Title: Composite quantum particles as ideal quantum clocks â€” operational approach to quantum aspects of time

Speakers: Magdalena Zych

Collection: Quantizing Time

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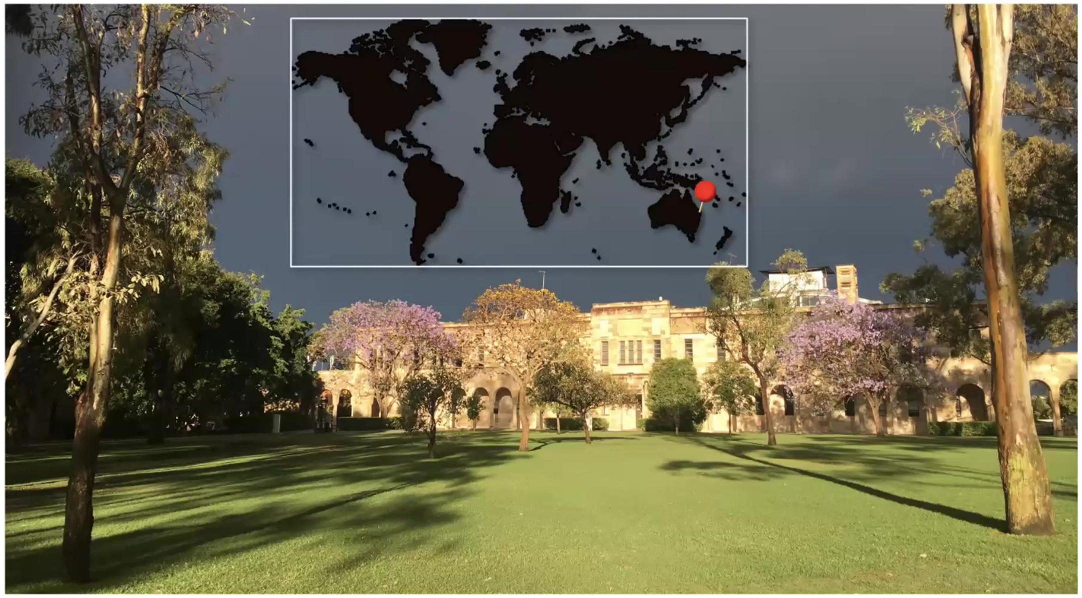
Abstract: In general relativity time requires an operational description, for example, associated with the reading of an idealised clock following some world line. I will show that in quantum physics idealised clocks can be modelled as composite quantum particles and discuss what foundational insights into the notion of time is enabled by this approach. Moreover, since quantum particles do do not follow classical trajectories a question arises to which extent idealised quantum clocks can be associated with semi-classical paths â€” in analogy with quantum particles in Gaussian states being associated with semi-classical trajectories? I will show that for quantum clocks semi-classical propagation is not described by Gaussian but by a new class of quantum states derived from a new uncertainty inequality for configuration space rather than for phase space variables of the quantum clock.

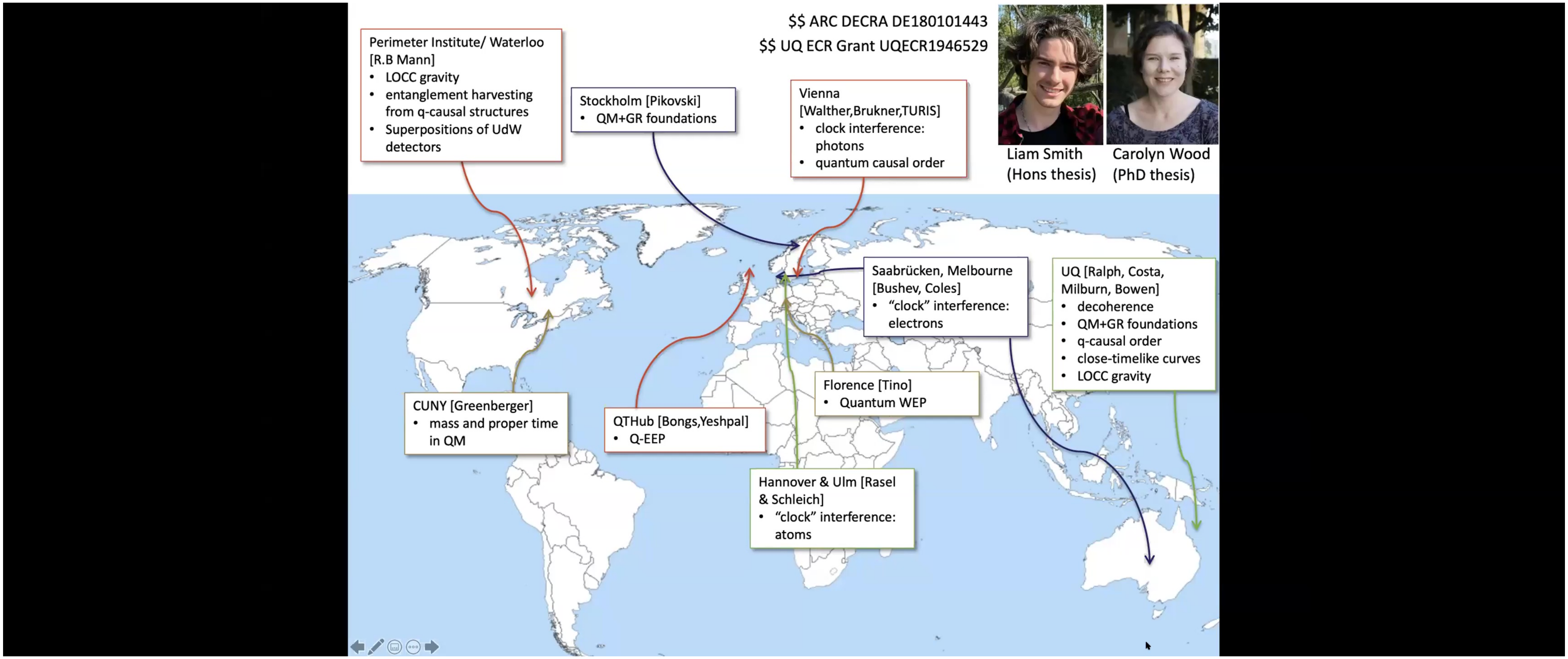
Composite particles as ideal quantum clocks

operational approach to quantum aspects of time

Magdalena Zych, UQ



14 June 2021


$$ ARC DECRA DE180101443
$$ UQ ECR Grant UQECR1946529
Liam Smith (Hons thesis)
Carolyn Wood (PhD thesis)
Perimeter Institute/ Waterloo [R.B Mann]
• LOCC gravity
• entanglement harvesting from q-causal structures
• Superpositions of UdW detectors
Stockholm [Pikovski]
• QM+GR foundations
Vienna [Walther,Brukner,TURIS]
• clock interference: photons
• quantum causal order
Saabrücken, Melbourne [Bushev, Coles]
• "clock" interference: electrons
UQ [Ralph, Costa, Milburn, Bowen]
• decoherence
• QM+GR foundations
• q-causal order
• close-timelike curves
• LOCC gravity
Florence [Tino]
• Quantum WEP
CUNY [Greenberger]
• mass and proper time in QM
QTHub [Bongs,Yeshpal]
• Q-EEP
Hannover & Ulm [Rasel & Schleich]
• "clock" interference: atoms

Motivation

general relativity:

- ☑ Events = operationally def., eg positon & proper time of a clock

quantum theory:

- ☑ clocks = quantum systems

Quantum clocks in curved spacetime

- [?] Quantum effects from proper time
- [?] Time as a new degree of freedom
- [?] Role of gravity in quantum-to-classical transition
- [?] Quantum causal structures/relations?
  Hardy, arxiv:0509120 [gr-qc] vs Penrose, Gen. Rel. Grav. ***28****, 581–600 (1996).*
- [?] Quantum combs/process-matrices as “toy models” of quantum gravity?
  Chiribella et al PRA ***86*** *040301(R) (2012); OCB, Nat. Commun.* ***3****, 1092 (2012)*
- [?] Equivalnce principle in QM?
- [?] Gravitational mass of composite systems? (puzzle even classically!)

Composite particles as idealized clocks

Hamiltonian $\hat{H} = \sqrt{-g_{00}\left(c^2 \hat{P}_i \hat{P}^i + \boxed{\hat{M}^2 c^4}\right)}$ rest energy

Relativistic composite quantum particles

N bosonic fields in curved space-time

$$S = \int d^4x \sqrt{-g} \left(\sum_J g^{\mu\nu} \partial_\mu \varphi_J \partial_\nu \varphi_J + \sum_{J,K} M^2_{JK} c^2 \varphi_J \varphi_K \right)$$

After some grinding: 1-particle Schrödinger equation:

$$\hat{H} = \sqrt{-g_{00} \left(c^2 \hat{P}_i \hat{P}^i + \hat{M}^2 c^4 \right)}$$

MZ *Quantum Systems Under Gravitational Time Dilation* (Springer, 2017),
MZ, Brukner, Nat. Phys 14, 1027–1031 (2018), MZ, Rudnicki, Pikovski, PRD 99, 104029 (2019)

Composite particles as idealized clocks

Hamiltonian $$\hat{H} = \sqrt{-g_{00}\left(c^2\hat{P}_i\hat{P}^i + \boxed{\hat{M}^2c^4}\right)}$$ rest energy

Lagrangian $$L = L_{rest}\boxed{\dot{\tau}} \qquad \dot{\tau} \propto \sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$$ proper time

Routhian $$R = H_{rest}\dot{\tau}$$

Composite particles as idealized clocks

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Routhian $R = \boxed{H_{rest}}\boxed{\dot{\tau}}$ energy & proper time

- Internal evolution measures proper time along the CoM trajectory → **CLOCK**

Composite particles as idealized clocks

Hamiltonian $\hat{H} = \sqrt{-g_{00}\left(c^2 \hat{P}_i \hat{P}^i + \boxed{\hat{M}^2 c^4}\right)}$ rest energy

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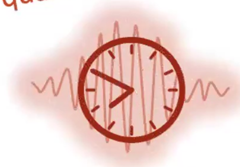
Routhian $R = \boxed{H_{rest}}\boxed{\dot{\tau}}$ energy & proper time

- Internal evolution measures proper time along the CoM trajectory → **CLOCK**
- Both internal and CoM quantized → **QUANTUM**

= quantum clocks!



Composite particles as idealized clocks

Hamiltonian $\hat{H} = \sqrt{-g_{00}\left(c^2 \hat{P}_i \hat{P}^i + \boxed{\hat{M}^2 c^4}\right)}$ rest energy

@ low energies $\hat{M}c^2 + \frac{\hat{p}^2}{2\hat{M}} + \hat{M}\Phi(\hat{x})$

$$\hat{M} = \boxed{m}\hat{\mathbb{1}}_{int} + \frac{\boxed{\hat{H}_{int}}}{c^2}$$

dynamical part => internal Hamiltonian

static part => effective mass parameter

$$H = H_{cm} + H_{int}\left(1 + \frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right)$$

$$H_{cm} = mc^2 + \frac{p^2}{2m} + m\Phi(x)$$

Composite particles as idealized clocks

$$H = H_{cm} + H_{int}\left(1 + \boxed{\frac{\Phi(x)}{c^2}} - \boxed{\frac{p^2}{2m^2c^2}}\right)$$

Gravitational time dilation

Special relativistic time dilation

Interference of clocks

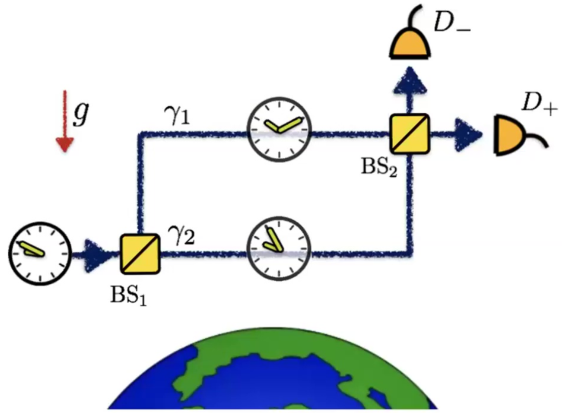
Quantum twin paradox:

Single “twin” in superposition of two paths with different proper times

- *MZ., Costa, Pikovski, Brukner Nature Commun.* ***2****, 505 (2011)*
- *Bushev et al , Single electron relativistic clock interferometer, NJP* ***18*** *093050 (2016)*
- *editorial: JD Franson, Quantum-mechanical twin paradox, NJP* ***18*** *101001 (2016)*
- *Loriani et al “Interference of Clocks: A Quantum Twin Paradox” Science Advances 5: eaax8966, (2019)*

With photons:

- *MZ., Costa, Pikovski, Ralph, Brukner Class. Quantum Grav.* ***29*** *224010 (2012)*



Quantum “twin“ in a superposition of being older-and-younger than itself...

Decoherence from time dilation

Decoherence from time dilation

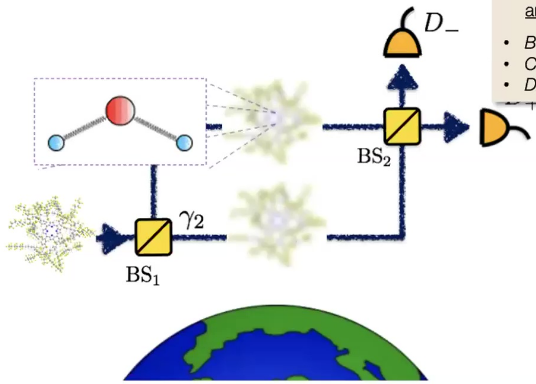
- *Pikovski, M.Z., Costa, Brukner. Nat Phys 11 668—672 (2015)*
- *M.Z., Pikovski, Costa, Brukner JPCS 723 012044 (2016)*
- *Pikovski, M.Z., Costa,Brukner. NJP 19 025011 (2017)*

see also

- *Gooding, Unruh PRD 90(4) 044071 (2014)*
- *Gooding, Unruh, Found.Physics 1-13 (2015)*

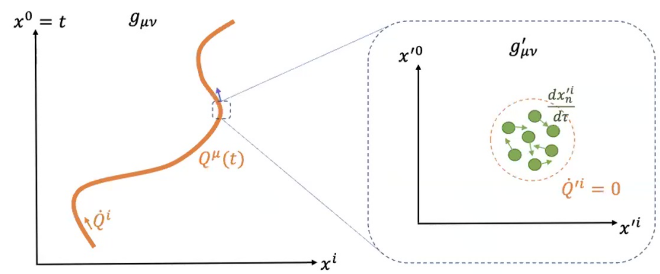
and more

- *Bonder, Okon, Sudarsky PRD 92, 124050 (2015)*
- *Carlesso, Bassi, Physics Letters A 380 (2016)*
- *Diósi J. Phys.: Conf. Ser. 880 012020 (2017)*



Gravitational mass of a composite system?

M.Z., Rudnicki, Pikovski
"Gravitational mass of composite systems",
PRD 99, 104029 (2019)



- Gravity couples to 3K+2T → universal violation of EP?!
- EP holds only "on average" (virial theorem)?!

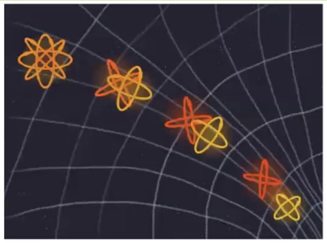
A. Eddington, G. Clark, *Proc. R. Soc. Lond.* A166, 465 (1938)
K. Nordtvedt, *Int. J. Theor. Phys.* 3, 133-139 (1970)
E. Fishbach et al., *Phys. Rev. D.* 23, 2157-2180 (1981)
S. Carlip, *Am. J. Phys.* 66, 409-413 (1998)
A. G. Lebed, *Cent. Eur. J. Phys.* 11, 969–976 (2013)

our resolution:

- "3K+2T" coupling : coordinate artefact
- wrong def. of gravitational mass!
- correct definition → explicit validity of EP

Einstein Equivalence Principle

MZ, Brukner, Nat Phys 14 1027–1031 (2018)



What does it mean to test Einstein Equivalence Principle in QM?

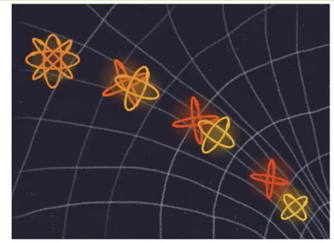
$$\hat{H} = \hat{M}_r c^2 + \frac{\hat{p}^2}{2\hat{M}_i} + \hat{M}_g \Phi(\hat{x})$$

$$\hat{M} = m\hat{\mathbb{1}}_{int} + \frac{\hat{H}_{int}}{c^2}$$

Quantum EEP = Classical EEP: Only when mass-energy operators commute

Einstein Equivalence Principle

MZ, Brukner, Nat Phys 14 1027–1031 (2018)



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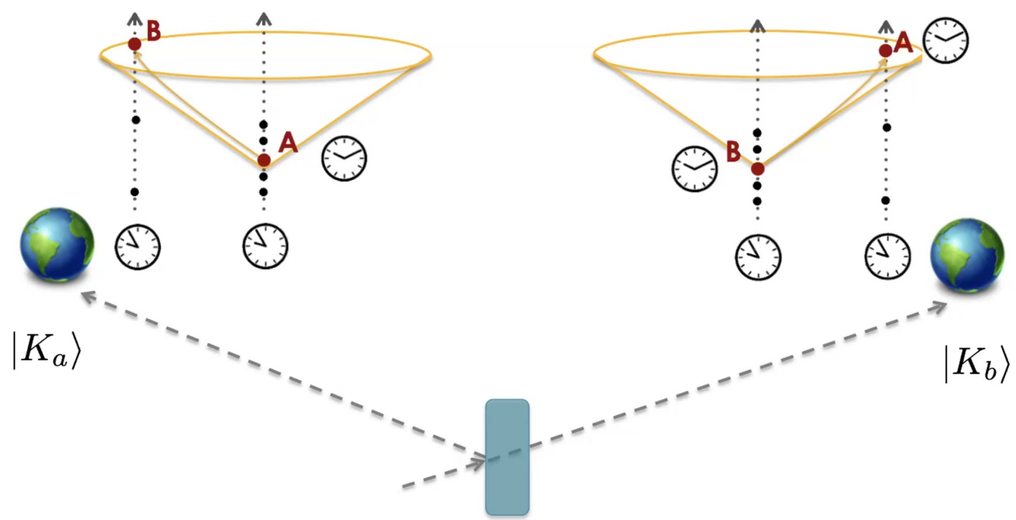
	CLASSICAL		QUANTUM
# parameters to test for N-level system	2N-1	<	$2N^2$-1

Quantum EEP = Classical EEP: Only when mass-energy operators commute

Testing EEP in QM requires more & different experiments

Quantum causal relations

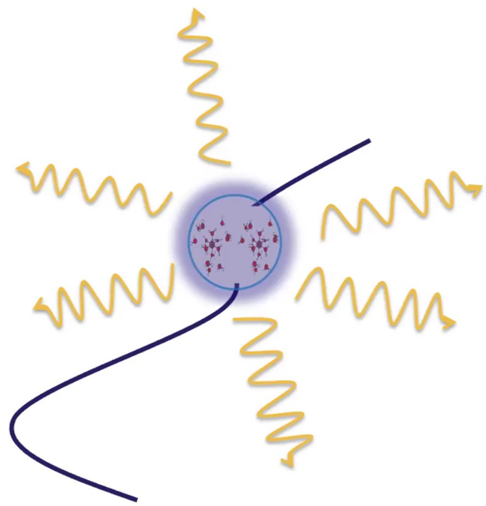
MZ, Costa, Pikovski, Brukner, *Nature Commun.* **10** 1, 3772 (2019)



- **Quantum gravitational realization of a quantum SWITCH**
- **Time-like events with no local classical time order**

Does a decaying atom experience friction?

- *Sonnleitner, Trautmann, Barnett, "Will a decaying atom feel a friction force?", PRL 118, 053601 (2017).*
- *Sonnleitner, Barnett, "Mass-energy and anomalous friction in quantum optics," PRA 98, 042106 (2018).*



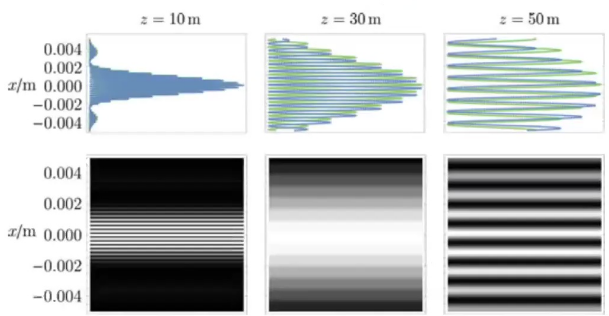
- emission of radiation allegedly causes "friction"
- paradox resolved by dynamical mass-energy:

→ emitted energy = decrease in mass, *not* in velocity

How does interference pattern fall?

Dispersion from mass-energy equiv.

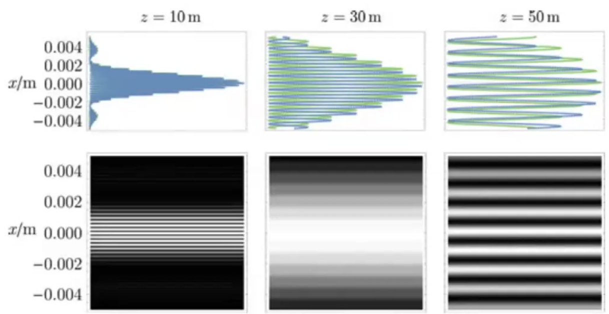
- Pang, Khalili, Chen "On universal decoherence under gravity: a perspective through the Equivalence Principle". PRL 117, 09040 (2016)
- Orlando, Pollock, Modi, How Does Interference Fall?, Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology (2017)



Orlando Pollock Modi, Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology (2017)

Quantum clocks with semiclassical trajectories?

initial state: Internal energy ⊗ Gaussian CoM



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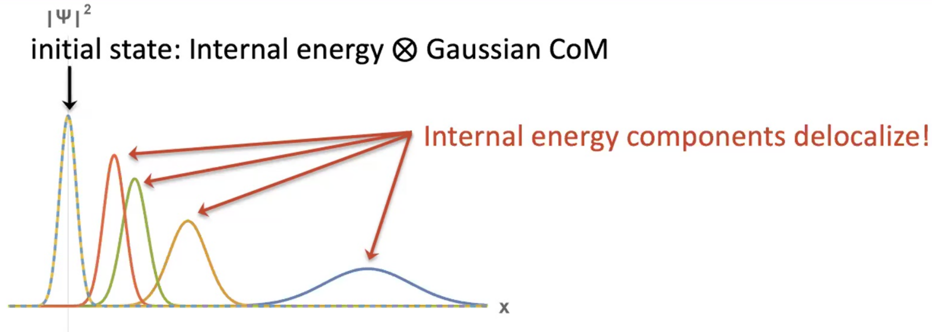
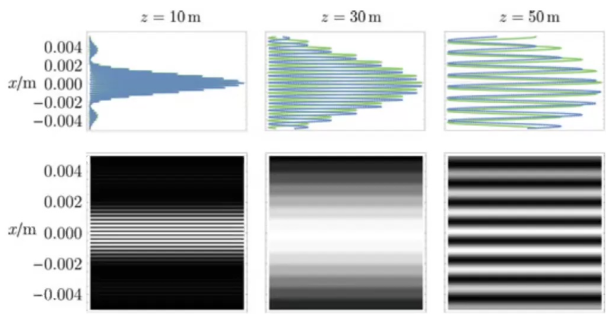


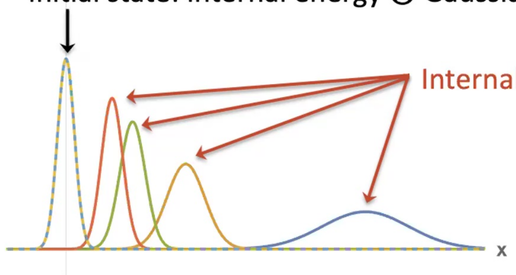


Orlando Pollock Modi, Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology (2017)

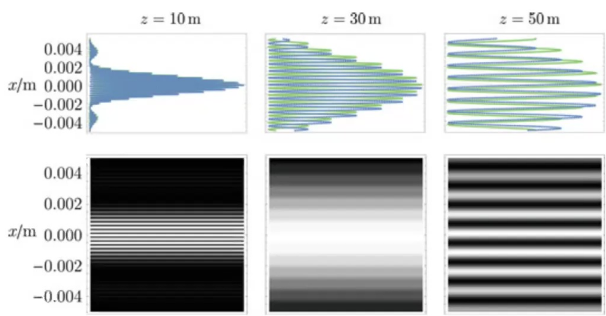
Quantum clocks with semiclassical trajectories?

$|\Psi|^2$

initial state: Internal energy ⊗ Gaussian CoM



Internal energy components delocalize!



Orlando Pollock Modi, Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology (2017)

Is this fundamental?

YES

- No clocks in QM?
- Atoms ≠ clocks?
- Detrimental to experiments

Configuration space (x-v) uncertainty principle

Gaussian: $[\hat{x}, \hat{p}] = i\hbar \quad \Delta x \Delta p \geq \hbar/2$ min. uncertainty in phase space

We need minimum uncertainty in configuration space

Velocity operator: $\hat{v} := -\frac{i}{\hbar}[\hat{x}, \hat{H}] = \frac{\hat{p}c^2}{\sqrt{\hat{p}^2c^2 + \hat{M}^2c^4}} \approx \hat{p}\hat{M}^{-1}$

Uncertainty $(\Delta x)^2(\Delta v)^2 - (\Delta xv)^2 \geq \frac{1}{4}\left|\langle[\hat{x}, \hat{v}]\rangle\right|^2$

Min Uncertainty State (MUS): Eigenvalue eqn.

$$(\mu\hat{A} + \nu\hat{A}^\dagger)|\Psi\rangle = z|\Psi\rangle \qquad \mu, \nu, z \in \mathbb{C}$$

$$|\mu|^2 - |\nu|^2 = 1$$

$$\hat{A} = \hat{x} + i\hat{v}/\Omega$$

x-v Minimum Uncertainty States

Min Uncertainty State (MUS): $|\Psi\rangle = \sum_m c_m |\psi_m\rangle |m\rangle$

$$\psi_m(p) = \frac{1}{\sqrt{N_m}} e^{-\frac{izp}{\hbar(\mu+\nu)}} e^{-\frac{\sqrt{p^2c^2+m^2c^4}(\mu-\nu)}{\hbar\Omega(\mu+\nu)}} e^{\frac{mc^2(\mu-\nu)}{\hbar\Omega(\mu+\nu)}}$$

Semi-classically propagating composite quantum particles

1) Minimise uncertainty inequality for position and velocity
2) CoM and internal states entangled
   (inital tensor product will entangle CoM and internal states even more!)

x-v Minimum Uncertainty States

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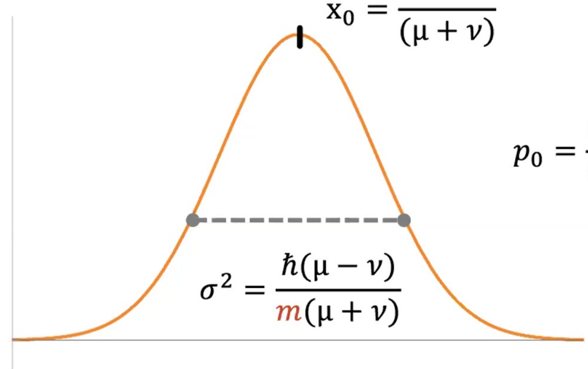
Semi-classically propagating composite quantum particles

1) Minimise uncertainty inequality for position and velocity
2) CoM and internal states entangled
   (inital tensor product will entangle CoM and internal states even more!)
3) Fully characterised properties and propagation including relativistic regime!

x-v Minimum Uncertainty States – low energies

Min Uncertainty State (MUS): $|\Psi\rangle = \sum_m c_m |\psi_m\rangle |m\rangle$

$$\psi_m(x) \approx \frac{1}{\sqrt{\mathcal{N}_m}} e^{\frac{m}{2\hbar}\left[-\frac{(\mu+\nu)}{(\mu-\nu)}\left(x-\frac{z}{(\mu+\nu)}\right)^2 + i\Im\left[\frac{z^2}{(\mu+\nu)(\mu-\nu)}\right]\right]}$$

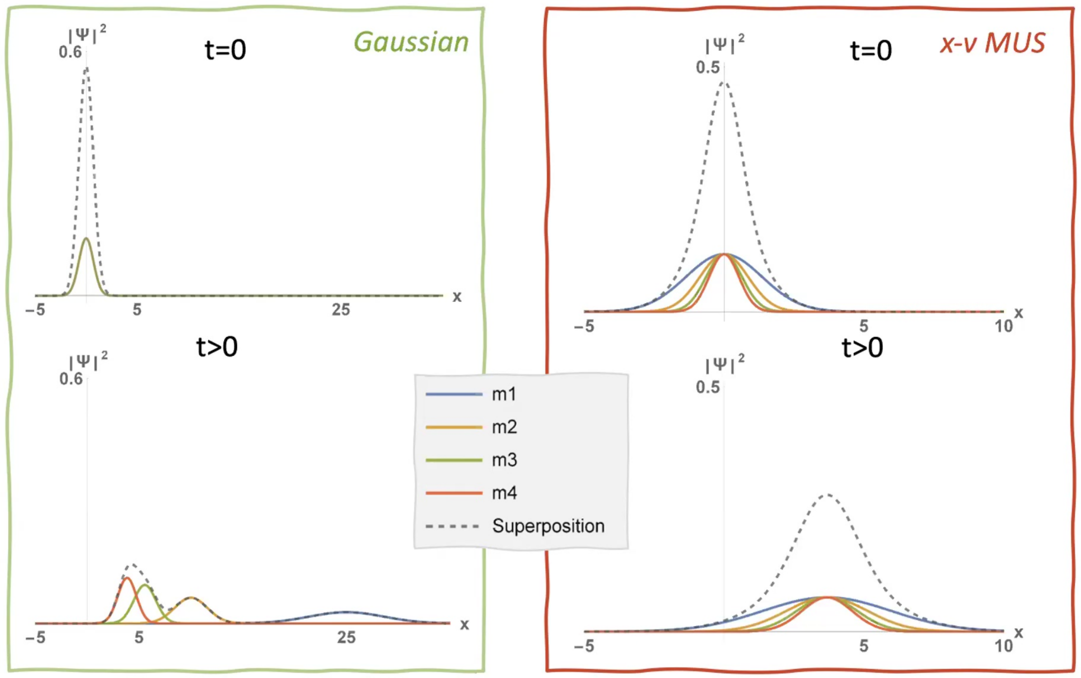


$$x_0 = \frac{z}{(\mu+\nu)}$$

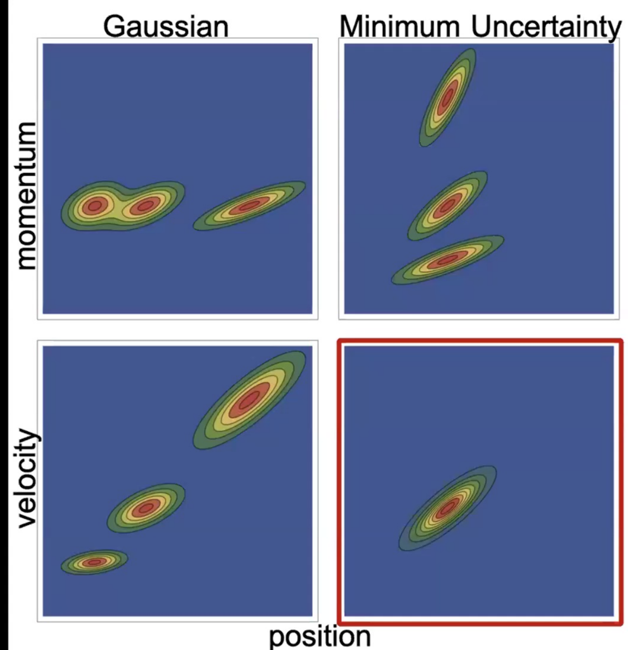
$$p_0 = \frac{m\,\Im[z]}{(\mu-\nu)}$$

$$\sigma^2 = \frac{\hbar(\mu-\nu)}{m(\mu+\nu)}$$

x-v Minimum Uncertainty States – low energies



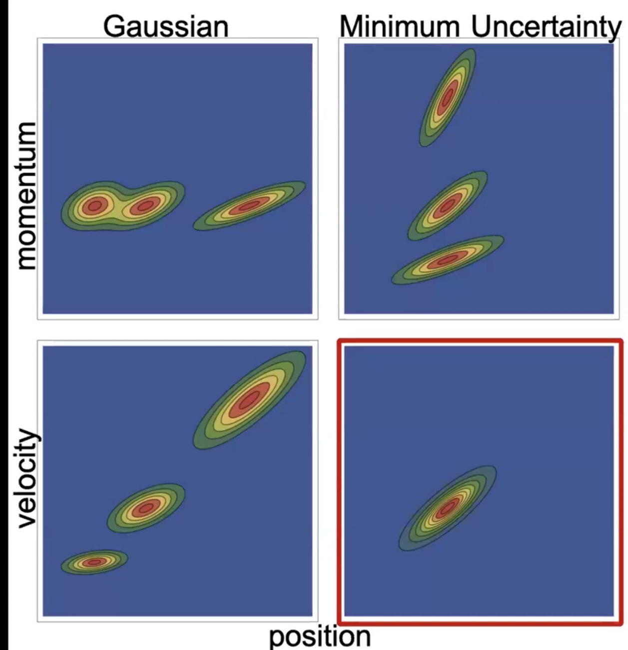
MUS properties: covariance



$$\psi_{MUS}(x,t) = \frac{e^{-\left[\frac{m\Omega}{2\hbar}\frac{e^{-2r}(x+vt)^2}{(1+e^{-4r}t^2\Omega^2)}+\frac{r}{2}+\frac{imc^2t}{\hbar}\mathcal{F}(\Omega,r,v,t)\right]}}{\sqrt[4]{\frac{\pi\hbar}{m\Omega}}\sqrt{1+ie^{-2r}t\Omega}}$$

For highly localised state (large r) $\approx 1 + \frac{vx}{c^2 t} + \frac{v^2}{2c^2}$

MUS properties: covariance



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For highly localised state (large r)

$$\approx 1+\frac{vx}{c^2t}+\frac{v^2}{2c^2}$$

$$\frac{t+\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz transformation to $O(1/c^2)$

MUS properties: localization

Quantifying localization – position variance

p_0=0 and $\Delta x^2_{MUS}(0) = \Delta^2_G(0)$

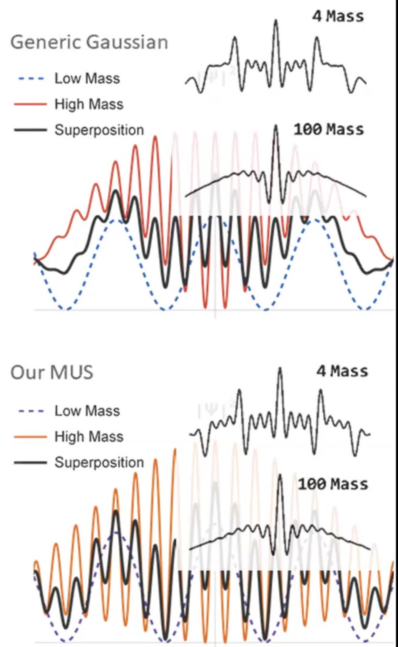
- Gaussian state: $\Delta x^2_G(t) = \Delta x^2_G(0)\left(1 + \frac{t^2\hbar^2}{m^2\sigma_0^2}\right)$
- Our MUS: $\Delta x^2_{MUS}(t) = \Delta^2_{MUS}(0)\left(1 + e^{-4r}t^2\right)$

$$\boxed{\Delta x^2_{MUS}(t) \leq \Delta x^2_G(t)}$$

Delocalization ⇔ time-dilation decoherence?

Is mass delocalization the physical reason for time dilation (gravitational) decoherence ?

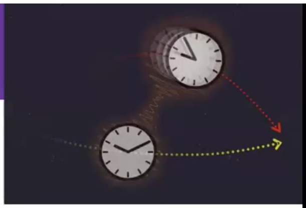
No! For MUS interference suppressed as predicted from time dilation



Removing differing velocities
(double-slit exp. with screen moving towards the particle)
→ Gaussian and MUS interference still differs!
→ MUS gives higher contrast than the Gaussian

Two different decoherence effects:
→ from time dilation
→ from CoM delocalisation

Conclusion & Outlook



Quantum clocks + relativity → new effects & experiments

Idealised quantum clocks:

- Phase space classicality ≠ spacetime classicality!
- New class of quantum states
- Minimize x-v uncertainty principle
- Covariant under Lorentz transformations
- Solve the mystery of delocalization-decoherence vs time-dilation decoherence

Further applications

- Fundamental limit to time associated with clocks as quantum particles?
- Time more precisely measured by MUS than Gaussian?
- Neutrino oscillations and propagation
- UdW: must change mass when absorbing/emitting radiation (to avoid spurious friction from *PRL 118, 053601 (2017)*)

THANK YOU!