Title: Non-causal Page-Wootters circuits

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Collection: Quantizing Time

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Abstract: "The process matrix framework was invented to capture a phenomenon known as indefinite or quantum causal structure. Due to the generality of that framework, however, for many process matrices there is no clear physical interpretation. A popular approach towards a quantum theory of gravity is the Page-Wootters formalism, which associates to time a Hilbert space structure similar to spatial position. By explicitly introducing a quantum clock, it allows to describe time-evolution of systems via correlations between this clock and said systems encoded in history states. We combine the process matrix framework with a generalization of the Page-Wootters formalism in which one considers several observers, each with their own discrete quantum clock.

This allows for implementing processes with indefinite casual order. The description via a history state with multiple clocks imposes constraints on the implementability of process matrices intros framework and on the perspectives of the observers. We describe how to to implement processes were the different definite causal orders are coherently controlled and explain why certain non-causal processes might not be implementable within this setting."

Non-causal Page Wootters circuits

The Page-Wootters formulation of indefinite causal order

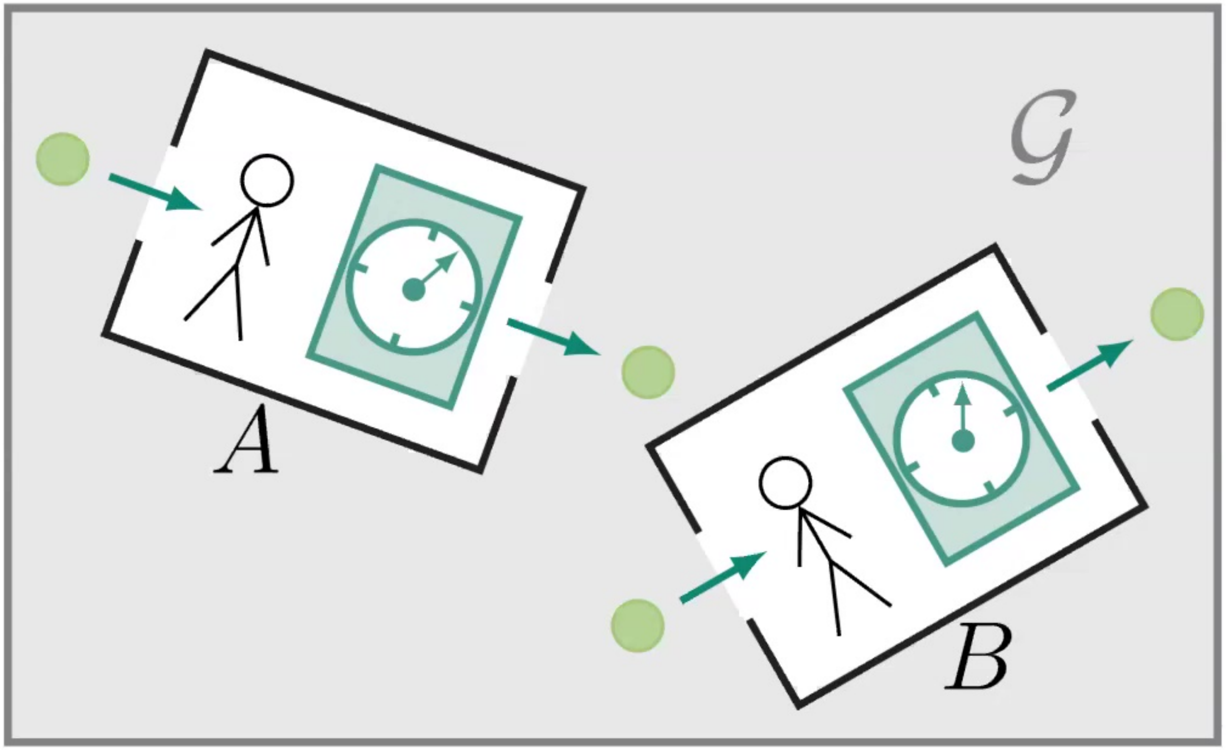
Veronika Baumann

with Marius Krumm, Philippe Allard Guèrin and Časlav Brukner

June 14, 2021

arXiv:2105.02304

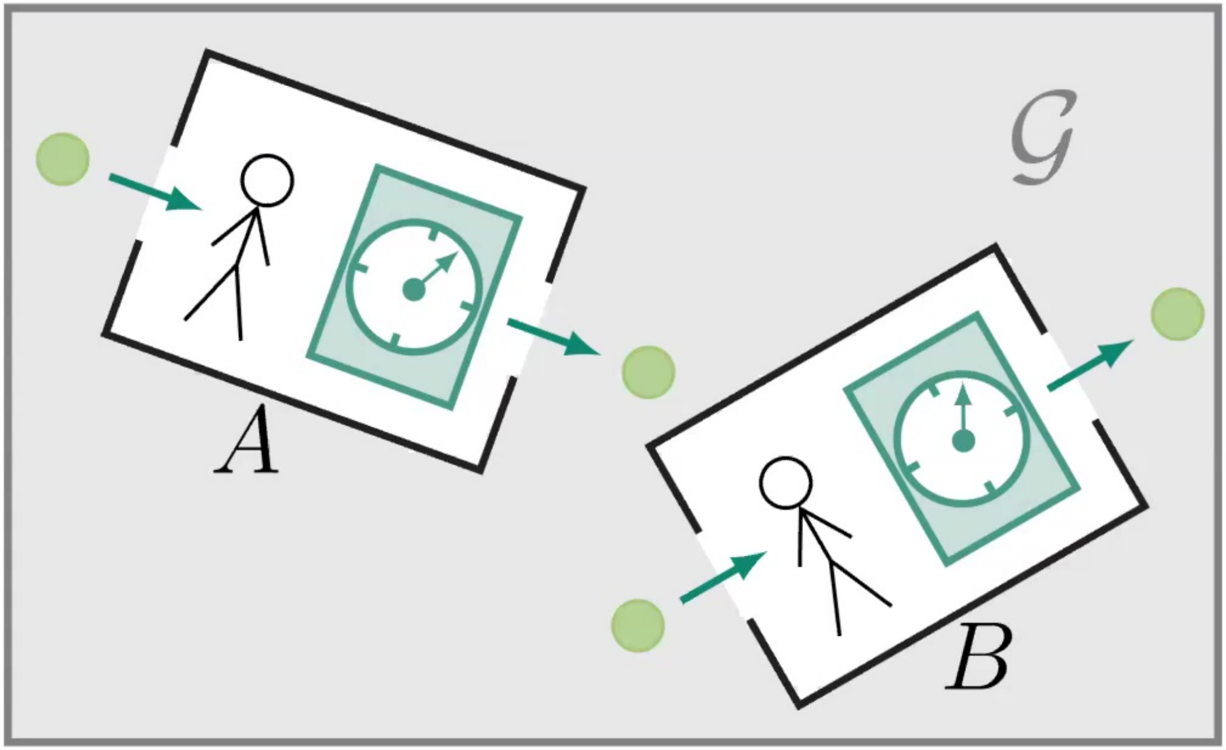
Indefinite causal order [1]



purified operations: U_A and U_B $\mathcal{G}(U_A, U_B)$

[1] O. Oreshkov, F. Costa, and C. Brukner. Nature Commun., 3:1092, 2012.

Indefinite causal order [1]



purified operations: $U_{A_1} \dots U_{A_N}$ $\qquad$ $\mathcal{G}(U_{A_1} \dots U_{A_N}) \dots$ unitary

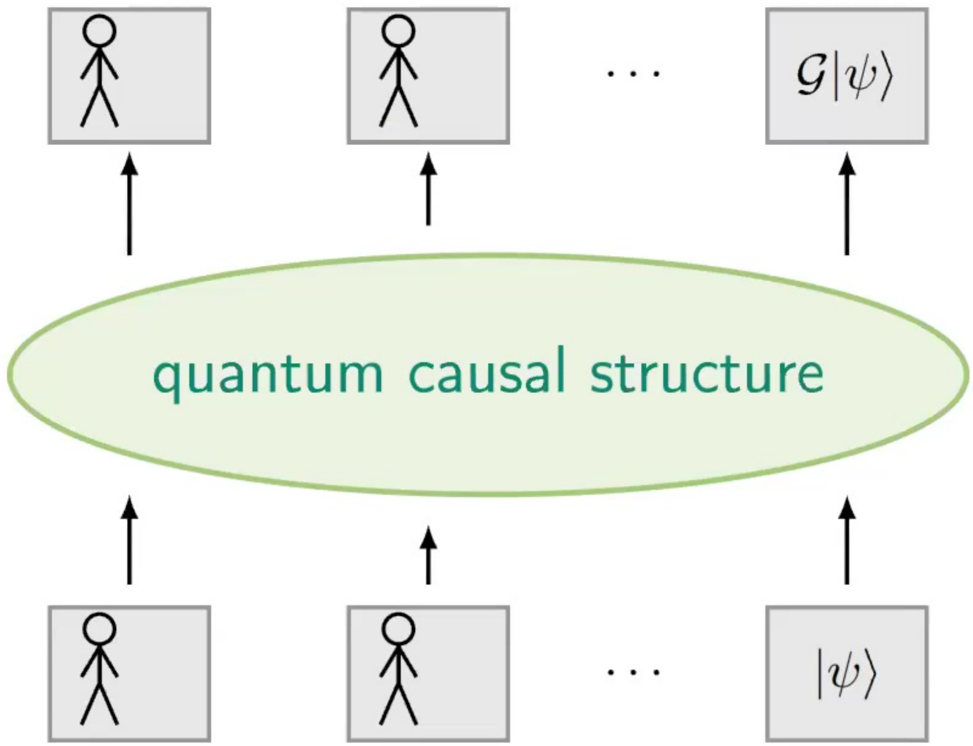
Pure process $\mathcal{G}$.

[1] O. Oreshkov, F. Costa, and C. Brukner. Nature Commun., 3:1092, 2012.

Non-causal and physical processes

$\mathcal{G}$ incompatible with a global causal order . . . non-causal processes

Only purifyable processes are physical [2], reversibly map a global causal past to a global causal future.



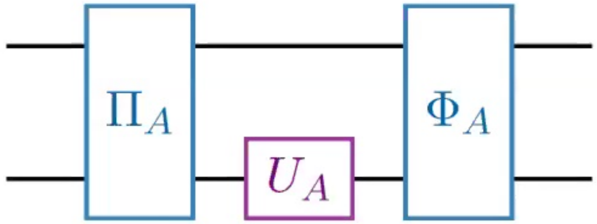
[2] M. Araújo, A. Feix, M. Navascués, and C. Brukner. Quantum, 1:10, 2017.

Causal reference frames [3]

An agent's perspective inside a (non-causal) pure process.

$$\mathcal{G}(U_A, U_B) = \Phi_A(U_B)\,(U_A \otimes \mathbb{1})\,\Pi_A(U_B)$$

Alice: causal future of A causal past of A



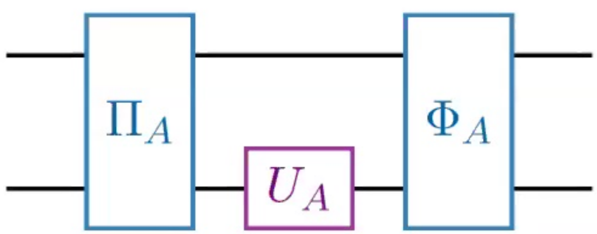
[3] P. Allard Guérin and C. Brukner. New Journal of Physics, 20(10):103031, 2018.

Causal reference frames [3]

An agent's perspective inside a (non-causal) pure process.

$$\mathcal{G}(U_A, U_B) = \Phi_B(U_A)\,(U_B \otimes \mathbb{1})\,\Pi_B(U_A)$$

Bob:



[3] P. Allard Guérin and C. Brukner. New Journal of Physics, 20(10):103031, 2018.

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$$\mathcal{G}(U_A, U_B) = \Phi_B(U_A)\,(U_B \otimes \mathbb{1})\,\Pi_B(U_A)$$

Bob: causal future of B causal past of B

[3] P. Allard Guérin and C. Brukner. New Journal of Physics, 20(10):103031, 2018.

Causal reference frames [3]

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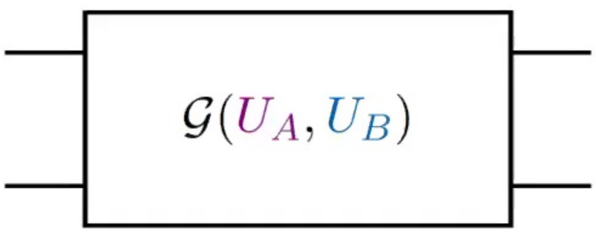
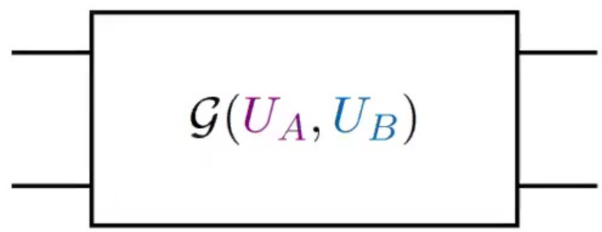




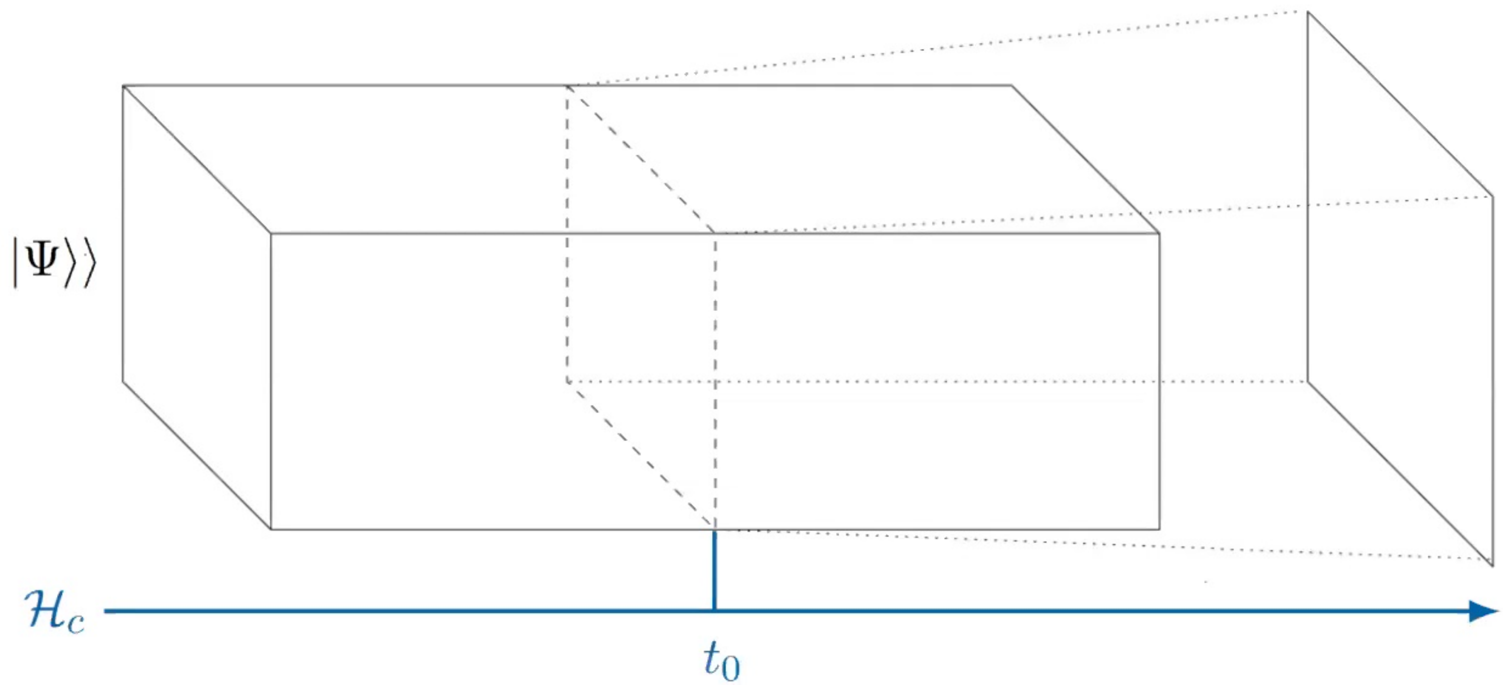
[3] P. Allard Guérin and C. Brukner. New Journal of Physics, 20(10):103031, 2018.

The Page-Wootters formalism [4,5]

Combined Hilbert space of system and clock $\mathcal{H}_c \otimes \mathcal{H}_S$.

Physical states satisfy a constraint equation: $\hat{C}|\Psi\rangle\rangle = 0$

$$\hat{C} = \hat{P}_t \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}_S : \quad |\Psi\rangle\rangle = \int \mathrm{d}t \; |t\rangle_c \otimes |\psi(t)\rangle_S$$



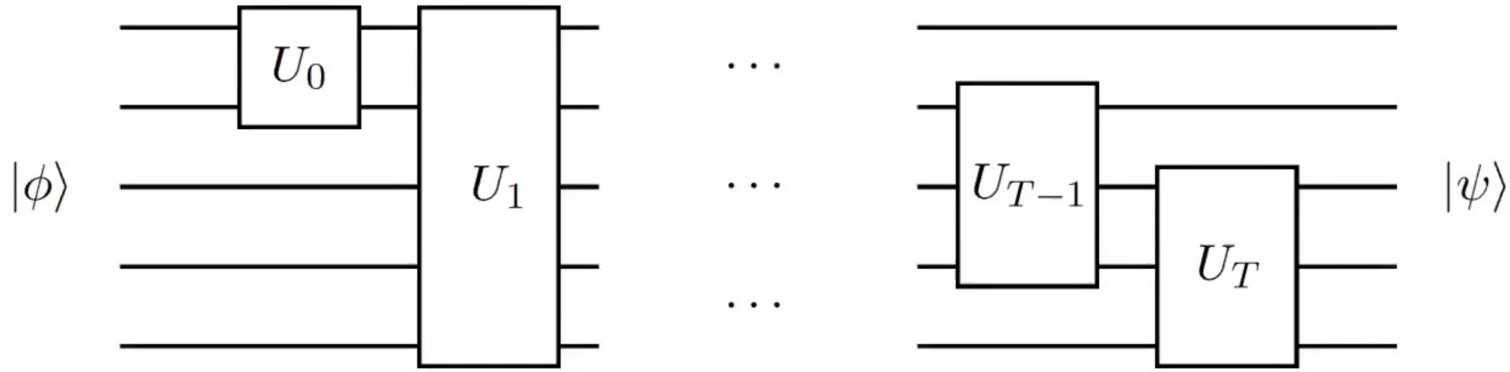
[4] D. N. Page and W. K. Wootters. Physical Review D, 27(12):2885–2892,1983.

[5] V.Giovannetti, S. Lloyd, and L. Maccone. Physical Review D, 92(4):045033, 2015.

Page Wootters circuits [6,7]

Clock system performs a finite number of ticks.

Model quantum circuites:



[6] R. P. Feynman. Optics news, 11(2):11–20, 1985.

[7] A. Yu. Kitaev, A. H. Shen, and M. N. Vyalyi. *Classical and Quantum Computation*, 2002.

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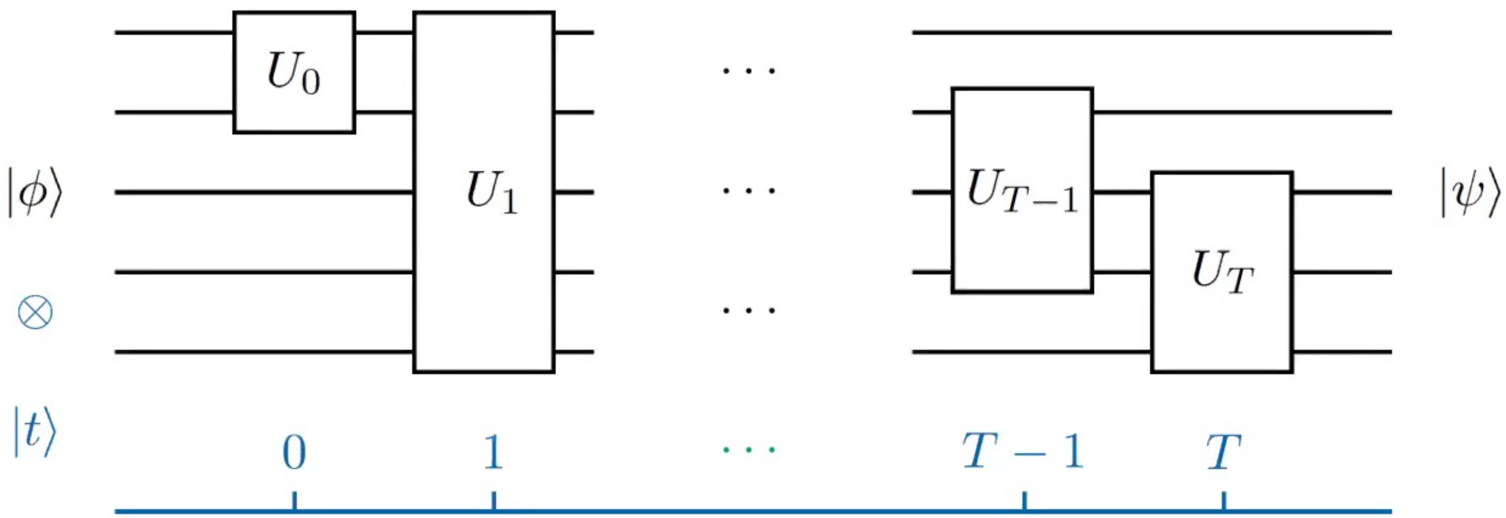


$$|\Psi\rangle\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |t\rangle_c \otimes U_t \dots U_0 |\phi\rangle_S = \sum_{t=0}^{T} |t\rangle_c \otimes |\psi(t)\rangle_S$$

[6] R. P. Feynman. Optics news, 11(2):11–20, 1985.

[7] A. Yu. Kitaev, A. H. Shen, and M. N. Vyalyi. *Classical and Quantum Computation*, 2002.

Pure processes in a Page-Wootters state

Associate a clock with each agent ... continuous case [8]

Consider discrete quantum clocks. $|\Psi\rangle\rangle \in \mathcal{H}_c \otimes \mathcal{H}_S \otimes \mathcal{H}_{S'}$

$$\mathcal{H}_c = \mathcal{H}_{c_{A_1}} \otimes \mathcal{H}_{c_{A_2}} \cdots \otimes \mathcal{H}_{c_{A_N}}$$

$$\mathcal{H}_{S'} = \mathcal{H}_{A'_1} \otimes \mathcal{H}_{A'_2} \cdots \otimes \mathcal{H}_{A'_N}$$

$$|\Psi\rangle\rangle = \sum_{t_{A_1}=0,\ldots t_{A_N}=0}^{T_{A_1}\ldots T_{A_N}} |t_{A_1}, \ldots t_{A_N}\rangle_c \otimes |\psi(t_{A_1} \ldots t_{A_N})\rangle\rangle_{SS'}.$$

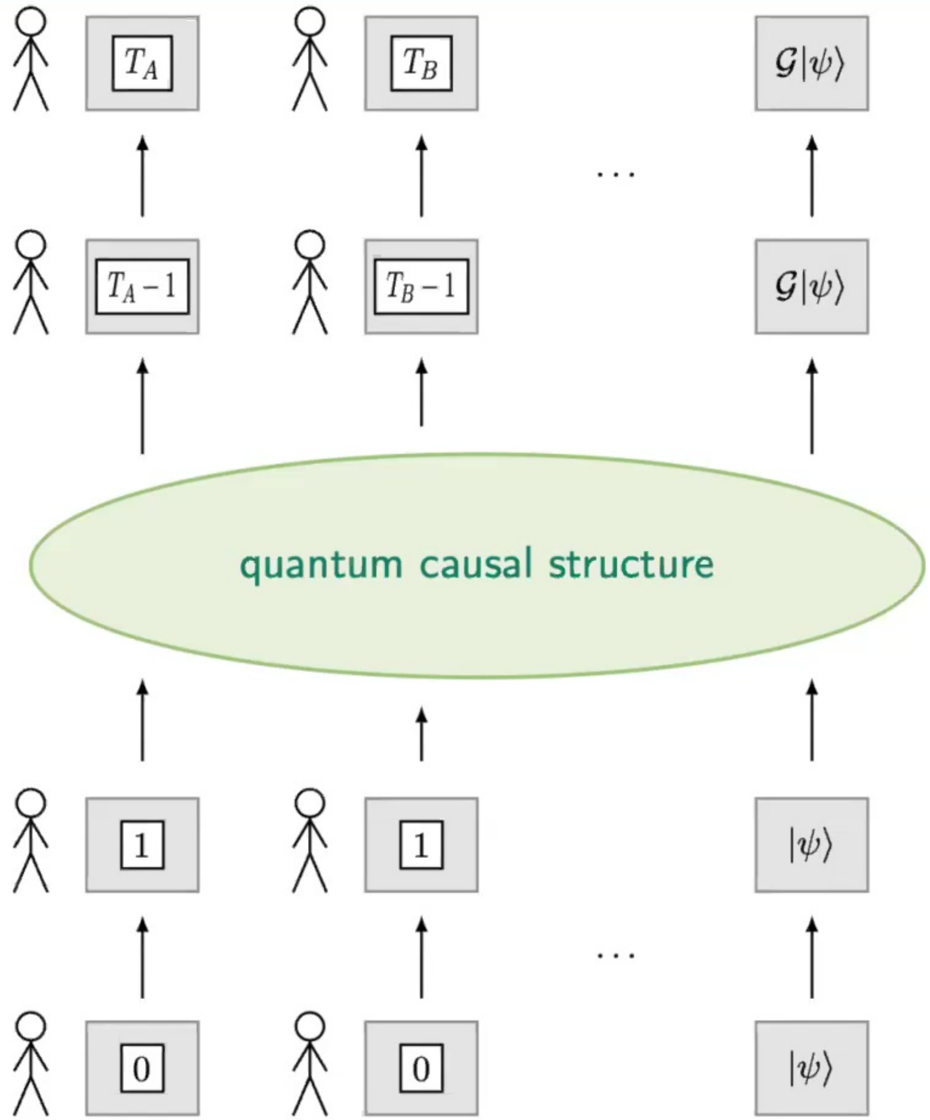
$|0 \ldots 0\rangle_c \otimes |\psi\rangle_S |0\rangle_{S'}$ $\qquad$ $|T_{A_1} \ldots T_{A_N}\rangle_c \otimes \mathcal{G}(U_{A_1} \ldots U_{A_N})|\psi\rangle_S |0\rangle_{S'}$

Definite causal order

One synchronized tick of all clocks at the start and the end.

[8] E. Castro-Ruiz, F. Giacomini, A. Belenchia, and C. Brukner. Nature Commun., 11(1):2672, 2020.

Pure processes in a Page-Wootters state



Pure processes in a Page-Wootters state

Perspectival states of agent X:

$$|\psi_X(t_X)\rangle = N_{t_X}^{(X)} \langle t_X|\Psi\rangle\rangle = \langle t_X|_{c_X} \otimes N_{t_X}^{(X)}|\Psi\rangle\rangle,$$

$N_{t_X}^{(X)} \in \mathcal{L}(\mathcal{H}_{c_{\setminus X}} \otimes \mathcal{H}_S \otimes \mathcal{H}_{S'})$ is the normalization operator.

- unitarily related:

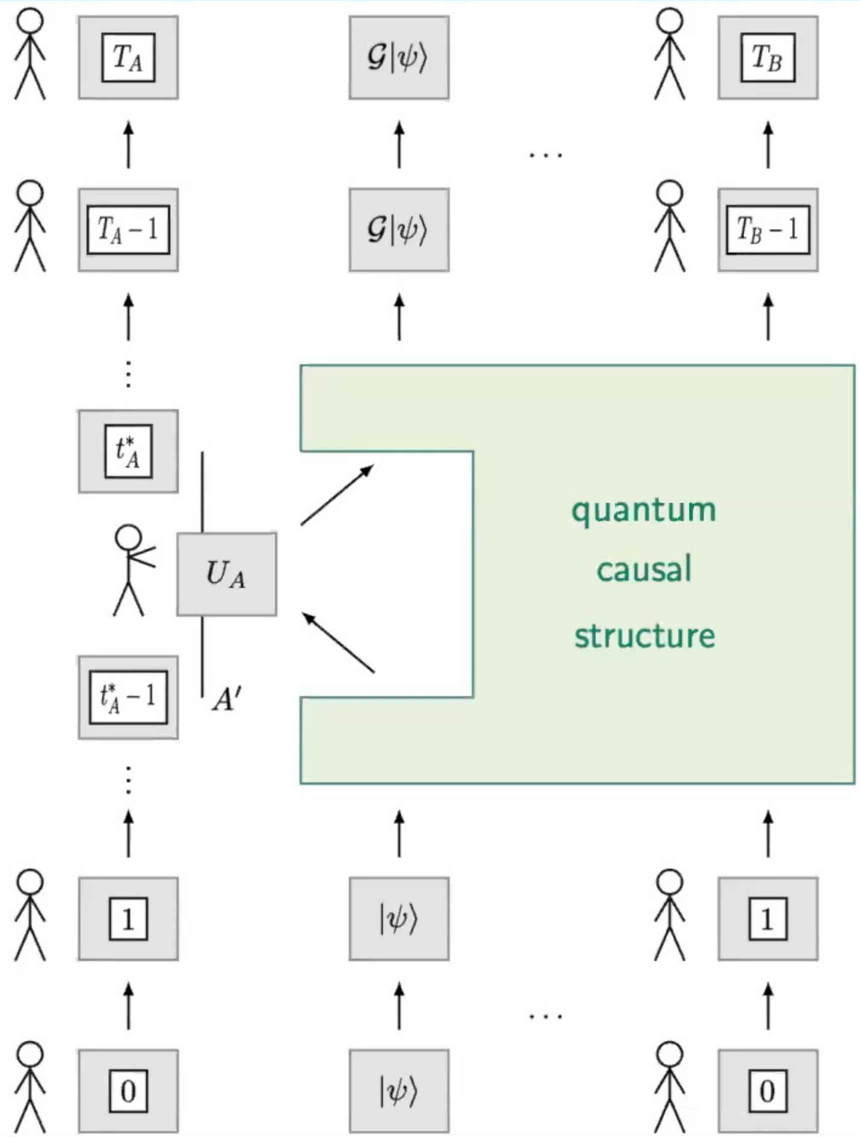
$$|\psi_X(t_X)\rangle = \mathcal{U}_X(t_X, t'_X)|\psi_X(t'_X)\rangle$$

  perspectival unitary $\mathcal{U}_X$.

- time of action t^*_X:

$$\mathcal{U}_X(t^*_X, t^*_X - 1) = U_X \otimes \mathrm{Rest}^{(X)}$$

Pure processes in a Page-Wootters state



Properties of non-causal Page-Wootters circuits

Perspectival states give refined causal reference frames

A_1:

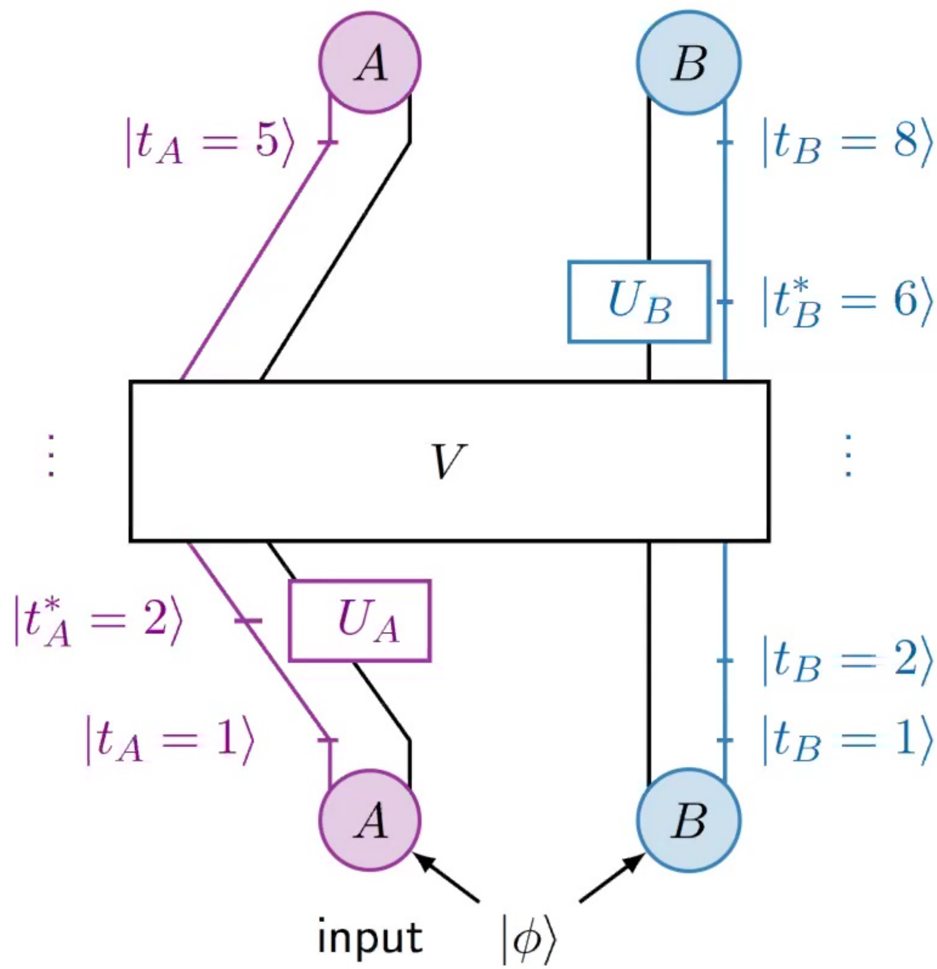
$$|T_{A_2} \dots T_{A_N}\rangle_{c\setminus A_1} \otimes |\psi(T_{A_1}, T_{A_2}, \dots T_{A_N})\rangle =$$

$$\Big(\underbrace{\mathcal{U}_{A_1}(T_{A_1}, t^*_{A_1})}_{\text{causal future of } A_1}(U_{A_1} \otimes \mathrm{Rest}^{(A_1)})\underbrace{\mathcal{U}_{A_1}(t^*_{A_1} - 1, 0)}_{\text{causal past of } A_1}\Big)\, |0, \dots 0\rangle_{c\setminus A_1} \otimes |\psi(0, 0, \dots 0)\rangle.$$

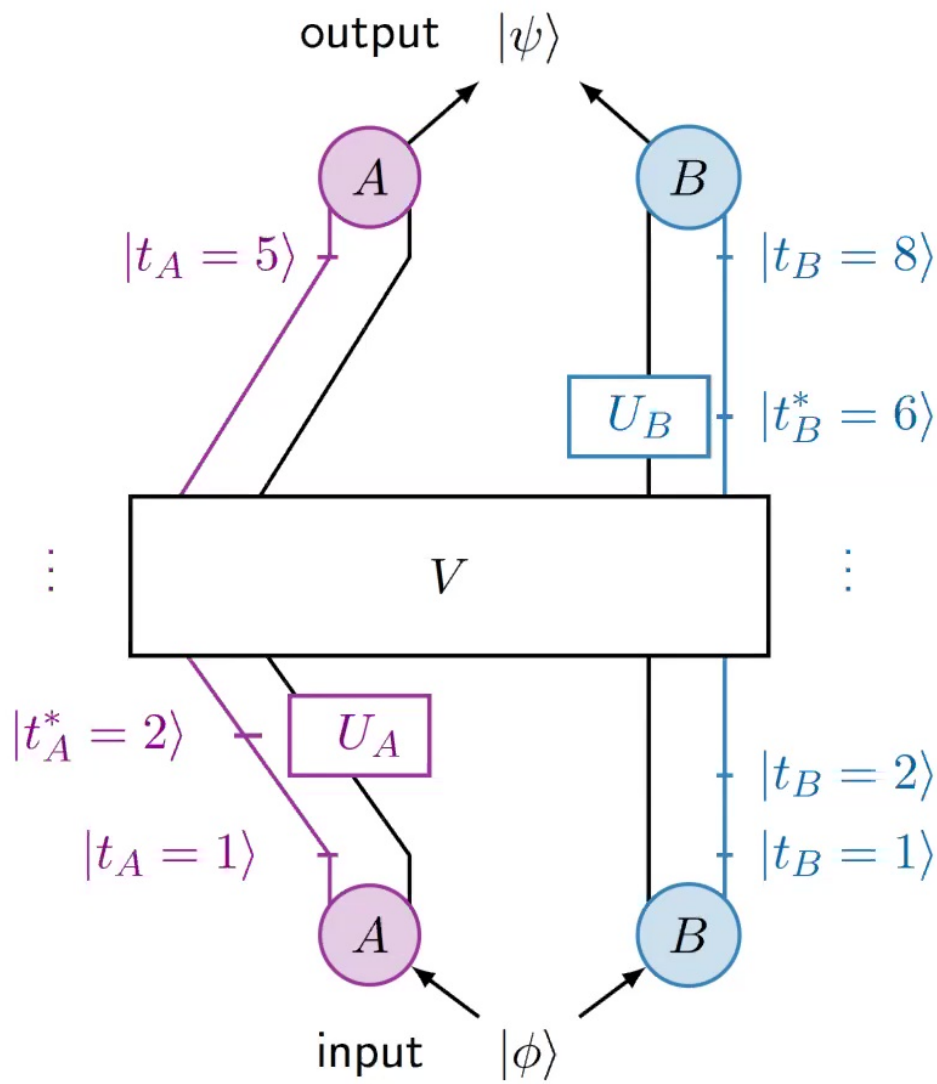
Affine-linearity:

The $\mathcal{U}_{A_X}$ consist of terms that are either *linear* or *constant* in all the operations $U_{A_1} \dots U_{A_N}$!

Twin-paradox circuit

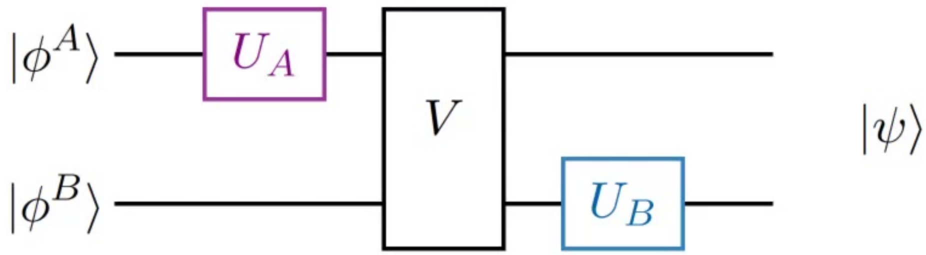


Twin-paradox circuit



Twin paradox circuit

Causal process: $\mathcal{G}(U_A, U_B) = (\mathbb{1} \otimes U_B) V (U_A \otimes \mathbb{1})$



$$
\begin{aligned}
|\Psi\rangle\rangle = & |0_A, 0_B\rangle_c \otimes |\phi\rangle + |1_A, 1_B\rangle_c \otimes |\phi\rangle \\
& + |2_A, 2_B\rangle_c \otimes (U_A \otimes \mathbb{1})|\phi\rangle + |2_A, 3_B\rangle_c \otimes (U_A \otimes \mathbb{1})|\phi\rangle \\
& + |3_A, 4_B\rangle_c \otimes V(U_A \otimes \mathbb{1})|\phi\rangle + |3_A, 5_B\rangle_c \otimes V(U_A \otimes \mathbb{1})|\phi\rangle \\
& + |4_A, 6_B\rangle_c \otimes (\mathbb{1} \otimes U_B) V (U_A \otimes \mathbb{1})|\phi\rangle + |4_A, 7_B\rangle_c \otimes |\psi\rangle \\
& + |5_A, 8_B\rangle_c \otimes |\psi\rangle + |6_A, 9_B\rangle_c \otimes |\psi\rangle
\end{aligned}
$$

Twin paradox circuit

Alice's perspective:

$$
\begin{aligned}
|\psi_A(0)\rangle &= |0_B\rangle_{c_B} \otimes |\phi\rangle, \\
|\psi_A(1)\rangle &= |1_B\rangle_{c_B} \otimes |\phi\rangle, \\
|\psi_A(2)\rangle &= \frac{1}{\sqrt{2}}(|2_B\rangle + |3_B\rangle)_{c_B} \otimes (U_A \otimes \mathbb{1})|\phi\rangle, \\
&\quad \text{with } N_2^{(A)} = \frac{1}{\sqrt{2}}\mathbb{1}_S, \\
|\psi_A(3)\rangle &= \frac{1}{\sqrt{2}}(|4_B\rangle + |5_B\rangle)_{c_B} \otimes V(U_A \otimes \mathbb{1})|\phi\rangle \\
&\quad \text{with } N_3^{(A)} = \frac{1}{\sqrt{2}}\mathbb{1}_S \\
|\psi_A(4)\rangle &= \frac{1}{\sqrt{2}}(|6_B\rangle + |7_B\rangle)_{c_B} \otimes (\mathbb{1} \otimes U_B)V(U_A \otimes \mathbb{1})|\phi\rangle \\
&\quad \text{with } N_4^{(A)} = \frac{1}{\sqrt{2}}\mathbb{1}_S, \\
|\psi_A(5)\rangle &= |8_B\rangle_{c_B} \otimes \mathcal{G}(U_A, U_B)|\phi\rangle, \\
|\psi_A(6)\rangle &= |9_B\rangle_{c_B} \otimes \mathcal{G}(U_A, U_B)|\phi\rangle
\end{aligned}
$$

Twin paradox circuit

Alice's perspective:

$$\mathcal{U}_A(1,0) = T_{c_B} \otimes \mathbb{1}_S,$$

$$\mathcal{U}_A(2,1) = (T_2')_{c_B} \otimes (U_A \otimes \mathbb{1})_S, \quad t_B^* = 2$$

$$\mathcal{U}_A(3,2) = (T^2)_{c_B} \otimes V_S,$$

$$\mathcal{U}_A(4,3) = (T^2)_{c_B} \otimes (\mathbb{1} \otimes U_B)_S,$$

$$\mathcal{U}_A(5,4) = (T_6')_{c_B} \otimes \mathbb{1}_S,$$

$$\mathcal{U}_A(6,5) = T_{c_B} \otimes \mathbb{1}_S$$

Twin paradox circuit

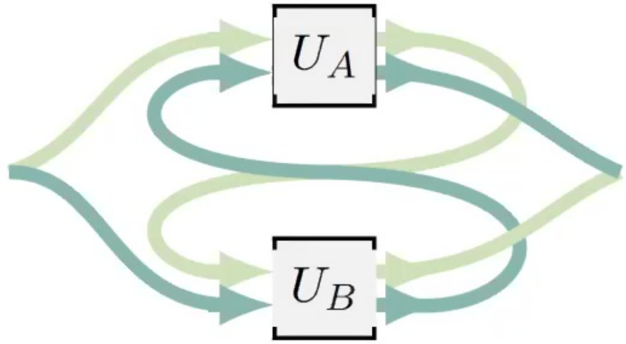
Bob's perspective:

$$
\begin{aligned}
\mathcal{U}_B(1,0) &= T_{c_A} \otimes \mathbb{1}_S, \\
\mathcal{U}_B(2,1) &= T_{c_A} \otimes (U_A \otimes \mathbb{1})_S, \\
\mathcal{U}_B(3,2) &= \mathbb{1}, \\
\mathcal{U}_B(4,3) &= T_{c_A} \otimes V_S, \\
\mathcal{U}_B(5,4) &= \mathbb{1}, \\
\mathcal{U}_B(6,5) &= T_{c_A} \otimes (\mathbb{1} \otimes U_B)_S, \quad t_B^* = 6 \\
\mathcal{U}_B(7,6) &= \mathbb{1}, \\
\mathcal{U}_B(8,7) &= T_{c_A} \otimes \mathbb{1}_S, \\
\mathcal{U}_B(9,8) &= T_{c_A} \otimes \mathbb{1}_S
\end{aligned}
$$

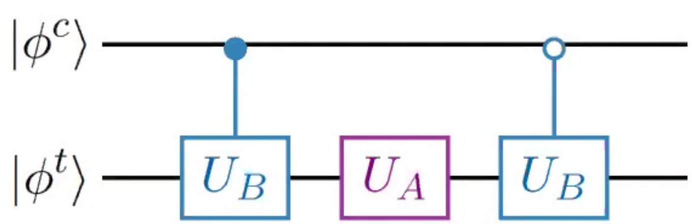
The quantum switch [9]

Coherent control of the order of operations:

$$\sqrt{\frac{1}{2}}(|0\rangle + |1\rangle)$$



Causal reference frames:



[9] G. Chiribella, G.M. D'Ariano, P. Perinotti, and B. Valiron. Physical Review A, 88(2):022318, 2013.

The quantum switch [9]

Indefinite causal order:
$\mathcal{G}(U_A, U_B) = |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$

$$
\begin{aligned}
|\Psi\rangle\rangle = {} & |0_A, 0_B\rangle_c \otimes |\phi\rangle + |1_A, 1_B\rangle_c \otimes |\phi\rangle + |2_A, 2_B\rangle_c \otimes |\phi\rangle \\
& + |3_A, 2_B\rangle_c \otimes (|0\rangle\langle 0| \otimes \mathbb{1})|\phi\rangle + |2_A, 3_B\rangle_c \otimes (|1\rangle\langle 1| \otimes \mathbb{1})|\phi\rangle \\
& + |4_A, 3_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_A)|\phi\rangle + |3_A, 4_B\rangle_c (|1\rangle\langle 1| \otimes U_B)|\phi\rangle \\
& + |5_A, 4_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_B U_A)|\phi\rangle + |4_A, 5_B\rangle_c \otimes (|1\rangle\langle 1| \otimes U_A U_B)|\phi\rangle \\
& + |5_A, 5_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B)|\phi\rangle \\
& + |6_A, 6_B\rangle_c \otimes \mathcal{G}(U_A, U_B)|\phi\rangle + |7_A, 7_B\rangle_c \otimes \mathcal{G}(U_A, U_B)|\phi\rangle,
\end{aligned}
$$

[9] G. Chiribella, G.M. D'Ariano, P. Perinotti, and B. Valiron. Physical Review A, 88(2):022318, 2013.

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$$\mathcal{G}(U_A, U_B) = |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$$

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& + |3_A, 2_B\rangle_c \otimes (|0\rangle\langle 0| \otimes \mathbb{1})|\phi\rangle + |2_A, 3_B\rangle_c \otimes (|1\rangle\langle 1| \otimes \mathbb{1})|\phi\rangle \\
& + |4_A, 3_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_A)|\phi\rangle + |3_A, 4_B\rangle_c (|1\rangle\langle 1| \otimes U_B)|\phi\rangle \\
& + |5_A, 4_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_B U_A)|\phi\rangle + |4_A, 5_B\rangle_c \otimes (|1\rangle\langle 1| \otimes U_A U_B)|\phi\rangle \\
& + |5_A, 5_B\rangle_c \otimes (|0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B)|\phi\rangle \\
& + |6_A, 6_B\rangle_c \otimes \mathcal{G}(U_A, U_B)|\phi\rangle + |7_A, 7_B\rangle_c \otimes \mathcal{G}(U_A, U_B)|\phi\rangle,
\end{aligned}$$

desynchronization

resynchronization

[9] G. Chiribella, G.M. D'Ariano, P. Perinotti, and B. Valiron. Physical Review A, 88(2):022318, 2013.

The quantum switch [9]

Alice's perspective:

$$\mathcal{U}_A(1,0) = T_{c_B} \otimes \mathbb{1}_S$$

$$\mathcal{U}_A(2,1) = T_{c_B} \otimes (|0\rangle\langle 0| \otimes \mathbb{1})_S + (T_2')_{c_B} \otimes (|1\rangle\langle 1| \otimes \mathbb{1})_S$$

$$\mathcal{U}_A(3,2) = \mathbb{1}_{c_B} \otimes (|0\rangle\langle 0| \otimes \mathbb{1})_S + (T_2')_{c_B} \otimes (|1\rangle\langle 1| \otimes U_B)_S$$

$$\mathcal{U}_A(4,3) = T_{c_B} \otimes (\mathbb{1} \otimes U_A)_S, \quad t_A^* = 4$$

$$\mathcal{U}_A(5,4) = (T_4')_{c_B} \otimes (|0\rangle\langle 0| \otimes U_B)_S + \mathbb{1}_{c_B} \otimes (|1\rangle\langle 1| \otimes \mathbb{1})_S$$

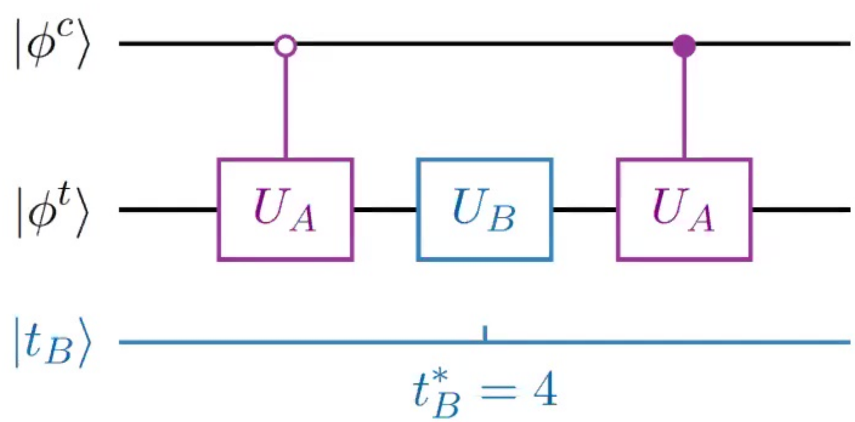
$$\mathcal{U}_A(6,5) = (T_4')_{c_B} \otimes (|0\rangle\langle 0| \otimes \mathbb{1})_S + T_{c_B} \otimes (|1\rangle\langle 1| \otimes \mathbb{1})_S$$

$$\mathcal{U}_A(7,6) = T_{c_B} \otimes \mathbb{1}_S$$

[9] G. Chiribella, G.M. D'Ariano, P. Perinotti, and B. Valiron. Physical Review A, 88(2):022318, 2013.

The quantum switch [9]

Bob's perspective is analogous. $|0\rangle_{S_c} \longleftrightarrow |1\rangle_{S_c}$



Similar procedure works for any coherent control of causal order.

[9] G. Chiribella, G.M. D'Ariano, P. Perinotti, and B. Valiron. Physical Review A, 88(2):022318, 2013.

The Lugano process [10]

A pure non-causal process that is not an example of coherent control of causal order.

...time-reversed version:

$$\begin{aligned}
\mathcal{G}(U_A, U_B, U_C)|jjj\rangle &= U_A \otimes U_B \otimes U_C|jjj\rangle \\
\mathcal{G}(U_A, U_B, U_C)|j01\rangle &= XU_A \otimes U_B \otimes U_C|j01\rangle \\
\mathcal{G}(U_A, U_B, U_C)|1j0\rangle &= U_A \otimes XU_B \otimes U_C|1j0\rangle \\
\mathcal{G}(U_A, U_B, U_C)|01j\rangle &= U_A \otimes U_B \otimes XU_C|01j\rangle
\end{aligned}$$

with $j \in \{0, 1\}$.

[10] Ä. Baumeler and S. Wolf. New Journal of Physics, 18(1):013036, 2016.

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\mathcal{G}(U_A, U_B, U_C)|01j\rangle &= U_A \otimes U_B \otimes XU_C |01j\rangle
\end{aligned}
$$

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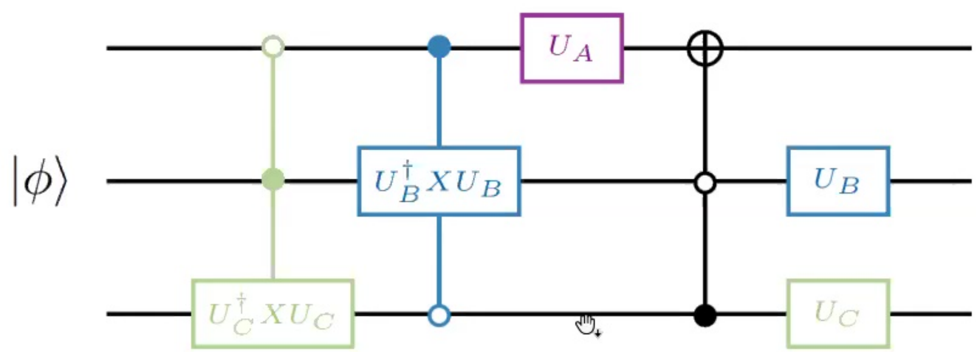

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Thank you!