

Title: Relative subsystems and quantum reference frame transformations.

Speakers: Esteban Castro Ruiz

Collection: Quantizing Time

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Abstract: Transformations between reference frames play a crucial role in our understanding of physical processes. In practice, reference frames are realised by physical systems, which are standardly treated as classical. However, assuming that every physical system is ultimately quantum, it is interesting to ask how a theory of transformations wrt quantum reference frames would look like, and what implications it would have for our description of spacetime. Recently, there has been a lot of effort towards developing a quantum generalisation of reference frame transformations, unveiling novel phenomena that are absent in the classical treatment of reference frames. Here, we develop a first-principles framework for quantum reference frame transformations which clarifies important conceptual issues of previous treatments. Based on the algebra of relative observables between a system and a reference frame, our operational perspective leads naturally to a mixed-state approach (incoherent twirling), in contrast to current pure-state approaches (coherent twirling). Within our framework, the full invariant quantum subsystem contains not only the algebra of relative observables between the system and the reference frame but also an “extra particle” related to the invariant degrees of freedom of the reference frame itself. Importantly, this extra particle contains information about the “quantumness” of the reference frame and is essential to the unitarity of quantum reference frame transformations. Our approach is general, in the sense that it can be applied to a vast set of symmetry groups and to any type of system. We illustrate the physical meaning of the concepts developed by analysing quantum reference frame transformations with respect to the (centrally extended) Galilei group.

Relative subsystems and quantum reference frame transformations

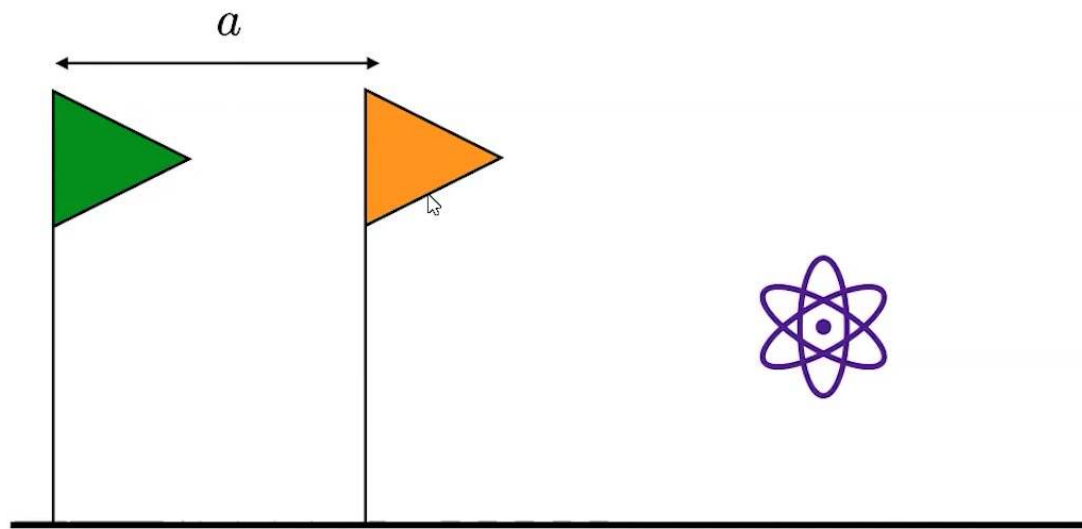
Esteban Castro & Ognjan Oreshkov



Quantizing Time Conference, Perimeter Institute, June 2021



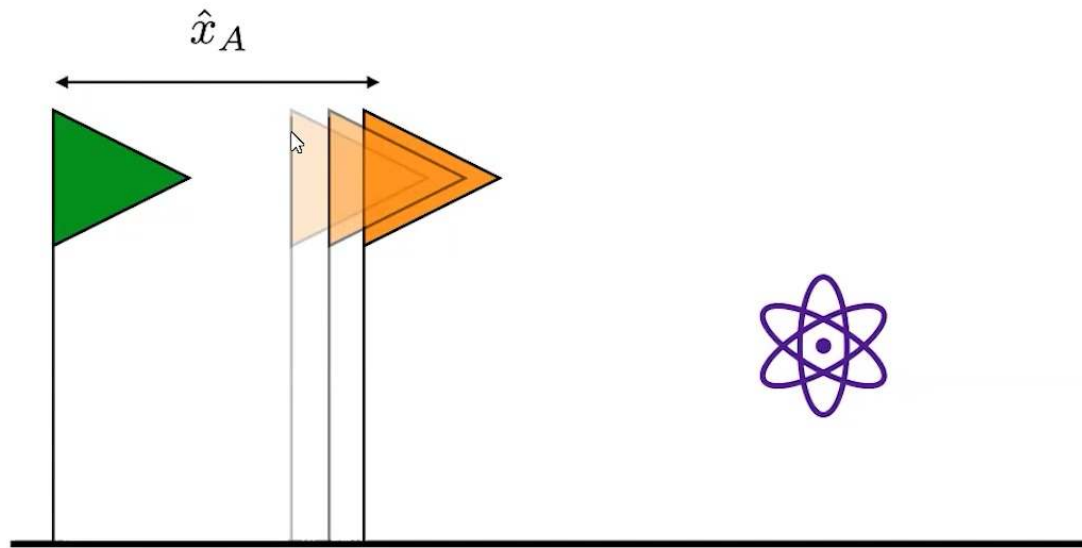
What is a QRF?



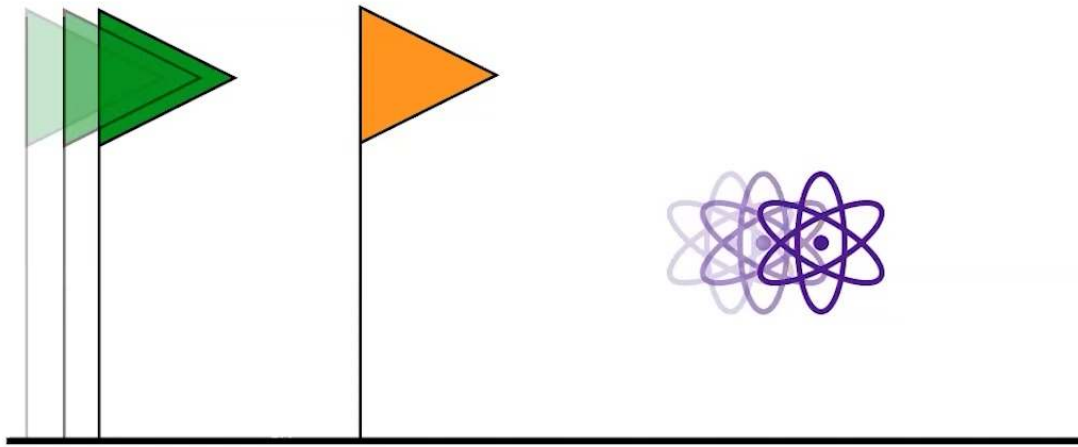
$$|\psi\rangle \longrightarrow e^{ia\hat{p}}|\psi\rangle$$



What is a QRF?



What is a QRF transformation?



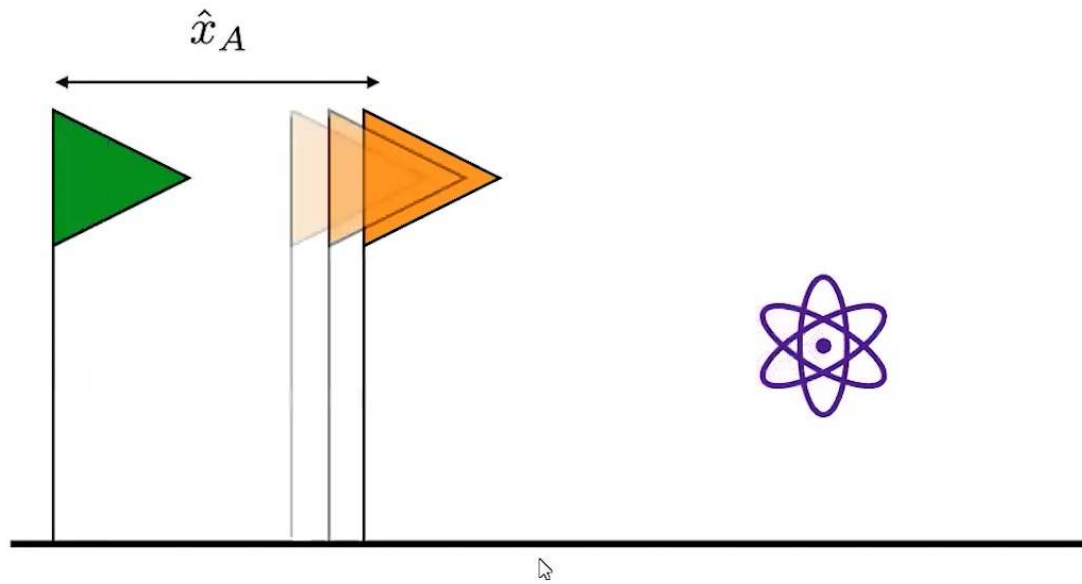
$$|\varphi\rangle_{AS} \longrightarrow \mathcal{P}_{AB} e^{i\hat{x}_A \hat{p}_S} |\varphi\rangle_{AS}$$

Giacomini, C-R, Brukner (2017)

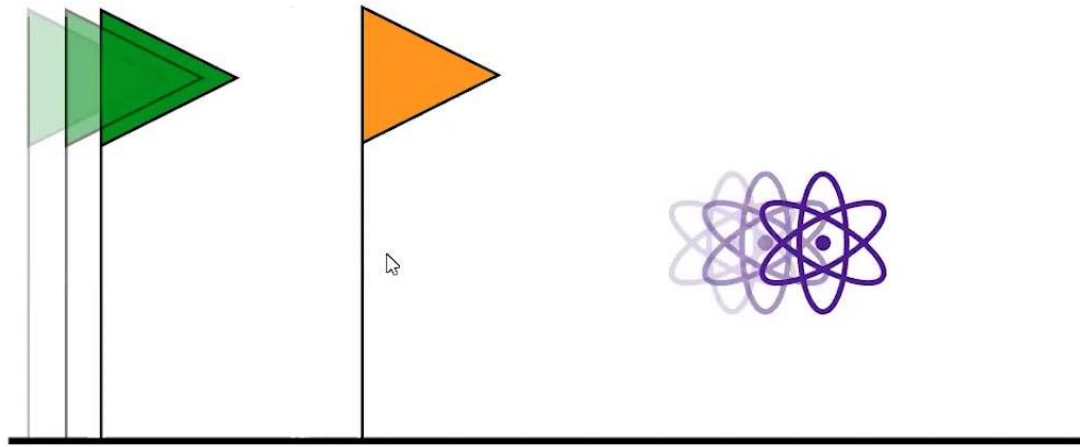


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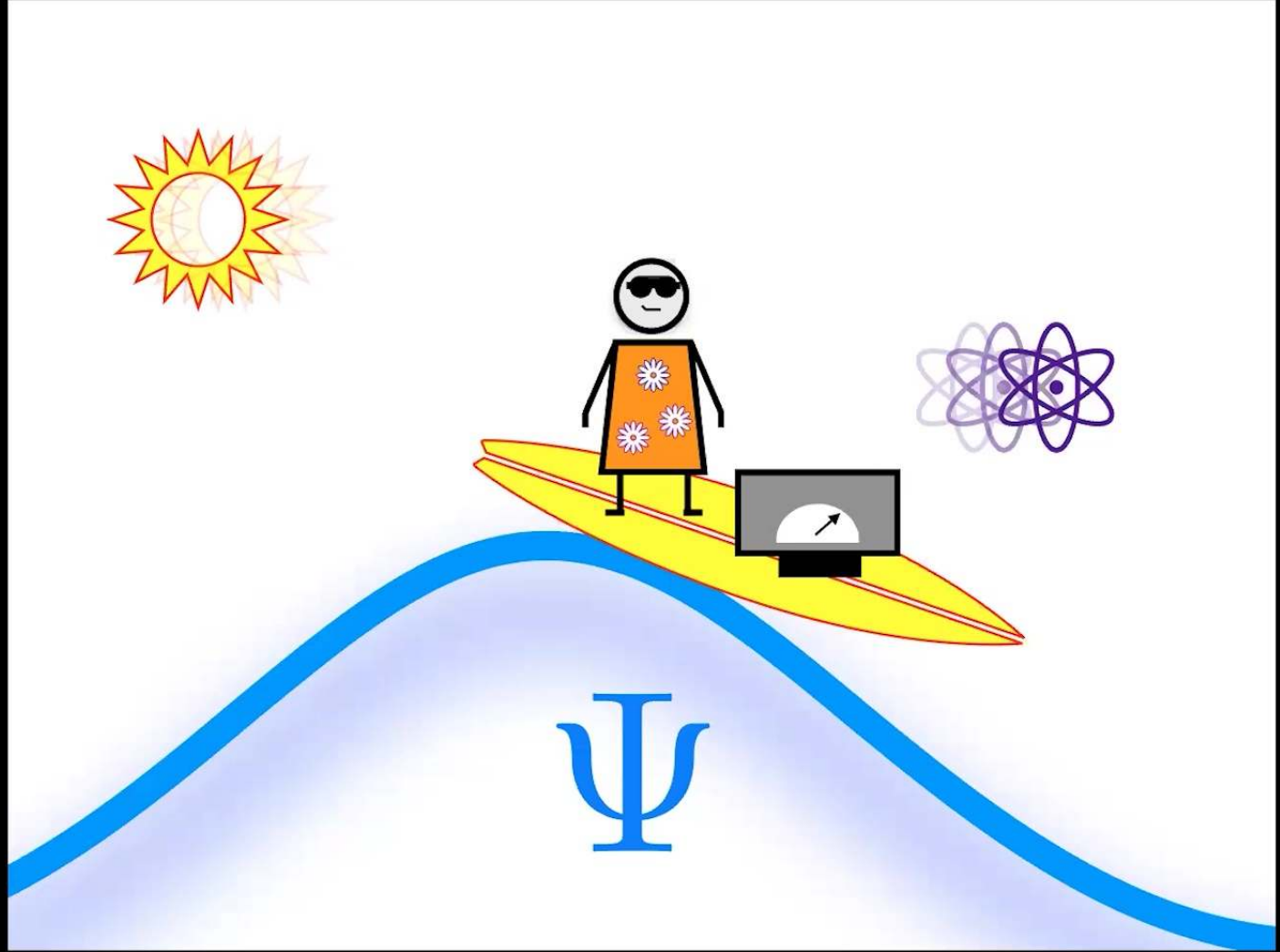
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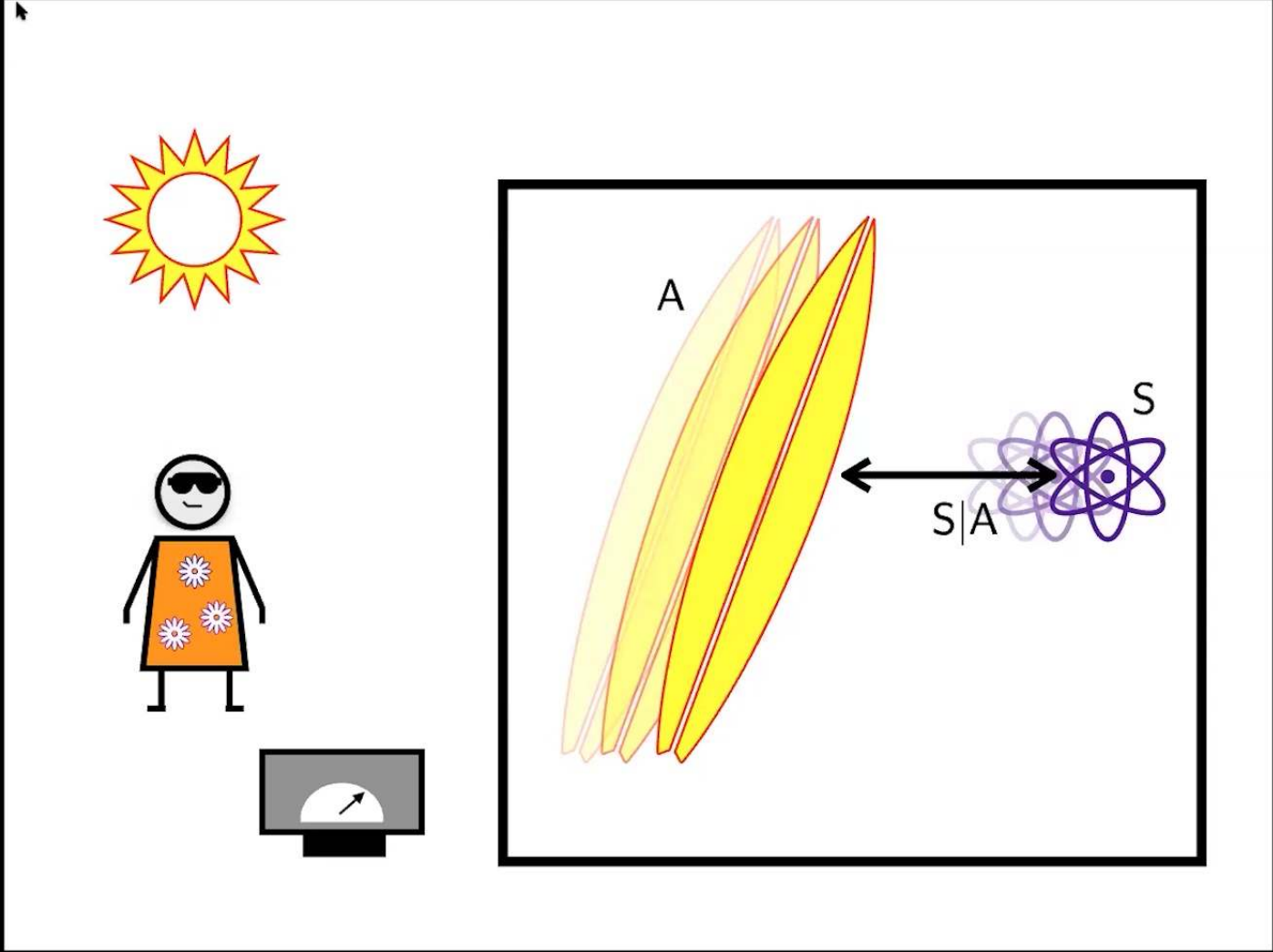


Why are QRF transformations interesting?

- Notion of symmetry and covariance wrt QRF transformations
- Rest frame for quantum systems in superposition (definition of spin)
- Reference frames for “classicalising” gravitating quantum systems
- Quantum foundational thought experiments







Frame and system

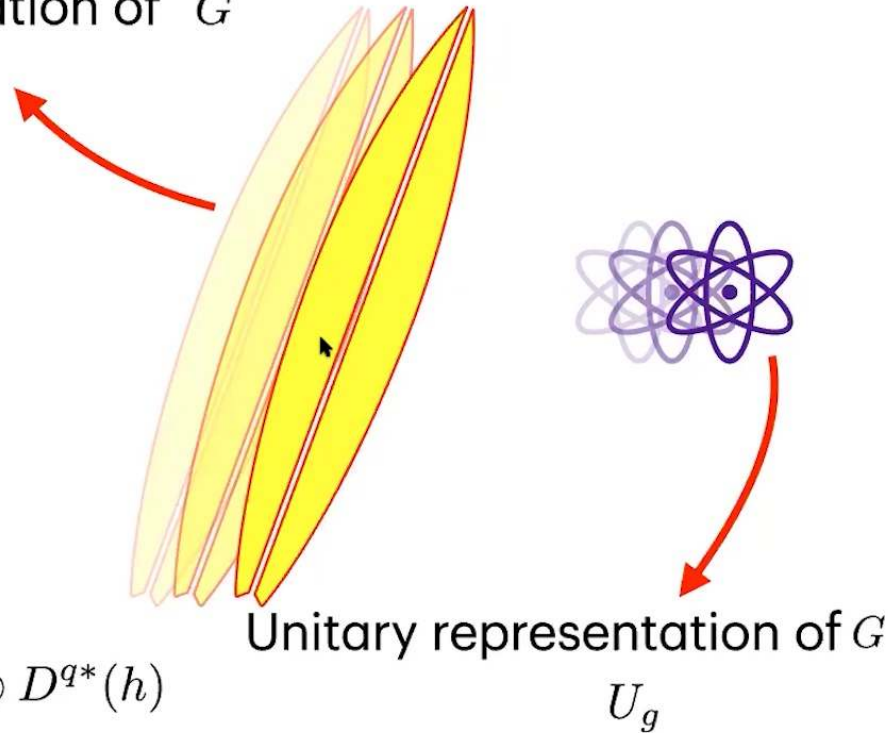
Regular representation of G

$$L_g |g'\rangle = |gg'\rangle$$

$$\langle g | g' \rangle = \delta(g^{-1}g')$$

Reducible:

$$L_g R_h = \bigoplus_q D^q(g) \otimes D^{q*}(h)$$



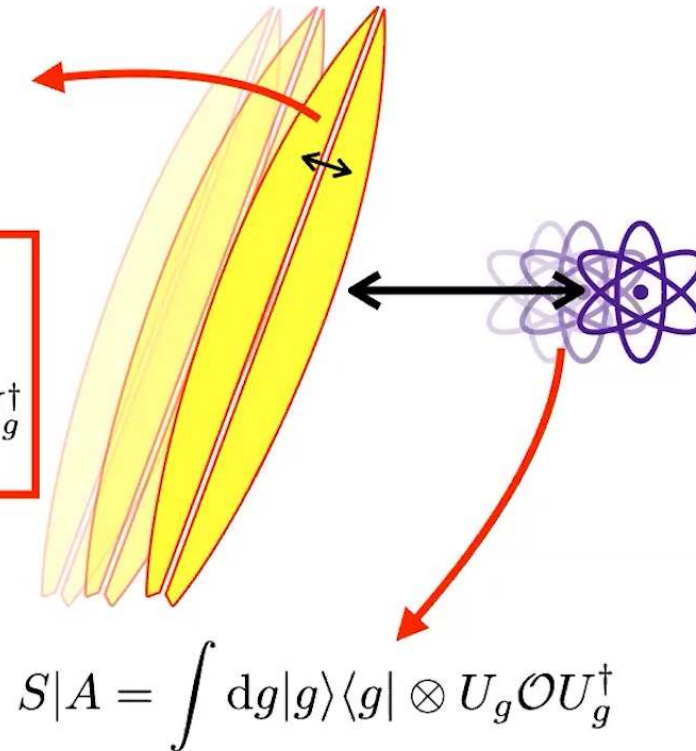
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Relative degrees of freedom (external point of view)

$$R = \bigoplus_q \mathbb{1}^q \otimes \mathcal{O}^{q*} \otimes \mathbb{1}$$

Invariant operators:

$$\mathcal{O} = \int dg L_g \otimes U_g \mathcal{O} L_g^\dagger \otimes U_g^\dagger$$



$$S|A = \int dg |g\rangle \langle g| \otimes U_g \mathcal{O} U_g^\dagger$$

Bartlett, Rudolph, Spekkens (2006)

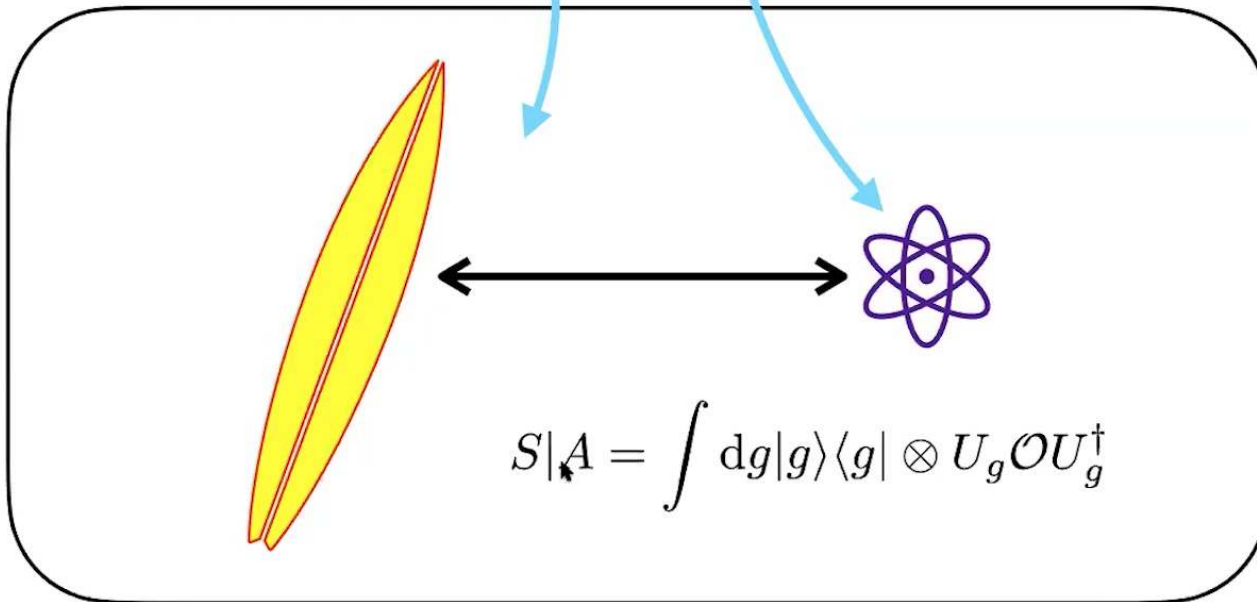


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QRF perspectives as subsystem structures



$$\mathcal{L}(\mathcal{H}) = \mathcal{L}(\mathcal{H}_A) \otimes \mathcal{L}(\mathcal{H}_S)$$



$$S|_A = \int dg |g\rangle\langle g| \otimes U_g O U_g^\dagger$$

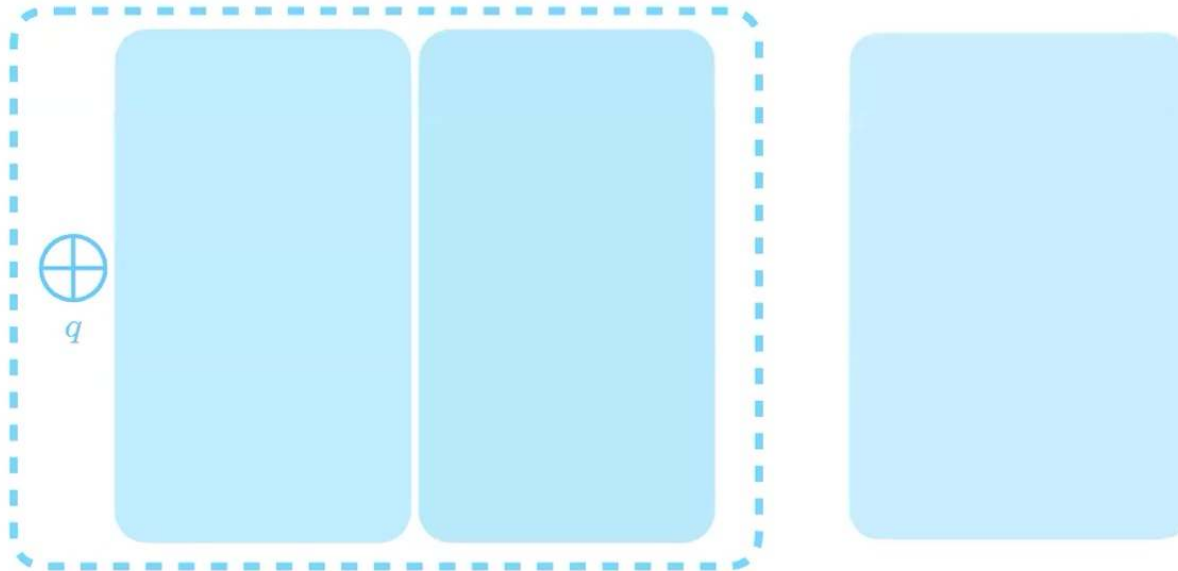


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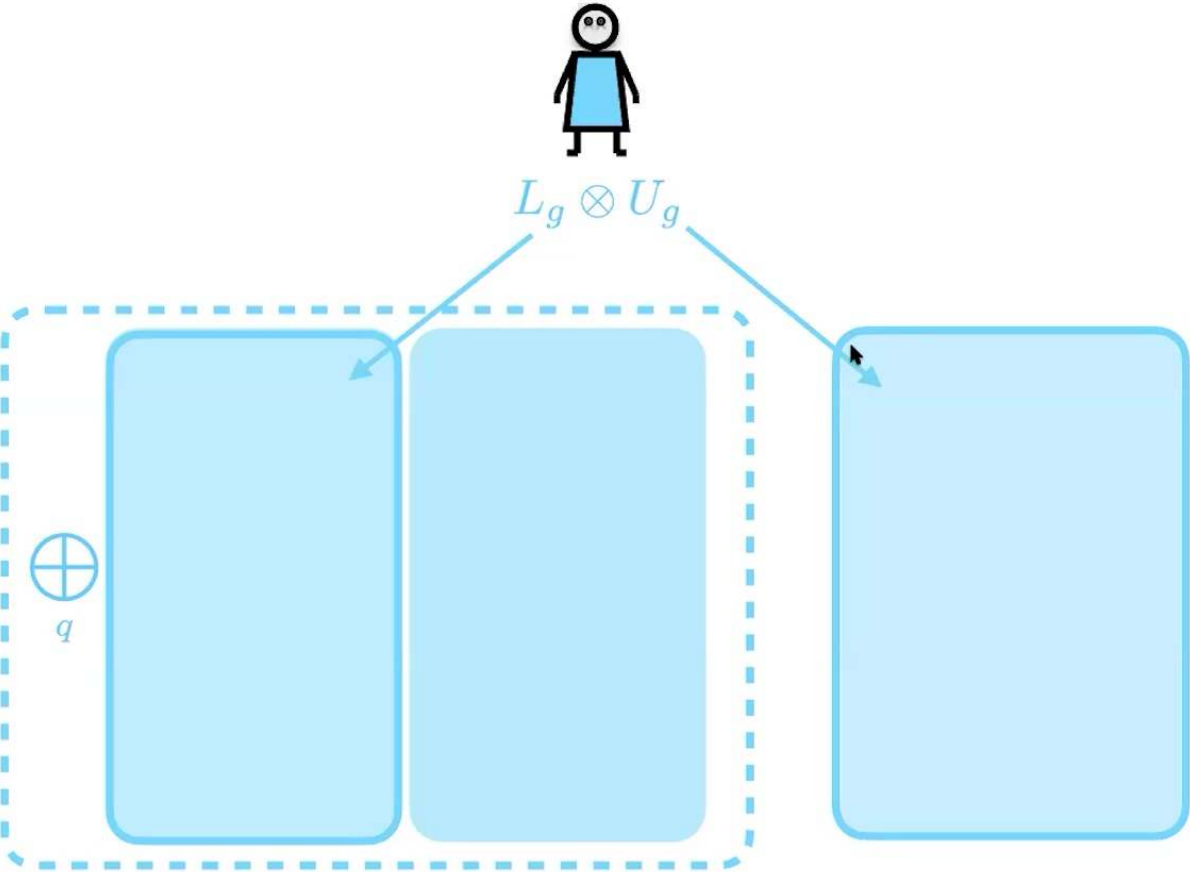
QRF perspectives as subsystem structures



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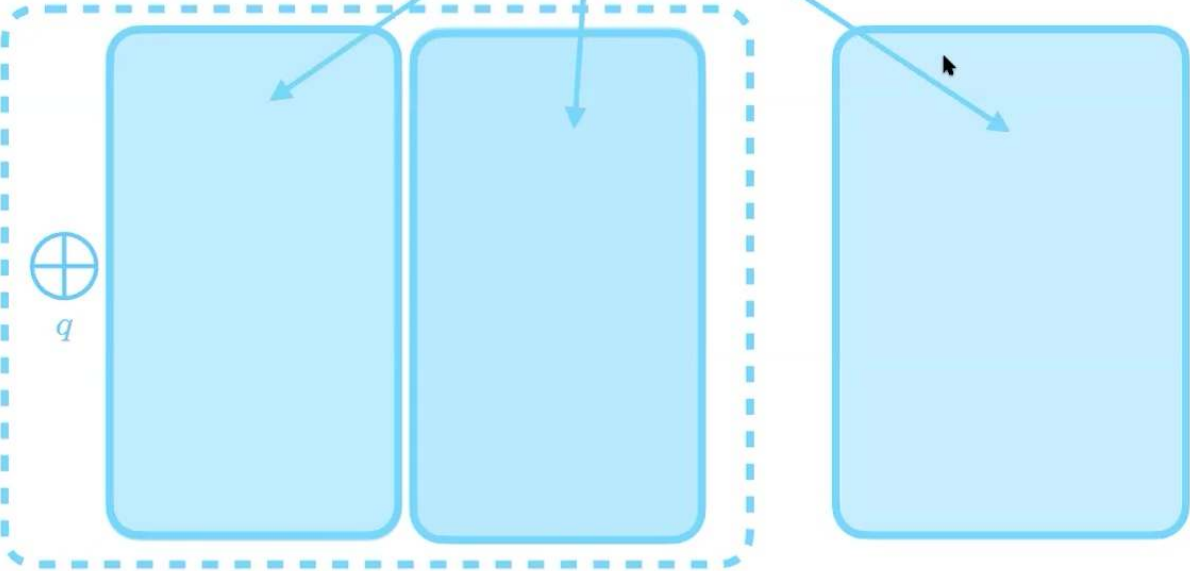
QRF perspectives as subsystem structures



QRF perspectives as subsystem structures



$$S|A = \int dg |g\rangle\langle g| \otimes U_g \mathcal{O} U_g^\dagger$$



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QRF perspectives as subsystem structures

$$U^\dagger(\hat{g}) = \int dg |g\rangle \langle g| \otimes U_g^\dagger$$

Hoehn, Lock, Ahmad, Smith, Galley (2021)

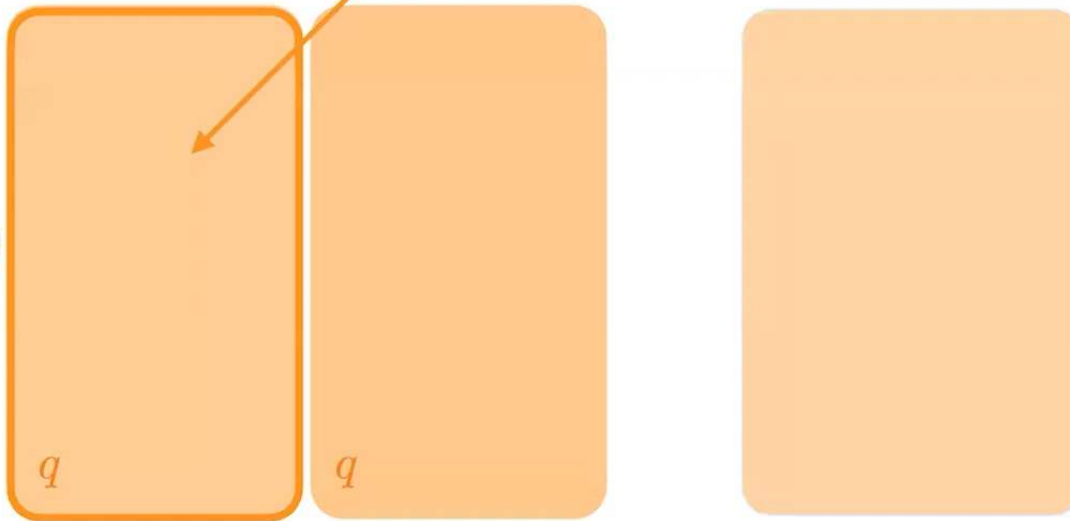


QRF perspectives as subsystem structures

$$U^\dagger(\hat{g})L_g \otimes U_g U(\hat{g}) = L_g \otimes \mathbb{1}$$



$$L_g \otimes \mathbb{1}$$

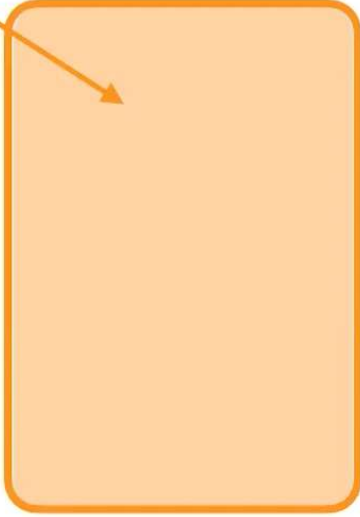
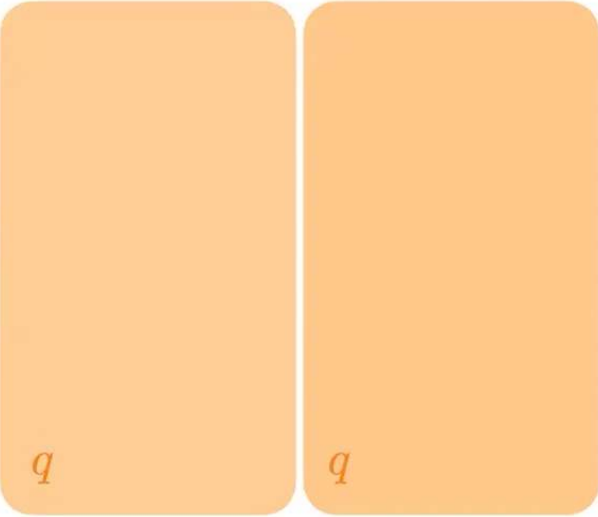


QRF perspectives as subsystem structures



S|A

\oplus
 q



QRF perspectives as subsystem structures

QRF is classical ($|g\rangle$)
 $\overline{S|A}$ is trivial!



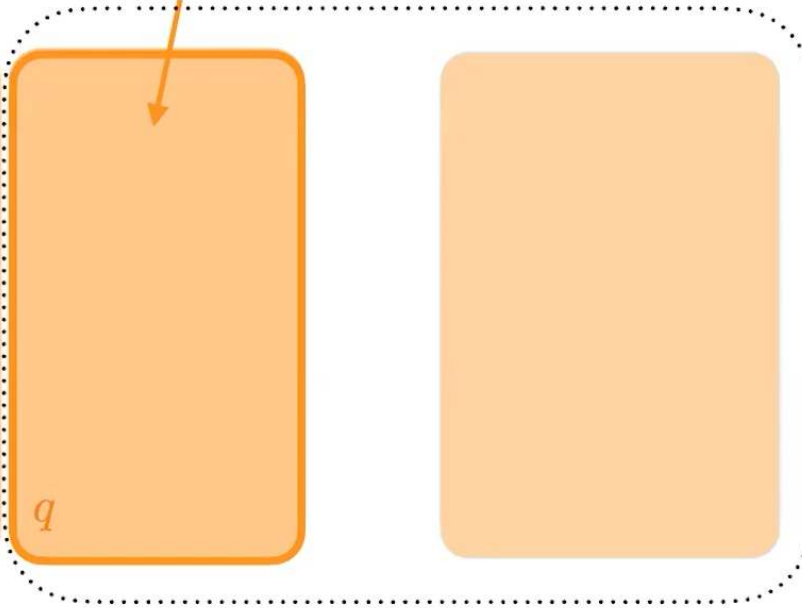
$\overline{S|A}$



q



q

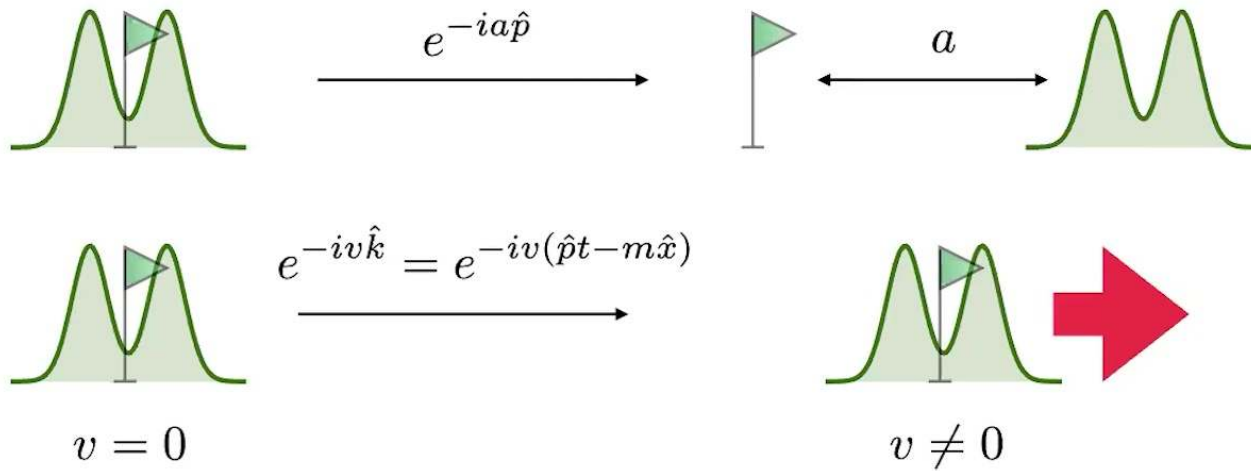


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Example: Centrally extended Galilei group

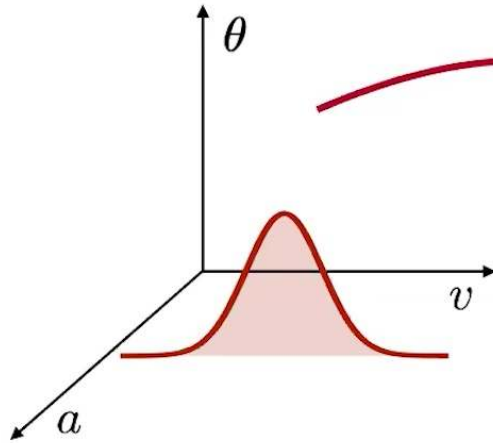


$$(\theta, a, v)(\theta', a', v') = \left(\theta + \theta' + \frac{1}{2}(a'v - av'), a + a', v + v'\right)$$

$$U^{(m)}(\theta, a, v) = e^{im\theta} e^{-i(a\hat{p} + v\hat{k})}$$



Example: Centrally extended Galilei group



$$\psi(\theta, a, v) = \langle (\theta, a, v) | \psi \rangle$$

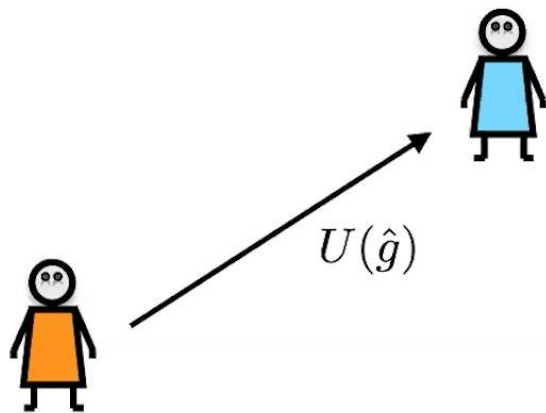
$$\hat{M}_L = i\partial_\theta$$

$$\hat{p}_L = \frac{i}{2}v\partial_\theta - i\partial_a$$

$$\hat{k}_L = -\frac{i}{2}a\partial_\theta - i\partial_v$$



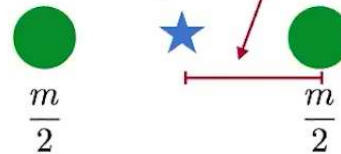
Example: Centrally extended Galilei group



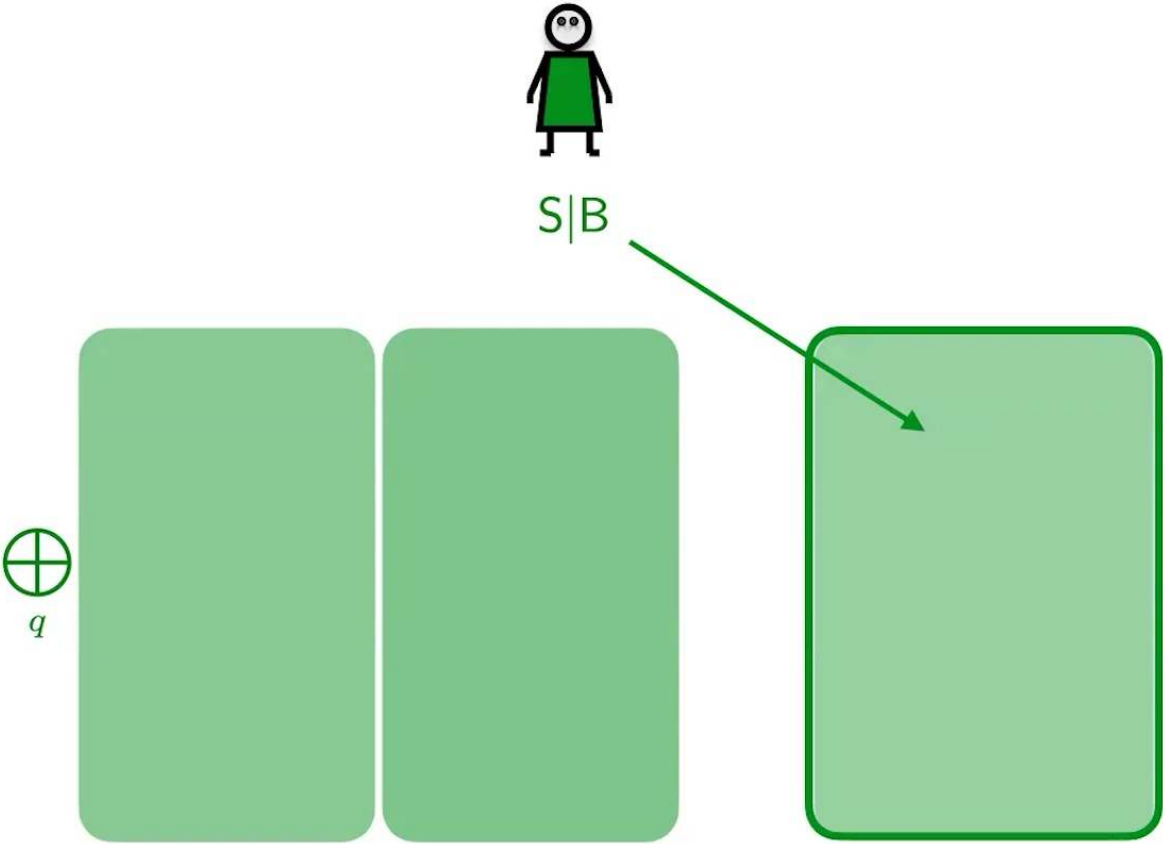
$$\hat{x}_S \mapsto \hat{x}_S - \hat{x}_A$$

$$\hat{p}_S \mapsto \hat{p}_S - \frac{2m_S}{m} \hat{p}_B$$

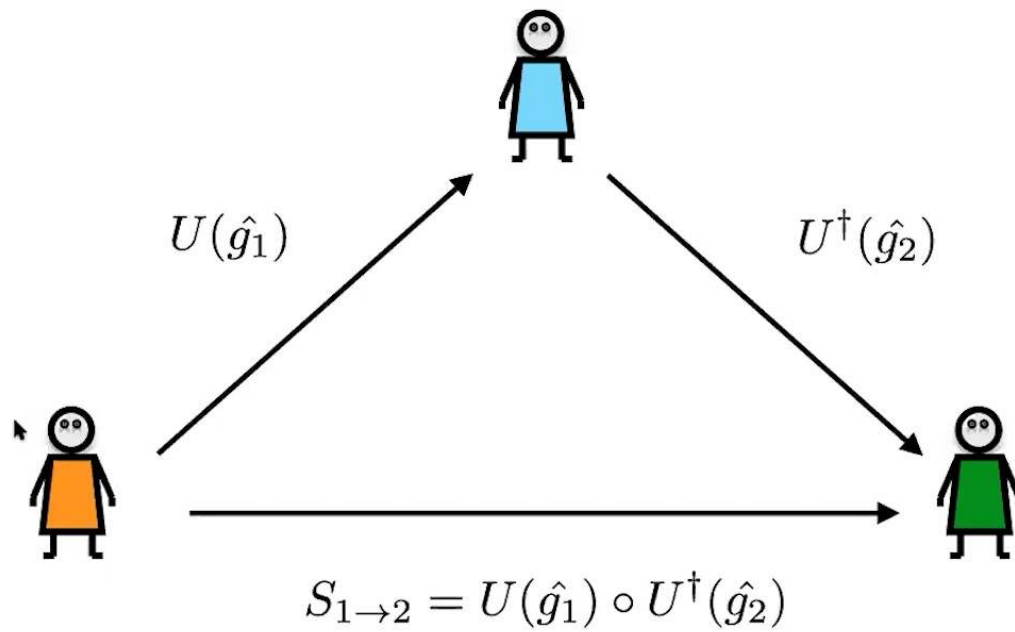
$$L_g R_h = \bigoplus_m D^m(g) \otimes D^{m^*}(h)$$



QRF perspectives as subsystem structures



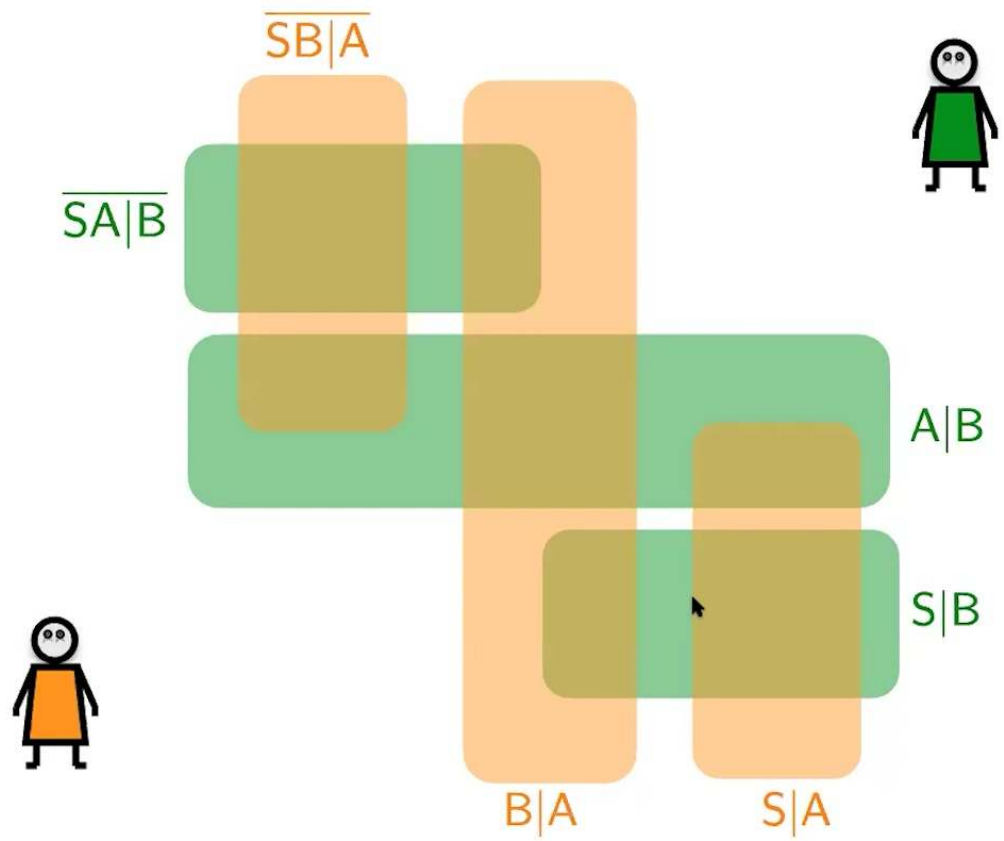
Example: Centrally extended Galilei group



Vanrietvelde, Hoehn, Giacomini, C-R (2018)



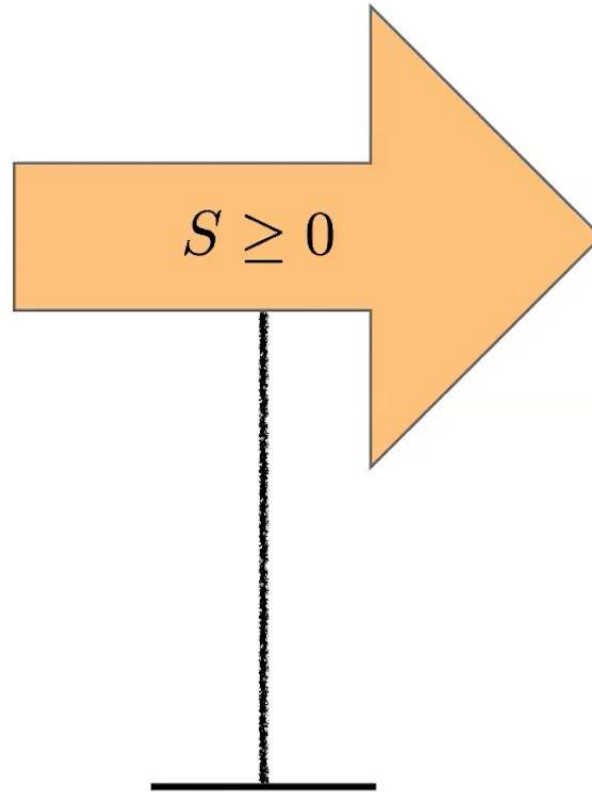
QRF perspectives as subsystem structures



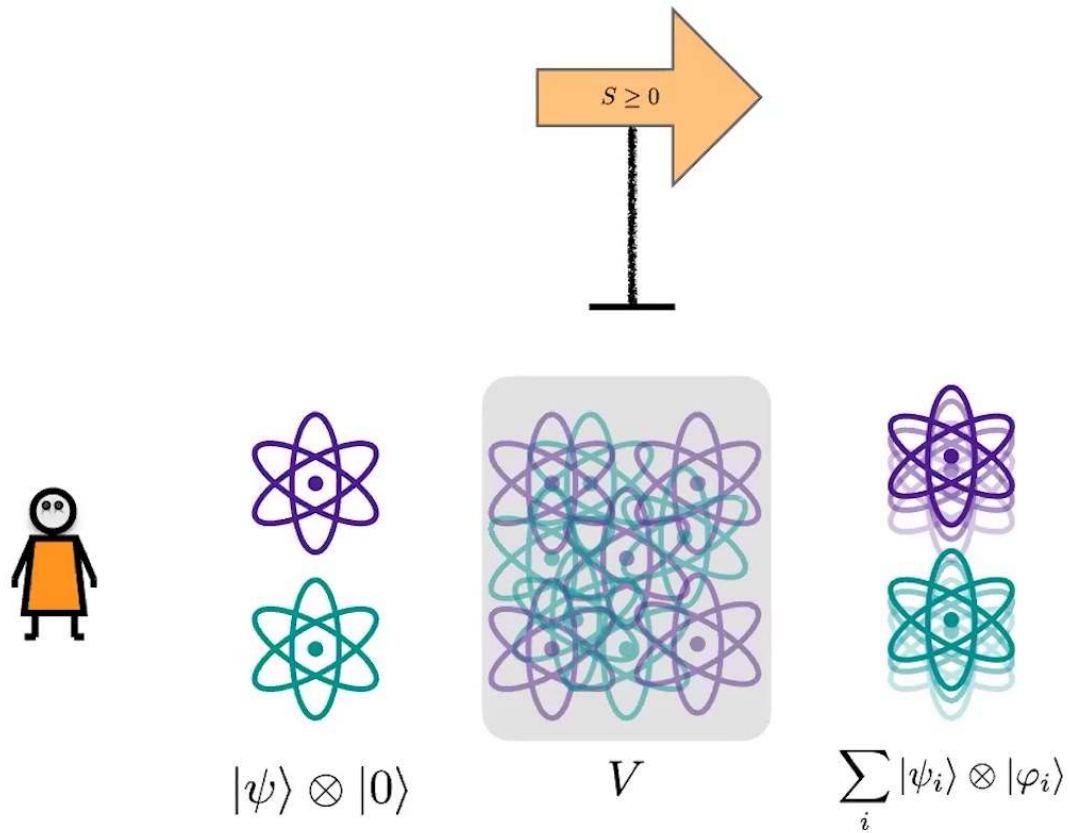
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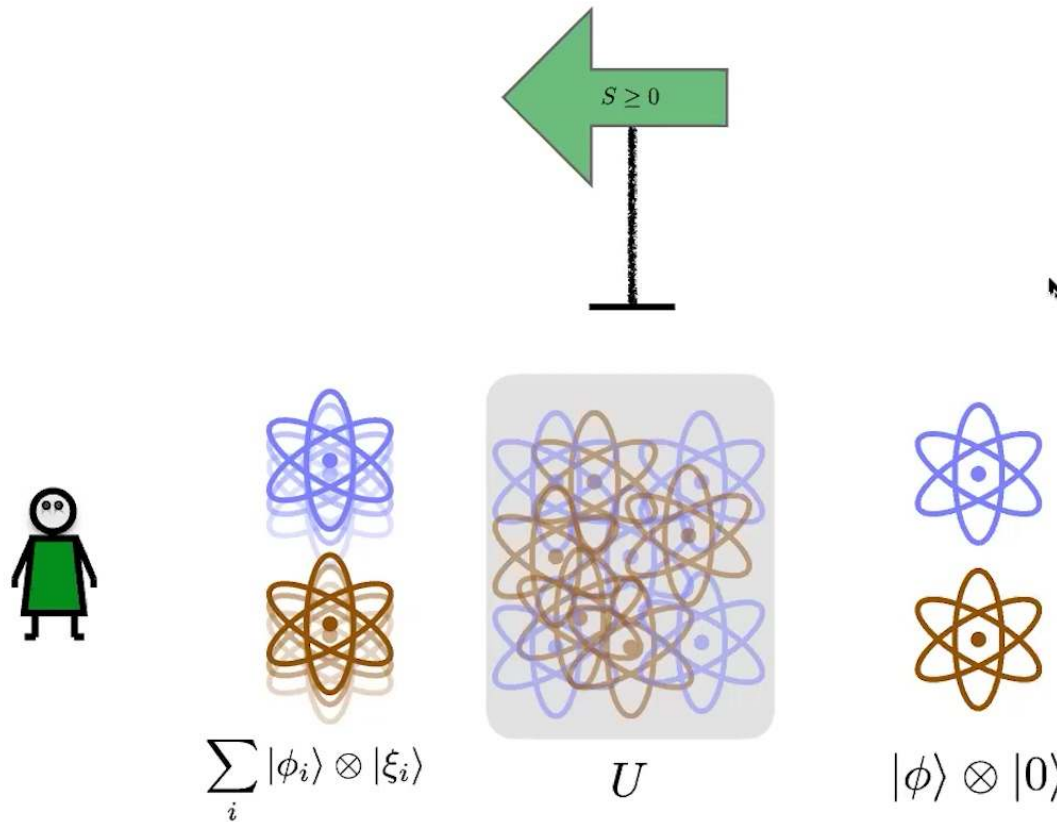
Relation to arrow of time



Relation to arrow of time



Relation to arrow of time



Overview: desirable features of a framework for QRFT

- Connects different approaches to QRFs
- General groups*
- “Algorithmic”
- Consistent with external observer
- General systems

*de la Hamette, Galley (2020)



Thank you!

Mischa Woods