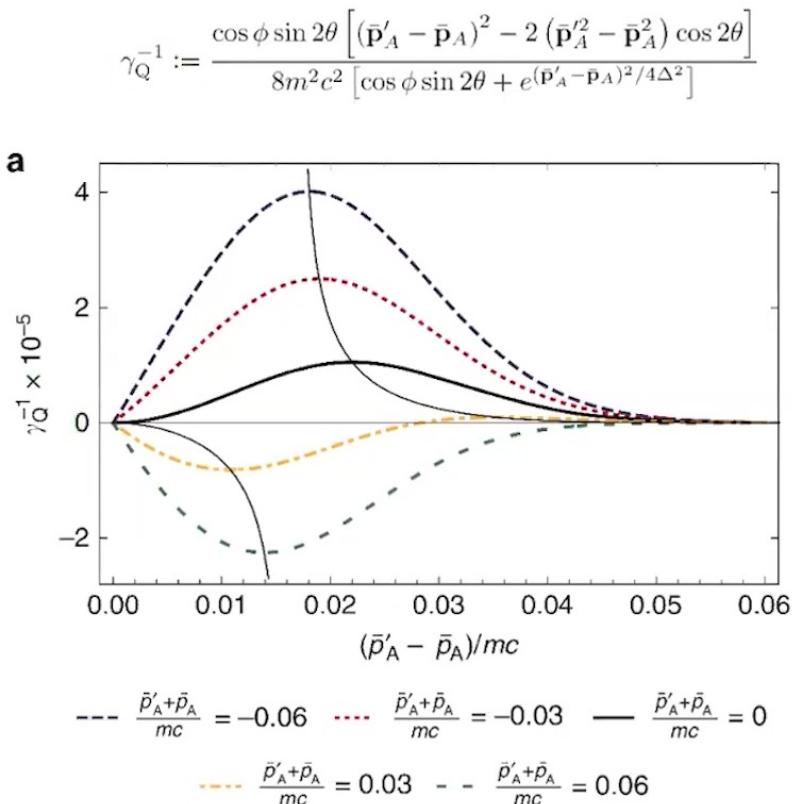


$$\langle T_A \rangle = (\gamma_C^{-1} + \gamma_Q^{-1}) \tau_B$$

$$\gamma_C^{-1} := \frac{\bar{\mathbf{p}}_A^2 \cos^2 \theta + \bar{\mathbf{p}}_A'^2 \sin^2 \theta}{2m^2 c^2}$$

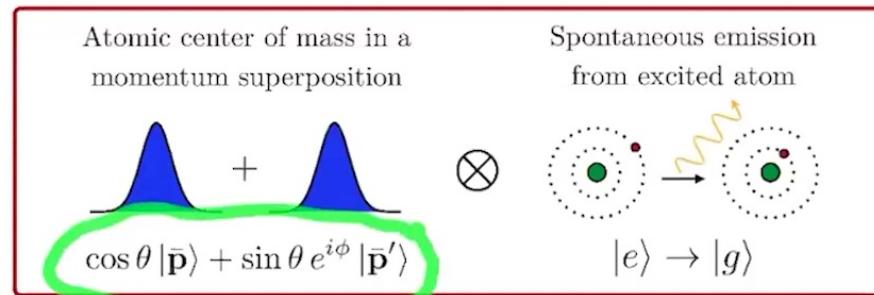
$$\gamma_Q^{-1} := \frac{\cos \phi \sin 2\theta \left[(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2 - 2(\bar{\mathbf{p}}_A'^2 - \bar{\mathbf{p}}_A^2) \cos 2\theta \right]}{8m^2 c^2 [\cos \phi \sin 2\theta + e^{(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2 / 4\Delta^2}]}$$







Quantum time dilation in atomic spectra



$$\hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{field}} + \hat{H}_{\text{af}}$$

$$\hat{H}_{\text{atom}} = \frac{\hat{\mathbf{p}}^2}{2m} - \frac{1}{8} \frac{\hat{\mathbf{p}}^4}{m^3 c^2} + \hbar \Omega \left(1 - \frac{1}{2} \frac{\hat{\mathbf{p}}^2}{m^2 c^2} \right) |\bar{e}\rangle \langle \bar{e}|$$

$$\hat{H}_{\text{af}} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}^\perp - \frac{1}{2m} \left[\hat{\mathbf{p}} \cdot (\hat{\mathbf{B}} \times \hat{\mathbf{d}}) + (\hat{\mathbf{B}} \times \hat{\mathbf{d}}) \cdot \hat{\mathbf{p}} \right]$$

P. T. Grochowski, A. R. H. Smith, A. Dragan, K. Dębski, *Quantum time dilation in atomic spectra*, Phys. Rev. Research 3, 023053 (2021)
 M. Sonnleitner and S. M. Barnett, *Mass-energy and anomalous friction in quantum optics*, Phys. Rev. A 98, 042106 (2018)

Piotr T. Grochowski ,^{1,*} Alexander R. H. Smith ,^{2,3,†} Andrzej Dragan ,^{4,5,‡} and Kacper Dębski ,^{4,§}

Spontaneous emission rate experiences quantum time dilation:

$$\frac{\Gamma_{\text{sup}} - \Gamma_{\text{cl}}}{\Gamma_0} = \gamma_Q^{-1}$$
$$\gamma_Q^{-1} := \frac{\cos \phi \sin 2\theta \left[(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2 - 2(\bar{\mathbf{p}}'^2_A - \bar{\mathbf{p}}^2_A) \cos 2\theta \right]}{8m^2c^2 [\cos \phi \sin 2\theta + e^{(\bar{\mathbf{p}}'_A - \bar{\mathbf{p}}_A)^2 / 4\Delta^2}]}$$

which suggests **quantum time dilation is universal!**



Effects of relativistic momentum coherence on atomic transitions

Kacper Dębski ,^{1,*} Piotr T. Grochowski ,^{2,3,†}
Alexander R. H. Smith ,^{4,5,‡} Mehdi Ahmadi ,^{6,§} and Andrzej Dragan ,^{1,7,¶}



Quantum time dilation in atomic spectra

Piotr T. Grochowski^{1,*}, Alexander R. H. Smith^{2,3,†}, Andrzej Dragan^{4,5,‡}, and Kacper Dębski^{4,§}

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Effects of relativistic momentum coherence on atomic transitions

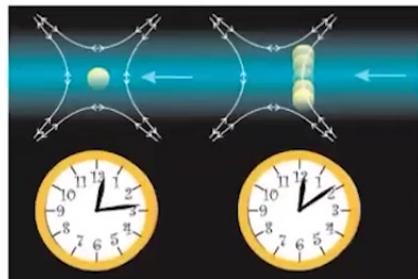
Kacper Dębski^{1,*}, Piotr T. Grochowski^{2,3,†},
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Is quantum time dilation observable?

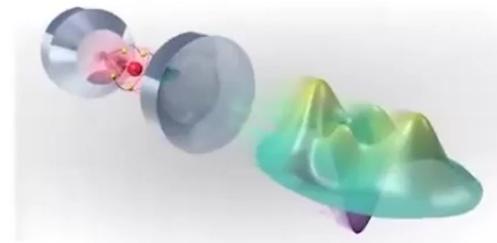
Time dilation observed for clocks moving several meters per second

C. W. Chou, D. B. Hume, T. Rosenband, D. J. Wineland,
Optical Clocks and Relativity, Science 329, 1630 (2010)



Atomic superpositions separated by several meters per second have been created.

P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben,
Phys. Rev. Lett. 102, 240402 (2009)



An estimate of the quantum contribution is $\gamma_Q^{-1} \approx 10^{-15}$

This quantum contribution is within the resolution of current atomic clock provided the coherence time of the superposition is longer than the atom's lifetime.



Mischa Woods

Thanks for your attention!