Title: Quantum reference frames for space and space-time

Speakers: ÄŒaslav Brukner

Collection: Quantizing Time

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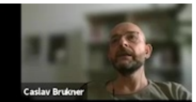
Abstract: In physics, a reference frame is an abstract coordinate system that specifies observations within that frame. While quantum states depend on the choice of reference frame, the form of physical laws is assumed to be covariant. Recently, it has been proposed to consider reference frames as physical systems and as such assume that they obey quantum mechanics. In my talk, I will present recent results in the field of "quantum reference frames" (QRF). In particular, I will formulate the covariance of dynamical physical laws with respect to non-relativistic QRF transformations and show how relativistic QRFs can be used to solve a long-standing problem in relativistic quantum information or to address typical quantum gravity scenarios.



Faculty of Physics, University of Vienna &
Institute for Quantum Optics and Quantum Information – Vienna

Quantum reference frames for space and space-time

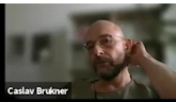
Flaminia Giacomini, Esteban Castro, Časlav Brukner

Nature Communications **10**, 494 (2019),
Phys. Rev. Lett. **123**, 090404 (2019)
arXiv: 2012.13754

"Quantizing Time" conference at the Perimeter Institute, June 14th, 2021

Outline

- Classical reference frames
- Quantum reference frames
- Extended covariance of physical laws
- “Superposition of translations & boosts”
- “Quantum rest frame”: definition of relativistic spin qubit



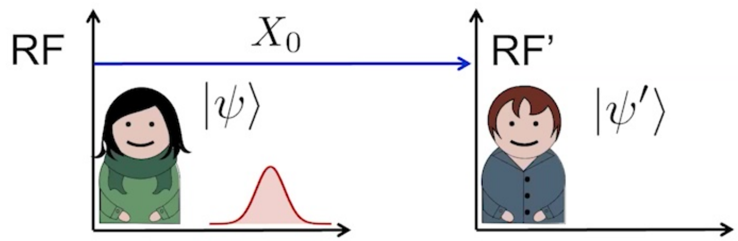
Outline

- Classical reference frames
- Quantum reference frames
- Extended covariance of physical laws
- “Superposition of translations & boosts”
- “Quantum rest frame”: definition of relativistic spin qubit
- “Quantum locally inertial frame”: Einstein’s equivalence principle for non-classical space-times

Covariance of physical laws

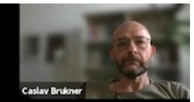
The laws of physics are of **"the same form"** regardless of the choice of the reference frame.

Translations



$$i\hbar\frac{d|\psi\rangle}{dt} = \frac{\hat{p}^2}{2m}|\psi\rangle \qquad i\hbar\frac{d|\psi'\rangle}{dt} = \frac{\hat{p}^2}{2m}|\psi'\rangle$$

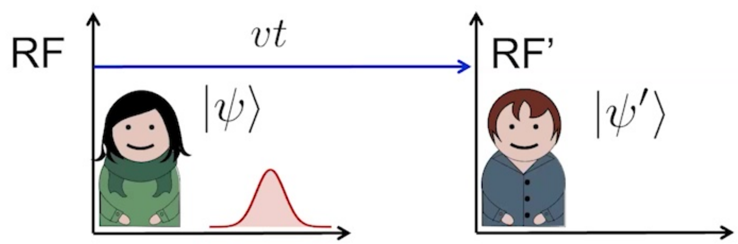
where $|\psi'\rangle = e^{\frac{i}{\hbar}X_0\hat{p}}|\psi\rangle$



Covariance of physical laws

The laws of physics are of **“the same form”** regardless of the choice of the reference frame.

Boosts



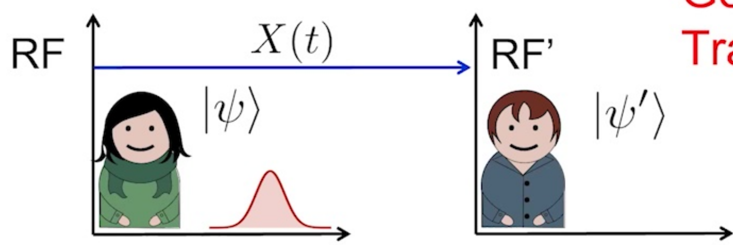
$$i\hbar \frac{d|\psi\rangle}{dt} = \frac{\hat{p}^2}{2m}|\psi\rangle \qquad i\hbar \frac{d|\psi'\rangle}{dt} = \frac{\hat{p}^2}{2m}|\psi'\rangle$$

where $|\psi'\rangle = e^{\frac{i}{\hbar} v \hat{G}}|\psi\rangle,\ \hat{G} = \hat{p}t - m\hat{x}$

Covariance of physical laws

The laws of physics are of **"the same form"** regardless of the choice of the reference frame.

General Transformation



$$i\hbar\frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle \qquad i\hbar\frac{d|\psi'\rangle}{dt} = \hat{H}'|\psi'\rangle$$

where $|\psi'\rangle = \hat{U}|\psi\rangle,\ \hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar\frac{d\hat{U}}{dt}\hat{U}^\dagger$

Covariance of physical laws

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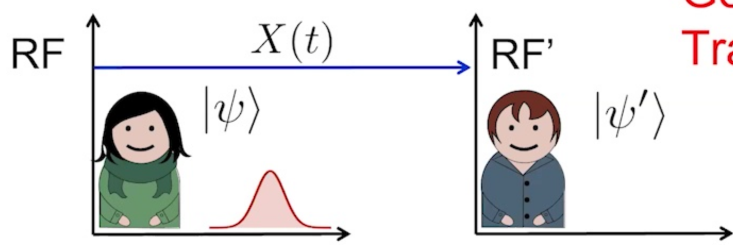


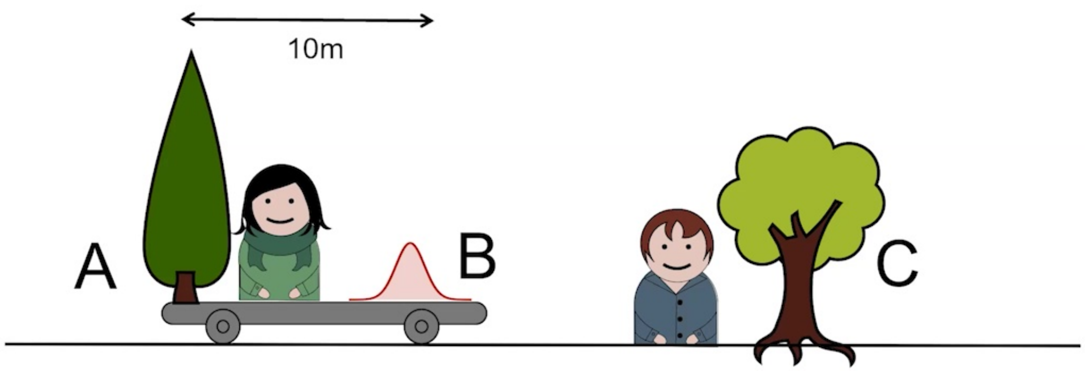
$$i\hbar\frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle \qquad i\hbar\frac{d|\psi'\rangle}{dt} = \hat{H}'|\psi'\rangle$$

where $|\psi'\rangle = \hat{U}|\psi\rangle,\ \hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar\frac{d\hat{U}}{dt}\hat{U}^\dagger$

A transformation is a **symmetry** if: $\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar\frac{d\hat{U}}{dt}\hat{U}^\dagger = \hat{H}$

Real reference frames

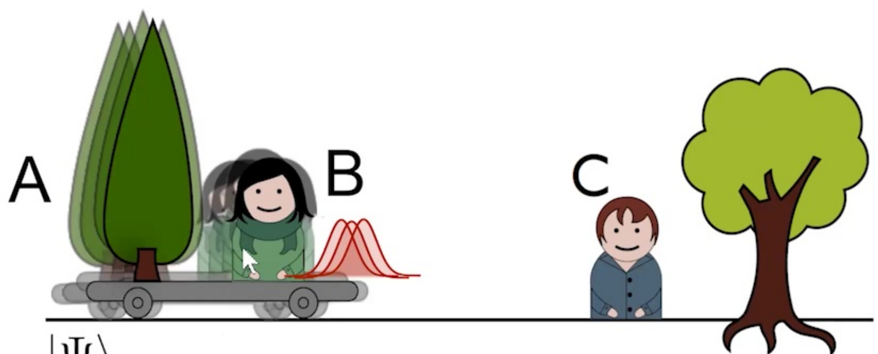
In practice, reference frames (RFs) are **physical systems**.



Statement from RF A: *"The system B is 10 m away from the tree".*

Real reference frames

In practice, reference frames (RFs) are **physical systems**.
They ultimately obey **quantum mechanical laws**.



$|\Psi\rangle_{\text{RF A + System B}}$

Quantum state as seen from RF C

Real reference frames

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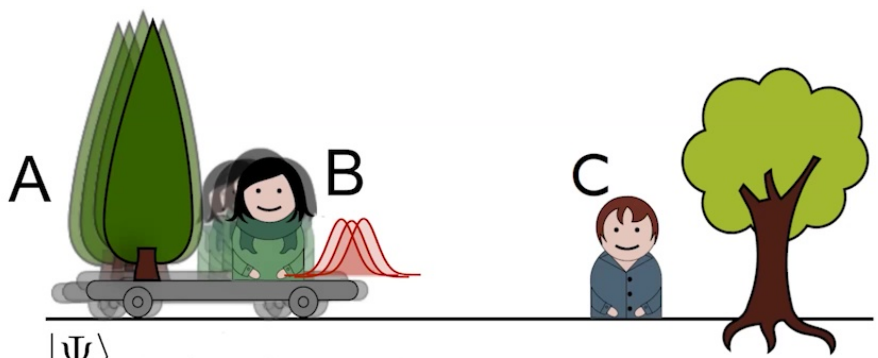

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Quantum state as seen from RF C

R. M. Angelo, N. Brunner, S. Popescu, A. Short, and P. Skrzypczyk, J. Phys. A: Math. Theor. **44**, 145304 (2011)

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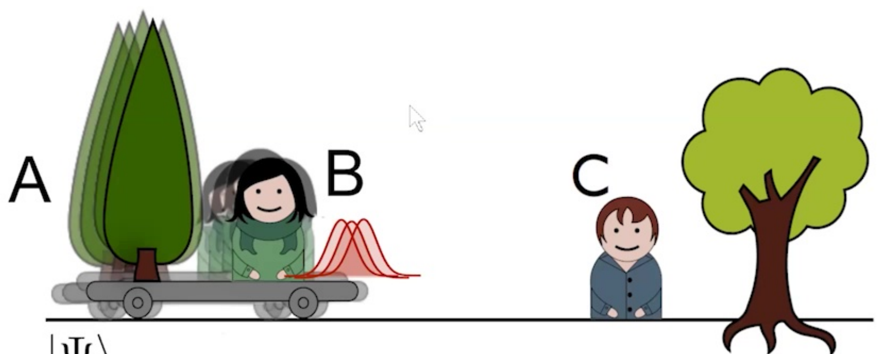
They ultimately obey **quantum mechanical laws**.



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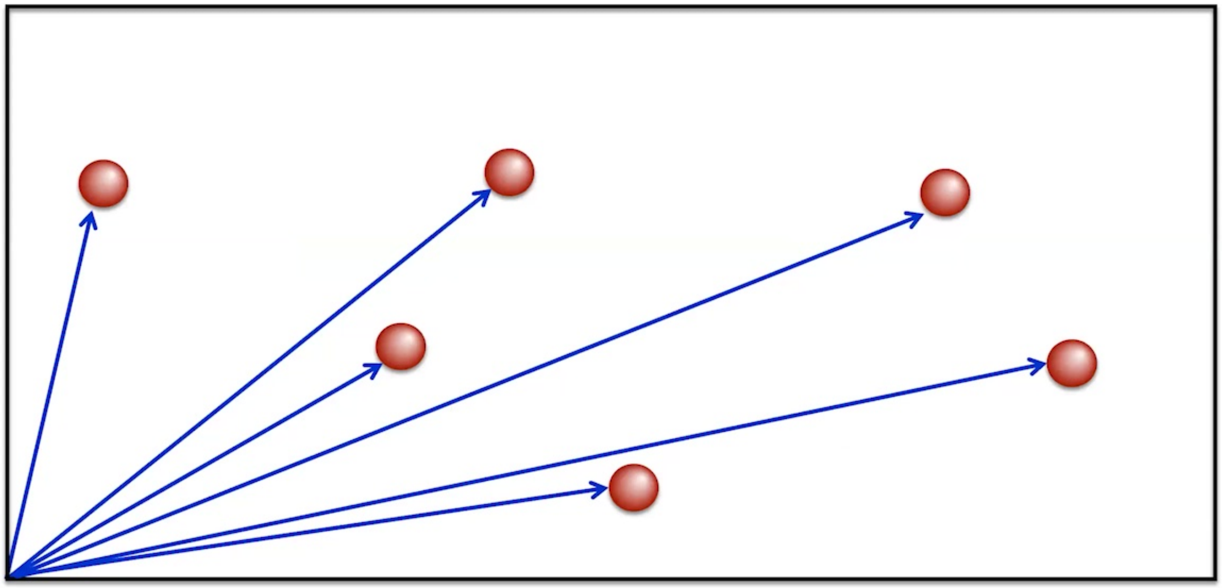
Quantum state as seen from RF C

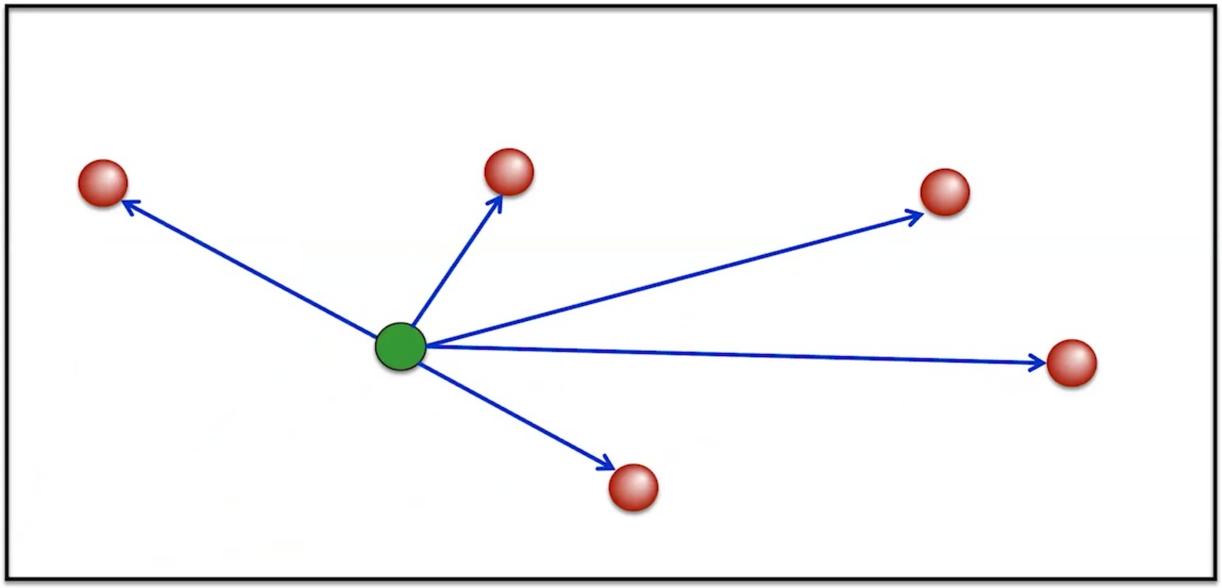
R. M. Angelo, N. Brunner, S. Popescu, A. Short, and P. Skrzypczyk, J. Phys. A: Math. Theor. **44**, 145304 (2011)

Are physical laws **covariant under the change of quantum RFs?**
Extended symmetries? How to formalize this idea?

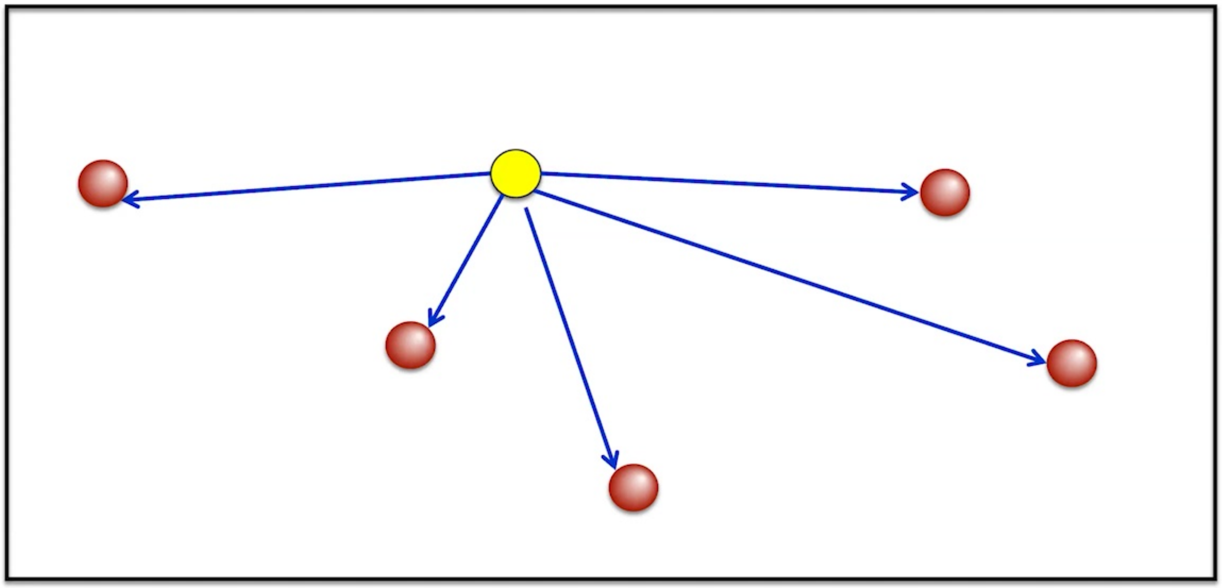
"Absolute space"
Absolute external RFs



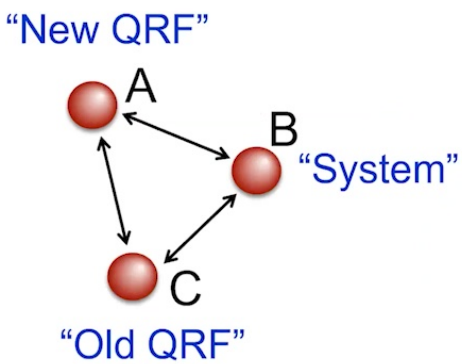
“Relational approach”



“Relational approach”
Transformations between RFs

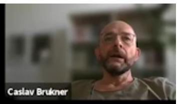


Relative quantities



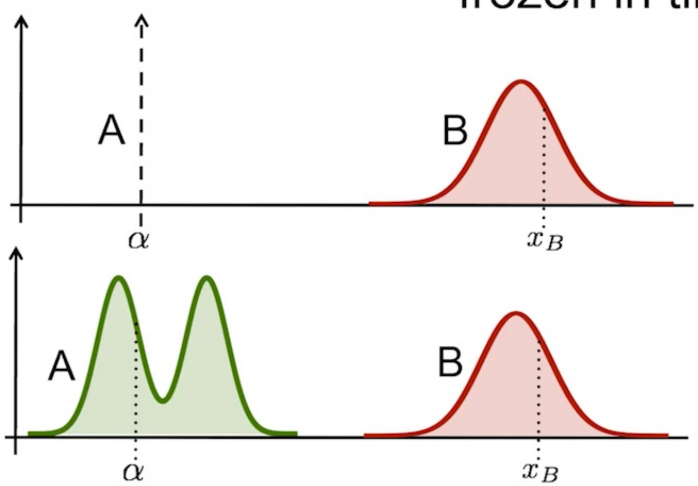
The description of the quantum state is given in terms of **relative** quantities.

From **C**: Systems are **A** and **B**
From **A**: Systems are **C** and **B**



"Superpositions of translations"

"frozen in time"



Classical RF:

$$e^{\frac{i}{\hbar}\alpha\hat{p}_B}|x_B\rangle_B = |x_B - \alpha\rangle_B$$

Quantum RF:

$$\int d\alpha \psi_A(\alpha) e^{\frac{i}{\hbar}\alpha\hat{p}_B}|\alpha\rangle_A|\phi\rangle_B =$$

$$= e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}|\psi\rangle_A|\phi\rangle_B$$

Parameter promoted to an operator

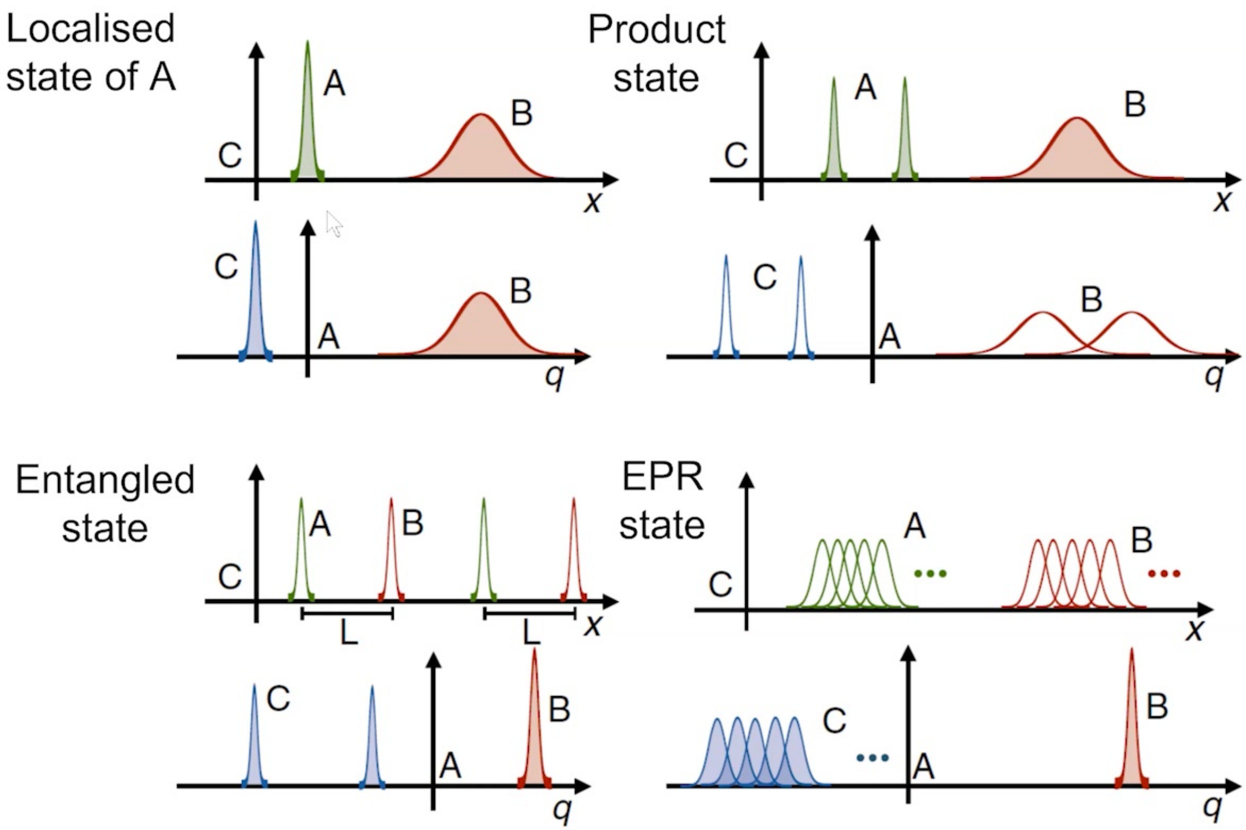
Because of $\hat{x}_A \to -\hat{q}_C$:

$$\hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

Parity-Swap Operator: $\mathcal{P}_{AC}\psi_A(x) = \psi_C(-x)$

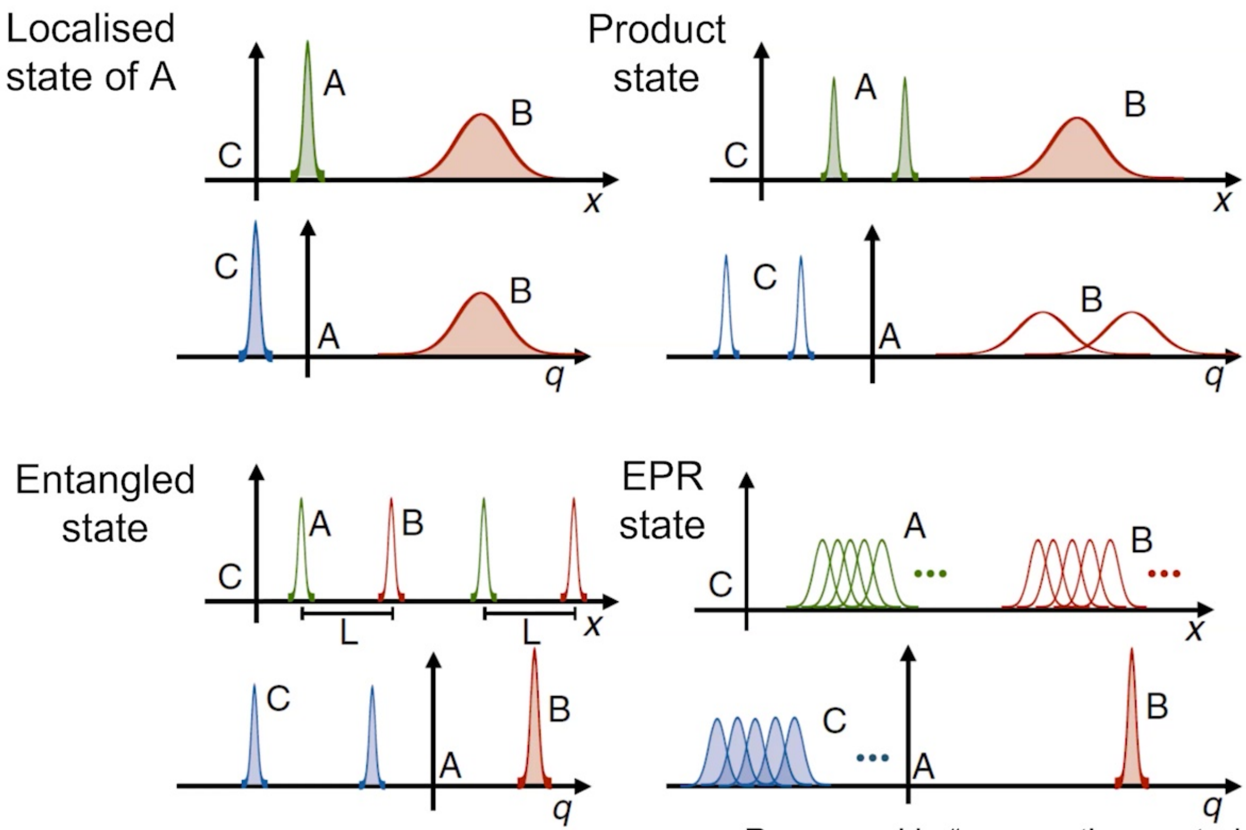
State of B & C as seen from A $\quad \hat{\rho}^{(A)}_{BC} = \hat{S}_x \hat{\rho}^{(C)}_{AB} \hat{S}^\dagger_x \quad$ State of A & B as seen from C

Relative States



F. Giacomini, E. Castro, and Č. Brukner, Nature Communications **10,** 494 (2019).

Relative States



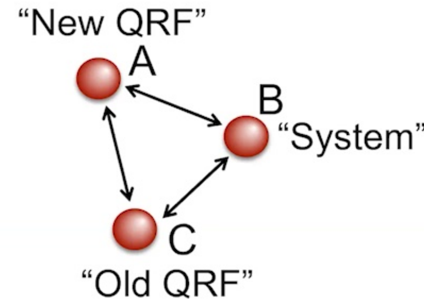
Recovered in “perspective neutral approach” in
A Vanrietvelde, P.A. Hoehn, F. Giacomini, E. Castro-Ruiz, Quantum **4**, 225 (2020)

Extended covariance of physical laws

A Hamiltonian is assigned to all systems "external" to a QRF.

Schrödinger equation in **C**'s reference frame

$$i\hbar \frac{d\hat{\rho}^{(C)}_{AB}}{dt} = [\hat{H}^{(C)}_{AB}, \hat{\rho}^{(C)}_{AB}]$$



Schrödinger equation in **A**'s reference frame

$$i\hbar \frac{d\hat{\rho}^{(A)}_{BC}}{dt} = [\hat{H}^{(A)}_{BC}, \hat{\rho}^{(A)}_{BC}]$$

$$\hat{H}^{(A)}_{BC} = \hat{S}\hat{H}^{(C)}_{AB}\hat{S}^{\dagger} + i\hbar \frac{d\hat{S}}{dt}\hat{S}^{\dagger}, \qquad \hat{\rho}^{(A)}_{BC} = \hat{S}\hat{\rho}^{(C)}_{AB}\hat{S}^{\dagger}$$

We define a **symmetry** transformation as:

$$\hat{S}\hat{H}(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B})\hat{S}^{\dagger} + i\hbar \frac{d\hat{S}}{dt}\hat{S}^{\dagger} = \hat{H}(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C})$$

It leaves the **functional form of the Hamiltonian invariant**.

Extended covariance
under "superpositions of translations"

Translation: We "jump" on A and translate B for the relative distance A had to B at time $t = 0$.

$$|\psi(0)\rangle_{AB} = \frac{1}{\sqrt{2}}(|x_1\rangle_A + |x_2\rangle_A)|\phi(0)\rangle_B$$

Parity + Swap:

$$\hat{S}_x = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\mathcal{P}_{AC}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

Move C forwards in time to t

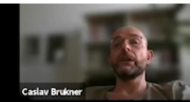
Translation conditioned by the position of A

Move A backwards in time to $t = 0$

The free Hamiltonian is **symmetric** under generalized boosts.

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} \quad \leftrightarrow \quad \hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_C^2}{2m_C} + \frac{\hat{\pi}_B^2}{2m_B}$$

F. Giacomini, E. Castro, and Č. Brukner, Nature Communications **10,** 494 (2019).



Extended covariance

under "superpositions of boosts"

Boost: We "jump" on A and boost B for the relative velocity between A and B at time t=0

$$|\psi(0)\rangle_{AB} = \frac{1}{\sqrt{2}}(|v_1\rangle_A + |v_2\rangle_A)|\phi(0)\rangle_B$$

Parity + Swap: Sets velocity of C to the opposite of velocity of A: $\hat{\pi}_C = -(m_C/m_A)\hat{p}_A$

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\mathcal{P}_{AC}^{(v)} \exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

Move C forwards in time to t

Galilean boost conditioned by the velocity of A

Move A backwards in time to $t=0$

The free Hamiltonian is **symmetric** under generalized boosts.

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} \quad \leftrightarrow \quad \hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_C^2}{2m_C} + \frac{\hat{\pi}_B^2}{2m_B}$$

F. Giacomini, E. Castro, and Č. Brukner, Nature Communications **10,** 494 (2019).

Relativistic Quantum Reference Frames

VOLUME 88, NUMBER 23 PHYSICAL REVIEW LETTERS 10 JUNE 2002

Quantum Entropy and Special Relativity

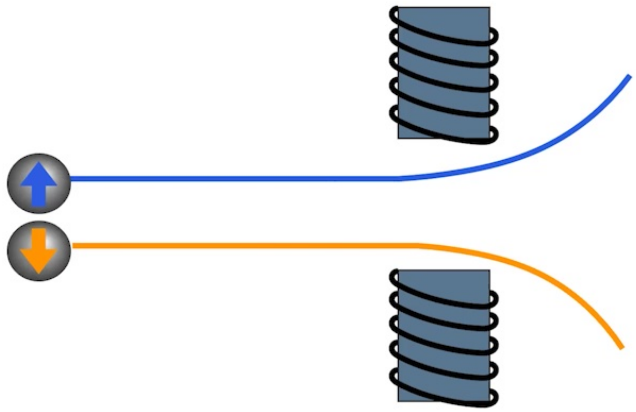
Asher Peres, Petra F. Scudo, and Daniel R. Terno
Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel
(Received 7 March 2002; published 22 May 2002)

We consider a single free spin-$\frac{1}{2}$ particle. The reduced density matrix for its spin is not covariant under Lorentz transformations. The spin entropy is not a relativistic scalar and has no invariant meaning.

DOI: 10.1103/PhysRevLett.88.230402 PACS numbers: 03.65.Ta, 03.30.+p

An open problem in relativistic quantum information: How to encode and decode qubit using the angular momentum of a relativistic particle?

Stern-Gerlach Experiment

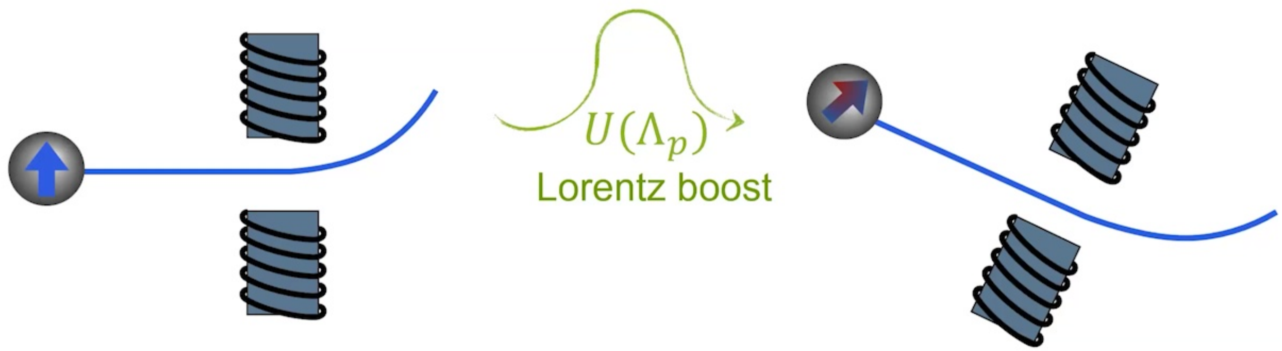


Operational definition of spin for low velocities

$$\vec{J} = \vec{L} + \vec{S}$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin in Special Relativity



The alignment of the magnet depends on the momentum of the particle.

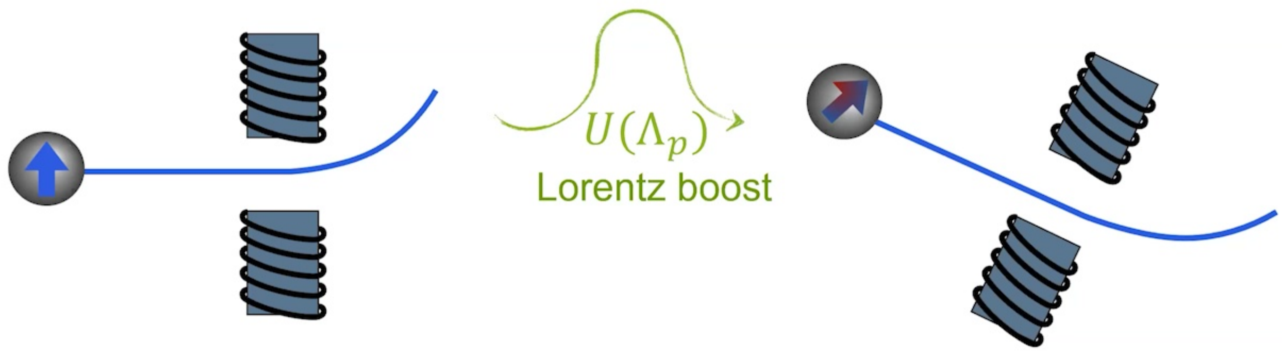
State of momentum and spin in the rest frame $|k;\vec{\sigma}\rangle \equiv |mc,0,0,0;\vec{\sigma}\rangle$

State of momentum and spin in the lab frame $|p;\Sigma_p\rangle = U(\Lambda_p)|k;\vec{\sigma}\rangle$

↑ Rotated spin

$$\vec{p} = m\gamma\vec{v},\ \gamma = \sqrt{1+\frac{p^2}{m^2c^2}}$$

Spin in Special Relativity



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State of momentum and spin in the lab frame $|p;\Sigma_p\rangle = U(\Lambda_p)|k;\vec{\sigma}\rangle$

↑ Rotated spin

How to measure spin if the particle is in a state of a superposition of momenta?

$$\vec{p} = m\gamma\vec{v},\ \gamma = \sqrt{1+\frac{p^2}{m^2c^2}}$$

Spin in Special Relativity

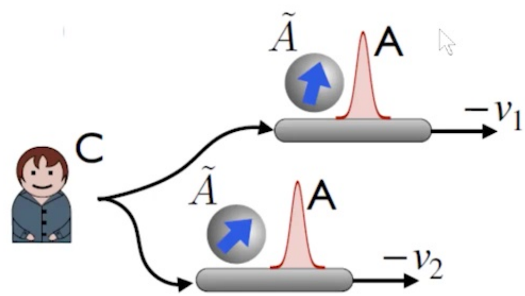
A general quantum state is written as

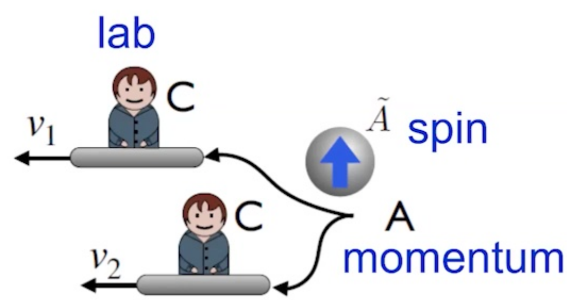
$$|\Psi\rangle = \int d\mu(p)\psi\,(p)\,|p;\Sigma_p\rangle$$

$$d\mu(p) = \frac{d^3p}{(2\pi)^{3/2}\sqrt{2(m^2c^2+\vec{p}^2)}}$$

Relativistic quantum reference frame:

1. Move to the rest frame
2. Define spin in the rest frame with the help of the Stern-Gerlach experiment
3. Move back to the lab frame





Lorentz boost in QRFs

In the rest frame of the particle (A): $|\psi\rangle^{(A)}_{\tilde{A}C} = |\vec{\sigma}\rangle_{\tilde{A}} |\psi\rangle_C$ (spin: $|\vec{\sigma}\rangle_{\tilde{A}}$; lab: $|\psi\rangle_C$)

$$|\psi\rangle_C = \int d\mu_C(\pi_C)\psi(\pi_C)|\pi_C\rangle_C$$

$$\hat{S}_L = \mathcal{P}^{(v)}_{AC} U_{\tilde{A}}(\Lambda_{\pi_C})$$

Lorentz boost by each velocity of the laboratory relative to the particle

$$\hat{S}_L |\vec{\sigma}\rangle_{\tilde{A}} |\pi\rangle_C = |-\frac{m_A}{m_C}\pi; \Sigma_\pi\rangle_{A\tilde{A}} \quad \text{where} \quad |p;\Sigma_p\rangle_{A\tilde{A}} = \hat{U}(\Lambda_p)|k;\vec{\sigma}\rangle_{A\tilde{A}}$$

$$\mathcal{P}^{(v)}_{AC} \hat{p}_A \mathcal{P}^{(v)\dagger}_{AC} = -\frac{m_A}{m_C}\hat{\pi}_C \quad \text{guarantees that } \vec{v}_A = -\vec{v}_C$$

In the laboratory frame (C):

$$|\psi\rangle^{(C)}_{A\tilde{A}} = \left(\frac{m_C}{m_A}\right)^2 \int d\mu_A(p_A)\psi\left(-\frac{m_C}{m_A}p_A\right)|p_A, \Sigma_{p_A}\rangle_{A\tilde{A}}$$

Spin Operator

With the QRF transformation, we can operationally identify a spin operator which gives the same results in the laboratory frame and in the rest frame.

$$\vec{\Xi} = \hat{S}_L(\vec{\sigma} \otimes \mathbb{1})\hat{S}_L^\dagger$$ (Foldy-Wouthuysen or Pryce spinoperator)

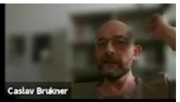
$$\vec{\Xi} \xrightarrow{v \to 0} \mathcal{P}_{CA}^{(v)}(\vec{\sigma} \otimes \mathbb{1})\mathcal{P}_{CA}^{(v)\dagger}$$ Correct non-relativistic limit

$$[\Xi_i, \Xi_j] = i\epsilon_{ijk}\Xi_k$$ su(2) algebra

$$\vec{\sigma}|\sigma_i\rangle = \lambda_i|\sigma_i\rangle$$ Correct eigenvalues

$$\vec{\Xi}\hat{S}_L|\sigma_i\rangle|\psi\rangle_C = \lambda_i \underbrace{\hat{S}_L|\sigma_i\rangle|\psi\rangle_C}_{\text{The state in the laboratory frame}}$$

F. Giacomini, E. Castro-Ruiz and Č.B., Phys. Rev. Lett. **123**, 090404 (2019).



Spin Operator

Partition of the total Hilbert space

$$\mathcal{H}_0 = \left\{ |\Psi\rangle_{A\tilde{A}} \in \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}} \text{ s.t. } |\Psi\rangle_{A\tilde{A}} \sim \hat{S}_L |0\rangle_{\tilde{A}} |\psi\rangle_C, \forall\, |\psi\rangle_C \in \mathcal{H}_C \right\},$$

$\rightarrow |0\rangle$

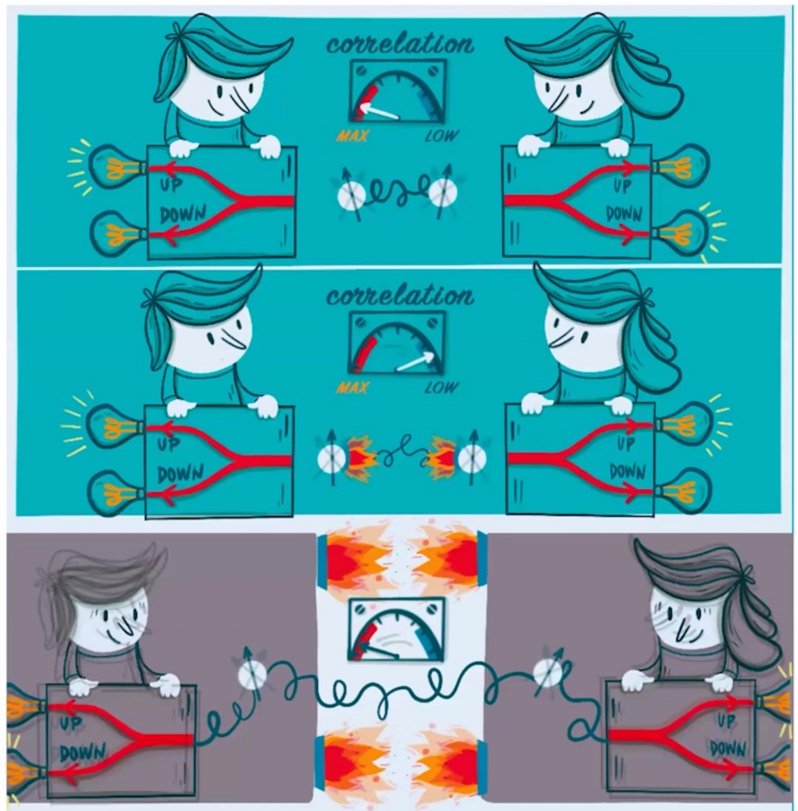
$$\mathcal{H}_1 = \left\{ |\Phi\rangle_{A\tilde{A}} \in \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}} \text{ s.t. } |\Phi\rangle_{A\tilde{A}} \sim \hat{S}_L |1\rangle_{\tilde{A}} |\phi\rangle_C, \forall\, |\phi\rangle_C \in \mathcal{H}_C \right\},$$

$\rightarrow |1\rangle$

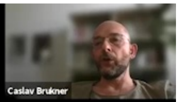
Effectively, we can treat the relativistic spin as a qubit
with the help of quantum reference frames.

F. Giacomini, E. Castro-Ruiz and Č.B., Phys. Rev. Lett. **123**, 090404 (2019).

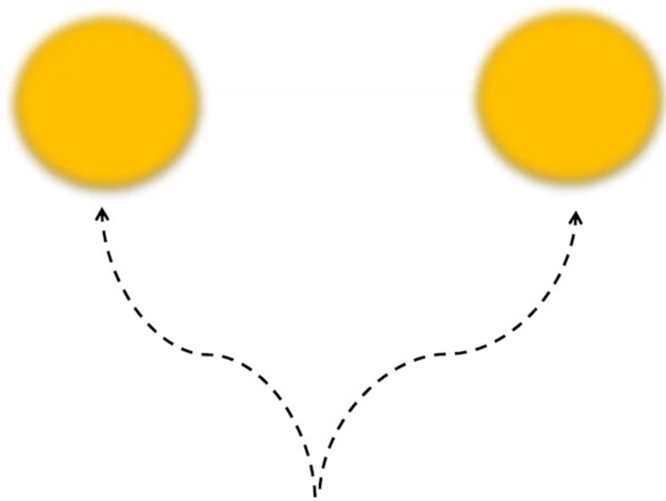
Relativistic Bell Test within QRFs



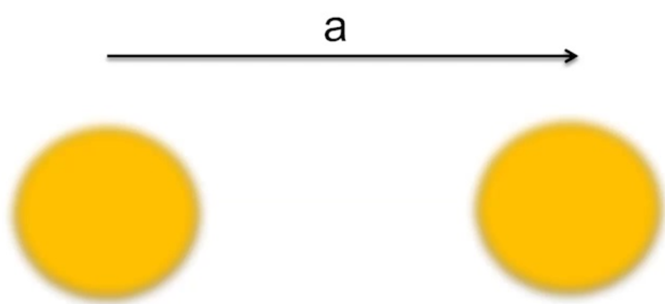
L. F. Streiter, F. Gacomini, and Č.B. Phys. Rev. Lett. **126**, 230403 (2021).



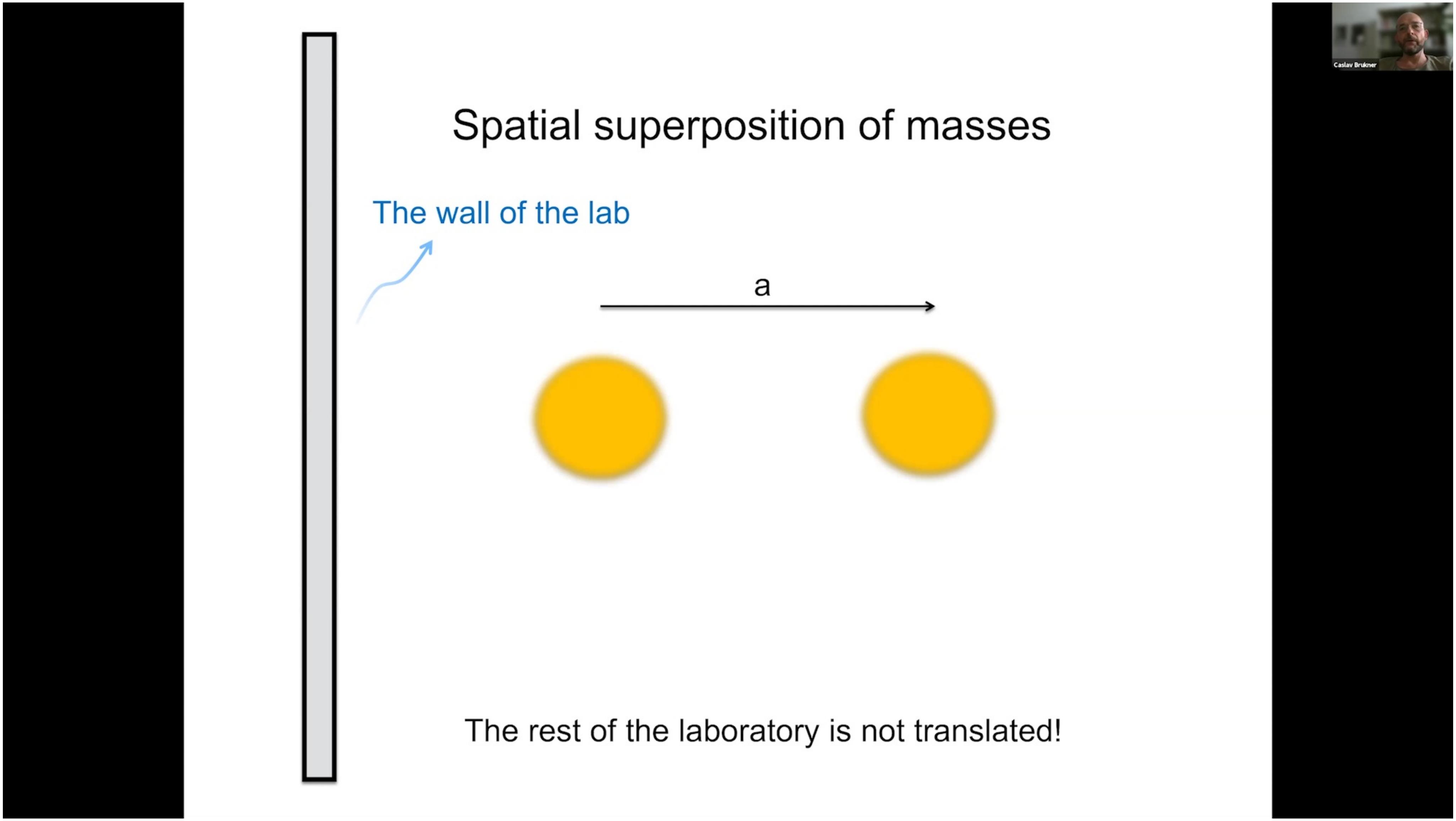
Spatial superposition of masses



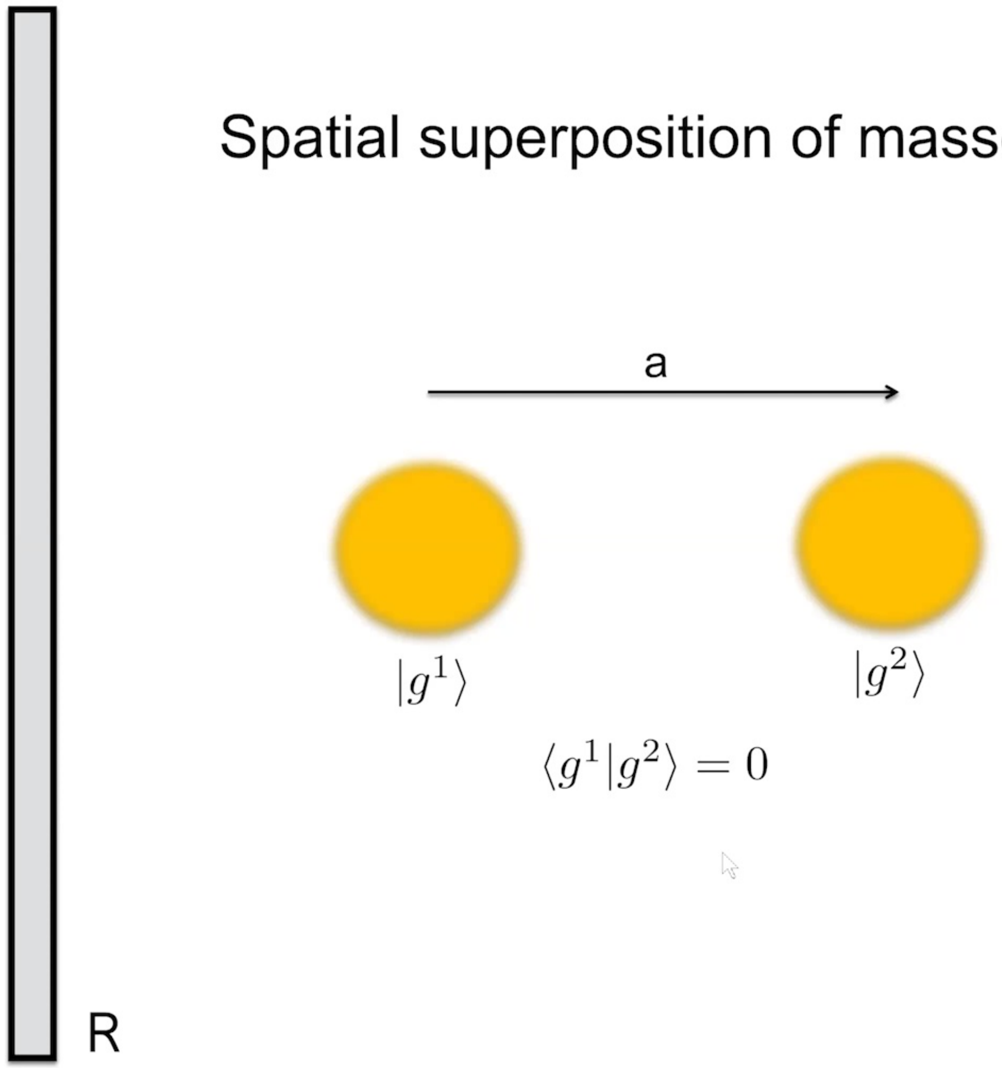
Spatial superposition of masses



Are the two space-times equivalent, since they are related to each other through a diffeomorphism (translation by a)?


Spatial superposition of masses
The wall of the lab
a
The rest of the laboratory is not translated!
Caslav Brukner

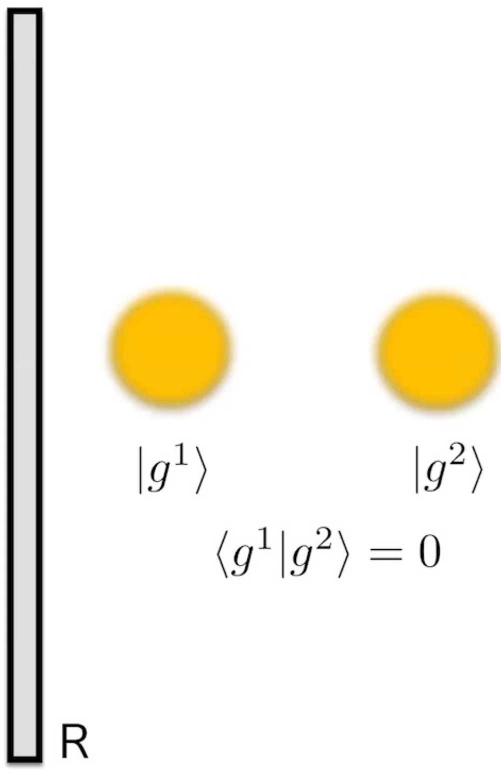
Spatial superposition of masses



$|g^1\rangle$ $|g^2\rangle$

$$\langle g^1|g^2\rangle = 0$$

Spatial superposition of masses



The regime considered:

1. gravitational fields produced by distinguishable mass distributions correspond to orthogonal quantum states
2. each well-defined gravitational field is described by GR
3. the quantum superposition principle holds for such gravitational fields

Valid when the distinguishability between the states is much larger than the "quantum fluctuations" of the individual states. **Far from the Planck scale**!

(Classical) locally inertial frame

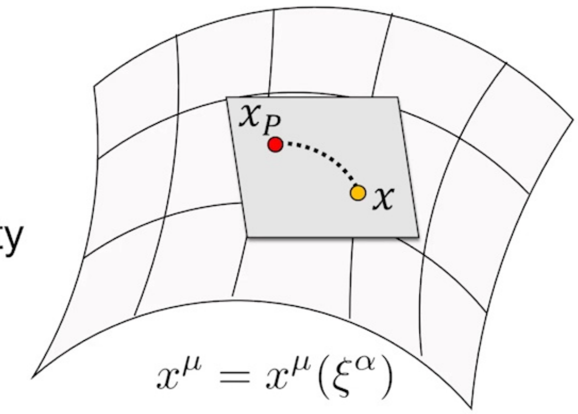
Step 1: Center the origin in P

$$x^\mu = x^\mu_P + \frac{\partial x^\mu}{\partial \xi^\alpha}|_{x_P}\xi^\alpha + ...$$

Step 2: „Straigthen" the metric in the vicinity

$$f^\mu_\alpha = \frac{\partial x^\mu}{\partial \xi^\alpha}|_{x_P}$$

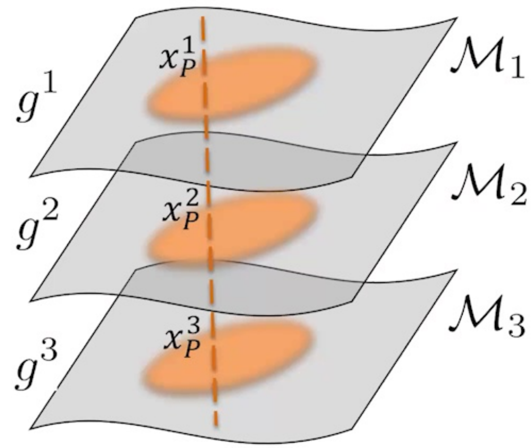
$$\tilde{g}_{\alpha\beta}(\xi) = f^\mu_\alpha f^\nu_\beta g_{\mu\nu}(x_P + f\xi)$$



Locally inertial frames for a family of space-times

Locally Inertial Frames transformations

1. Within each spacetime (for all positions)
2. Across different spacetimes (metrics)



At each point x_P^i in each spacetime g^i

$$(f^{(i)})^\mu_\alpha = \frac{\partial x^\mu}{\partial \xi^\alpha}\Big|^{(i)}_{x_P}$$

$$\tilde{g}^i_{\alpha\beta}(\xi^i) = (f^{(i)})^\mu_\alpha (f^{(i)})^\nu_\beta g_{\mu\nu}(x^i_P + f^i \xi^i)$$

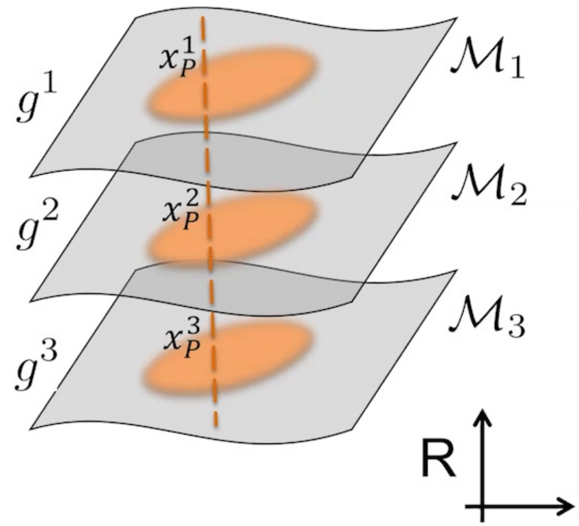
$$\Longrightarrow \; \tilde{g}^i_{\alpha\beta} = \eta_{\alpha\beta} + O((\xi^i)^2)$$ Locally flat

Quantum locally inertial frames

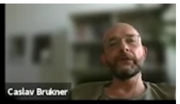
All details omitted

From QRF R:

$$\sum_i c_i \int d^4x \sqrt{-g_i(x)}\,\psi_i(x)|g_i(x)\rangle|x\rangle_P$$



F. Giacomini, C.B. arXiv: 2012.13754



Quantum locally inertial frames

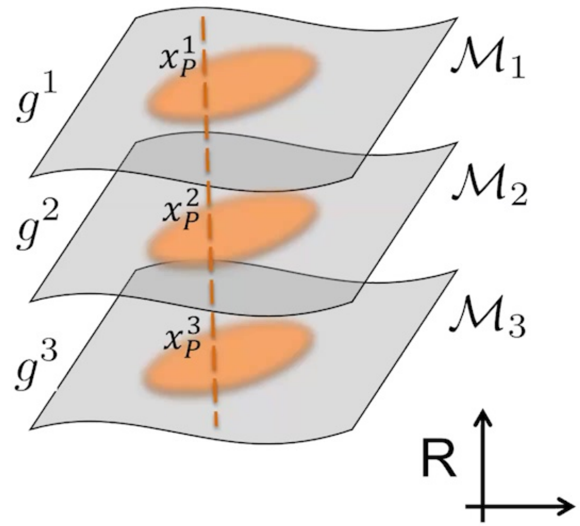
All details omitted

From QRF R:

$$\sum_i c_i \int d^4x \sqrt{-g_i(x)}\psi_i(x)|g_i(x)\rangle|x\rangle_P$$

From QRF P:

$$\sum_i c_i \int d^4x \sqrt{-g_i(x)}\psi_i(x)|-x^{(i)}\rangle_R|\eta\rangle$$



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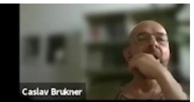


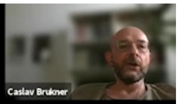
Quantum Einstein's equivalence principle

*In any and every **quantum locally inertial frame**, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special-relativistic form*.*

* Adapted from C. W. Misner, K. Thorne, and J. Wheeler, Gravitation. San Francisco: W. H. Freeman, 1973

Summary

- **Operational approach**: reference frames as physical systems obeying quantum mechanical laws
- Superposition and entanglement are **frame-dependent notions**



Summary

- **Operational approach**: reference frames as physical systems obeying quantum mechanical laws
- Superposition and entanglement are **frame-dependent notions**
- Generalization of the **covariance law** and **symmetries** to quantum reference frames: “superpositions of spatial translations & boosts”
- **Quantum rest frame**: a **relativistic spin operator** satisfying the spin algebra (enabling maximal violation of Bell’s inequality in any quantum relativistic frame)
- **Quantum locally inertial frame**: Extension of the Einstein’s equivalence principle to quantum reference frames and non-classical space-times

Thank you!

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