Title: (Quantum) frame covariance: from foundations via gauge theories to gravity

Speakers: Philipp Hoehn

Collection: Quantizing Time

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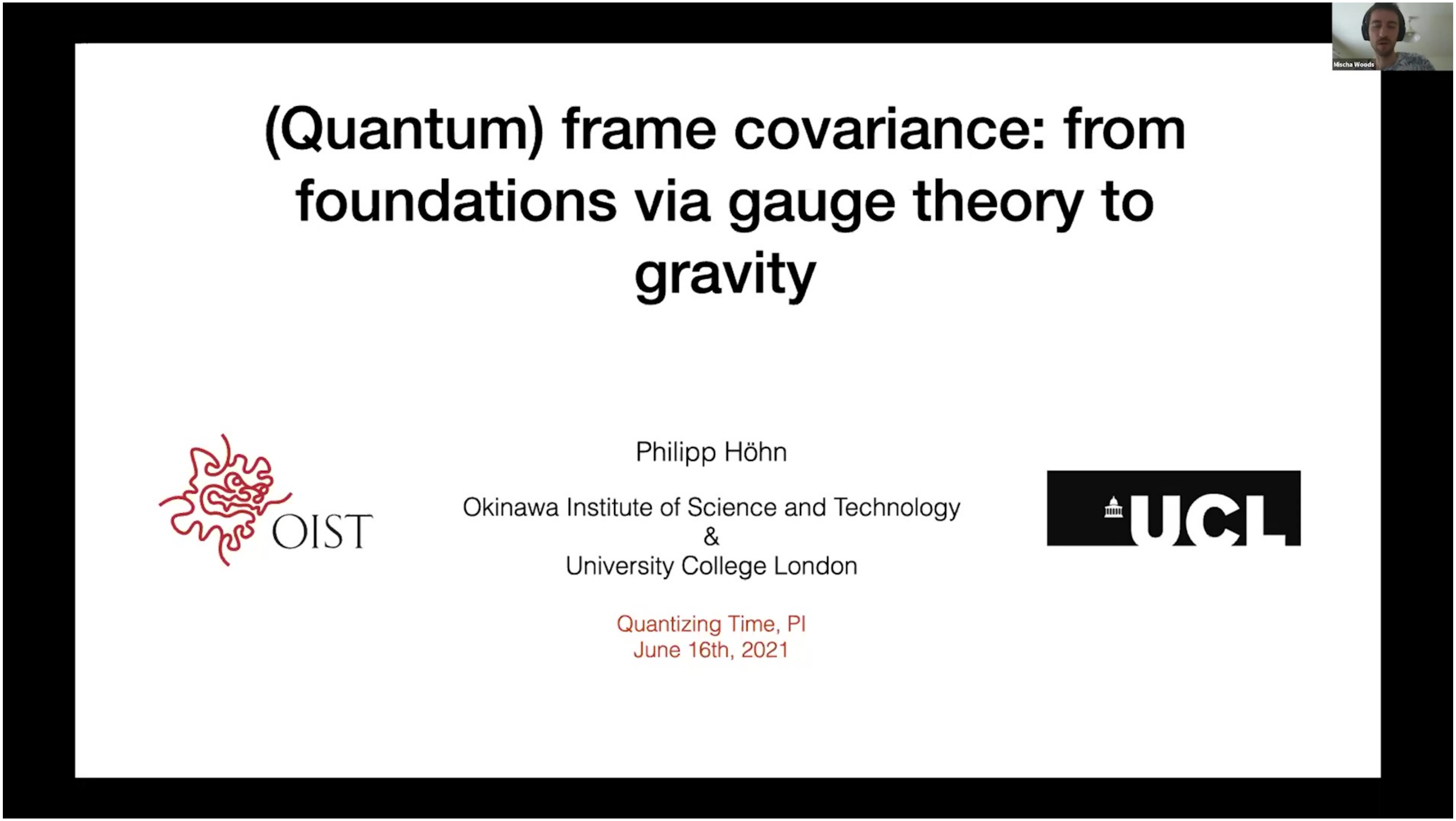
Abstract: I will sketch how the perspective-neutral approach to (quantum) frame covariance brings together some recent developments on dynamical reference frames in quantum foundations, gauge theories and gravity. The survey will touch on spatial frames, quantum clocks and the problem of time, edge modes, and the relativity of subsystems.



(Quantum) frame covariance: from foundations via gauge theory to gravity

Philipp Höhn

Okinawa Institute of Science and Technology
&
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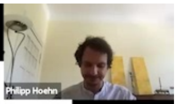
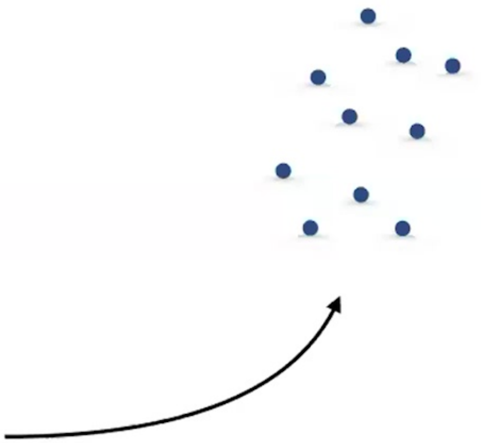
Quantizing Time, PI
June 16th, 2021



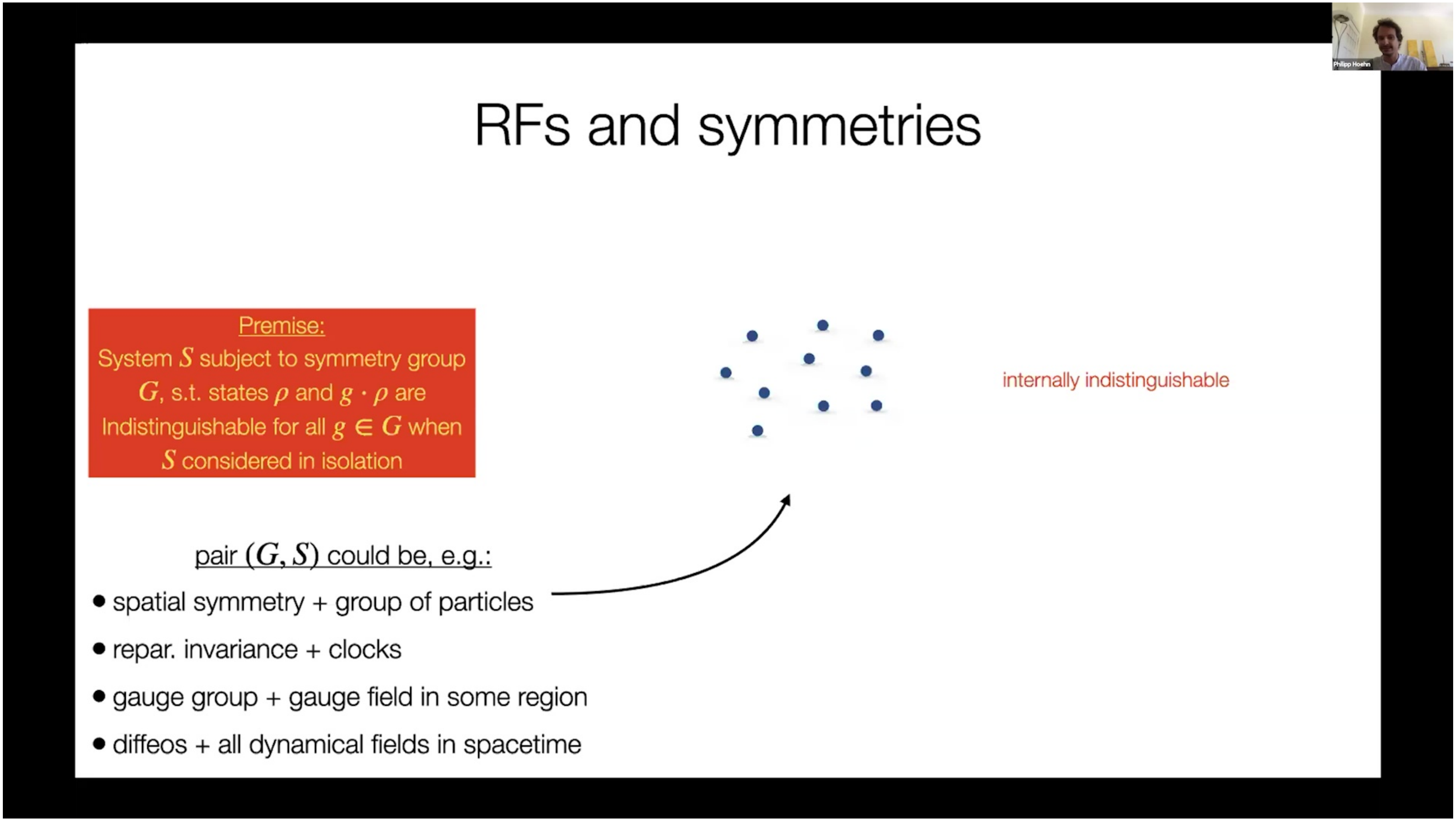
RFs and symmetries

Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



pair (G, S) could be, e.g.:

- spatial symmetry + group of particles
- repar. invariance + clocks
- gauge group + gauge field in some region
- diffeos + all dynamical fields in spacetime



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System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

internally indistinguishable

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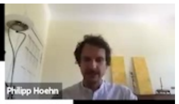

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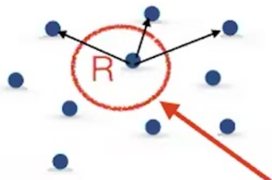

internally indistinguishable

Describe S relative to internal reference subsystem R

pair (G, S) could be, e.g.:

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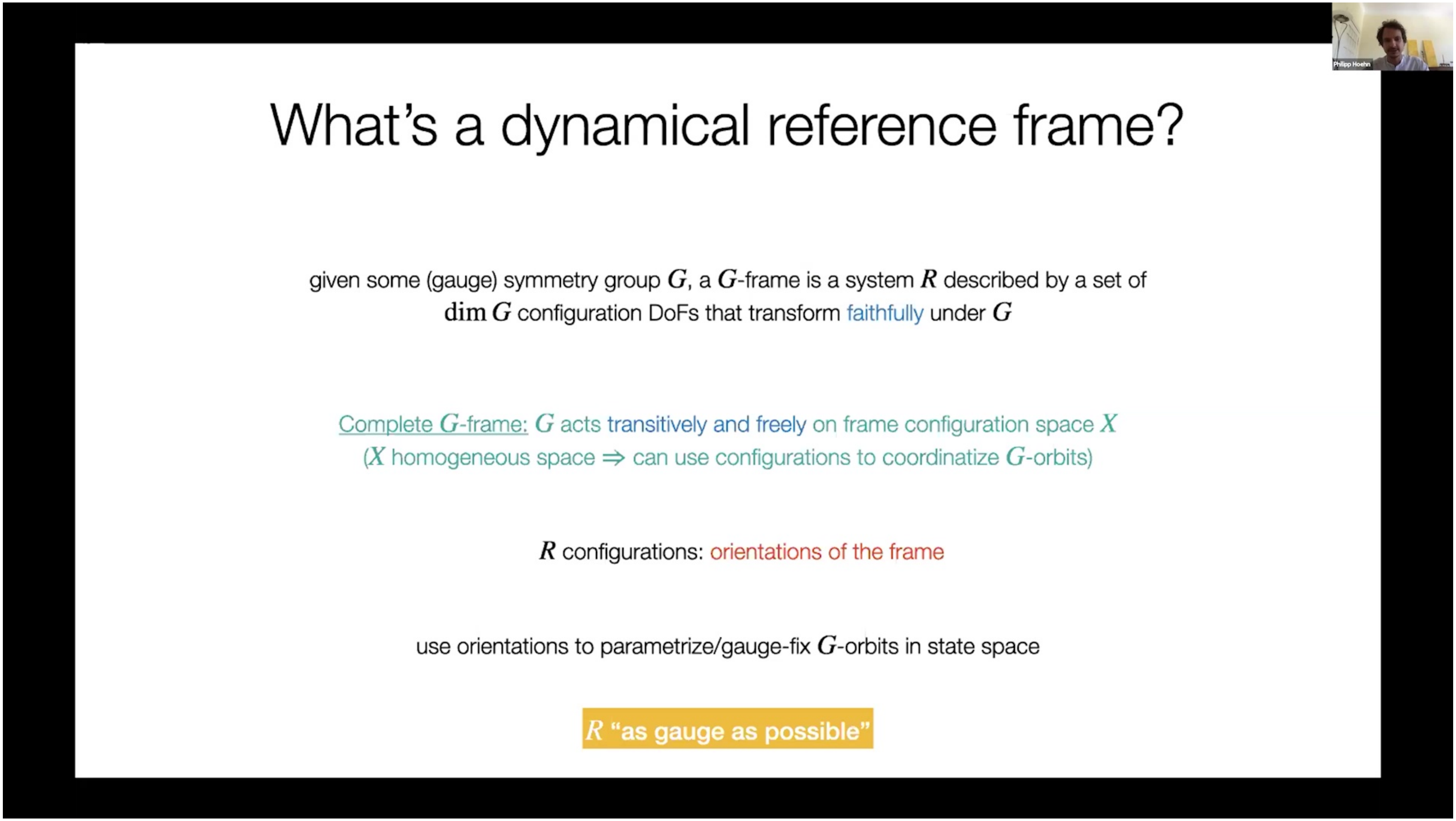


What's a dynamical reference frame?

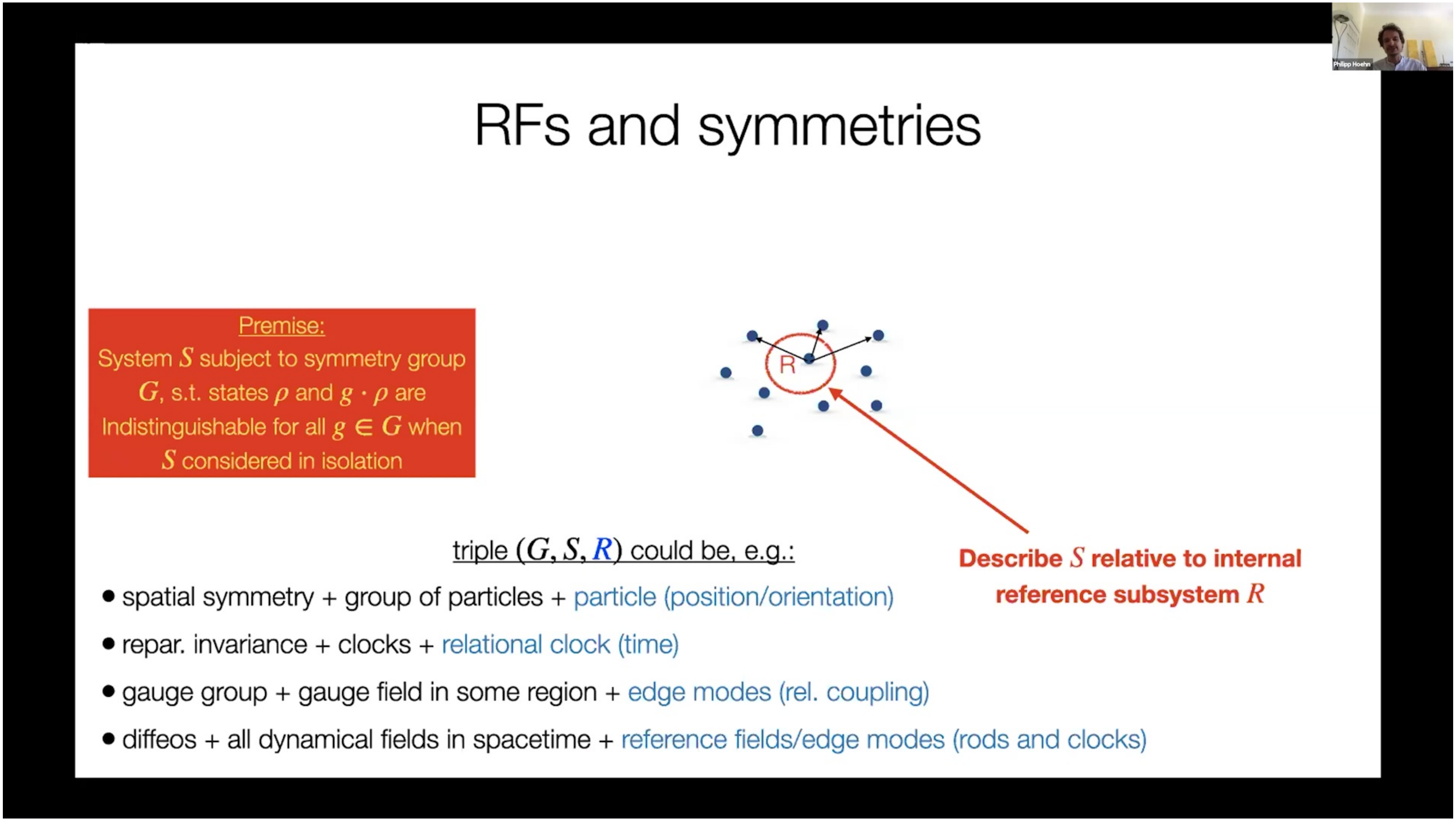
given some (gauge) symmetry group G, a G-frame is a system R described by a set of $\dim G$ configuration DoFs that transform faithfully under G

Complete G-frame: G acts transitively and freely on frame configuration space X
(X homogeneous space $\Rightarrow$ can use configurations to coordinatize G-orbits)

R configurations: orientations of the frame

use orientations to parametrize/gauge-fix G-orbits in state space

R "as gauge as possible"



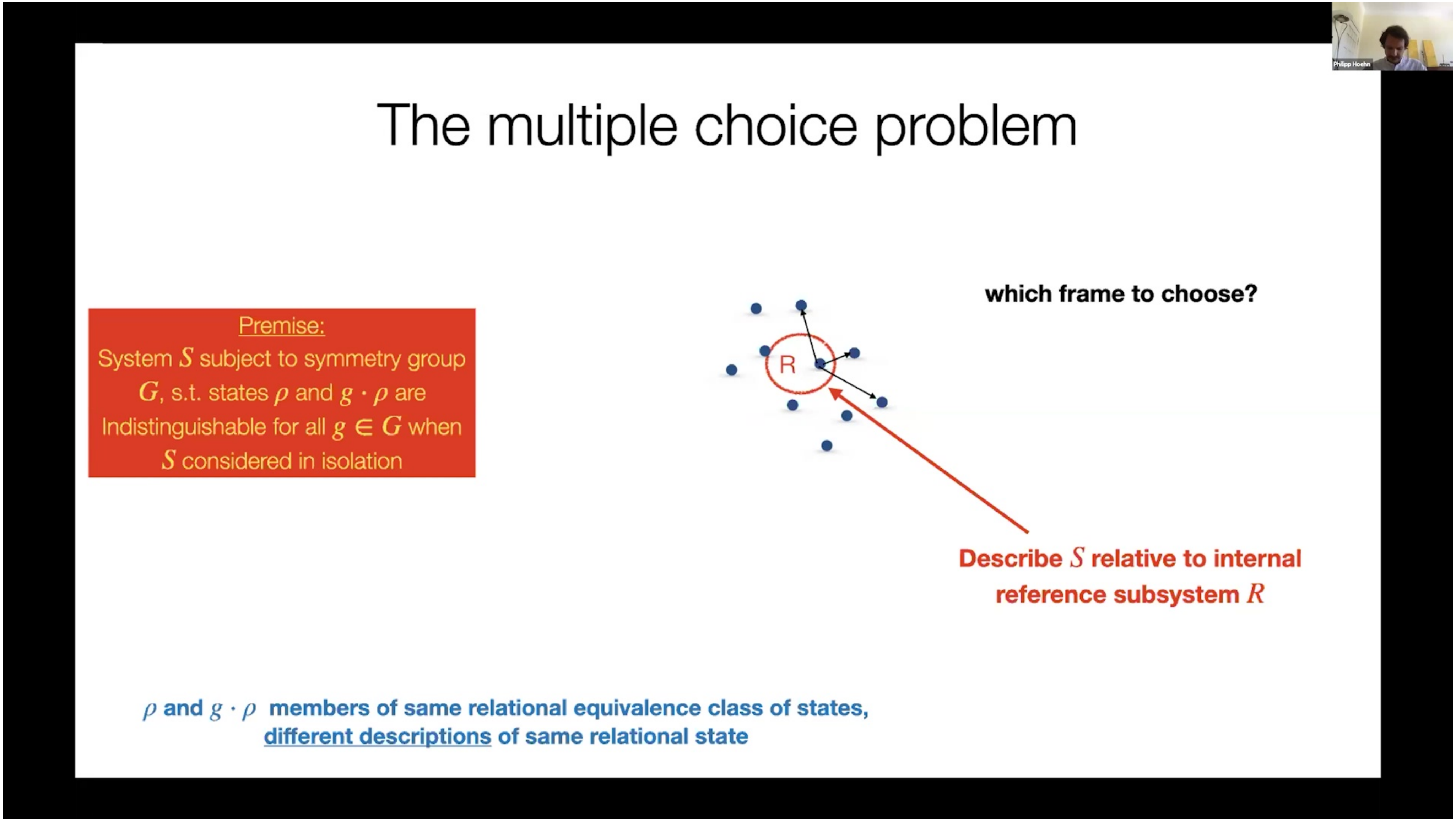
RFs and symmetries

Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

Describe S relative to internal reference subsystem R

triple (G, S, R) could be, e.g.:

- spatial symmetry + group of particles + particle (position/orientation)
- repar. invariance + clocks + relational clock (time)
- gauge group + gauge field in some region + edge modes (rel. coupling)
- diffeos + all dynamical fields in spacetime + reference fields/edge modes (rods and clocks)



The multiple choice problem

<u>Premise:</u>
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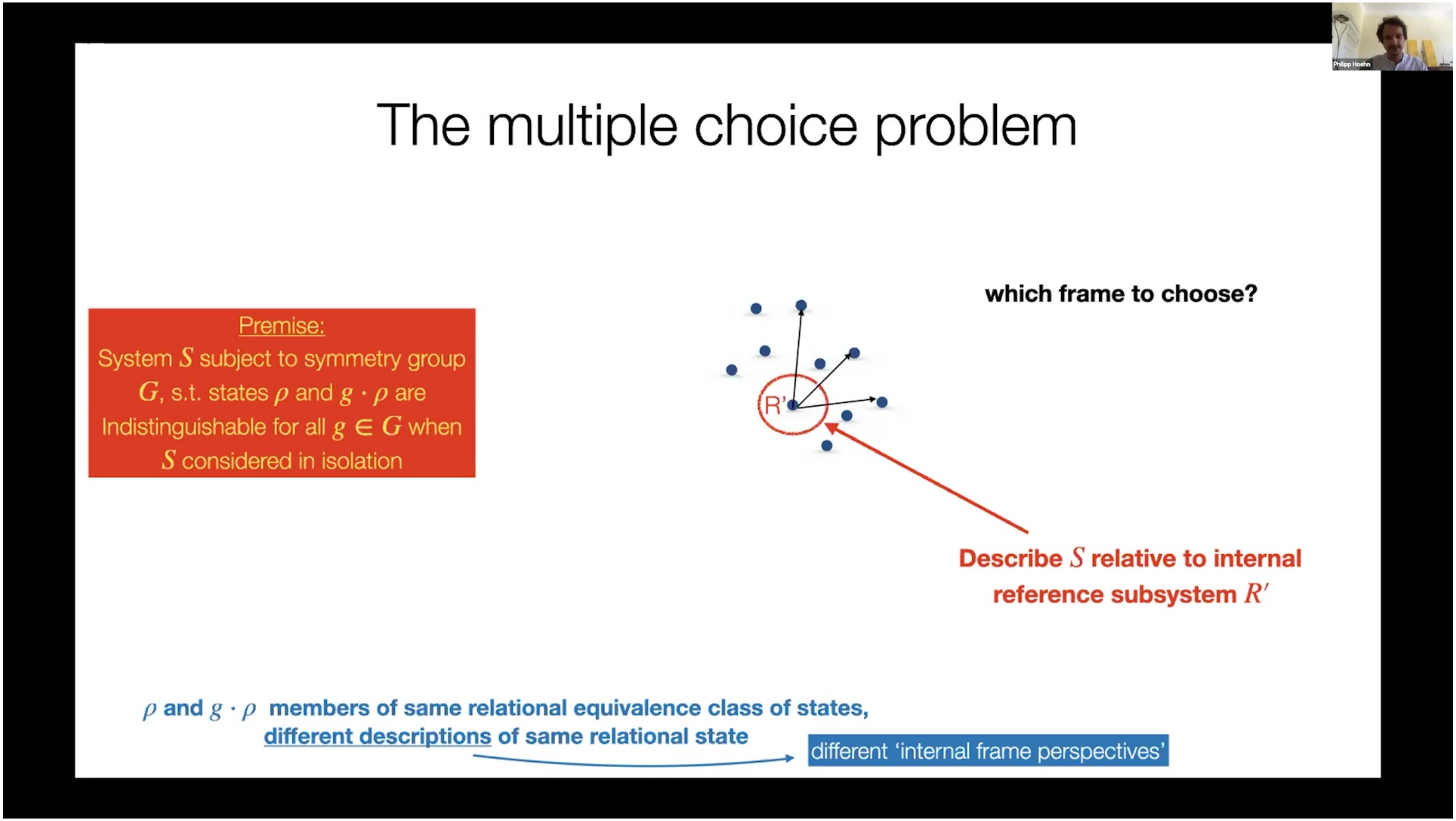
which frame to choose?

Describe S relative to internal reference subsystem R

ρ and $g \cdot \rho$ members of same relational equivalence class of states, <u>different descriptions</u> of same relational state



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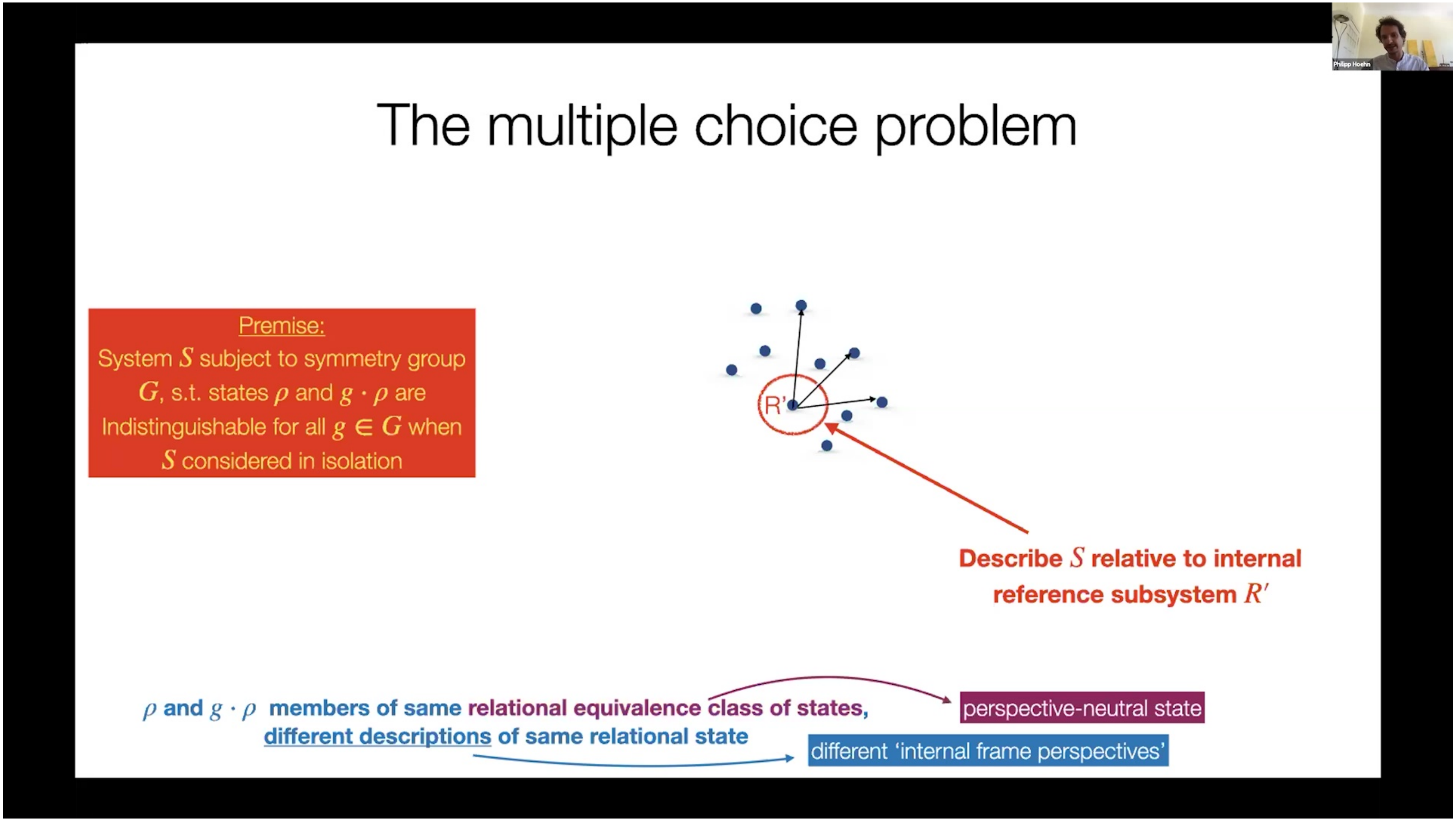
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

which frame to choose?

Describe S relative to internal reference subsystem R'

ρ and $g \cdot \rho$ members of same relational equivalence class of states, different descriptions of same relational state

different 'internal frame perspectives'



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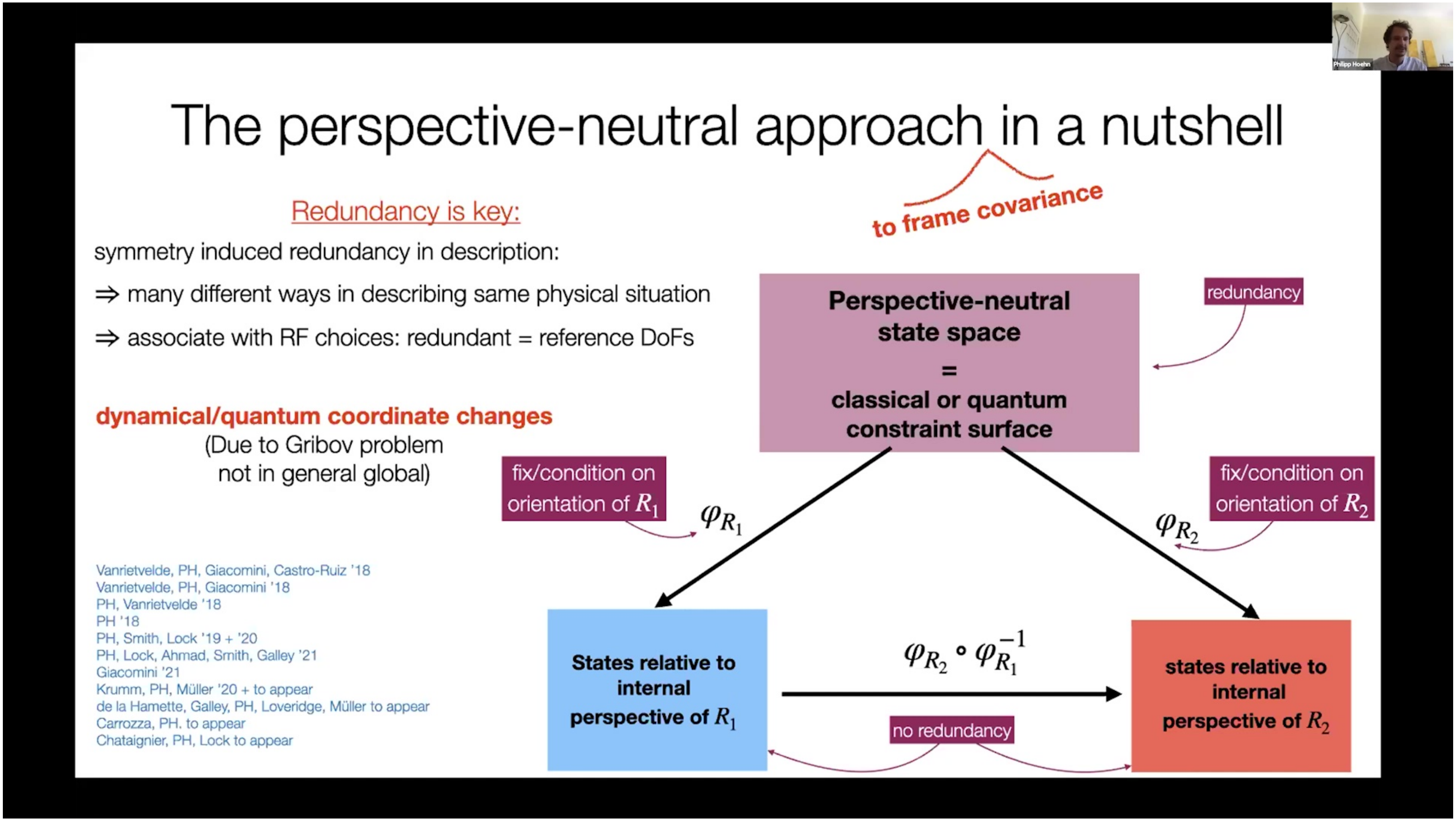
Describe S relative to internal reference subsystem R'

ρ and $g \cdot \rho$ **members of same relational equivalence class of states,** **different descriptions of same relational state**

perspective-neutral state

different 'internal frame perspectives'



The perspective-neutral approach in a nutshell

to frame covariance

Redundancy is key:

symmetry induced redundancy in description:

⇒ many different ways in describing same physical situation

⇒ associate with RF choices: redundant = reference DoFs

dynamical/quantum coordinate changes

(Due to Gribov problem
not in general global)

Vanrietvelde, PH, Giacomini, Castro-Ruiz '18
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PH, Vanrietvelde '18
PH '18
PH, Smith, Lock '19 + '20
PH, Lock, Ahmad, Smith, Galley '21
Giacomini '21
Krumm, PH, Müller '20 + to appear
de la Hamette, Galley, PH, Loveridge, Müller to appear
Carrozza, PH. to appear
Chataignier, PH, Lock to appear

Perspective-neutral state space = classical or quantum constraint surface

redundancy

fix/condition on orientation of R_1

φ_{R_1}

fix/condition on orientation of R_2

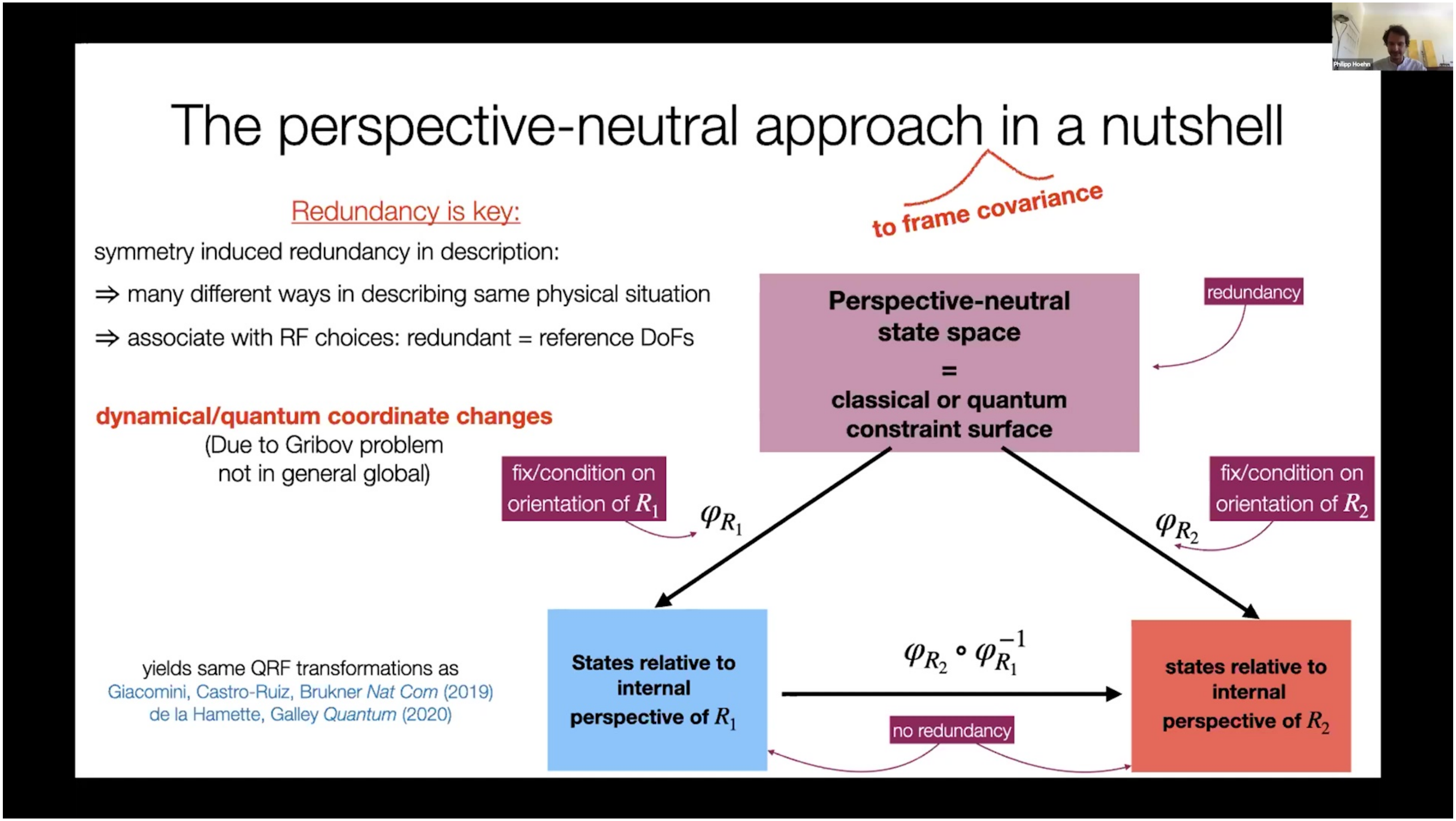
φ_{R_2}

States relative to internal perspective of R_1

$\varphi_{R_2} \circ \varphi_{R_1}^{-1}$

states relative to internal perspective of R_2

no redundancy



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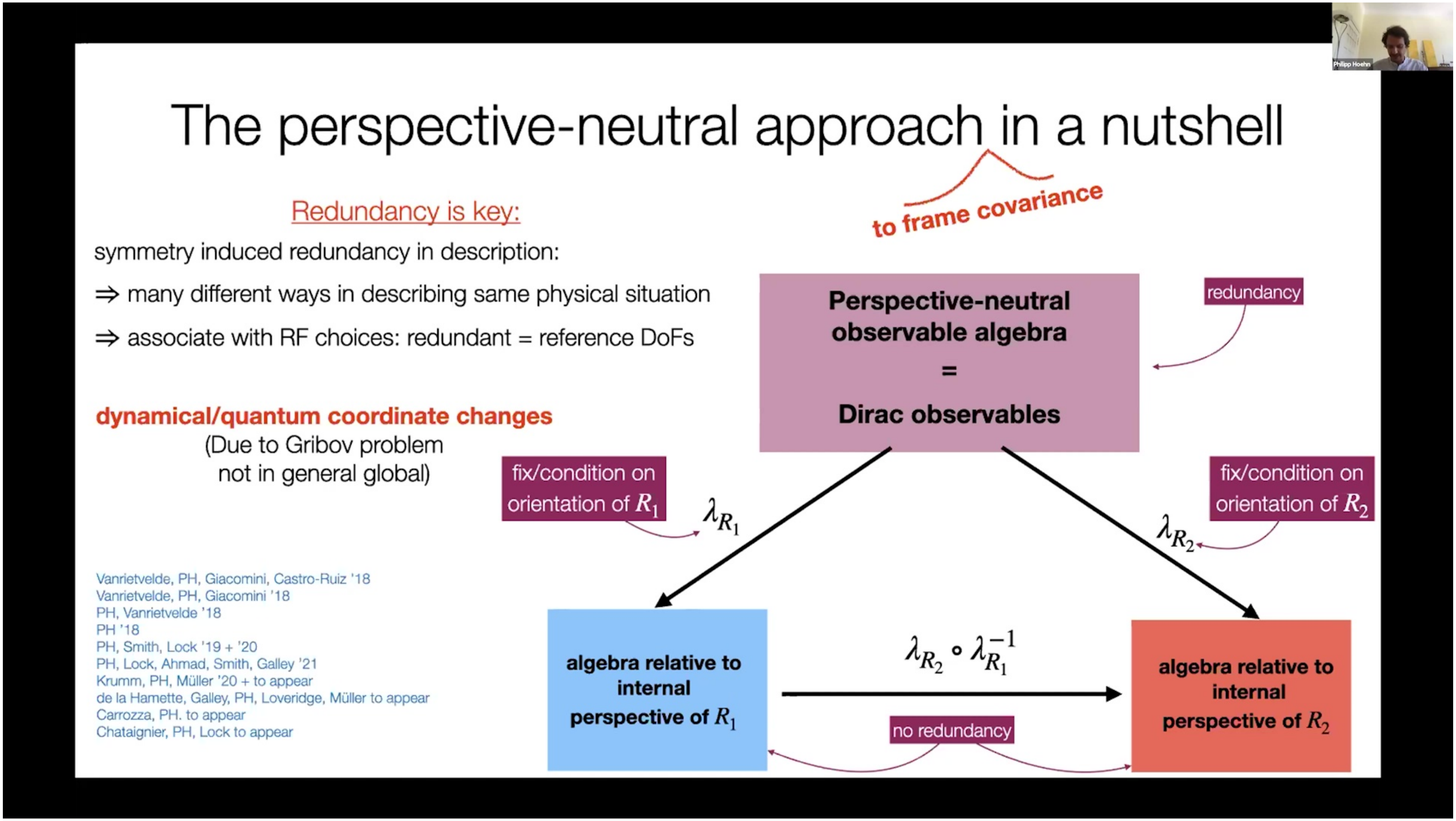
states relative to internal perspective of R_2

no redundancy

yields same QRF transformations as
Giacomini, Castro-Ruiz, Brukner *Nat Com* (2019)
de la Hamette, Galley *Quantum* (2020)



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Perspective-neutral observable algebra
=
Dirac observables

redundancy

fix/condition on orientation of R_1

λ_{R_1}

fix/condition on orientation of R_2

λ_{R_2}

algebra relative to internal perspective of R_1

$\lambda_{R_2} \circ \lambda_{R_1}^{-1}$

algebra relative to internal perspective of R_2

no redundancy

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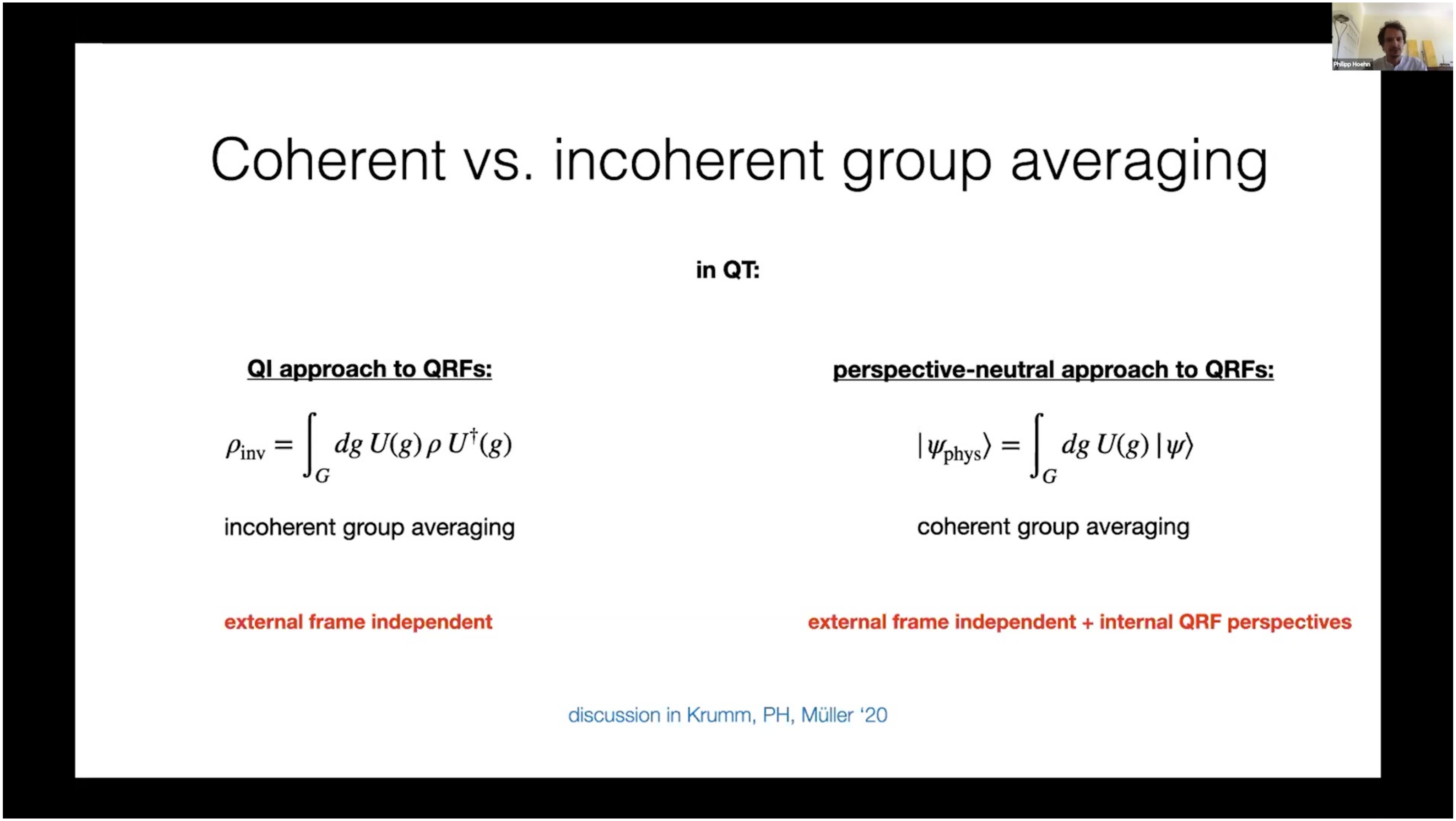


Coherent vs. incoherent group averaging

in QT:

QI approach to QRFs:

$$\rho_{\text{inv}} = \int_G dg\, U(g)\, \rho\, U^\dagger(g)$$

incoherent group averaging

external frame independent

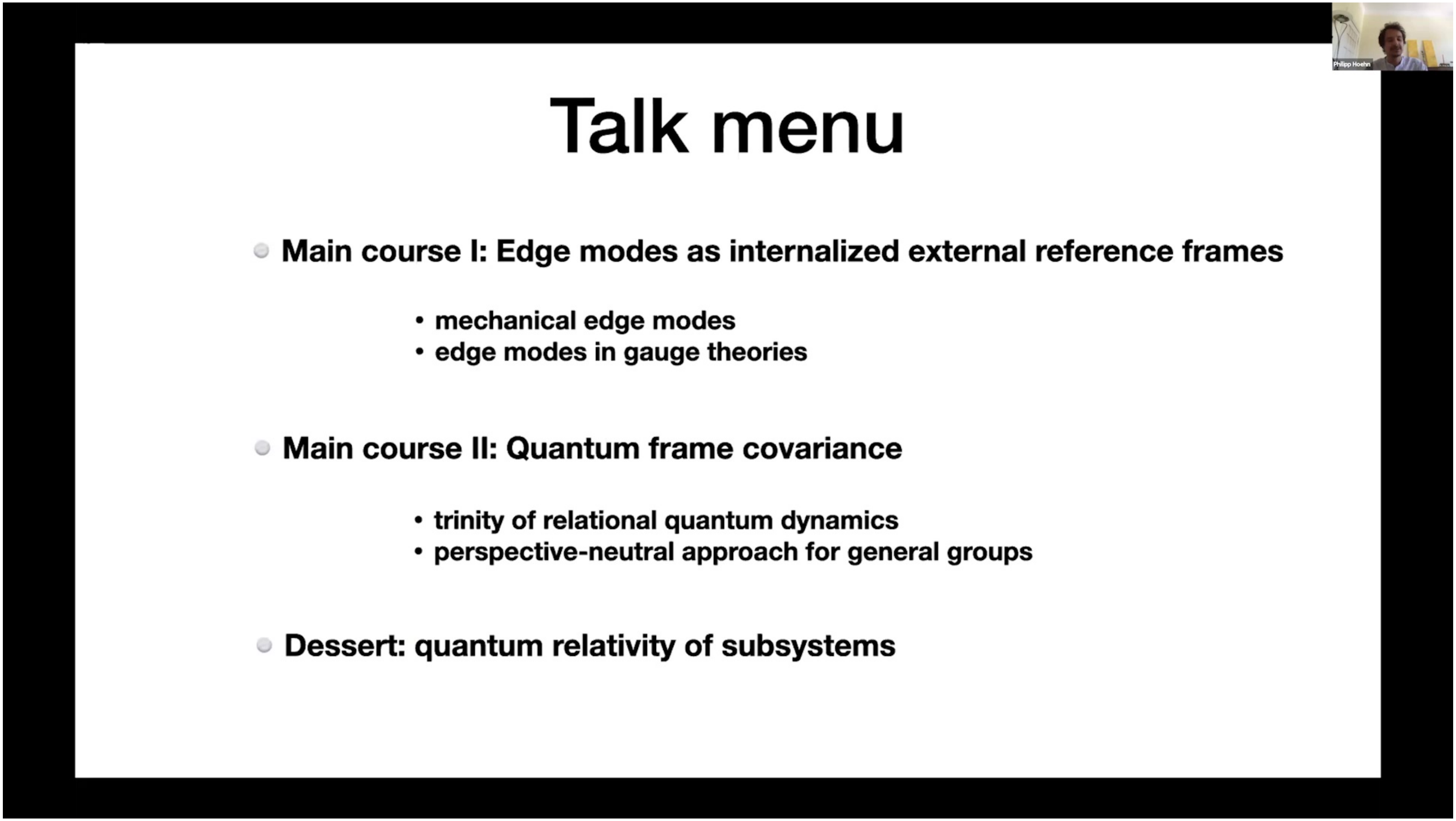
perspective-neutral approach to QRFs:

$$|\psi_{\text{phys}}\rangle = \int_G dg\, U(g)\, |\psi\rangle$$

coherent group averaging

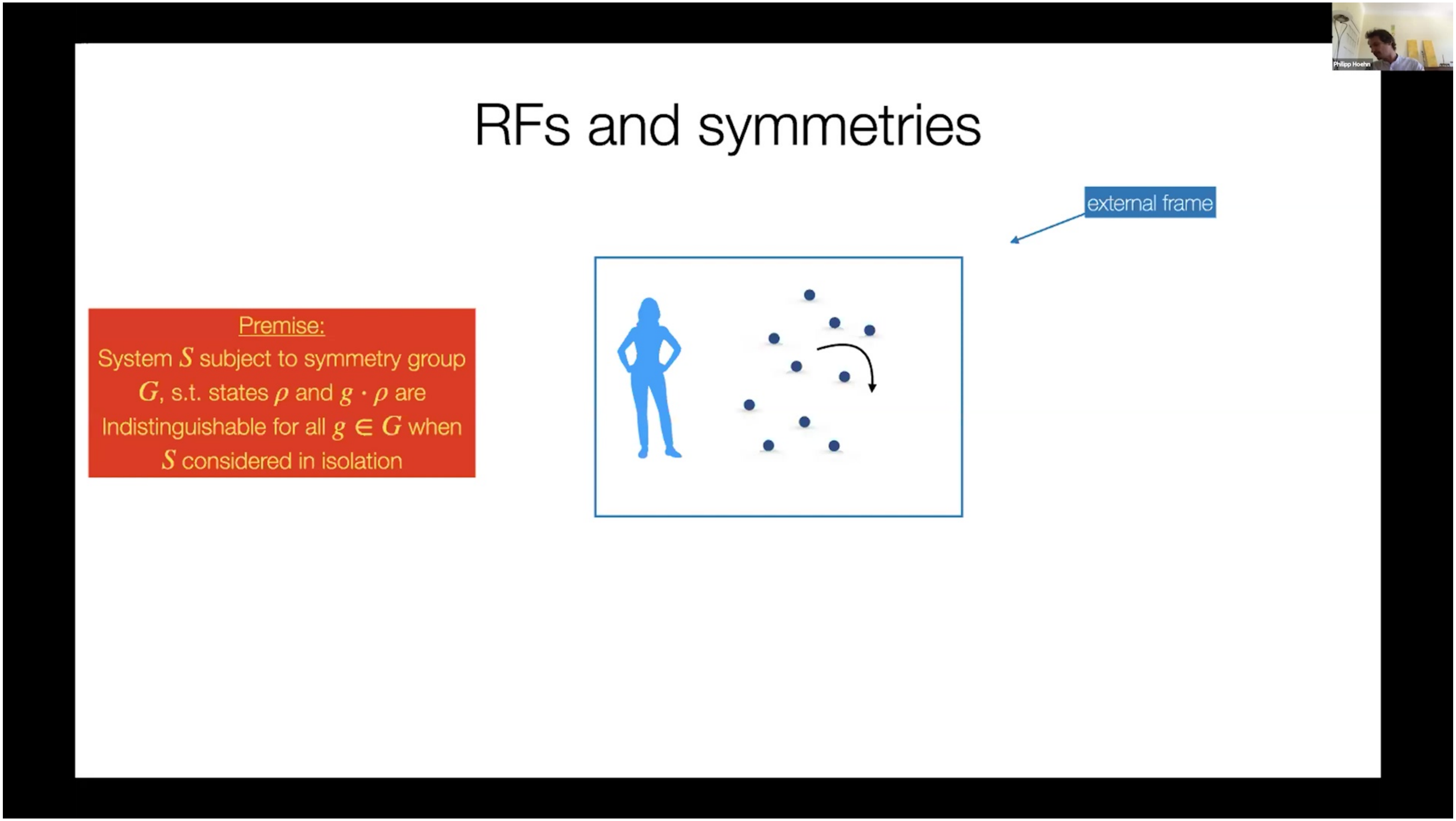
external frame independent + internal QRF perspectives

discussion in Krumm, PH, Müller '20



Talk menu

- **Main course I: Edge modes as internalized external reference frames**
  - mechanical edge modes
  - edge modes in gauge theories
- **Main course II: Quantum frame covariance**
  - trinity of relational quantum dynamics
  - perspective-neutral approach for general groups
- **Dessert: quantum relativity of subsystems**



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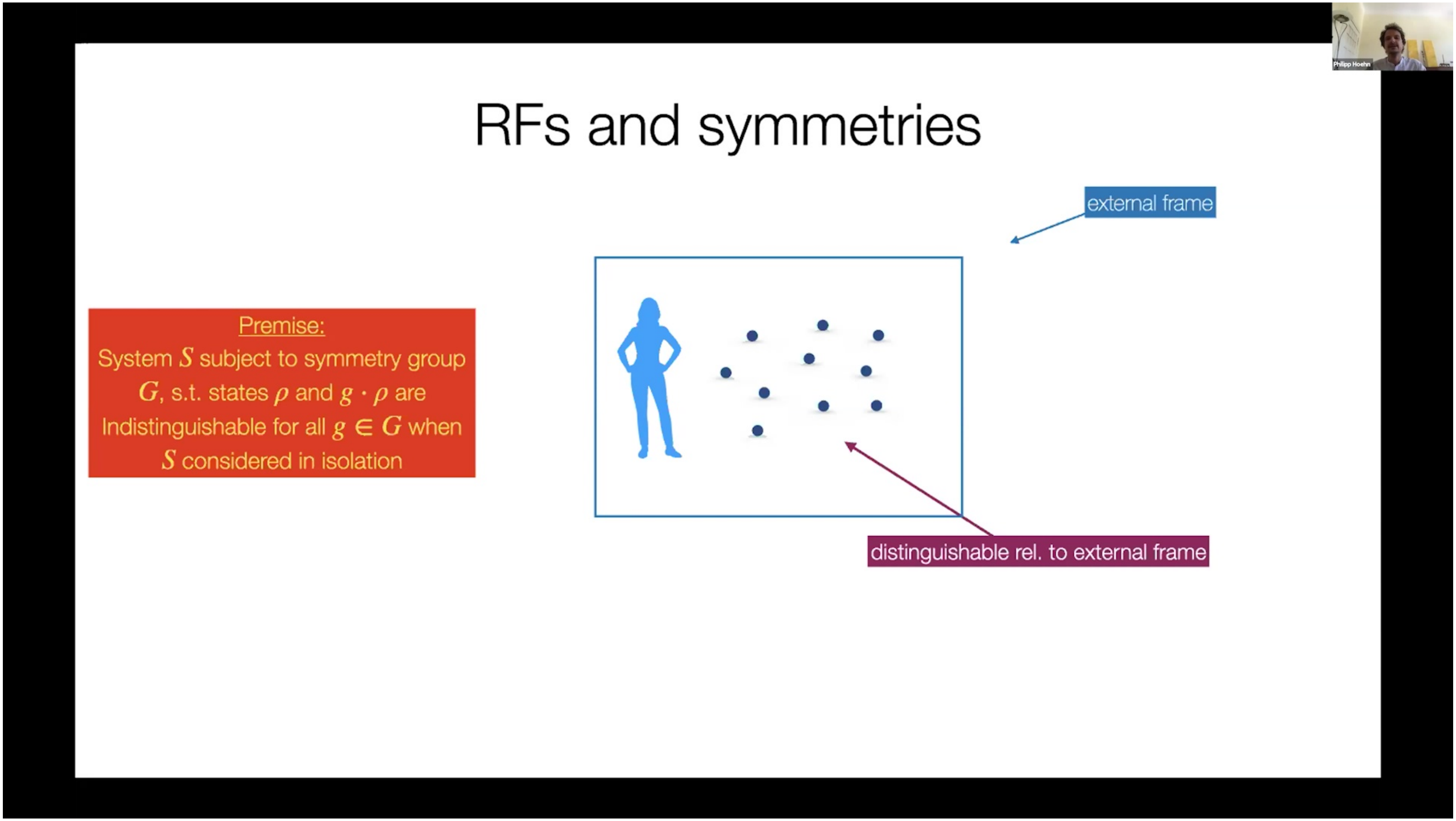
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

external frame



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external frame

distinguishable rel. to external frame

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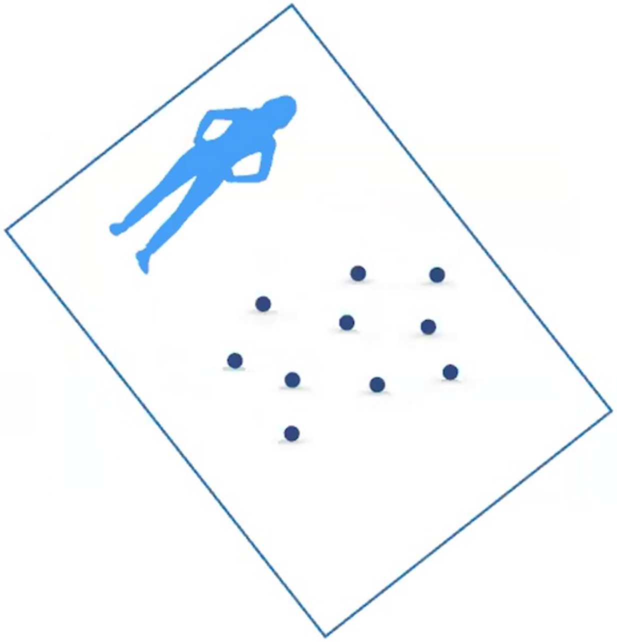
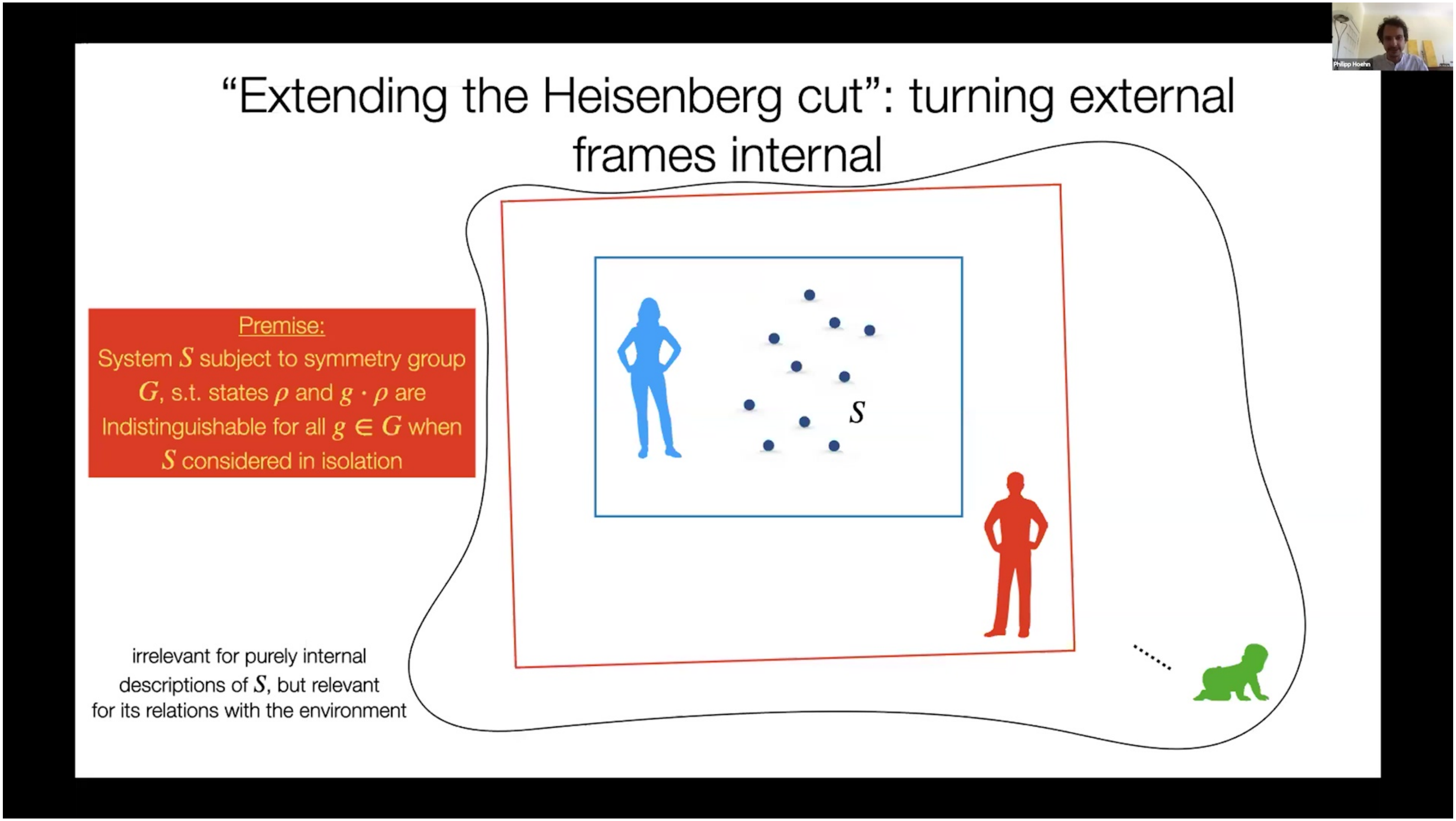


external frame

indistinguishable rel. to external frame

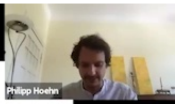


“Extending the Heisenberg cut”: turning external frames internal

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

irrelevant for purely internal descriptions of S, but relevant for its relations with the environment

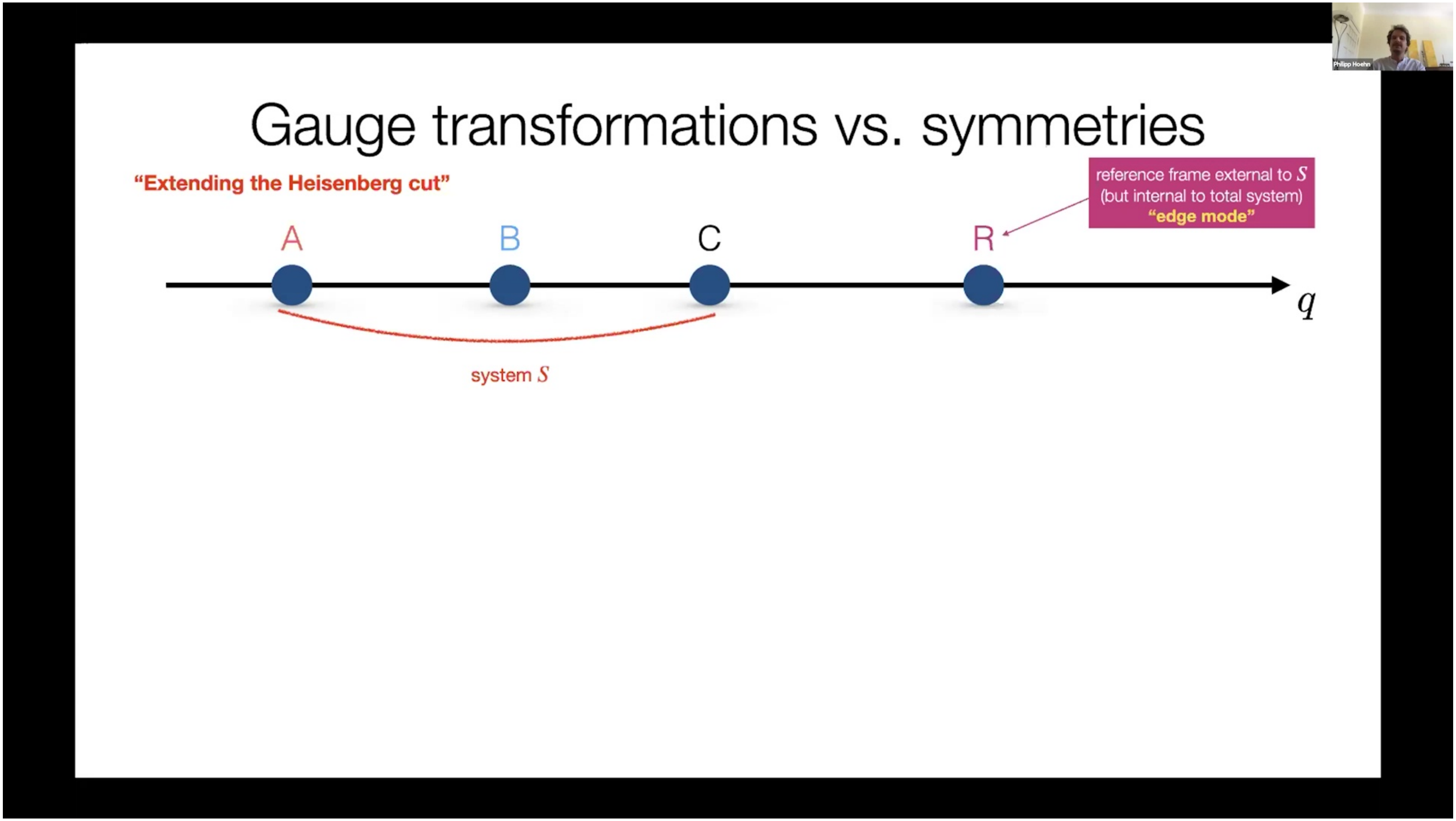


Gauge transformations vs. symmetries

Illustration for RFs associated with 1D translation invariance

internally indistinguishable: relational data $q_i - q_j$ within S invariant

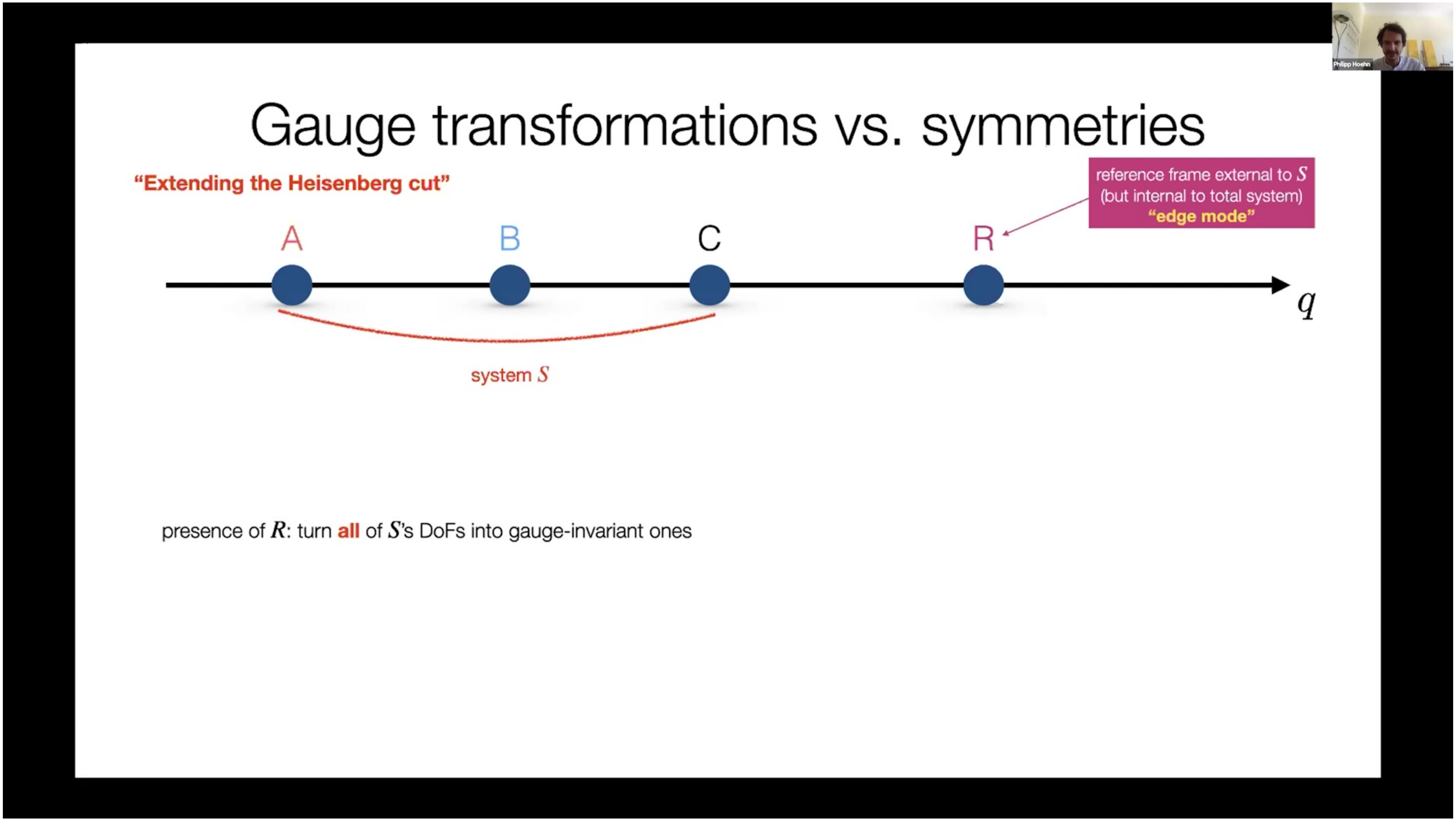
translation generator $P_S = p_A + p_B + p_C \approx 0$



Gauge transformations vs. symmetries

“Extending the Heisenberg cut”

reference frame external to S
(but internal to total system)
“edge mode”



Gauge transformations vs. symmetries

“Extending the Heisenberg cut”

reference frame external to S
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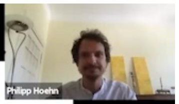
“edge mode”

presence of R: turn **all** of S's DoFs into gauge-invariant ones



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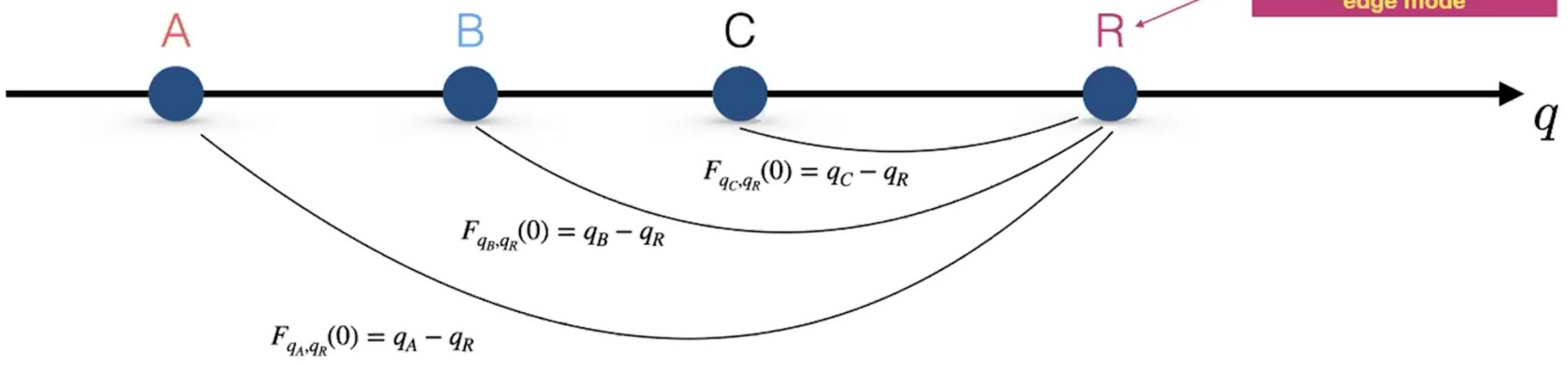


$F_{q_C,q_R}(0) = q_C - q_R$

$F_{q_B,q_R}(0) = q_B - q_R$

$F_{q_A,q_R}(0) = q_A - q_R$

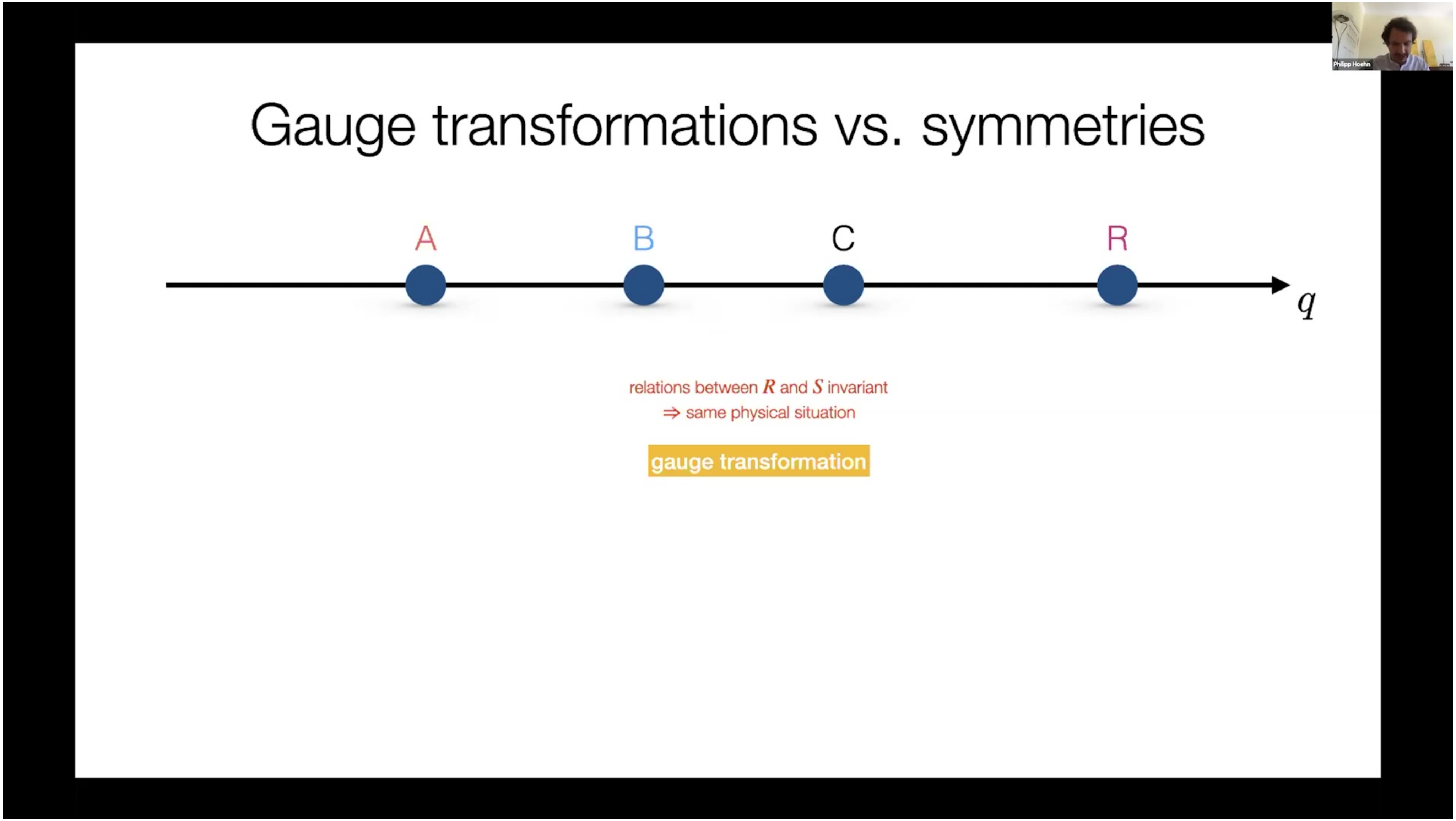
presence of R: turn **all** of S’s DoFs into gauge-invariant ones

“frame orientation”

$$F_{q_i,q_R}(X) = \sum_{n=0} \frac{(X-q_R)^n}{n!}\{q_i, P_{\text{tot}}\}_n = q_i - q_R + X$$

relational observable power series: Dittrich ’04; ‘05

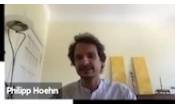
relational observable: “what’s the position of particle i when R is in position X?”



Gauge transformations vs. symmetries

relations between R and S invariant
⇒ same physical situation

gauge transformation



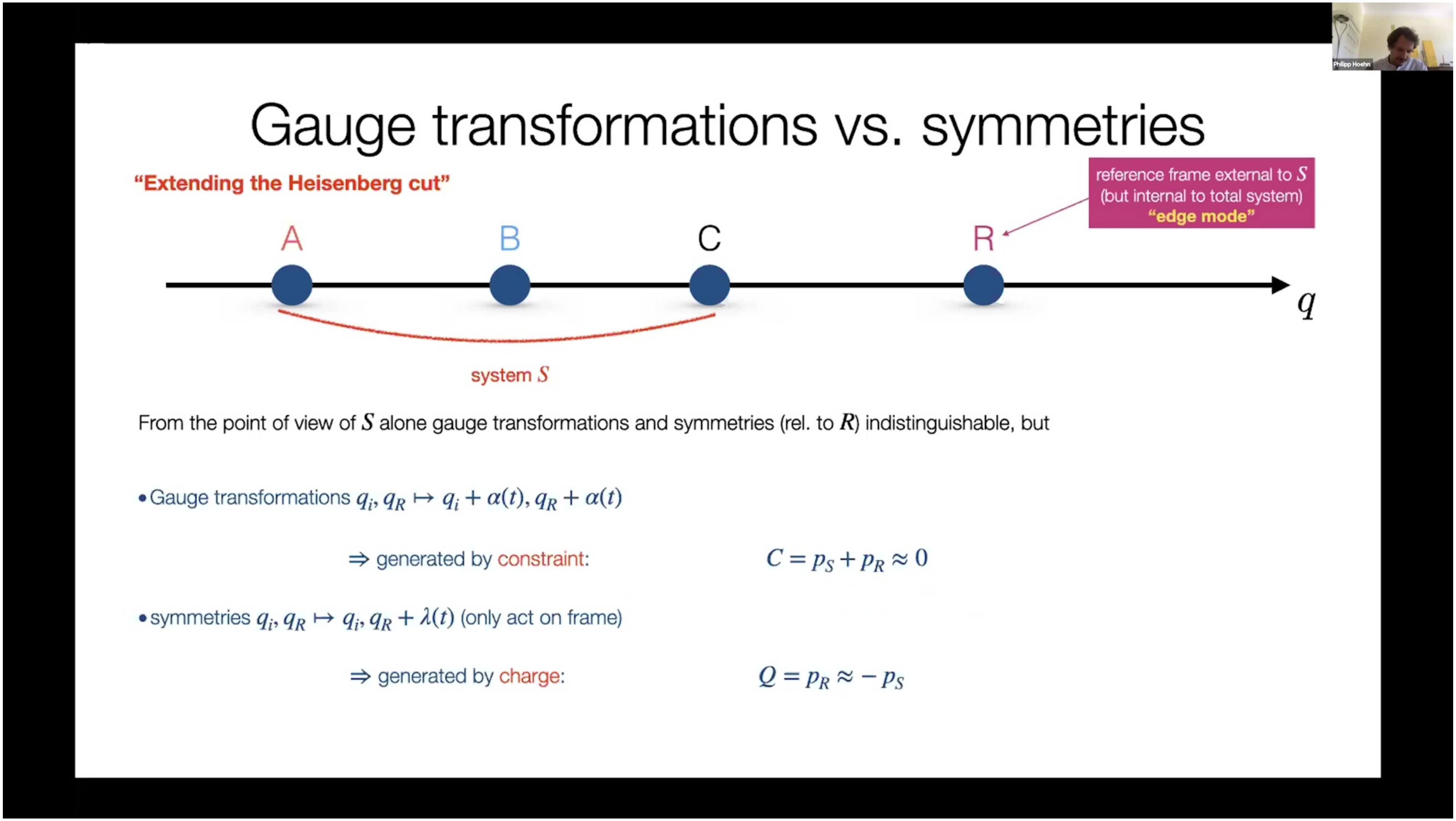
Gauge transformations vs. symmetries

relation between R and S changed

⇒ internally indistinguishable for S, but distinguishable for reference frame

⇒ physical situation changed

symmetry



Gauge transformations vs. symmetries

"Extending the Heisenberg cut"

reference frame external to S
(but internal to total system)
"edge mode"

system S

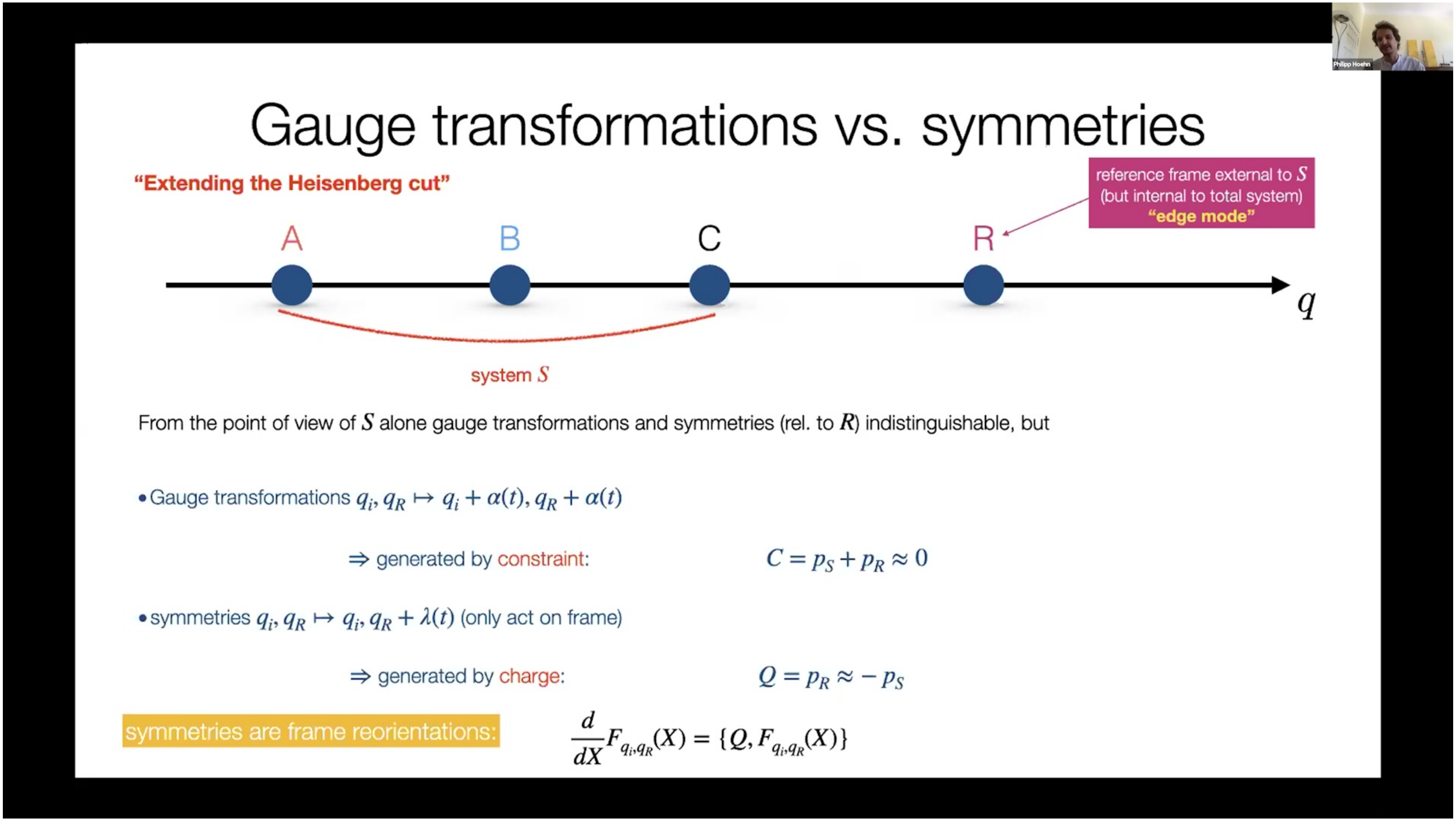
From the point of view of S alone gauge transformations and symmetries (rel. to R) indistinguishable, but

- Gauge transformations $q_i, q_R \mapsto q_i + \alpha(t), q_R + \alpha(t)$

  $\Rightarrow$ generated by constraint: $C = p_S + p_R \approx 0$

- symmetries $q_i, q_R \mapsto q_i, q_R + \lambda(t)$ (only act on frame)

  $\Rightarrow$ generated by charge: $Q = p_R \approx - p_S$



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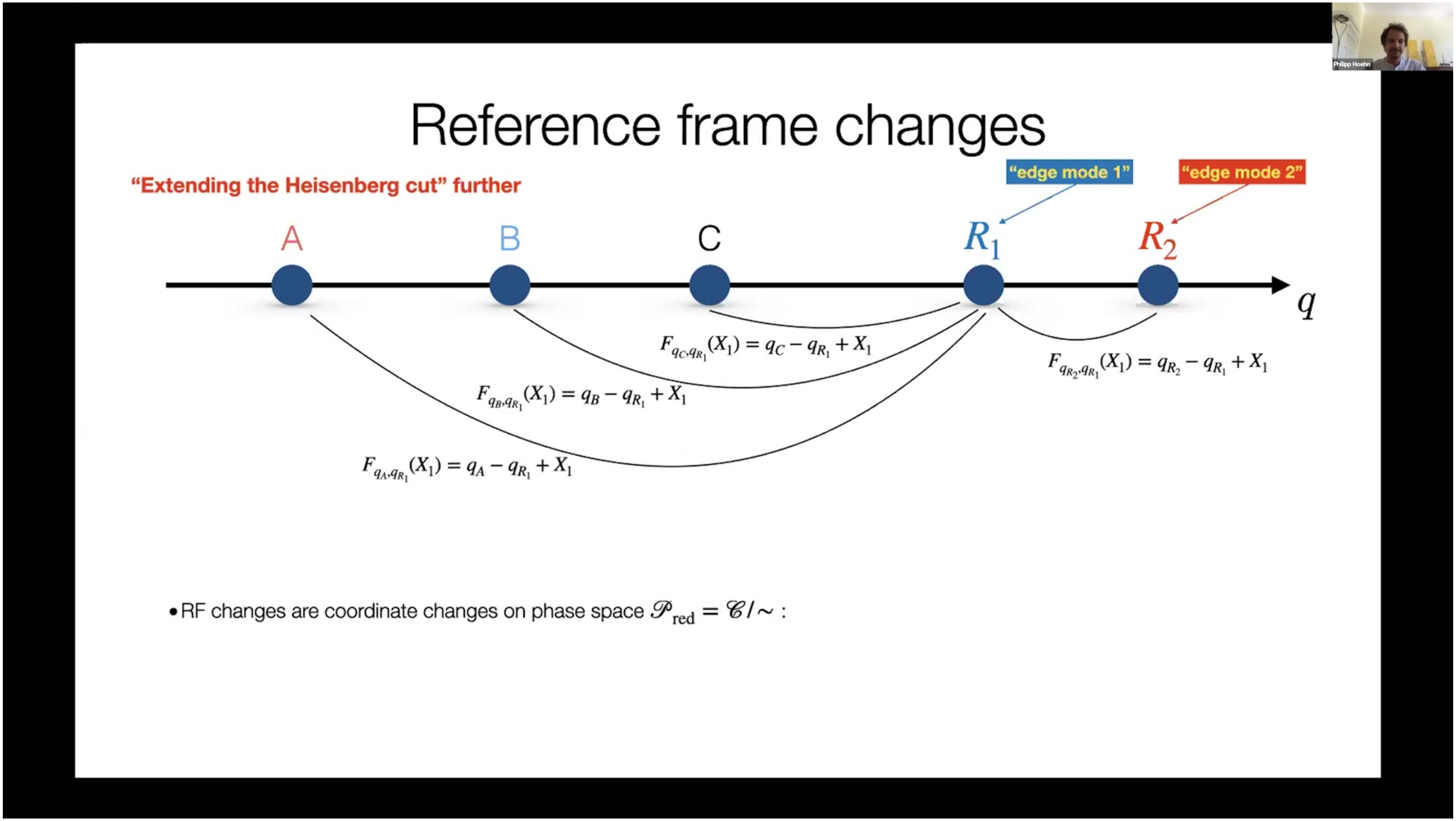
symmetries are frame reorientations:

$$\frac{d}{dX} F_{q_i,q_R}(X) = \{Q, F_{q_i,q_R}(X)\}$$



Reference frame changes

“Extending the Heisenberg cut” further

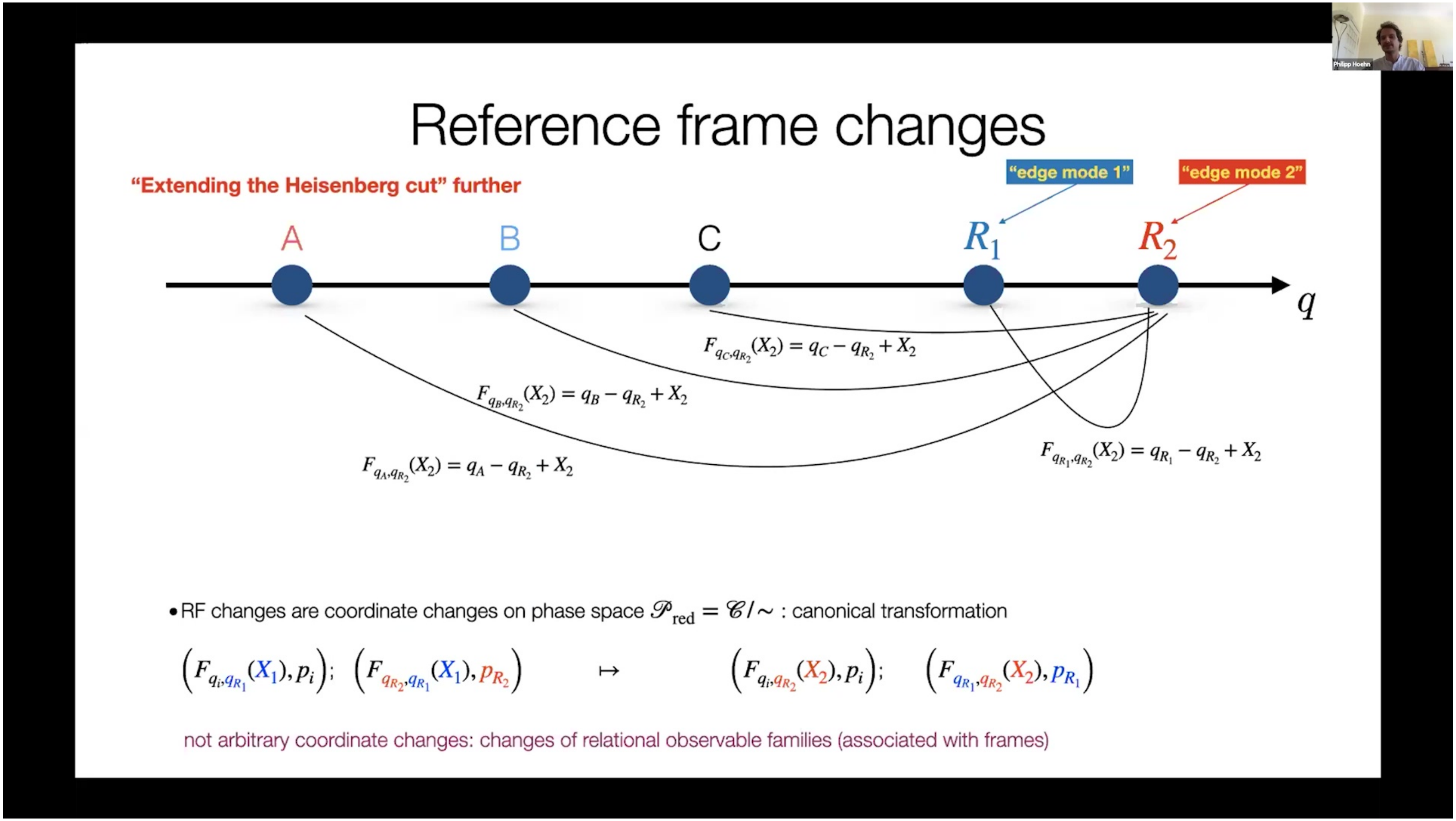
$F_{q_C,q_{R_1}}(X_1) = q_C - q_{R_1} + X_1$

$F_{q_{R_2},q_{R_1}}(X_1) = q_{R_2} - q_{R_1} + X_1$

$F_{q_B,q_{R_1}}(X_1) = q_B - q_{R_1} + X_1$

$F_{q_A,q_{R_1}}(X_1) = q_A - q_{R_1} + X_1$

- RF changes are coordinate changes on phase space $\mathscr{P}_{\text{red}} = \mathscr{C}/\sim$:



Reference frame changes

"Extending the Heisenberg cut" further

$F_{q_C,q_{R_2}}(X_2) = q_C - q_{R_2} + X_2$

$F_{q_B,q_{R_2}}(X_2) = q_B - q_{R_2} + X_2$

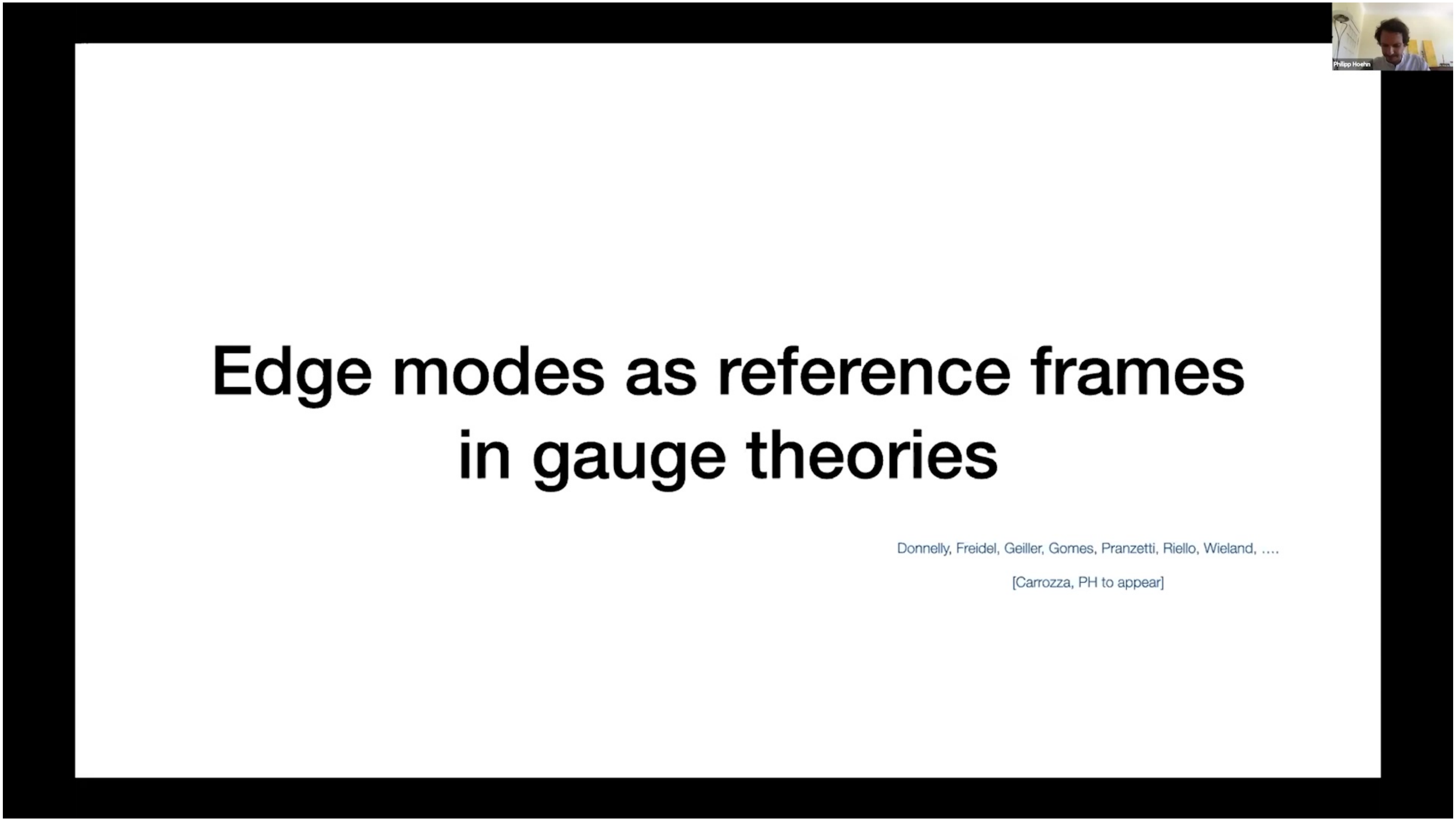
$F_{q_A,q_{R_2}}(X_2) = q_A - q_{R_2} + X_2$

$F_{q_{R_1},q_{R_2}}(X_2) = q_{R_1} - q_{R_2} + X_2$

- RF changes are coordinate changes on phase space $\mathscr{P}_{\text{red}} = \mathscr{C}/\sim$: canonical transformation

$$\left(F_{q_i,q_{R_1}}(X_1), p_i\right); \quad \left(F_{q_{R_2},q_{R_1}}(X_1), p_{R_2}\right) \qquad \mapsto \qquad \left(F_{q_i,q_{R_2}}(X_2), p_i\right); \quad \left(F_{q_{R_1},q_{R_2}}(X_2), p_{R_1}\right)$$

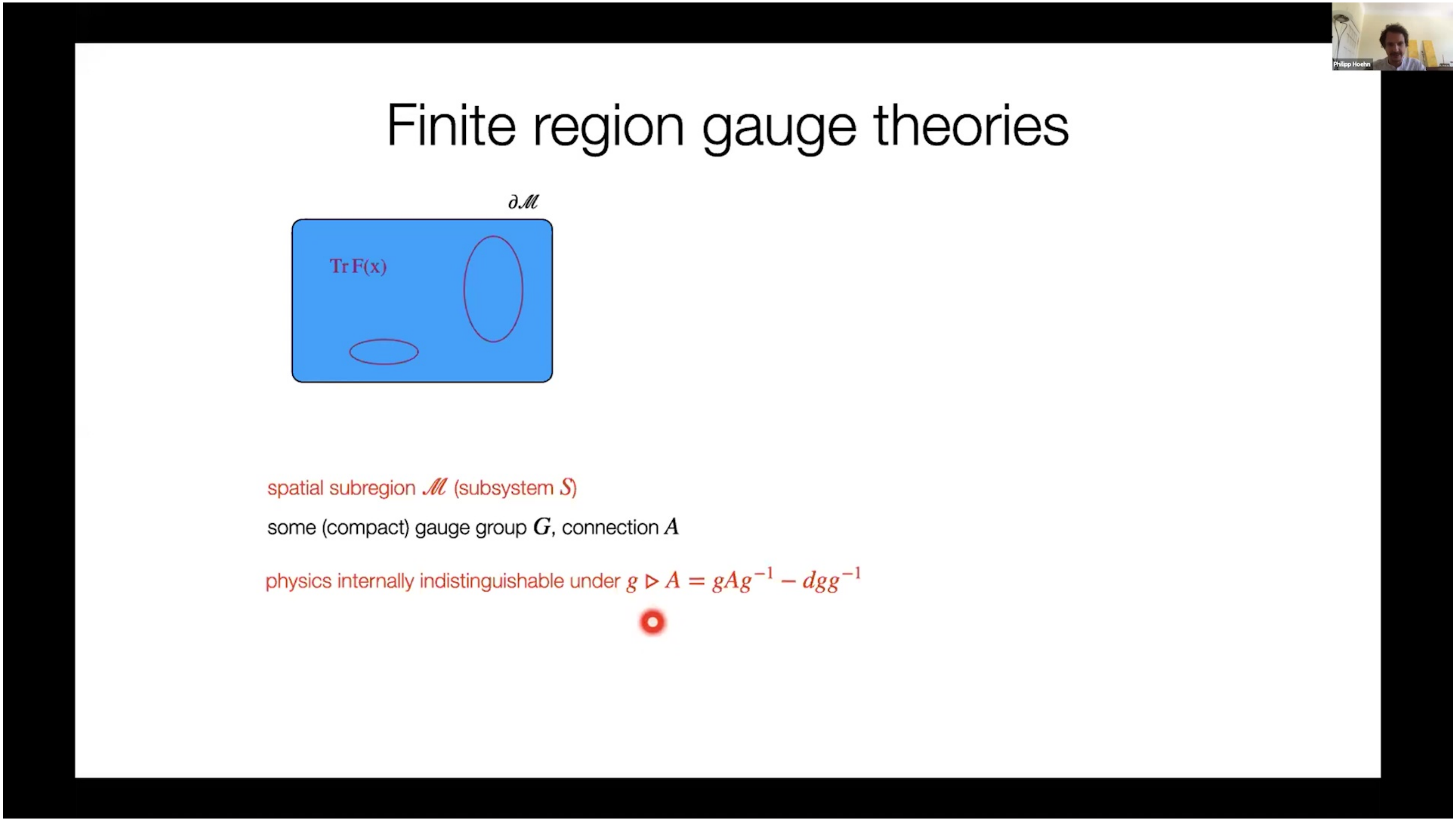
not arbitrary coordinate changes: changes of relational observable families (associated with frames)



Edge modes as reference frames in gauge theories

Donnelly, Freidel, Geiller, Gomes, Pranzetti, Riello, Wieland, ….

[Carrozza, PH to appear]

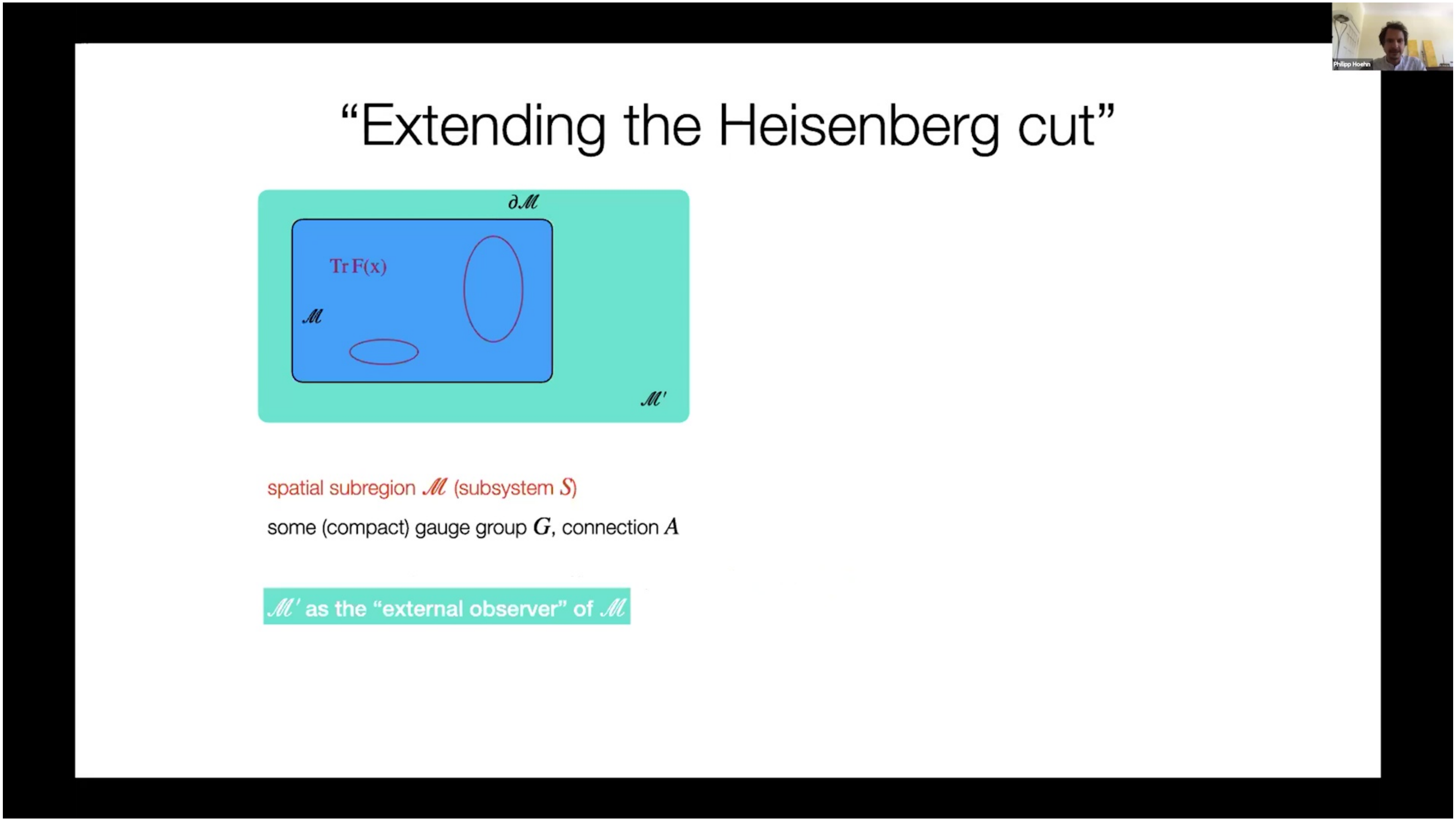


Finite region gauge theories

spatial subregion $\mathscr{M}$ (subsystem S)

some (compact) gauge group G, connection A

physics internally indistinguishable under $g \triangleright A = gAg^{-1} - dgg^{-1}$

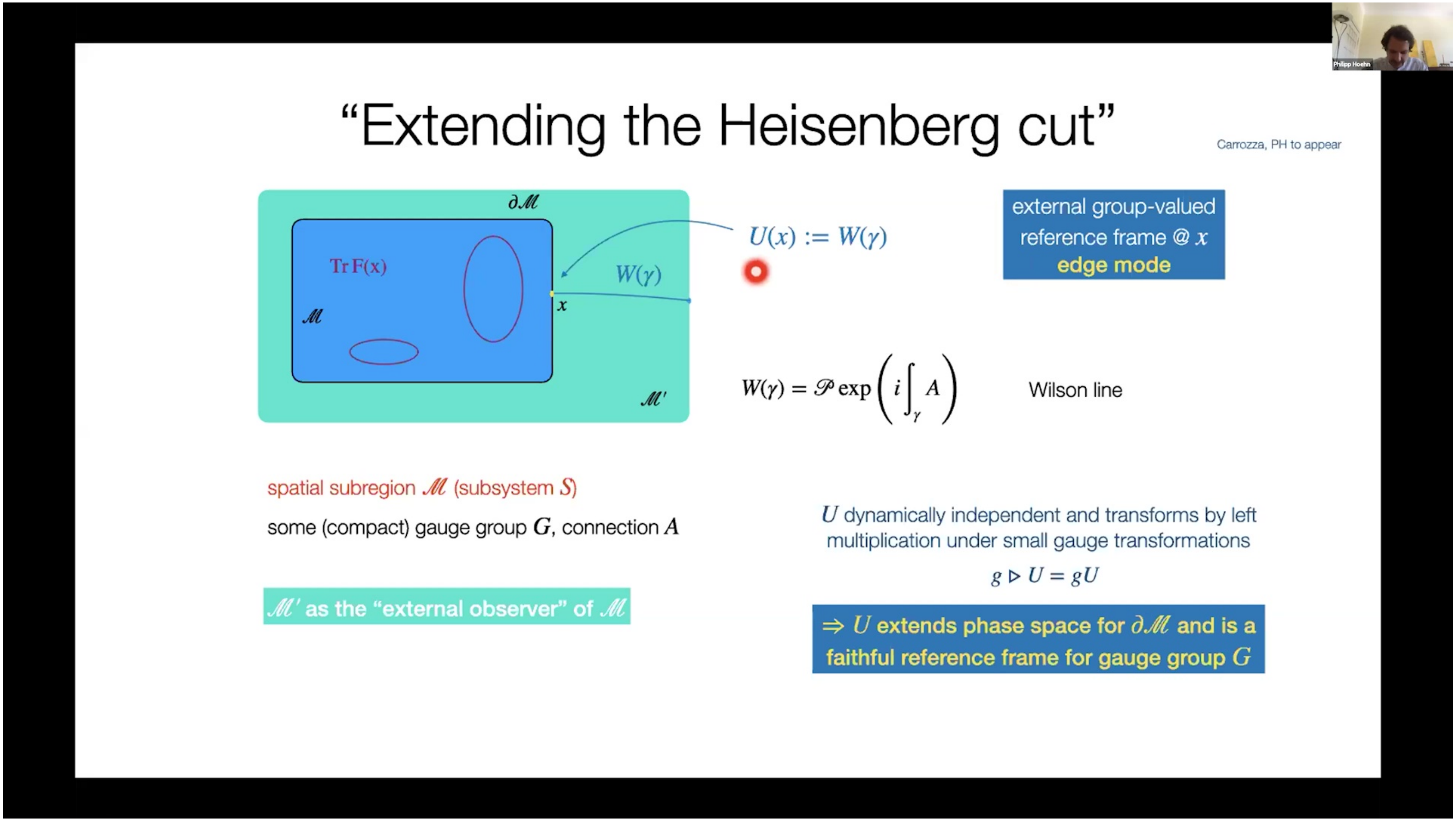


“Extending the Heisenberg cut”

spatial subregion $\mathscr{M}$ (subsystem S)

some (compact) gauge group G, connection A

$\mathscr{M}'$ as the “external observer” of $\mathscr{M}$



“Extending the Heisenberg cut”

Carrozza, PH to appear

$U(x) := W(\gamma)$

external group-valued reference frame @ x
edge mode

$$W(\gamma) = \mathscr{P} \exp\left(i \int_\gamma A \right)$$

Wilson line

spatial subregion $\mathscr{M}$ (subsystem S)

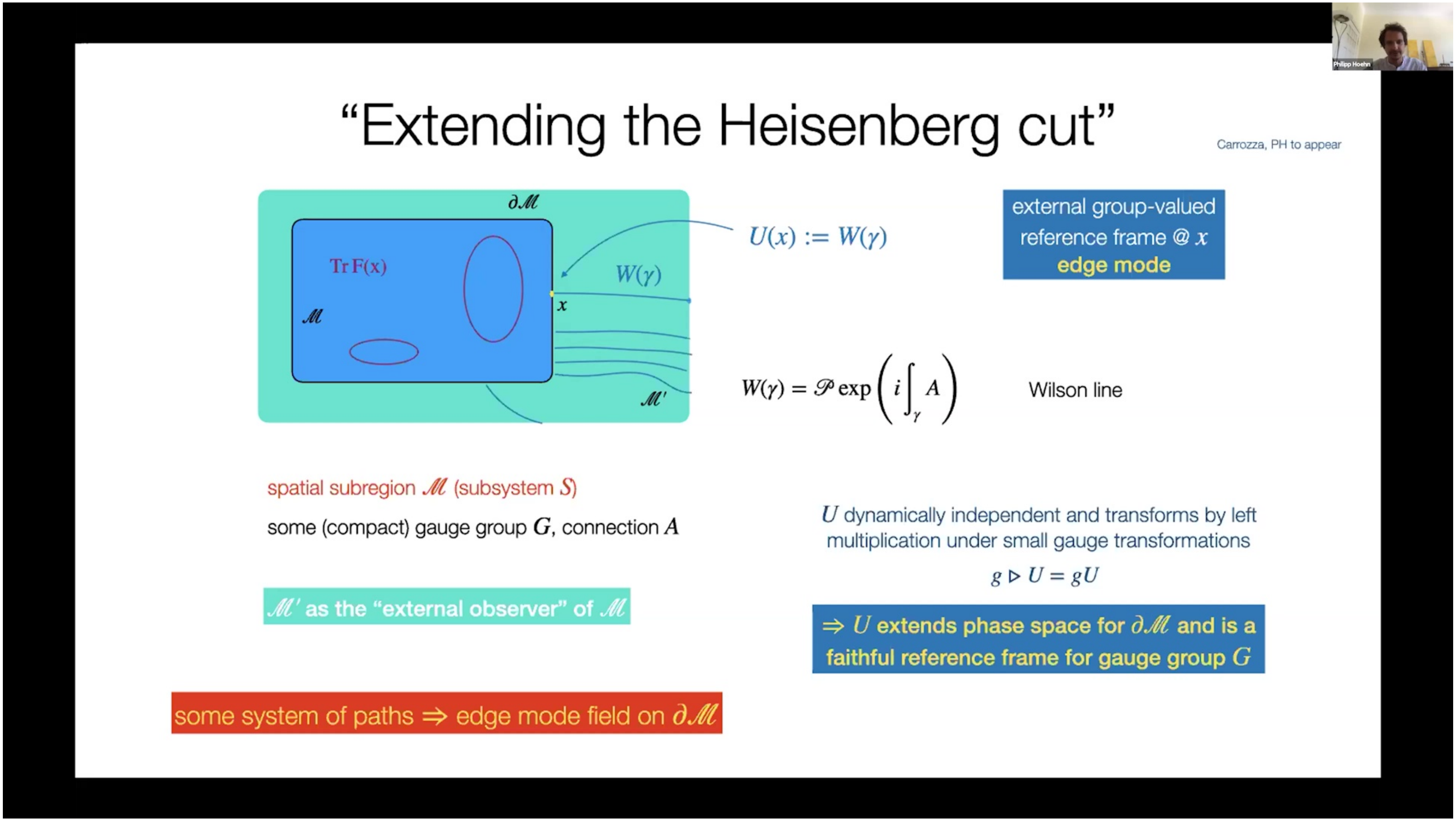
some (compact) gauge group G, connection A

$\mathscr{M}'$ as the “external observer” of $\mathscr{M}$

U dynamically independent and transforms by left multiplication under small gauge transformations

$$g \triangleright U = gU$$

$\Rightarrow$ U extends phase space for $\partial\mathscr{M}$ and is a faithful reference frame for gauge group G



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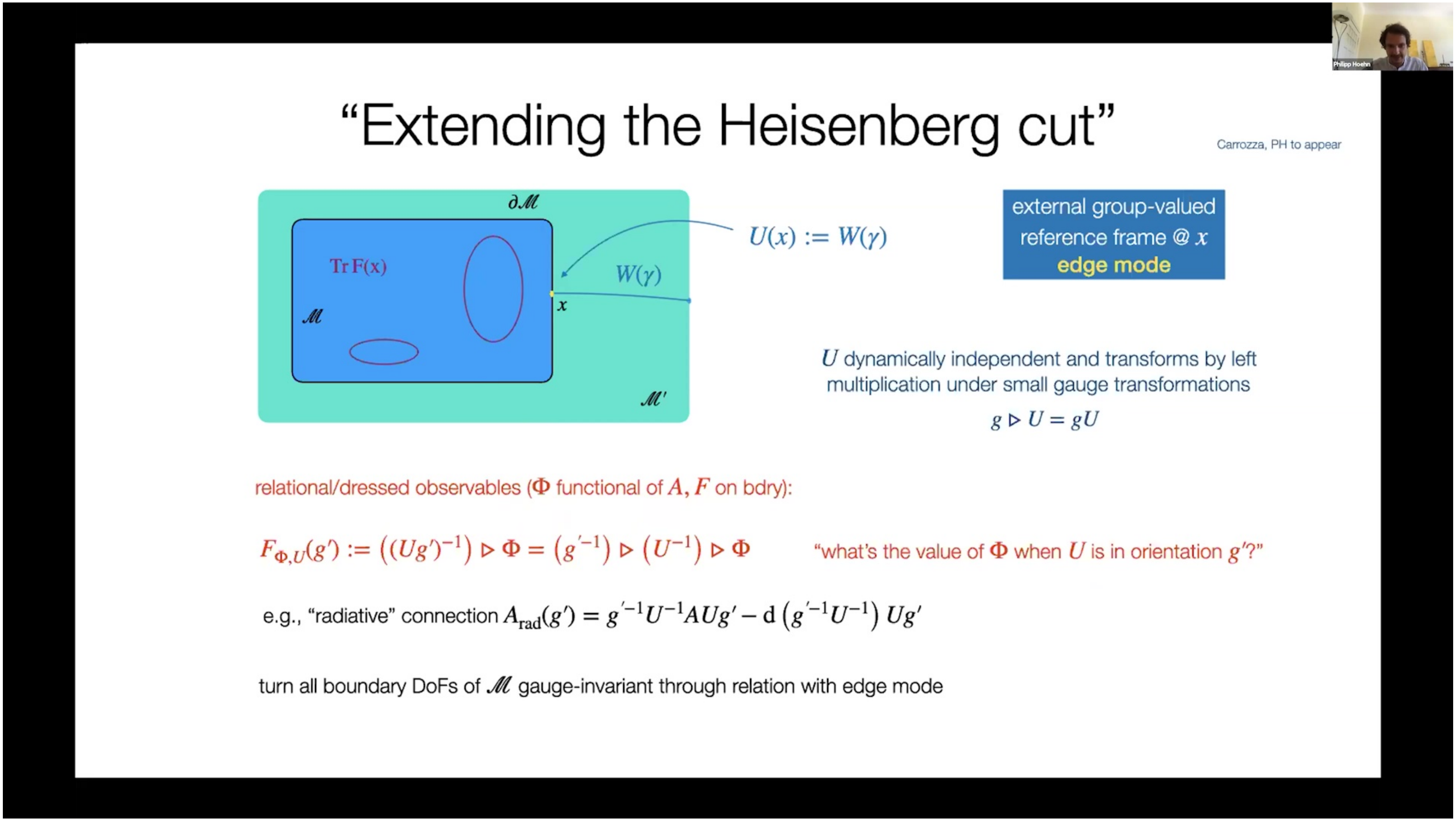
U dynamically independent and transforms by left multiplication under small gauge transformations

$$g \triangleright U = gU$$

⇒ U extends phase space for $\partial\mathscr{M}$ and is a faithful reference frame for gauge group G

some system of paths ⇒ edge mode field on $\partial\mathscr{M}$



“Extending the Heisenberg cut”

Carrozza, PH to appear

$U(x) := W(\gamma)$

external group-valued reference frame @ x
edge mode

U dynamically independent and transforms by left multiplication under small gauge transformations

$$g \triangleright U = gU$$

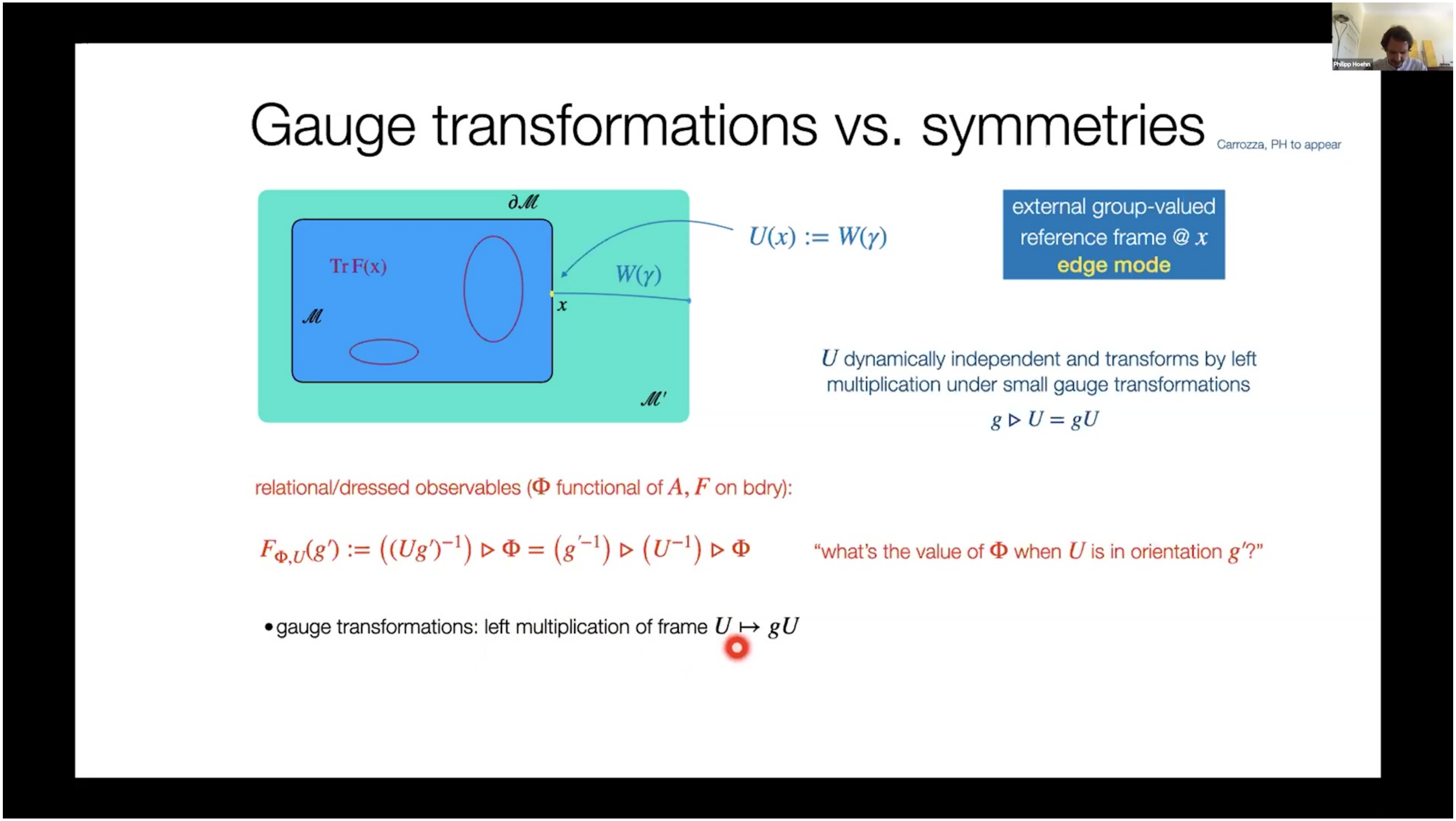
relational/dressed observables (Φ functional of A, F on bdry):

$$F_{\Phi,U}(g') := \left((Ug')^{-1}\right) \triangleright \Phi = \left(g'^{-1}\right) \triangleright \left(U^{-1}\right) \triangleright \Phi$$

“what’s the value of Φ when U is in orientation g'?”

e.g., “radiative” connection $A_{\text{rad}}(g') = g'^{-1}U^{-1}AUg' - \mathrm{d}\left(g'^{-1}U^{-1}\right)Ug'$

turn all boundary DoFs of $\mathscr{M}$ gauge-invariant through relation with edge mode



Gauge transformations vs. symmetries

Carrozza, PH to appear

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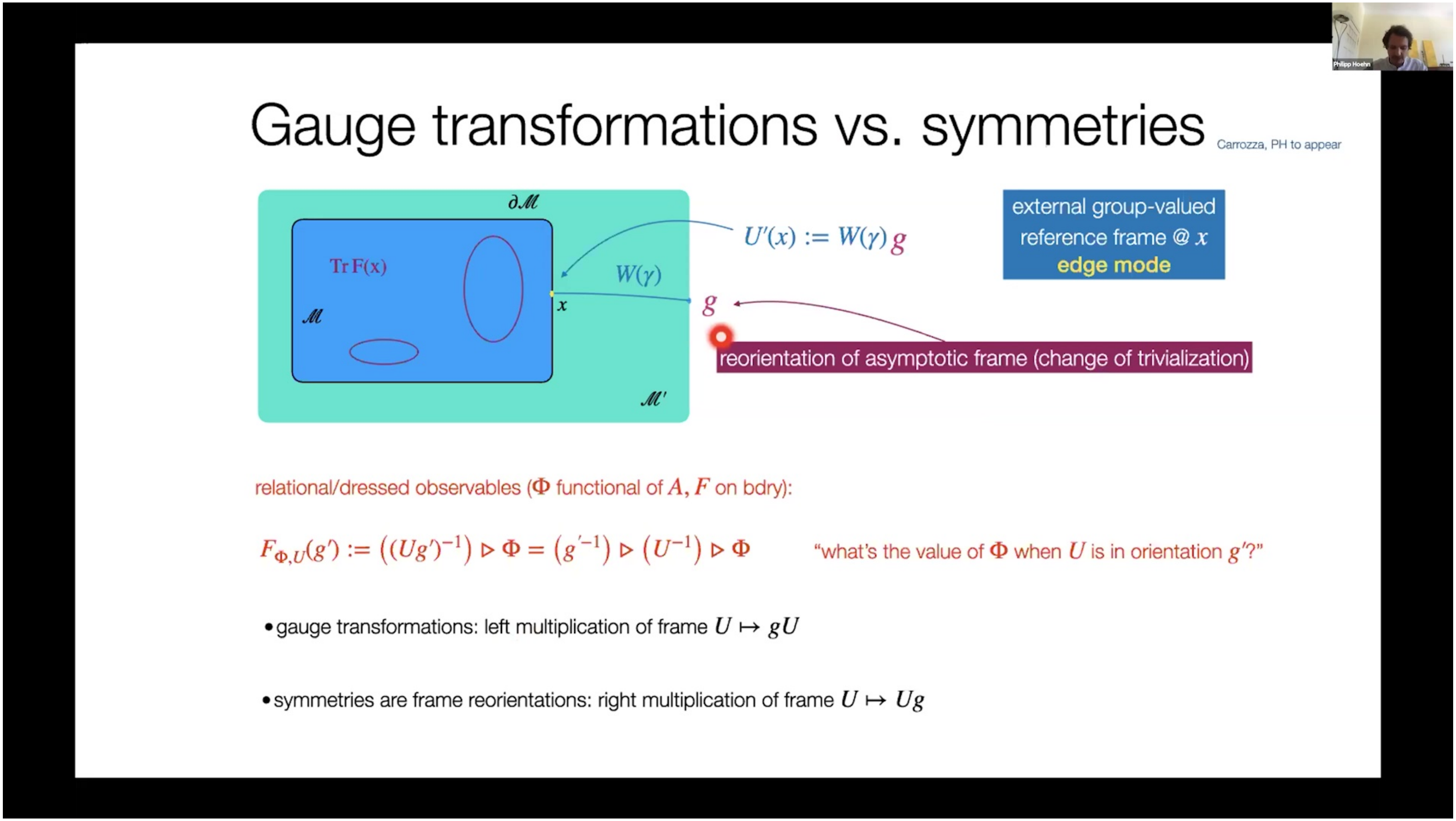
- gauge transformations: left multiplication of frame $U \mapsto gU$



Gauge transformations vs. symmetries

Carrozza, PH to appear

external group-valued reference frame @ x
edge mode

reorientation of asymptotic frame (change of trivialization)

relational/dressed observables (Φ functional of A, F on bdry):

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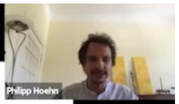
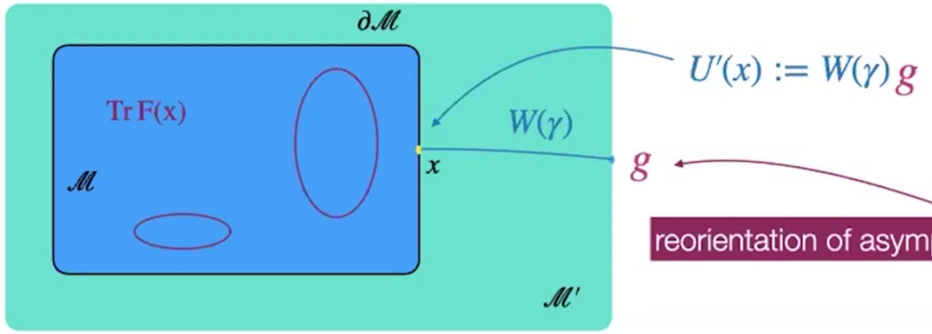
"what's the value of Φ when U is in orientation g'?"

- gauge transformations: left multiplication of frame $U \mapsto gU$
- symmetries are frame reorientations: right multiplication of frame $U \mapsto Ug$



Gauge transformations vs. symmetries

Carrozza, PH to appear



external group-valued reference frame @ x
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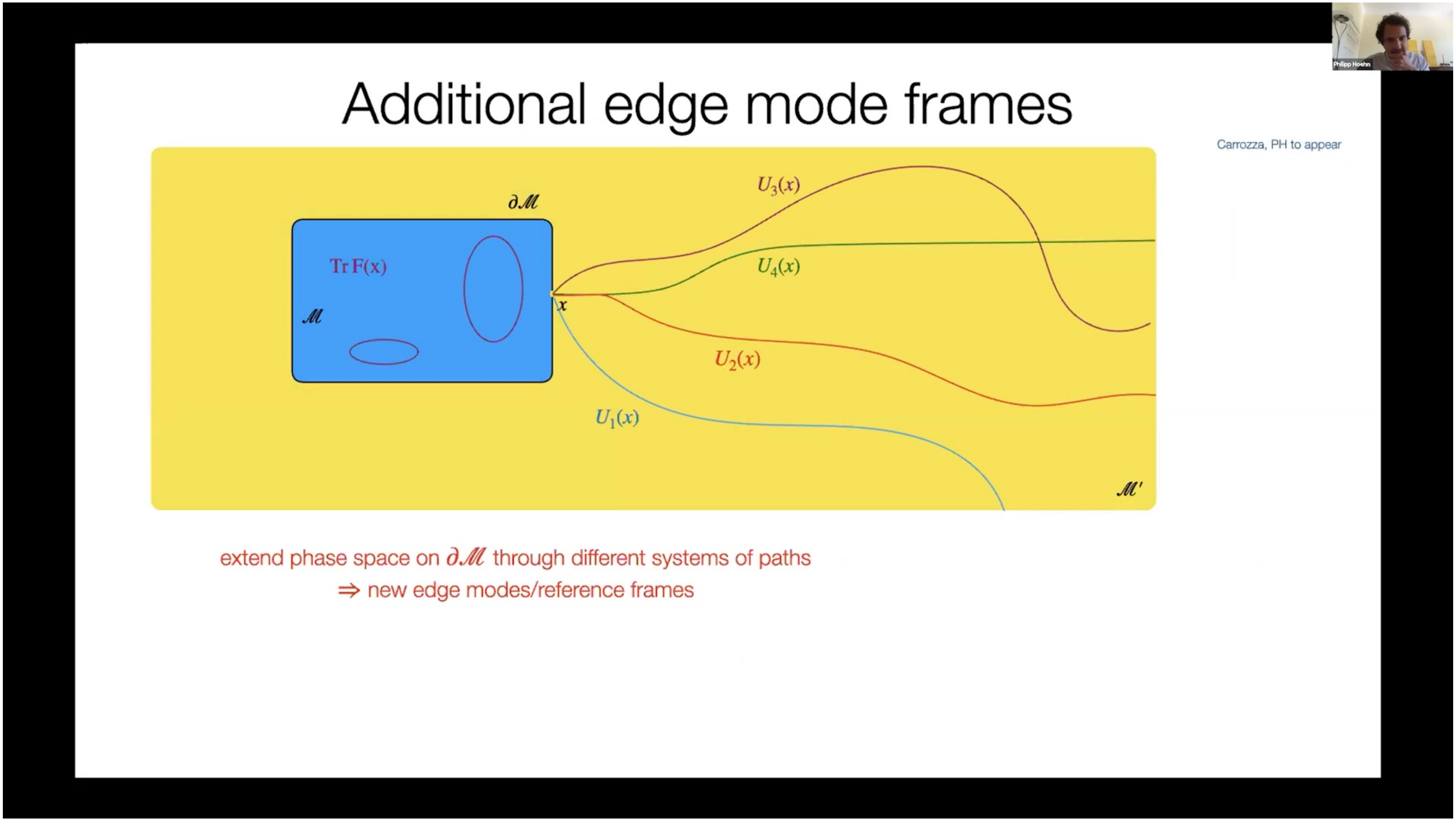
- gauge transformations: left multiplication of frame $U \mapsto gU$

  generated by constraints [Donnelly, Freidel '16; Geiller, Jai-akson '19; .....]

- symmetries are frame reorientations: right multiplication of frame $U \mapsto Ug$

  generated by charges [Donnelly, Freidel '16; Geiller, Jai-akson '19; .....]

$$F_{\Phi,Ug}(g') = F_{\Phi,U}(gg')$$

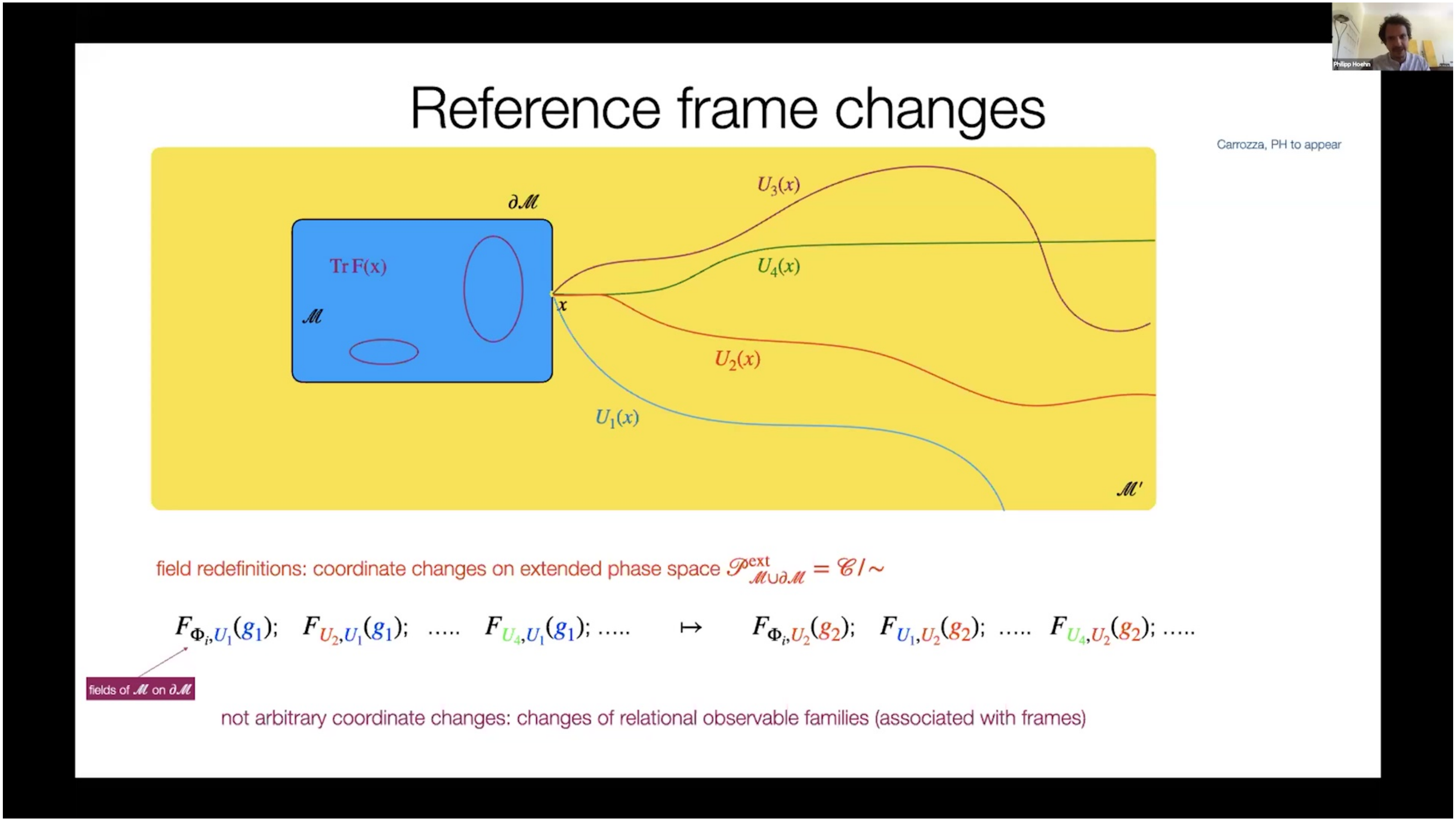


Additional edge mode frames

Carrozza, PH to appear

extend phase space on $\partial\mathscr{M}$ through different systems of paths

⇒ new edge modes/reference frames



Reference frame changes

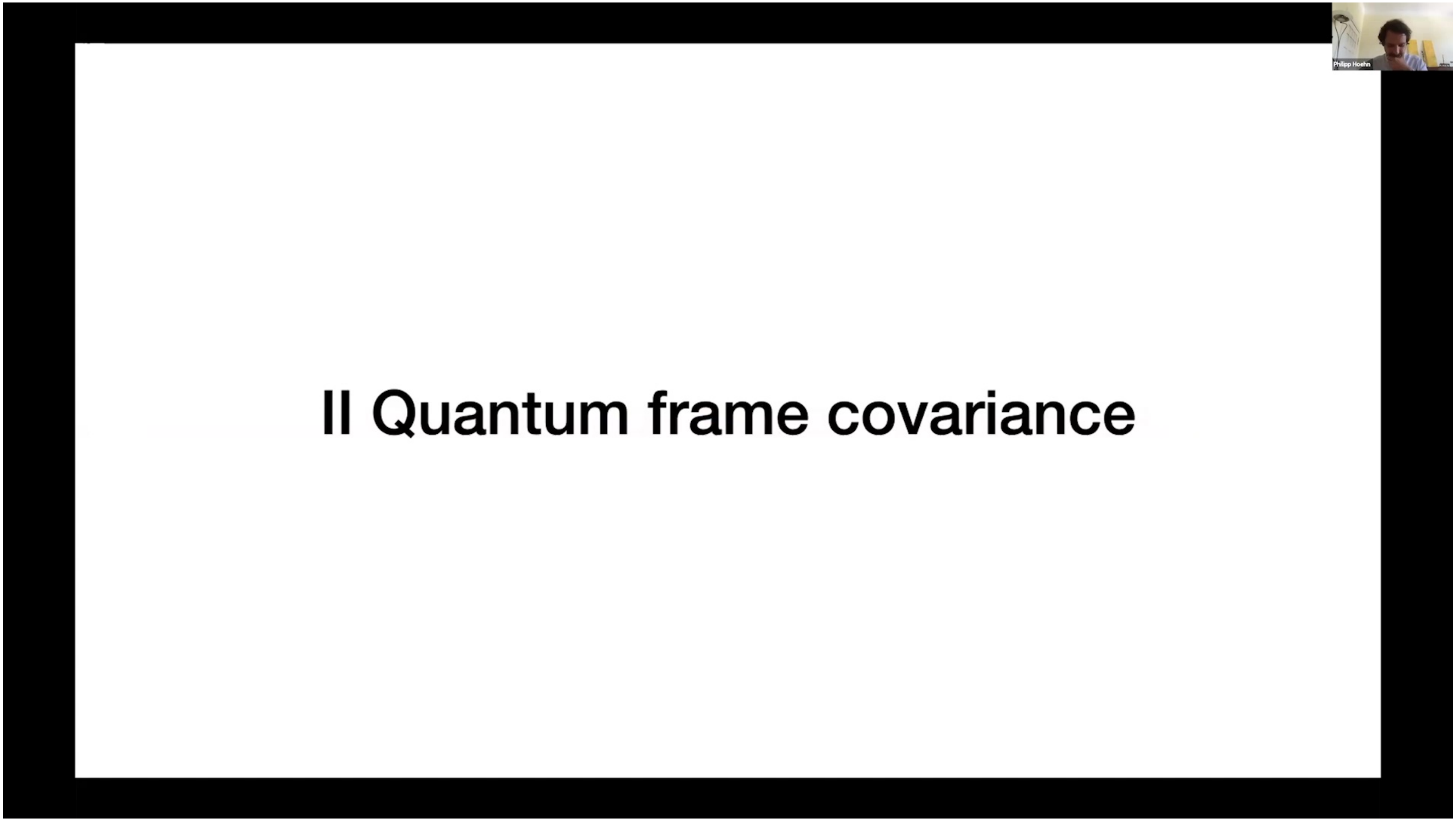
Carrozza, PH to appear

field redefinitions: coordinate changes on extended phase space $\mathscr{P}^{\text{ext}}_{\mathcal{M}\cup\partial\mathcal{M}} = \mathscr{C}/\sim$

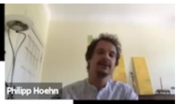
$$F_{\Phi_i,U_1}(g_1);\quad F_{U_2,U_1}(g_1);\quad \ldots\ldots\quad F_{U_4,U_1}(g_1);\ \ldots\ldots\qquad \mapsto\qquad F_{\Phi_i,U_2}(g_2);\quad F_{U_1,U_2}(g_2);\quad \ldots\ldots\quad F_{U_4,U_2}(g_2);\ \ldots\ldots$$

fields of $\mathcal{M}$ on $\partial\mathcal{M}$

not arbitrary coordinate changes: changes of relational observable families (associated with frames)



II Quantum frame covariance



Temporal QRFs: quantum clocks

Hamiltonian constraint

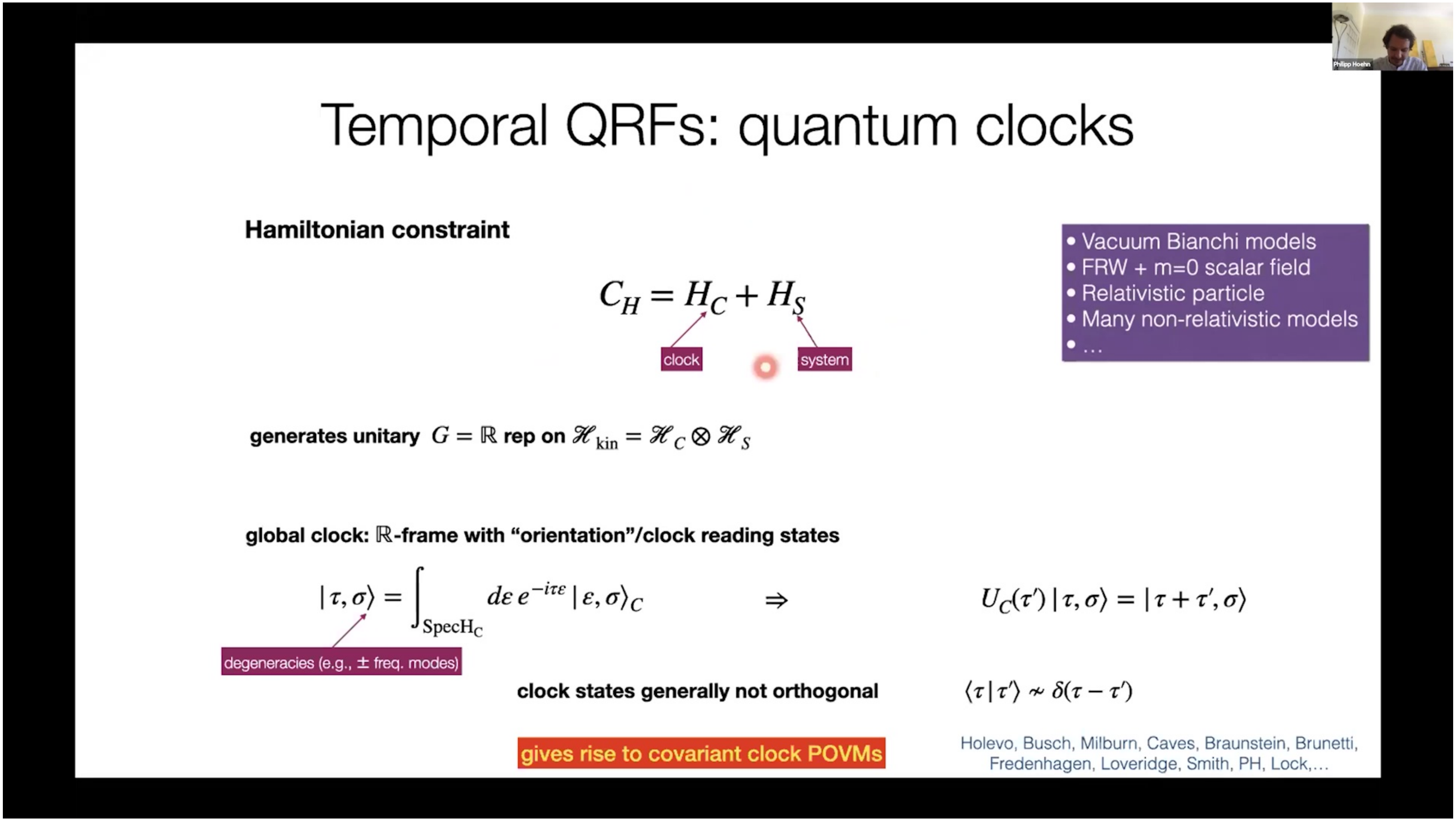
$$C_H = H_C + H_S$$

clock

system

- Vacuum Bianchi models
- FRW + m=0 scalar field
- Relativistic particle
- Many non-relativistic models
- …

generates unitary $G = \mathbb{R}$ **rep on** $\mathscr{H}_{\text{kin}} = \mathscr{H}_C \otimes \mathscr{H}_S$



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generates unitary $G = \mathbb{R}$ **rep on** $\mathscr{H}_{\text{kin}} = \mathscr{H}_C \otimes \mathscr{H}_S$

global clock: $\mathbb{R}$**-frame with "orientation"/clock reading states**

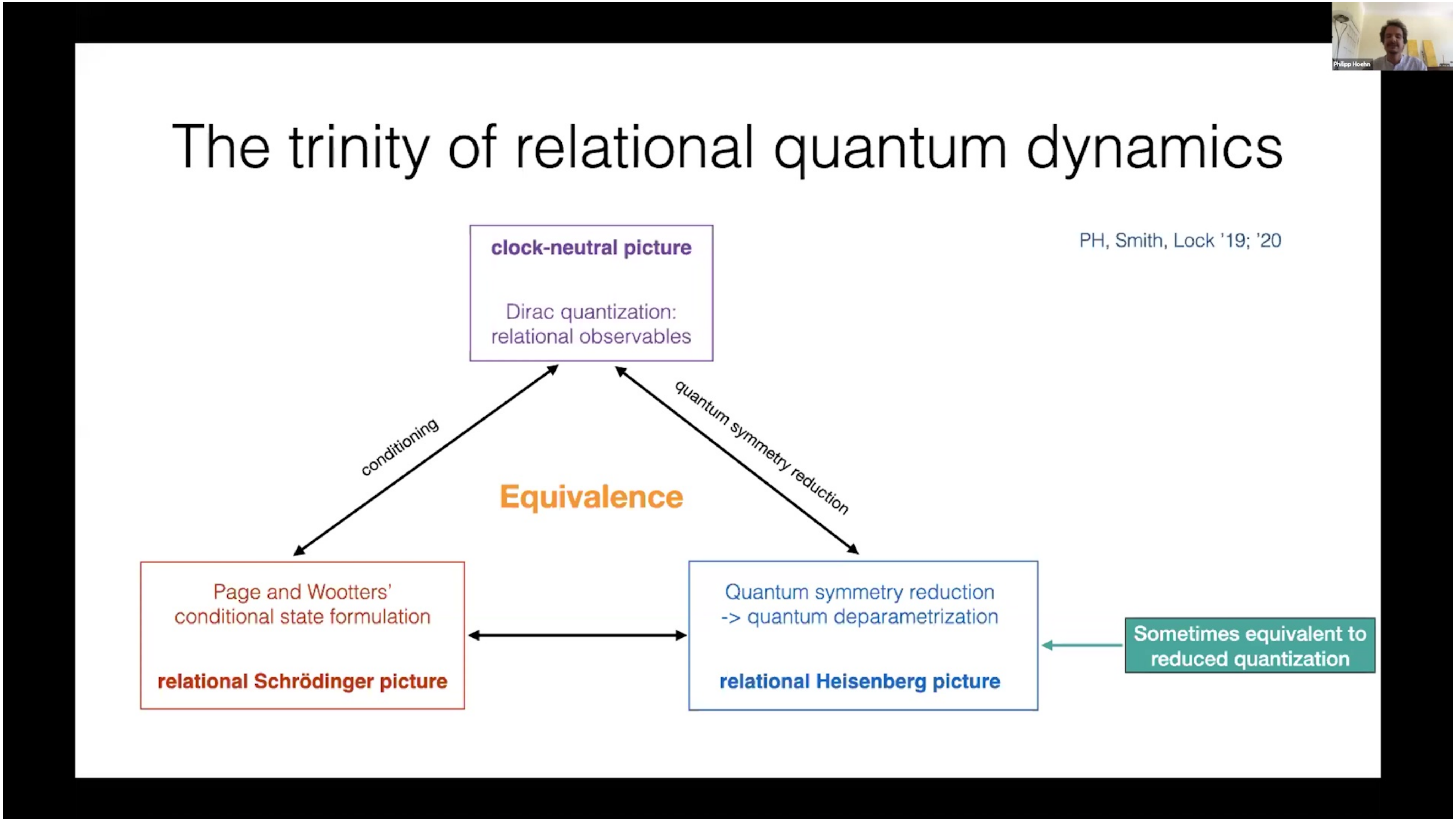
$$|\tau, \sigma\rangle = \int_{\text{SpecH}_C} d\varepsilon \, e^{-i\tau\varepsilon} \, |\varepsilon, \sigma\rangle_C \qquad \Rightarrow \qquad U_C(\tau') \, |\tau, \sigma\rangle = |\tau + \tau', \sigma\rangle$$

degeneracies (e.g., ± freq. modes)

clock states generally not orthogonal $\langle \tau | \tau' \rangle \nsim \delta(\tau - \tau')$

gives rise to covariant clock POVMs

Holevo, Busch, Milburn, Caves, Braunstein, Brunetti, Fredenhagen, Loveridge, Smith, PH, Lock,…



The trinity of relational quantum dynamics

PH, Smith, Lock '19; '20

clock-neutral picture

Dirac quantization:
relational observables

conditioning

quantum symmetry reduction

Equivalence

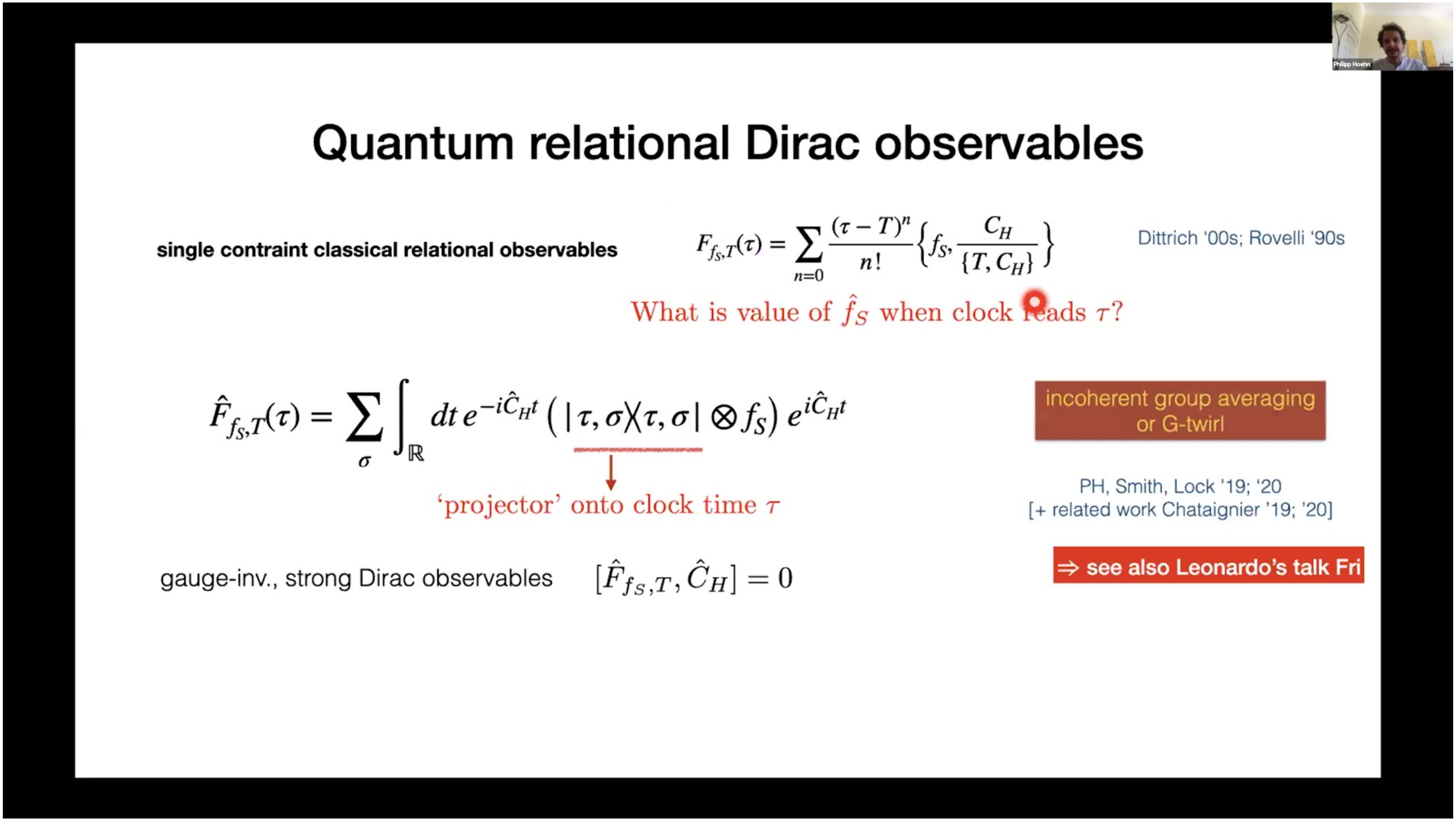
Page and Wootters'
conditional state formulation

relational Schrödinger picture

Quantum symmetry reduction
-> quantum deparametrization

relational Heisenberg picture

Sometimes equivalent to reduced quantization



Quantum relational Dirac observables

single contraint classical relational observables

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(\tau - T)^n}{n!} \left\{ f_S, \frac{C_H}{\{T, C_H\}} \right\}$$

Dittrich '00s; Rovelli '90s

What is value of $\hat{f}_S$ when clock reads τ?

$$\hat{F}_{f_S,T}(\tau) = \sum_{\sigma} \int_{\mathbb{R}} dt\, e^{-i\hat{C}_H t} \left(\underline{|\tau,\sigma\rangle\langle\tau,\sigma|} \otimes f_S \right) e^{i\hat{C}_H t}$$

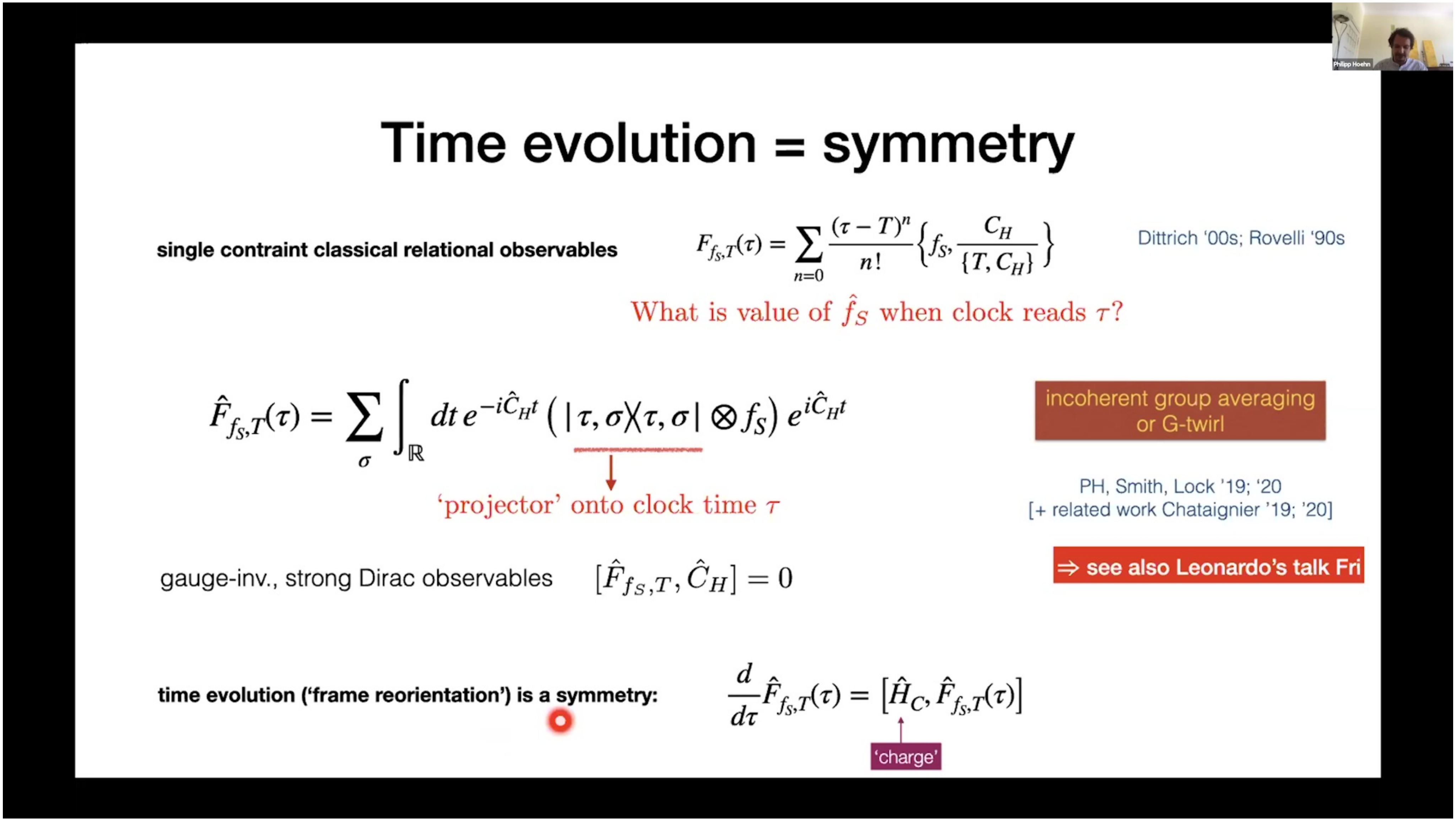
'projector' onto clock time τ

incoherent group averaging or G-twirl

PH, Smith, Lock '19; '20
[+ related work Chataignier '19; '20]

gauge-inv., strong Dirac observables $[\hat{F}_{f_S,T}, \hat{C}_H] = 0$

⇒ see also Leonardo's talk Fri



Time evolution = symmetry

single contraint classical relational observables

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(\tau - T)^n}{n!}\left\{f_S, \frac{C_H}{\{T, C_H\}}\right\}$$

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What is value of $\hat{f}_S$ when clock reads τ?

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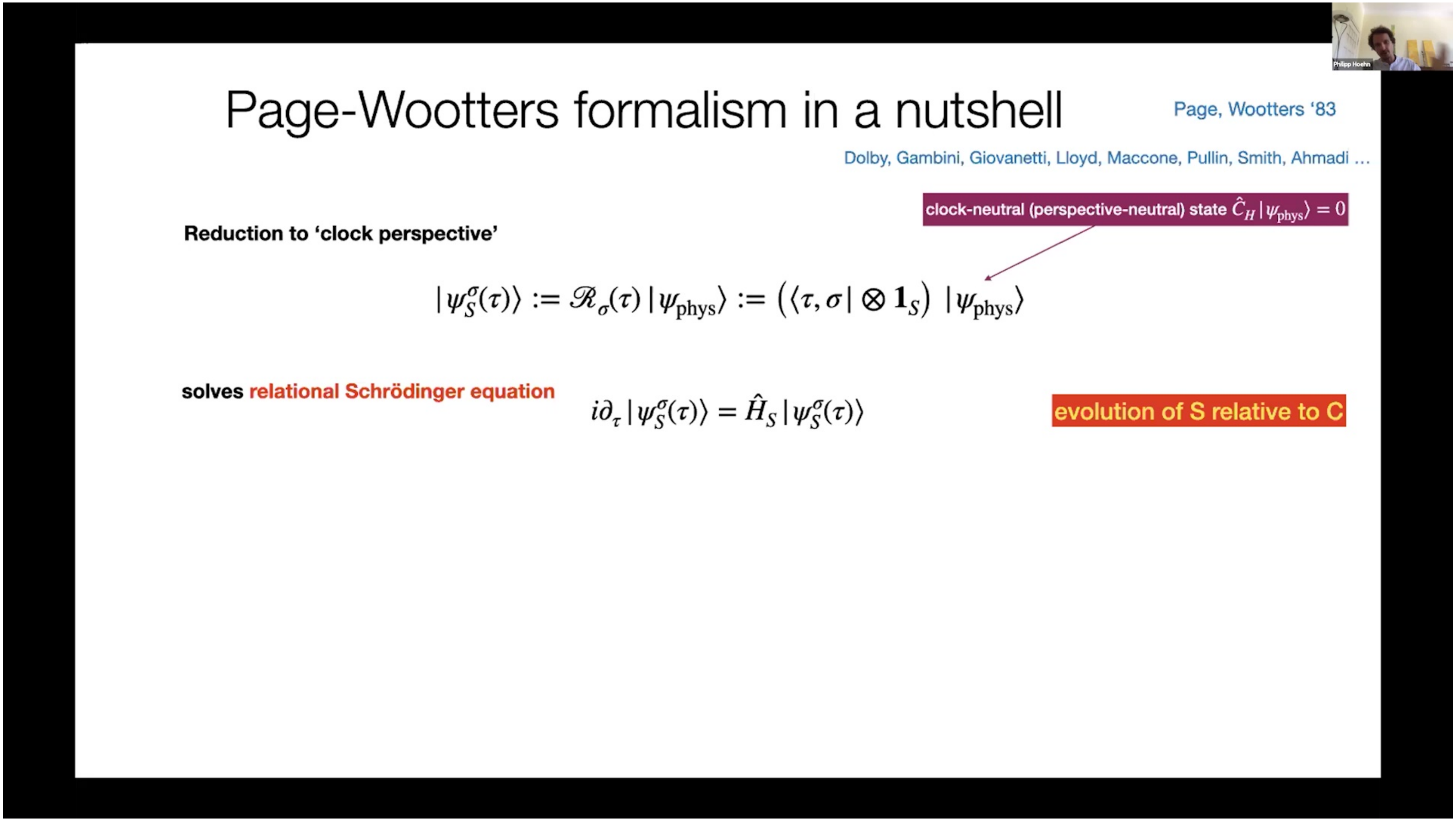
gauge-inv., strong Dirac observables $[\hat{F}_{f_S,T}, \hat{C}_H] = 0$

⇒ see also Leonardo's talk Fri

time evolution ('frame reorientation') is a symmetry:

$$\frac{d}{d\tau}\hat{F}_{f_S,T}(\tau) = [\underset{\substack{\uparrow \\ \text{'charge'}}}{\hat{H}_C}, \hat{F}_{f_S,T}(\tau)]$$



Page-Wootters formalism in a nutshell

Page, Wootters '83

Dolby, Gambini, Giovanetti, Lloyd, Maccone, Pullin, Smith, Ahmadi …

clock-neutral (perspective-neutral) state $\hat{C}_H\,|\,\psi_{\text{phys}}\rangle = 0$

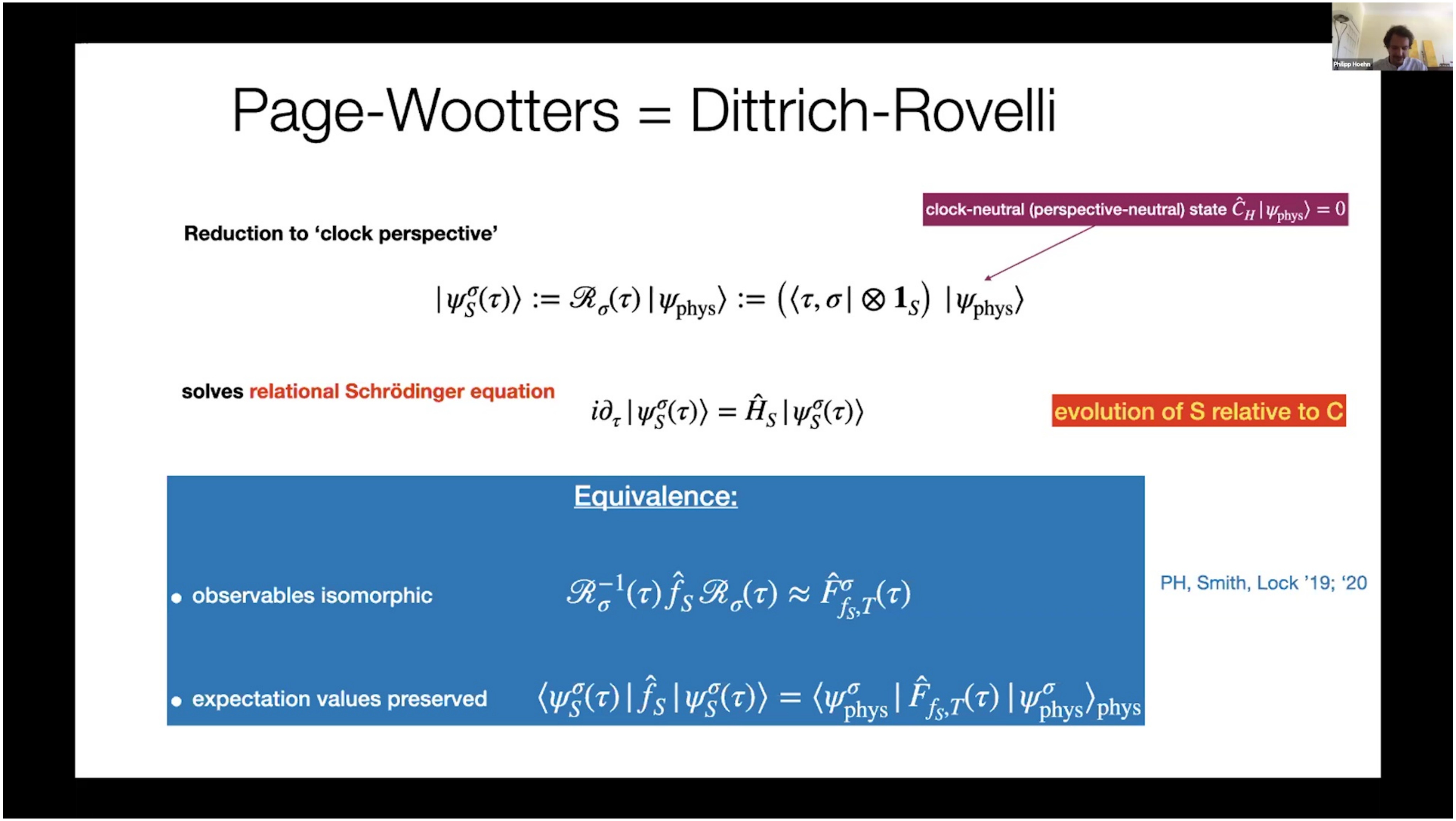
Reduction to 'clock perspective'

$$|\,\psi_S^\sigma(\tau)\rangle := \mathscr{R}_\sigma(\tau)\,|\,\psi_{\text{phys}}\rangle := \big(\langle\tau,\sigma\,|\otimes\mathbf{1}_S\big)\;|\,\psi_{\text{phys}}\rangle$$

solves relational Schrödinger equation

$$i\partial_\tau\,|\,\psi_S^\sigma(\tau)\rangle = \hat{H}_S\,|\,\psi_S^\sigma(\tau)\rangle$$

evolution of S relative to C



Page-Wootters = Dittrich-Rovelli

clock-neutral (perspective-neutral) state $\hat{C}_H \,|\, \psi_{\text{phys}}\rangle = 0$

Reduction to 'clock perspective'

$$|\,\psi_S^\sigma(\tau)\rangle := \mathscr{R}_\sigma(\tau)\,|\,\psi_{\text{phys}}\rangle := \left(\langle\tau,\sigma\,|\otimes \mathbf{1}_S\right)\,|\,\psi_{\text{phys}}\rangle$$

solves relational Schrödinger equation

$$i\partial_\tau\,|\,\psi_S^\sigma(\tau)\rangle = \hat{H}_S\,|\,\psi_S^\sigma(\tau)\rangle$$

evolution of S relative to C

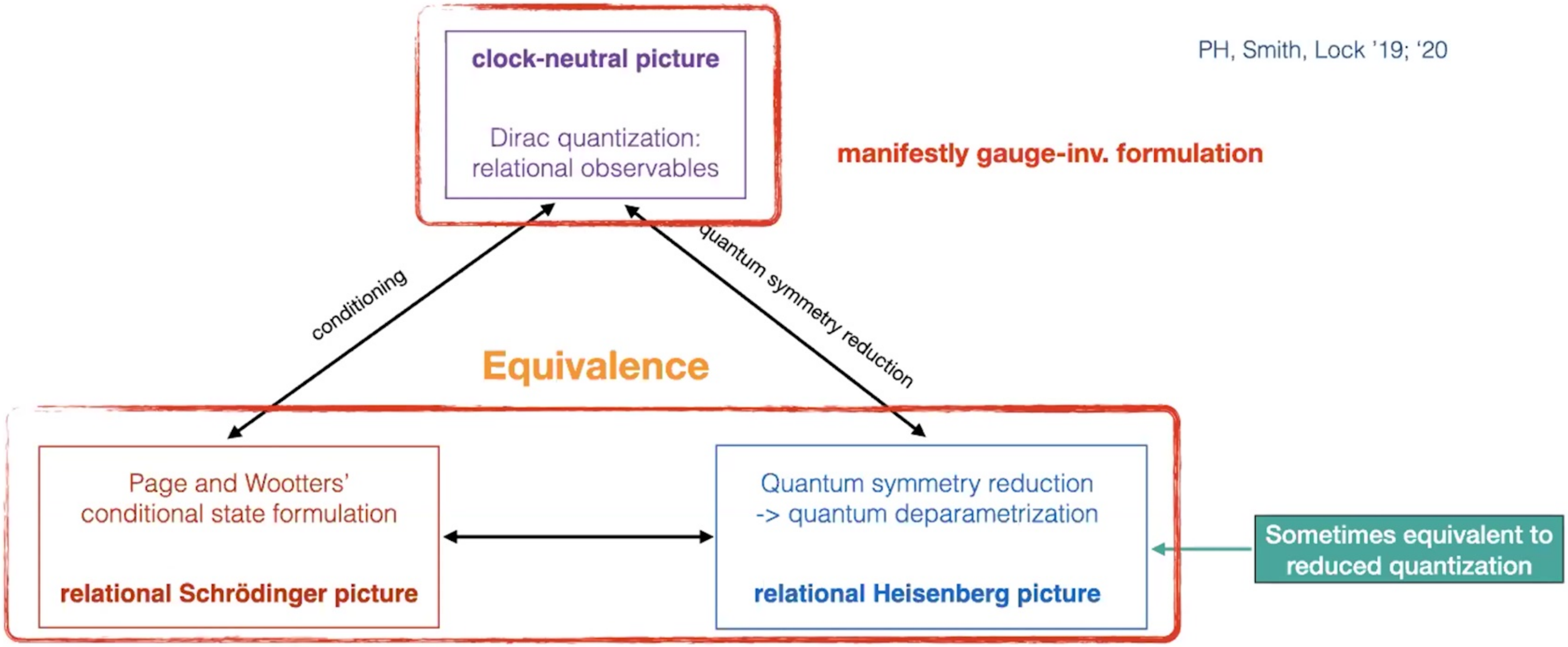
Equivalence:

- observables isomorphic $\quad \mathscr{R}_\sigma^{-1}(\tau)\,\hat{f}_S\,\mathscr{R}_\sigma(\tau) \approx \hat{F}^\sigma_{f_S,T}(\tau)$
- expectation values preserved $\quad \langle\psi_S^\sigma(\tau)\,|\,\hat{f}_S\,|\,\psi_S^\sigma(\tau)\rangle = \langle\psi^\sigma_{\text{phys}}\,|\,\hat{F}_{f_S,T}(\tau)\,|\,\psi^\sigma_{\text{phys}}\rangle_{\text{phys}}$

PH, Smith, Lock '19; '20

The trinity of relational quantum dynamics

PH, Smith, Lock '19; '20



clock-neutral picture

Dirac quantization:
relational observables

manifestly gauge-inv. formulation

conditioning

quantum symmetry reduction

Equivalence

Page and Wootters'
conditional state formulation

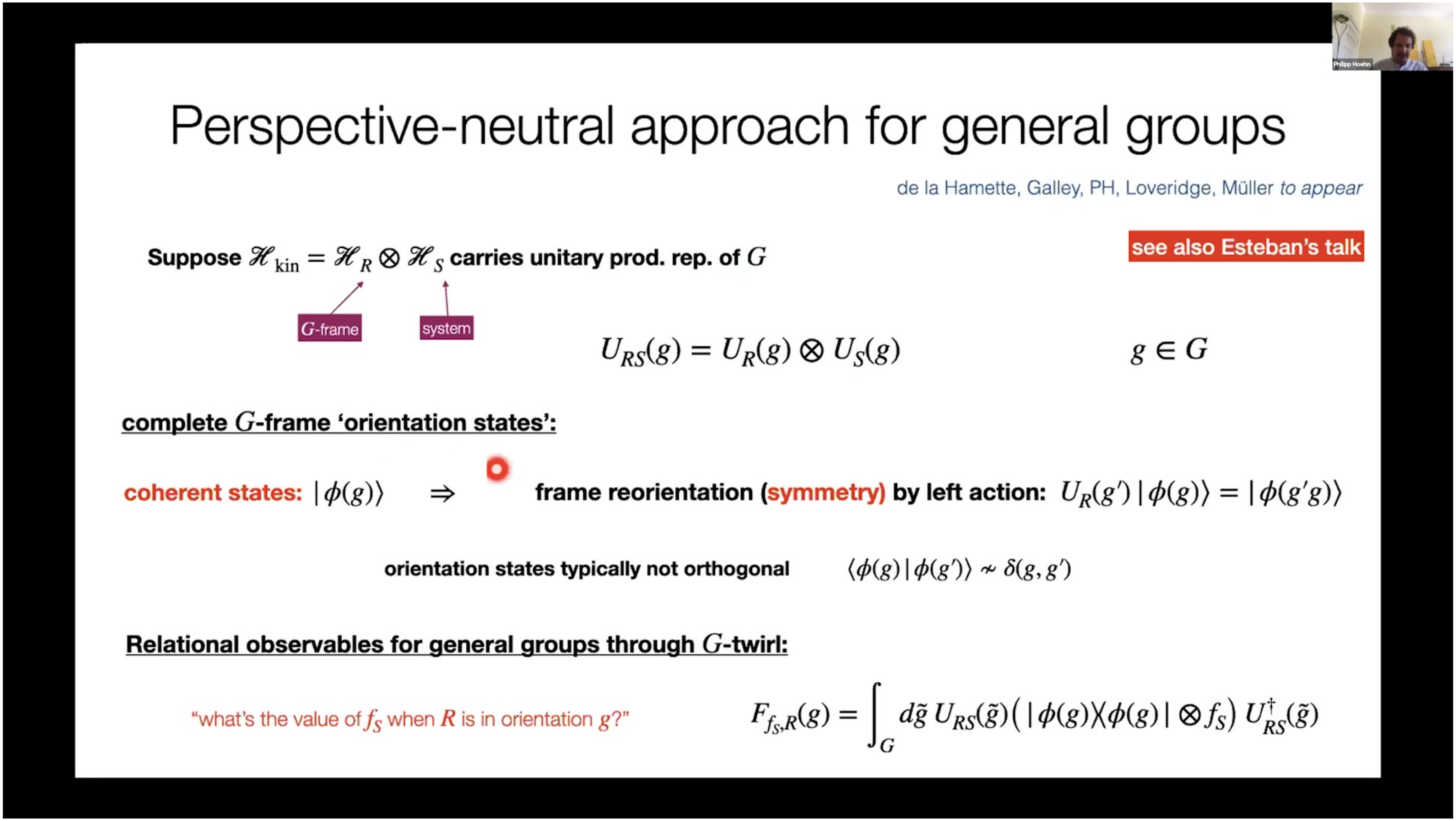
relational Schrödinger picture

Quantum symmetry reduction
-> quantum deparametrization

relational Heisenberg picture

Sometimes equivalent to reduced quantization

"gauge-fixed" formulations



Perspective-neutral approach for general groups

de la Hamette, Galley, PH, Loveridge, Müller *to appear*

see also Esteban's talk

Suppose $\mathscr{H}_{\text{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$ carries unitary prod. rep. of G

G-frame

system

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \qquad\qquad g \in G$$

complete G-frame 'orientation states':

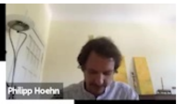
coherent states: $|\phi(g)\rangle$ $\Rightarrow$ **frame reorientation (symmetry) by left action:** $U_R(g')\,|\phi(g)\rangle = |\phi(g'g)\rangle$

orientation states typically not orthogonal $\langle\phi(g)\,|\,\phi(g')\rangle \nsim \delta(g,g')$

Relational observables for general groups through G-twirl:

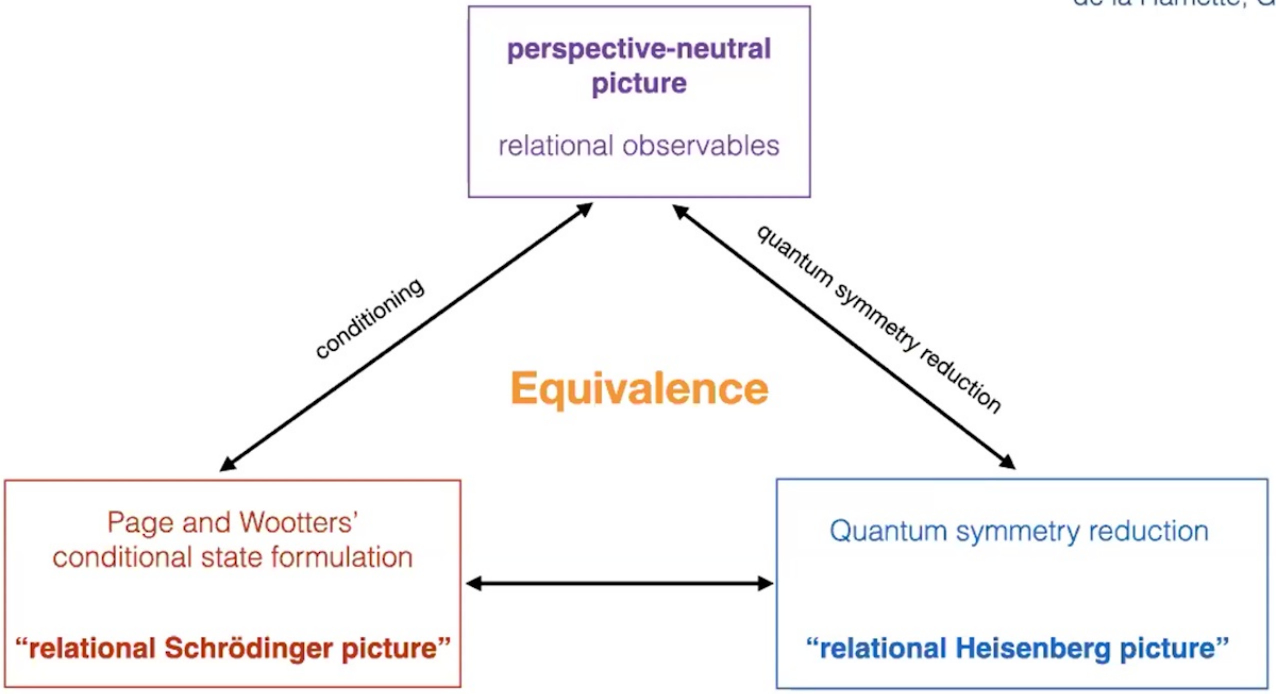
"what's the value of f_S when R is in orientation g?"

$$F_{f_S,R}(g) = \int_G d\tilde{g}\; U_{RS}(\tilde{g})\big(\,|\phi(g)\rangle\langle\phi(g)|\otimes f_S\big)\; U^\dagger_{RS}(\tilde{g})$$



The trinity for complete G-frames

de la Hamette, Galley, PH, Loveridge, Müller *to appear*



perspective-neutral picture

relational observables

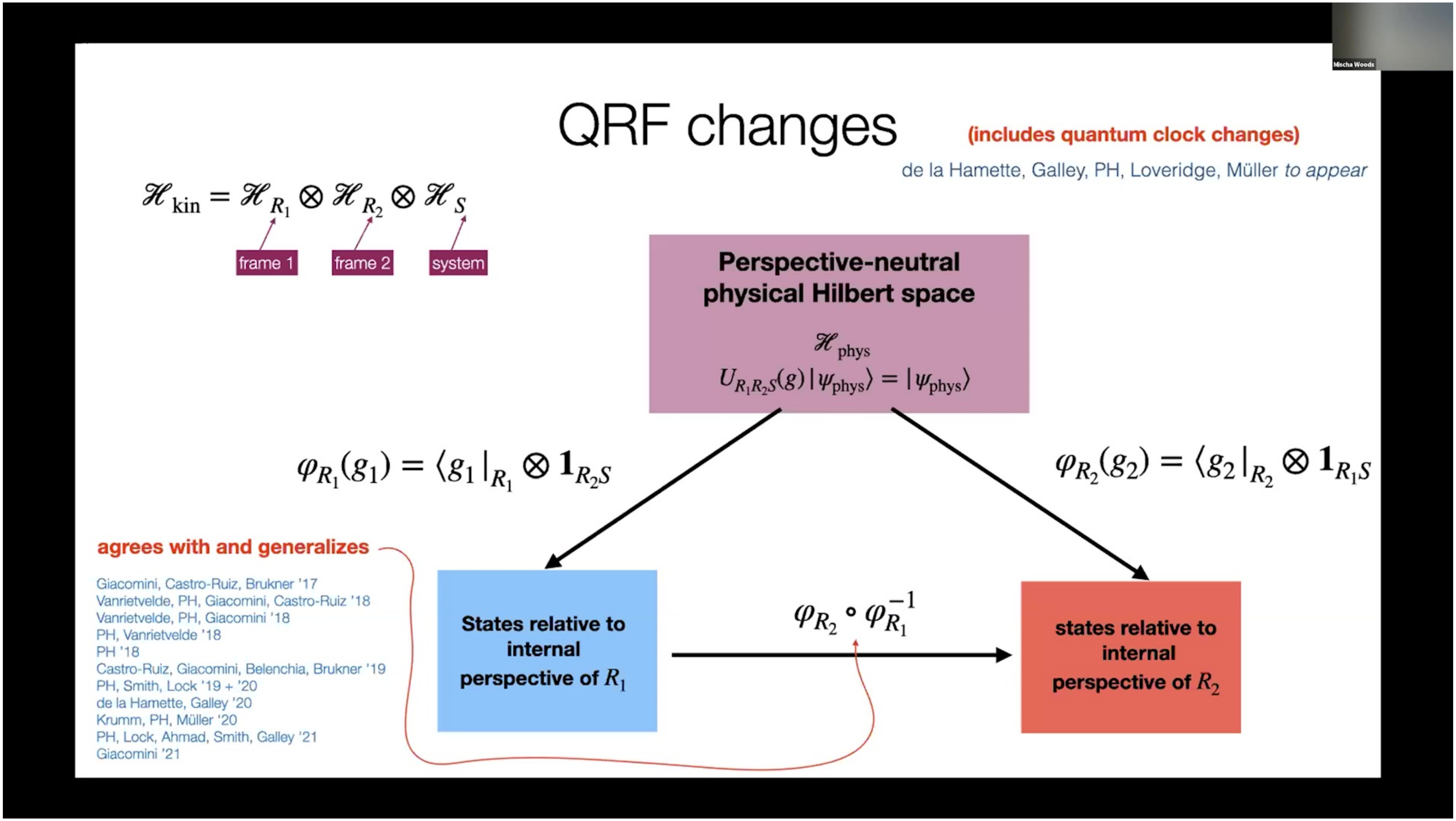
Equivalence

Page and Wootters' conditional state formulation

"relational Schrödinger picture"

Quantum symmetry reduction

"relational Heisenberg picture"



QRF changes

(includes quantum clock changes)

de la Hamette, Galley, PH, Loveridge, Müller *to appear*

$$\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$

frame 1
frame 2
system

Perspective-neutral physical Hilbert space

$$\mathscr{H}_{\text{phys}}$$

$$U_{R_1R_2S}(g)\,|\,\psi_{\text{phys}}\rangle = |\,\psi_{\text{phys}}\rangle$$

$$\varphi_{R_1}(g_1) = \langle g_1\,|_{R_1} \otimes \mathbf{1}_{R_2S}$$

$$\varphi_{R_2}(g_2) = \langle g_2\,|_{R_2} \otimes \mathbf{1}_{R_1S}$$

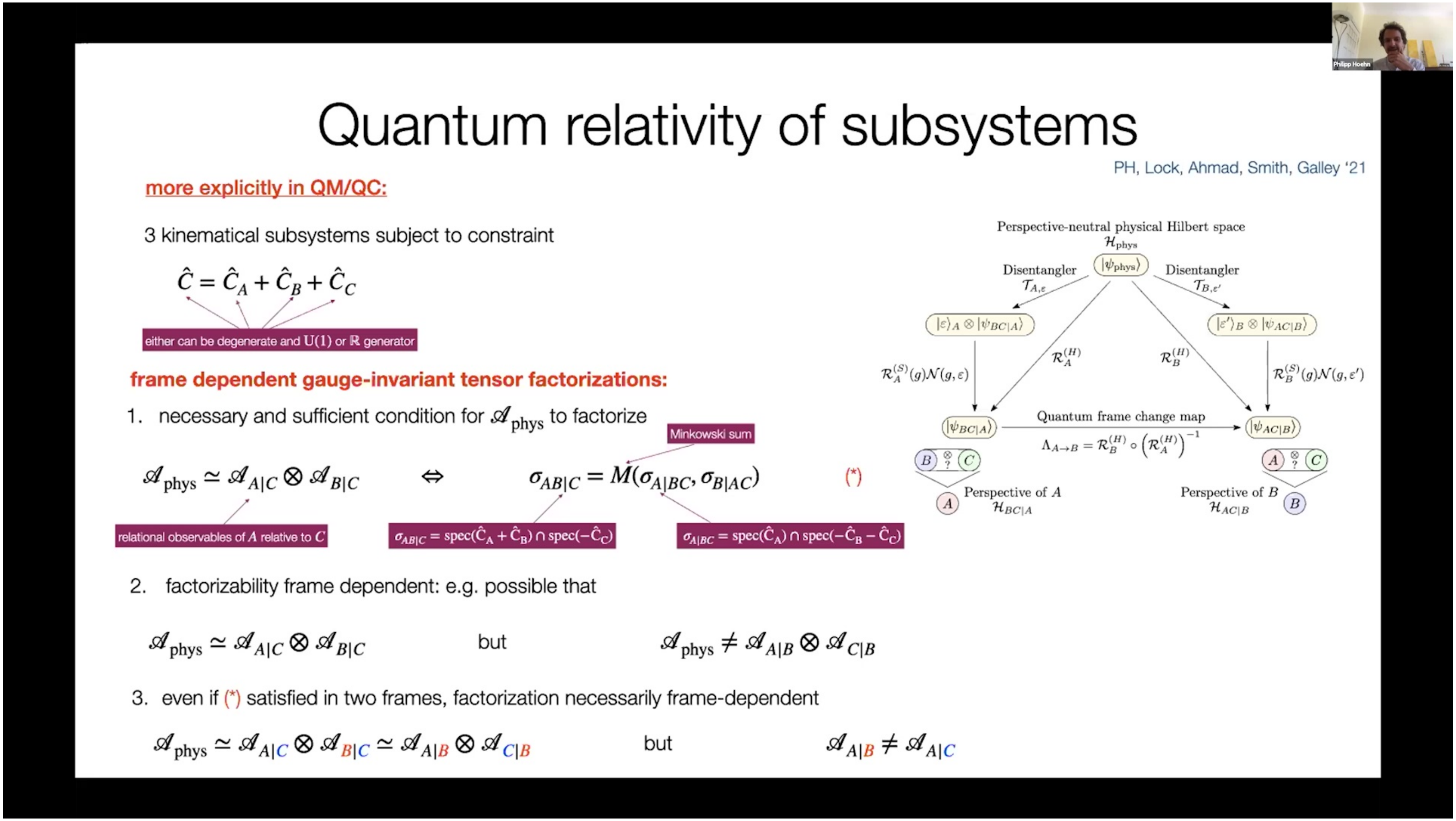
States relative to internal perspective of R_1

$$\varphi_{R_2} \circ \varphi_{R_1}^{-1}$$

states relative to internal perspective of R_2

agrees with and generalizes

Giacomini, Castro-Ruiz, Brukner '17
Vanrietvelde, PH, Giacomini, Castro-Ruiz '18
Vanrietvelde, PH, Giacomini '18
PH, Vanrietvelde '18
PH '18
Castro-Ruiz, Giacomini, Belenchia, Brukner '19
PH, Smith, Lock '19 + '20
de la Hamette, Galley '20
Krumm, PH, Müller '20
PH, Lock, Ahmad, Smith, Galley '21
Giacomini '21



Quantum relativity of subsystems

PH, Lock, Ahmad, Smith, Galley '21

more explicitly in QM/QC:

3 kinematical subsystems subject to constraint

$$\hat{C} = \hat{C}_A + \hat{C}_B + \hat{C}_C$$

either can be degenerate and $\mathrm{U}(1)$ or $\mathbb{R}$ generator

frame dependent gauge-invariant tensor factorizations:

1. necessary and sufficient condition for $\mathscr{A}_{\text{phys}}$ to factorize

Minkowski sum

$$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \quad \Leftrightarrow \quad \sigma_{AB|C} = M(\sigma_{A|BC}, \sigma_{B|AC}) \qquad (*)$$

relational observables of A relative to C

$\sigma_{AB|C} = \mathrm{spec}(\hat{C}_A + \hat{C}_B) \cap \mathrm{spec}(-\hat{C}_C)$

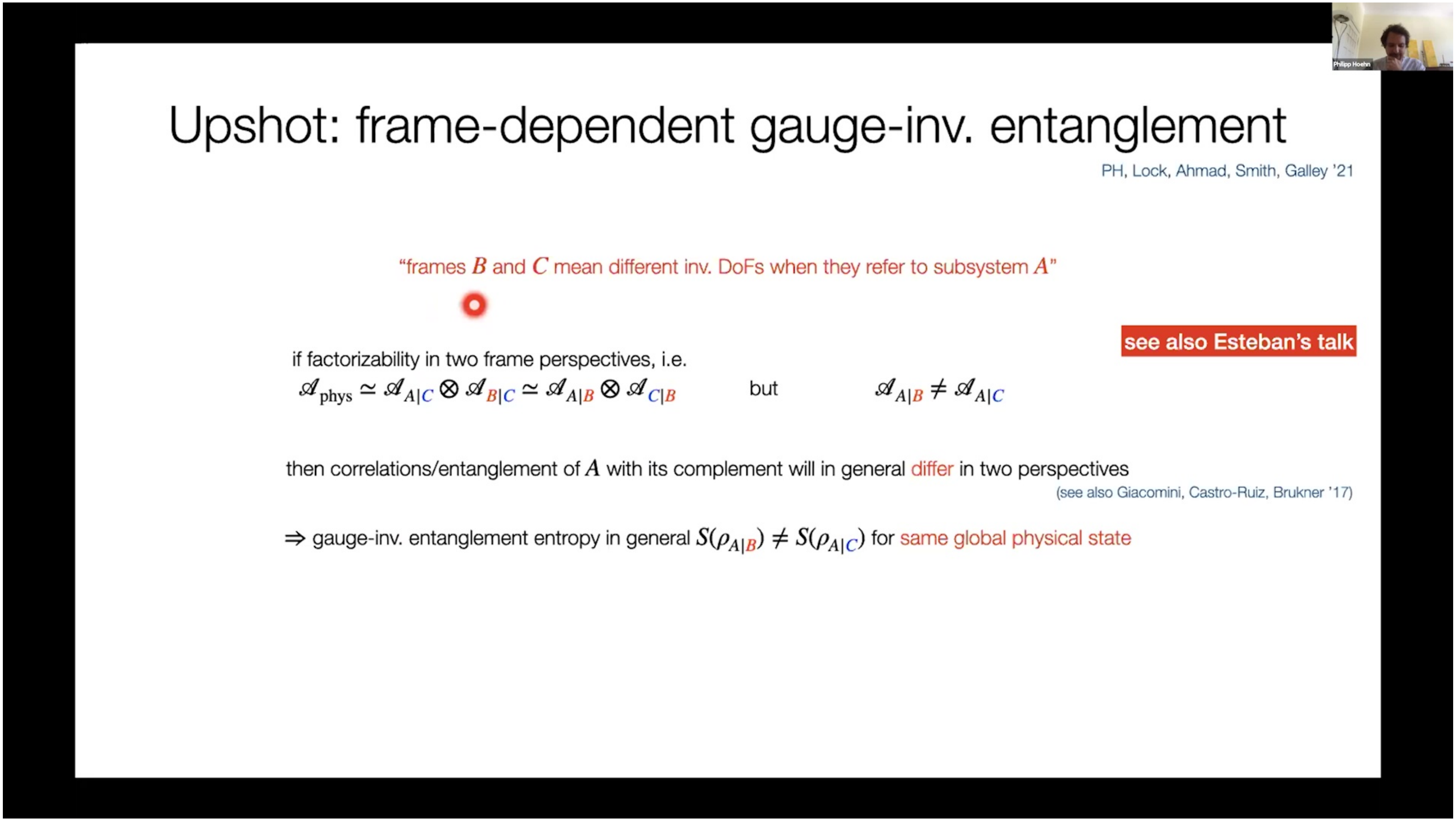
$\sigma_{A|BC} = \mathrm{spec}(\hat{C}_A) \cap \mathrm{spec}(-\hat{C}_B - \hat{C}_C)$

2. factorizability frame dependent: e.g. possible that

$$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \qquad \text{but} \qquad \mathscr{A}_{\text{phys}} \neq \mathscr{A}_{A|B} \otimes \mathscr{A}_{C|B}$$

3. even if (*) satisfied in two frames, factorization necessarily frame-dependent

$$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \simeq \mathscr{A}_{A|B} \otimes \mathscr{A}_{C|B} \qquad \text{but} \qquad \mathscr{A}_{A|B} \neq \mathscr{A}_{A|C}$$



Upshot: frame-dependent gauge-inv. entanglement

PH, Lock, Ahmad, Smith, Galley '21

"frames B and C mean different inv. DoFs when they refer to subsystem A"

see also Esteban's talk

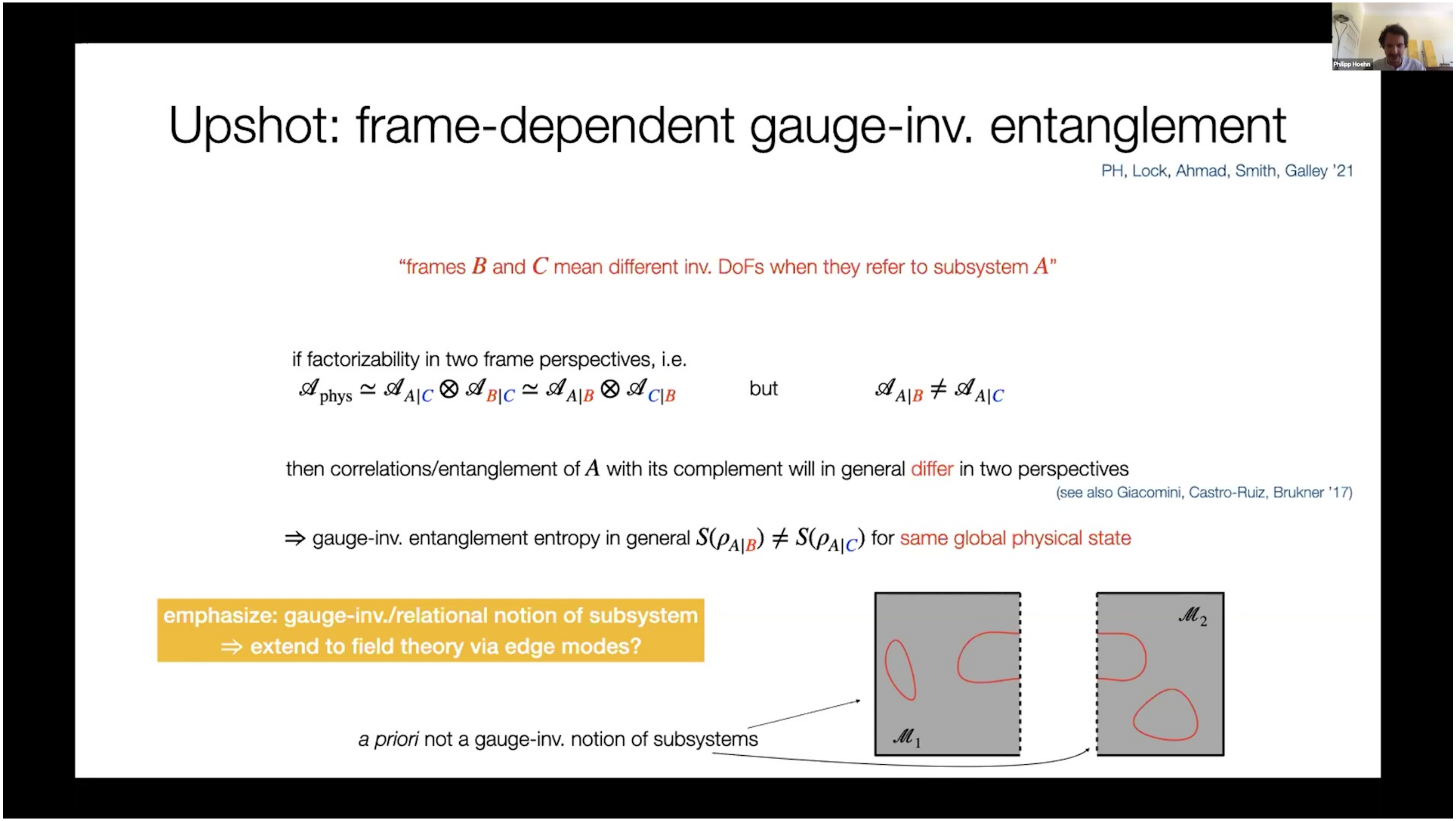
if factorizability in two frame perspectives, i.e.

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \simeq \mathscr{A}_{A|B} \otimes \mathscr{A}_{C|B}$ but $\mathscr{A}_{A|B} \neq \mathscr{A}_{A|C}$

then correlations/entanglement of A with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '17)

⇒ gauge-inv. entanglement entropy in general $S(\rho_{A|B}) \neq S(\rho_{A|C})$ for same global physical state



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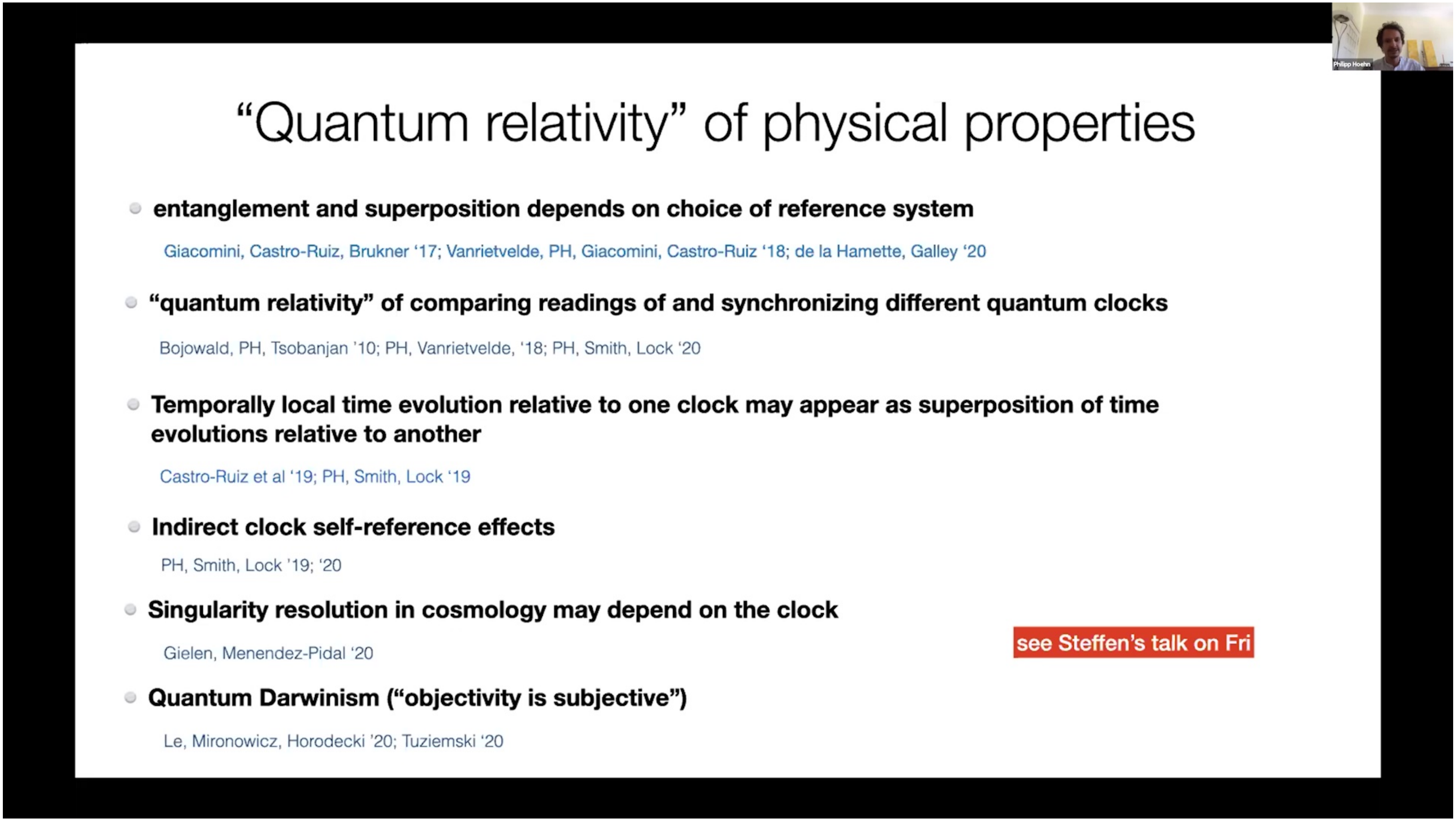
emphasize: gauge-inv./relational notion of subsystem
⇒ extend to field theory via edge modes?

a priori not a gauge-inv. notion of subsystems



“Quantum relativity” of physical properties

- **entanglement and superposition depends on choice of reference system**

  Giacomini, Castro-Ruiz, Brukner ‘17; Vanrietvelde, PH, Giacomini, Castro-Ruiz ‘18; de la Hamette, Galley ‘20

- **“quantum relativity” of comparing readings of and synchronizing different quantum clocks**

  Bojowald, PH, Tsobanjan ’10; PH, Vanrietvelde, ‘18; PH, Smith, Lock ‘20

- **Temporally local time evolution relative to one clock may appear as superposition of time evolutions relative to another**

  Castro-Ruiz et al ‘19; PH, Smith, Lock ‘19

- **Indirect clock self-reference effects**

  PH, Smith, Lock ’19; ‘20

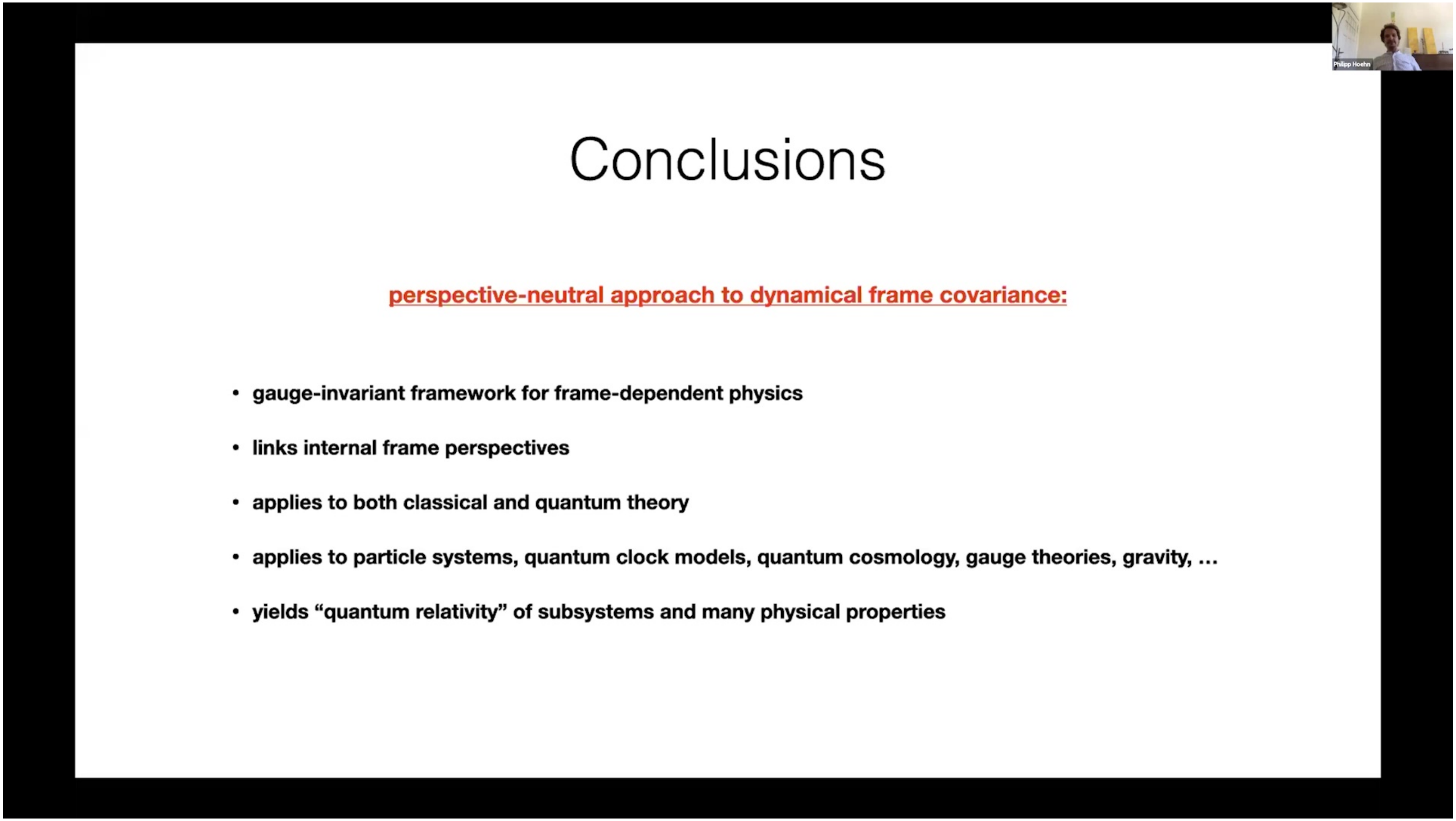
- **Singularity resolution in cosmology may depend on the clock**

  Gielen, Menendez-Pidal ‘20

  see Steffen’s talk on Fri

- **Quantum Darwinism (“objectivity is subjective”)**

  Le, Mironowicz, Horodecki ’20; Tuziemski ‘20



Conclusions

perspective-neutral approach to dynamical frame covariance:

- **gauge-invariant framework for frame-dependent physics**
- **links internal frame perspectives**
- **applies to both classical and quantum theory**
- **applies to particle systems, quantum clock models, quantum cosmology, gauge theories, gravity, …**
- **yields “quantum relativity” of subsystems and many physical properties**