Title: Space and Time in a Lorentzian path integral

Speakers: Bianca Dittrich

Collection: Quantizing Time

Date: June 15, 2021 - 9:40 AM

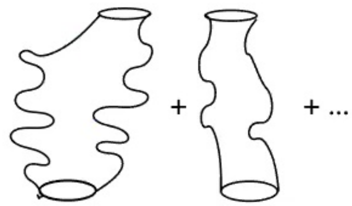
URL: http://pirsa.org/21060095

Abstract: I will present a quantum gravity approach based on a Lorentzian path integral for quantum geometries. The properties of quantum space time can be measured using geometric operators. This allows also to discuss fluctuations of causal structure as well as violations of (micro-) causality. I will explain how the Lorentzian path integral comes with various options regarding which quantum space times to sum over: e.g. whether to include causality violations or not, or whether to allow also for space times with Euclidean signatures in Lorentzian path integrals. I will sketch some consequences for the resulting theories.


Space and Time in a Lorentzian path integral
Bianca Dittrich,
Perimeter Institute
Quantizing Time Workshop, Perimeter, June 2021

Path integral over spacetimes

Path integral for quantum gravity =



$\exp(iS(\text{geom})) + \exp(iS(\text{geom})) + \dots$

sum over spacetime geometries

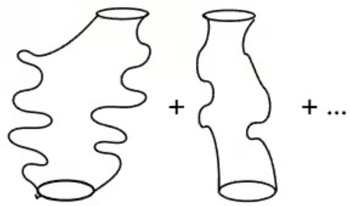
Time: reconstructed from this path integral

What goes into the path integral?

What are allowed "spacetimes" in the path integral?

Path integral over spacetimes

Path integral for quantum gravity =



sum over spacetime geometries

$$\exp(iS(\text{geom})) + \exp(iS(\text{geom})) + \ldots$$

Time: reconstructed from this path integral

What goes into the path integral?

What are allowed "spacetimes" in the path integral?

Path integral over geometries

Path integral:

$$Z = \int \mathscr{D}\mathbf{geom} \;\; \exp(iS(\mathbf{geom}))$$

Lots of different choices:
- -Space of "geometries"
- -signature of geometry
- -causal irregularities
- -Measure on this space:
  discrete, continuous, measure terms, …

(generalized versions of)
Einstein Hilbert action

Questions:

a) How to make this path integral well-defined?
E.g. Use discretization as regulator, remove regulator dependence.

b) How to make this path integral computable?
Often: Wick rotation in time lead to "Euclidean Quantum Gravity"
This obstructed a lot the investigation of the role of Lorentzian structures.

Wick rotation

For quantum field theory on fixed Lorentzian background:
rotating $t \rightarrow it$ leads to an Euclideanized action, defined on Euclidean background

Gravity: Can Wick rotate (certain) geometries, and define an Euclideanized version of the Einstein-Hilbert action S_E

Euclidean Quantum Gravity: Path integral over Euclidean geometries, with amplitude $\exp(-S_E)$.
Advantage: Can apply Monte-Carlo simulations and other standard QFT techniques.

Disadvantage:

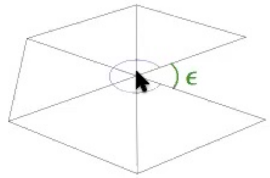
Practical: The Euclidean Einstein-Hilbert action is unbounded from below: Conformal factor problem.
The system will be driven to configurations where $-S_E$ is very large.

Conceptual: Space of Lorentzian geometries very different from space of Euclidean geometries,
no precise sense how these can be related via a (coordinate dependent) Wick rotation.

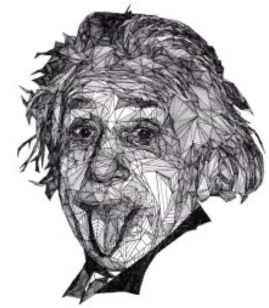
These problems killed (almost) all "Euclidean quantum gravity" non-perturbative approaches:
Regge calculus, (almost) dynamical triangulations, (Euclidean) lattice gauge formulations of gravity, …

Example: (Euclidean) Dynamical Triangulations

Path integral sums over geometries build from gluing equilateral simplices to triangulations.
Geometry is encoded in the combinatorics of the triangulation.



E.g. 2D: curvature around a vertex is given by the number of adjacent triangless.

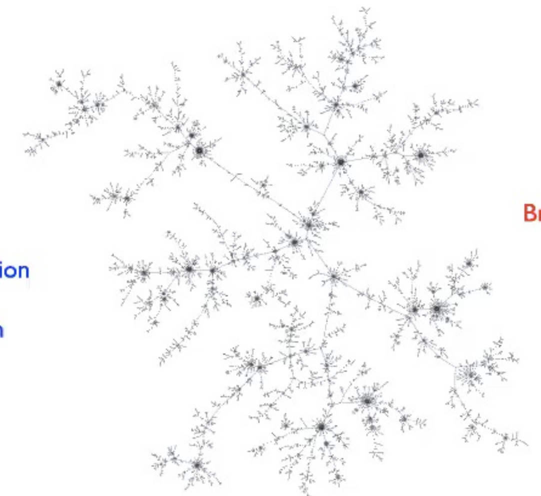


[Weingarten 1982, Ambjorn, Jurkiewicz 1992: first 4D simulations]

Results from Monte-Carlo simulations:

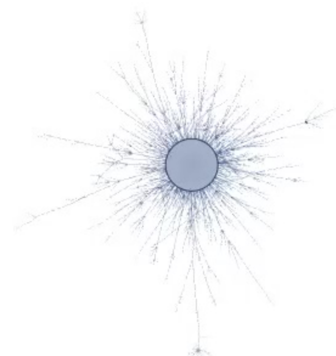
[from Rindlisbacher, de Forcrand 2015]

typical 4D triangulation in elongated phase, Hausdorff dimension around 2



Branching of "baby universes".

crumpled phase, Hausdorff dimension infinity



First order phase transition.
(No suitable continuum limit.)

Causal Dynamical Triangulations

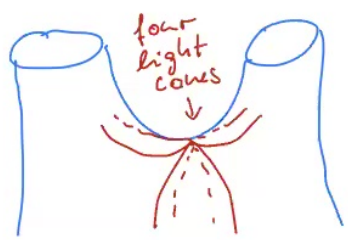
[Ambjorn, Loll 1998, …,
4D simulations: 2004 w/ Jurkiewicz, …]

Key input: A regular causal structure suppresses baby universes.
(For such causally regular (dynamical) triangulations one can also define a Wick rotation.)

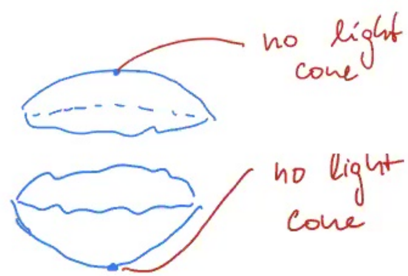
Still use Monte Carlo simulations, based on the Euclideanized action on causally regular triangulations.
(Similar philosophy: Causal sets.)

[Jordan, Loll 2013] Implement causality condition: Exactly two light cones at each point.
This prevents topology change of space during time evolution.

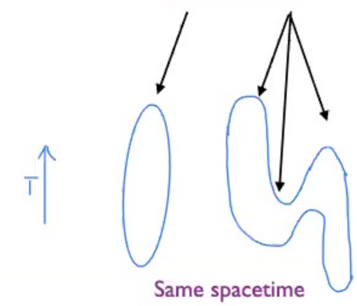
Configuration with more than two light cones describe branching of baby universes:



Configuration with no light cone at a point describe birth or end of space-time:



Irregular causal structure.



Same spacetime topology.

Causal Dynamical Triangulations

[Ambjorn, Loll 1998, …,
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Implement causality condition: Exactly two light cones at each point.
This prevents topology change of space during time evolution.

Very encouraging results:

- phase with 'smooth' geometry: deSitter phase
- dimensional reduction of spectral dimension from approx. 4 to 2
- strong indications of 2nd order phase transition

Questions:

- Horava-Lifshitz or GR (at end point of phase transition line)
- Interpretation of (inverse) Wick rotation, in particular for black hole space times

Spatial topology change in the Lorentzian path integral

$$Z = \int \mathscr{D}\mathbf{geom} \;\; \exp(iS(\mathbf{geom}))$$

The path integral over Lorentzian geometries provides a mechanism for suppressing baby universes.

[Sorkin 1975, Sorkin 2019]

In the context of Regge calculus.

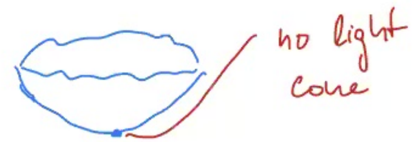
Classical action: Causal irregularities lead to imaginary parts in the action.

QM path integral: This leads either to an exponential suppression or enhancement of such configurations.

(With a choice of root for -1:)

Exponentially suppressed:

Exponentially enhanced:



But few calculations available, which would test Lorentzian path integral, in particular wrt topology change.

[Asante, BD, Padua-Arguelles 2021]

First explicit 4D path integral calculation for small triangulations, based on (effective) spin foams:

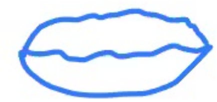
Indication: Configurations with exponential enhancement should be explicitly forbidden.

Euclidean configurations in the Lorentzian path integral

Lorentzian geometry:

$\neq$

Euclidean geometry:



[Hartle, Hawking, Halliwell, … Feldbrugge, Lehners, Turok, …, Hartle, Hertog, …]

- geometries with Euclidean region can emerge from a path integral over purely Lorentzian geometries (even when restricting to regular causal configurations)
- when critical points for the path integral lie in the complex plane
- e.g. critical point for a solution with imaginary time: leads to a contribution $\exp(\pm S_E)$
- Example: describing wave-function of the universe

[BD, Gielen, Schander, to appear]

[Asante, BD, Padua-Arguelles 2021]

[Barrett-Foxon 1994, Han, Liu 2021]

- For path integrals based on triangulations as regulator:
  - Such Euclidean solutions also appear. Triangulation acts as a natural regulator for no-boundary proposal.
  - Due to the existence of Euclidean solutions for sub-divided building blocks one can argue that the integral should include a sum over Euclidean building blocks with amplitude $\exp(-S_E)$.

- Such configurations due appear in the (asymptotics of the) spin foam path integral.

Euclidean configurations might appear as critical points or are directly included in the path integral, but with exponentially suppressed amplitudes.

How to construct the Lorentzian path integral ?

- What kind of (almost) Lorentzian geometries should we allow in the path integral?
- Should we allow for space-times with causal irregularities? That is allow for topology change of spatial hypersurfaces?
- Should we allow for space-times with Euclidean regions?

Test: Which microscopic inputs lead to what macroscopic pictures? This can exclude certain choices.
We need computable models: Relation between microscopic inputs and macroscopic outputs often very surprising.

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• It is typically very hard to impose restrictions in the path integral, e.g. to include only (discrete) space times, which define a regular Lorentzian geometry.
• e.g. metric tensor components need to satisfy intricate conditions to define regular geometry of a certain signature: eigenvalues need to be (-,+,+,+)
• so far there is no suitable quantum representation of metric tensor

• Loop quantum gravity: based on a reformulation of gravity using tetrads (encoding the metric) and the Ashtekar-(Barbero) connection
• Leads to a quantum representation of geometric variables generalizing the tetrads

Path integral over geometries

Path integral: $$Z = \int \mathscr{D}\mathbf{geom} \ \exp(iS(\mathbf{geom}))$$

Lots of different choices:
-Space of "geometries"
-signature of geometry
-causal irregularities
-Measure on this space:
discrete, continuous, measure terms, …

- These choices can be informed from a canonical quantization scheme: Loop quantum gravity → spin foams

Lorentzian quantum geometries

- Loop quantum gravity: (phase space) and Hilbert space for quantum geometries describing a 3D hyper-surfaces embedded in a 4D space time
- Standard choice: hypersurface is spatial, but techniques can be (at least formally) extended to time-like hypersurfaces (technically: exchange SO(3) with SO(2,1))

  - Areas appear as fundamental variables (instead of lengths). Quantum configurations can be interpreted to have not only curvature but also torsion. [BD, Ryan 2008, BD 2021]

- Results in operators for:
  - space-like and time-like areas, volumes, lengths
  - curvature angles, …

- spectra for space-like and time-like area operators are discrete [Rovelli-Smolin, Ashtekar-Lewandowski] [Conrady-Hnybida]

Areas ↔ Curvature Angles

Spin foam path integral: sum over discrete all possible discrete area assignments to triangles of a triangulation.

[Barrett-Crane 1996, Engle-Pereira-Rovelli-Livine, 2007 Freidel-Krasnov 2007…]

- derive from gauge (connection) formulation: amplitudes encoded in complicated recoupling symbols

[Asante, BD, Haggard 2020, Asante, BD, Padua-Arguelles 2021]

- work directly with discrete areas: effective spin foams, much more numerical accessible
- does in particular allow to include sum over time-like triangles, tetrahedra (more easily)

Expectation values for geometric operators

[Asante, BD, Haggard PRL 2020, Asante, BD, Haggard CQG 2020, Asante, BD, Padua-Arguelles 2021]

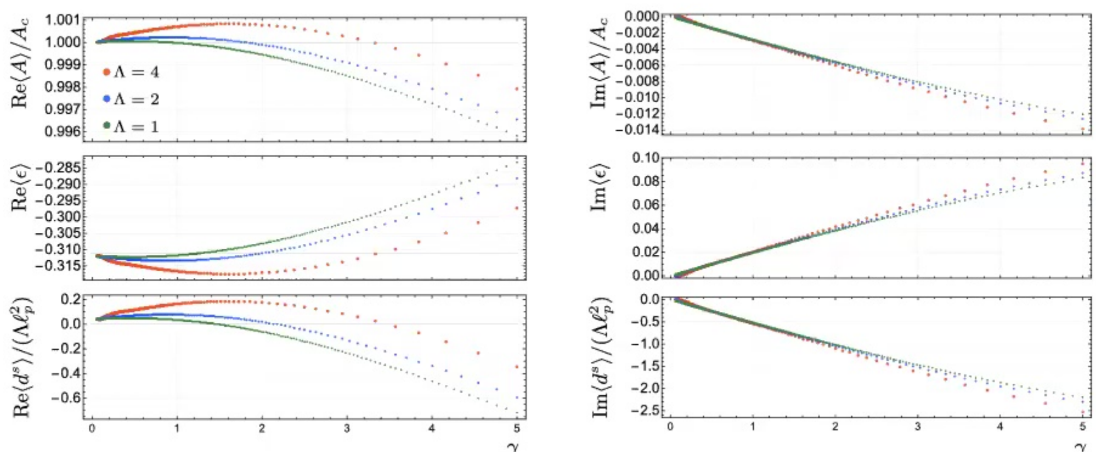
- Effective spin foam models allow direct evaluation of path integral as sum over (discrete) areas, at least for configurations involving only a few areas

- Allows the computation of expectation values for areas, lengths, curvature angles

- First explicit proof that spin foams can implement equations of motions for discrete gravity [Asante, BD, Haggard CQG 2020]
  (To show: the extension to generalized geometries does not interfere)

- First computation of discrete path integral and expectation values with time-like and space-like triangles [Asante, BD, Padua-Arguelles 2021]
  - Sum over areas can include sum over different causal structures
    e.g. edges or triangle can be space-like or time-like or null

- Fluctuations for area peaked on null value can be made small, although small areas are associated with "deep quantum regime"



Triangulations with many building blocks?

- Needed to disentangle the regulator from actual physics, restore diffeomorphism symmetry [BD 2014]
- direct evaluation of Lorentzian path integral out of reach
- but examples with small triangulations suggest that saddle point approximation is valid for solutions with small curvature

- first perturbative evaluation of effective spin foams on infinite (hyper-cubical) lattice: [BD 2021 + to appear] [Asante, BD to appear]
  to linear order: reproduces general relativity + correction of higher order in the lattice constant

- To show: 'non-metric' configurations appearing in path integral do not interfere.
  In fact, these non-metric degrees of freedom appear all as "heavy" degrees of freedom in the spin foam action in the limit of small lattice constant (compared to wave-length of graviton excitaitons).

- Surprise: This also holds for the Barrett-Crane Model, which was thought not to lead to General Relativity.
  Sign of universal behaviour.

- Shows that one can use a quantum configuration space based on (discrete) areas, and that one can nevertheless obtain GR, which is based on a much smaller configuration space.

Configuration space of quantum geometries quite different from classical, metric geometries.

- Areas appear also as more fundamental variables in holography, (black hole) thermo-dynamics.

Towards studying fluctuations of light cones

- Areas, volumes etc. are not diffeomorphism invariant observables in itself.

- But can use 'relational observables': localize fields with either matter (rods and clocks) or gravitational degrees of freedom

[Einstein, Bergman, …, Rovelli, BD, …]

- Gravitational degrees of freedoms can be used to fix coordinate system in unexpected ways: [BD, Tambornino 2006]
  Eg: for configurations around flat space: choose coordinates in which metric is "as near to Minkowski metric as possible".
  (Technically ADM gauge: longitudinal modes of metric vanish, trace mode of momentum vanishes)

- This allows recovery of usual quantum field theory formalism with coordinate dependent, local field observables from a theory of quantum gravity, where no such observables exist.

- Allows also the study of light cone fluctuations: encoded in (gauge invariant version of) commutator-function for gravitons

[BD, Tambornino 2006]

Can (perturbatively) evaluate such observables and commutators in effective spin foam path integral on infinite lattice.

Summary and Outlook

- Path integral over Lorentzian geometries:
  - connection between microscopic building blocks for space time and macroscopic quantum space times
- Many unexpected results: getting smooth space-times is hard
- Open question: Allowing causality violations or not?
- First studies of impact of causality violations — much more to explore

- Spin foams: one of few approaches to quantum gravity based on Lorentzian geometries
- based on a rigorous notion of quantum geometry
- configuration space much more general than metric geometries

- Challenge: evaluating the path integral
- Effective spin foams: eases this tasks
- has led to a number of interesting results about the properties of this path integral
- perturbative evaluation allows first studies of macroscopic quantum space times