Title: Time symmetry in operational theories

Speakers: Lucien Hardy

Series: Quantum Foundations

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Abstract: The standard operational probabilistic framework (within which Quantum Theory can be formulated) is time asymmetric. This is clear because the conditions on allowed operations include a time asymmetric causality condition. This causality condition enforces that future choices do not influence past events. To formulate operational theories in a time symmetric way I modify the basic notion of an operation allowing classical incomes as well as classical outcomes. I provide a new time symmetric causality condition which I call double causality. I apply these ideas to Quantum Theory proving, along the way, a time symmetric version of the Stinespring extension theorem using double causality. I also propose the idea of a conditional frame of reference. We can transform from the time symmetric frame of reference to a forward or a backward frame of reference. This talk is based on arXiv:2104.00071.

Zoom Link: https://pitp.zoom.us/j/96566316311?pwd=Q1dMM0lWRUdMQWFpZ3NUZ0g5TExSQT09

Time Symmetry in Operational Theories

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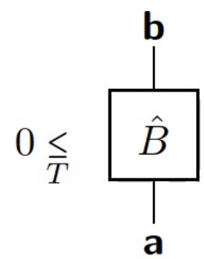
Talk based on arXiv:2104.00071

Standard Operational Quantum Theory

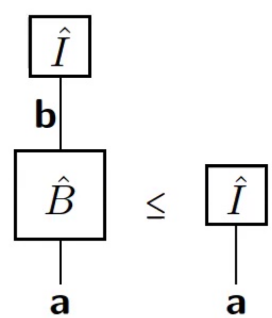
In standard operational quantum theory operations are

- Completely positive.



time symmetric

- Trace non-increasing (Pavia causality).



time asymmetric

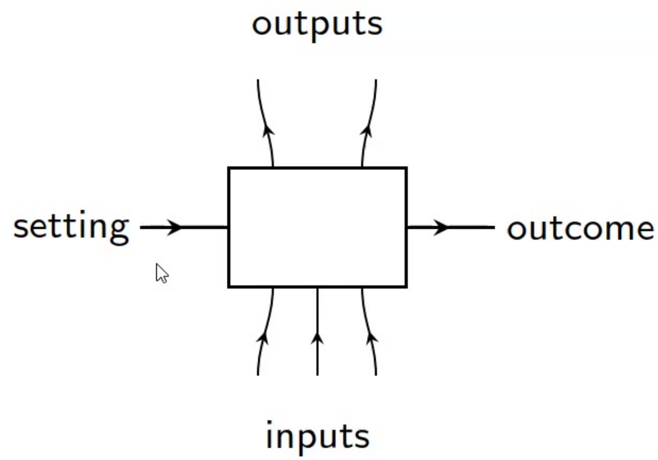
This is odd because. . .

1. Abstract probability theory concerns calculation of such objects as prob$(x|y)$ and, as such, knows nothing of time.
2. Quantum Theory, at the level of the Schrödinger equation, is time symmetric.
3. The Quantum Theory of measurement can be treated very simply without reference to the second law (the von Neumann model for example).

We will see how to fix this.

Operations and Circuits

Standard notion of an operation

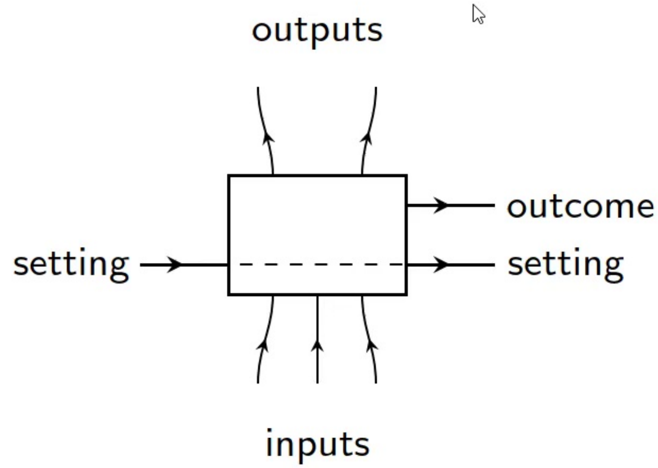


Settings are controlled by knobs, etc. These are classical.

Outcomes correspond to pointer readings, etc. These are classical.

Inputs and outputs are for the physical systems (e.g. Quantum Systems).

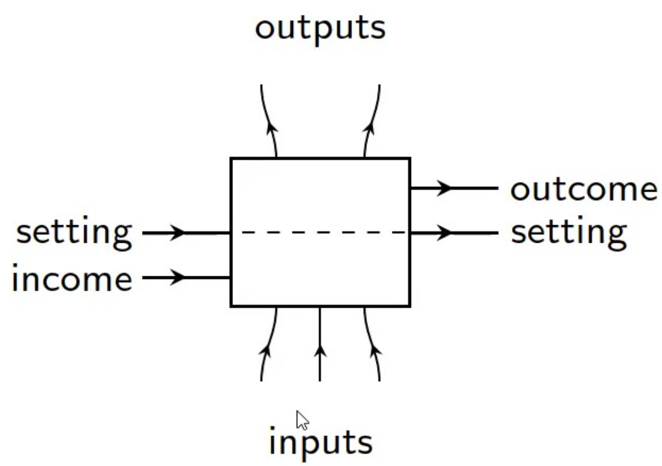
Standard notion of an operation



Settings are available before and after.

Outcomes are available afterwards but not before.

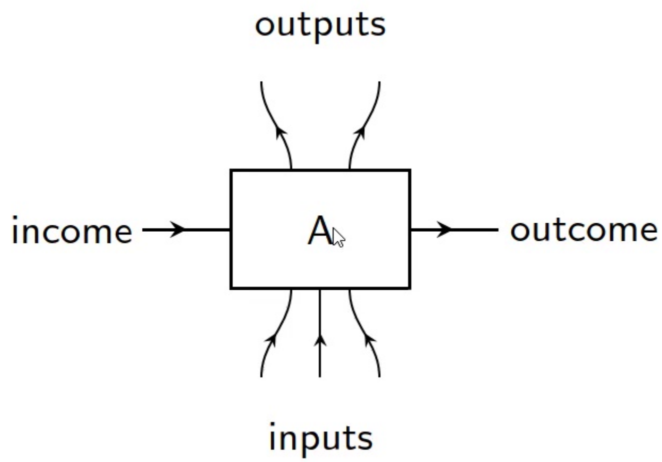
Time symmetric operation



Now have incomes as well as outcomes.

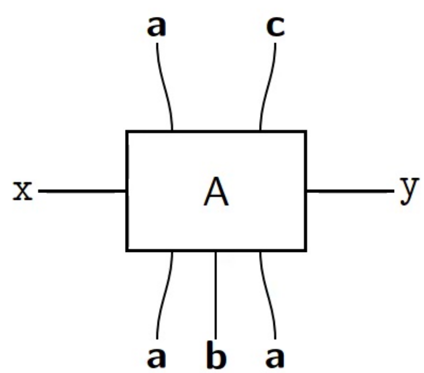
Incomes are available before but not after. These are classical.

Time symmetric operation



Since we will be primarily concerned with incomes and outcomes we will take the setting arrow to be implicit.

Time symmetric operation



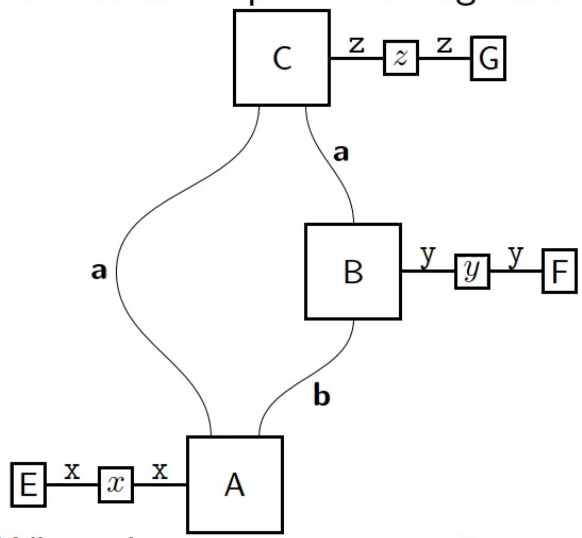
We use x, y, etc. to denote income/outcome (pointer) types. They are classical and take values

$$x = 1, 2, \dots N_{\mathrm{x}} \qquad y = 1, 2, \dots N_{\mathrm{y}}$$

We use **a**, **b**, etc. to denote the physical system types. Could be quantum systems.

Circuits

We can wire operations together



When there are no open wires we have a *circuit*.

Interested in the joint probability $p(x, y, z)$.

This is joint probability $p(\text{incomes}, \text{outcomes})$.

Points of view

Consider the following points of view

- Time symmetric point of view concerns $p(\text{incomes}, \text{outcomes})$.
- Time forward point of view concerns $p(\text{outcomes}|\text{incomes})$.
- Time backward point of view concerns $p(\text{incomes}|\text{outcomes})$.

The standard operational framework is in the time forward point of view. We wish to work in the time symmetric point of view. Given answers in one point of view, can convert to others.

In the time symmetric point of view, we demand that we get the same answer for

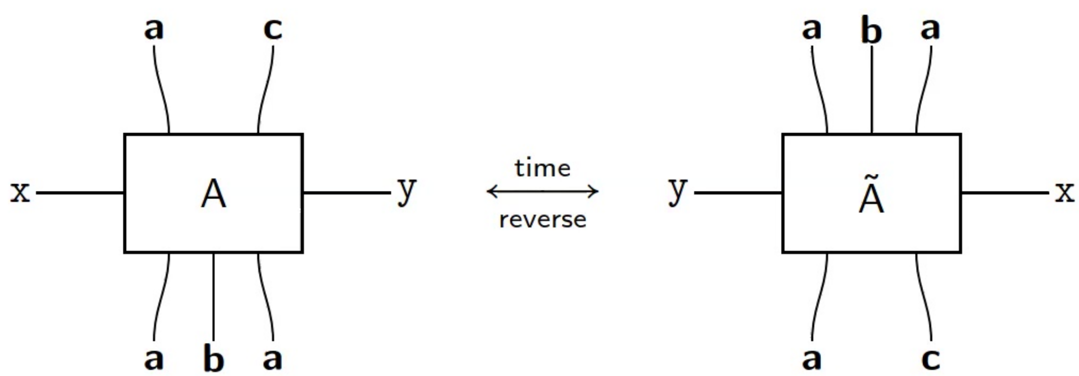
$$p(\text{incomes}, \text{outcomes})$$

whether we do the calculation forward or backwards in time.

Definition of time symmetric theory

We will say an operational theory is time symmetric if,

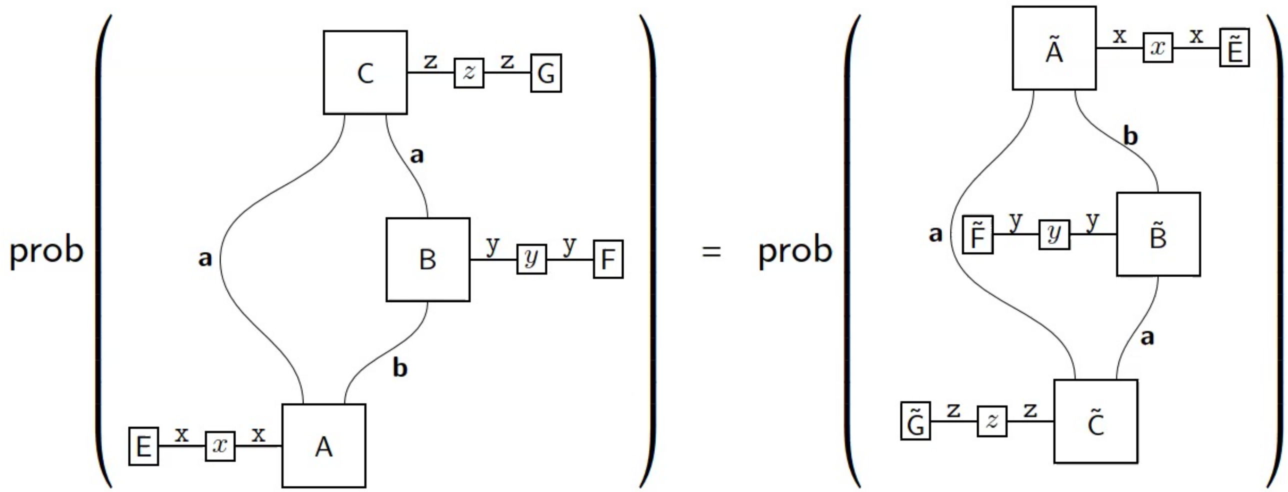
1. Every allowed operation, A, can be mapped to an allowed "time reversed" operation, $\tilde{A}$. For example,



(1)

2. Given any circuit, E, we can obtain the time reversed circuit, $\tilde{E}$, by inverting the graph replacing operations with their time reversed counterparts. We require that $\text{prob}(E) = \text{prob}(\tilde{E})$.

For example,



The circuit on the right is the time reverse of the circuit on the left and has the same probability.

This means that, for every process described in the forward direction, there is a corresponding process described in the backwards direction.

Contents of talk going forward

1. Double properties for a simple classical situation.
2. Physicality:
    - complete positivity
    - double causality
3. Quantum Theory:
    - operator tensors and physicality
    - gauge parameters
    - time symmetric extension theorem
    - Time reverse of teleportation

Previous work

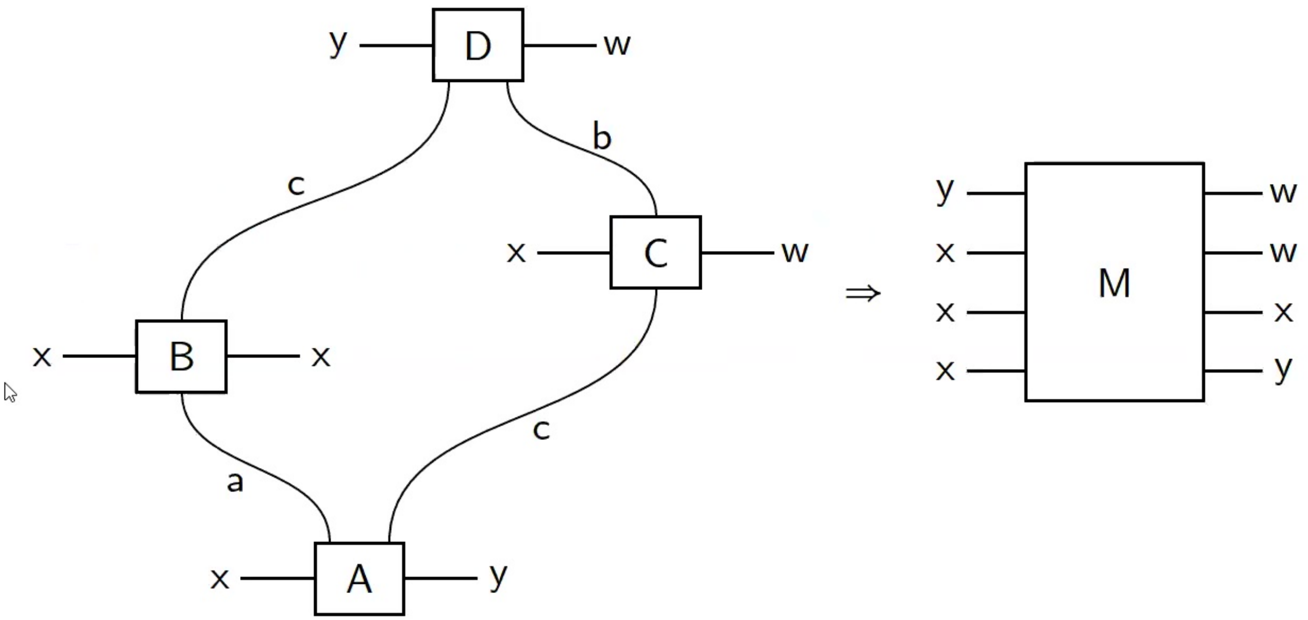
On time symmetry - small sample.

1. Aharonov, Bergmann, Lebowitz 1964.
2. Chiribella, D'Ariano, Perinoti 2010 (the Pavia causality condition).
3. Operational formulation of time reversal in quantum theory. Oreshkov, Cerf. arxiv:1507.07745
4. Quantum Information and the arrow of time. Biagio, Dona, Rovelli. arXiv:2010.05734
5. Quantum operations with indefinite time direction. Chiribella and Liu, arXiv:2012.03859
6. Symmetries of quantum evolutions, Chiribella. Aurell, Życzkowski arXiv:2101.04962

Also relevant to this work are the process theory and GPT approaches. My paper makes much use of the pictorial approach of Coecke, Selinger, etc (see book by Coecke and Kissinger).

Double properties for simple classical situation

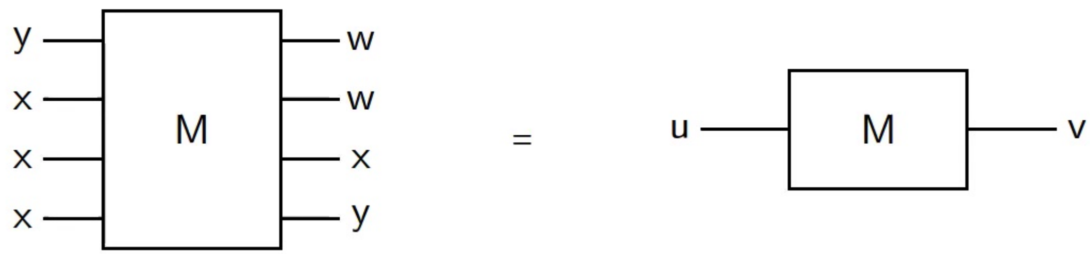
Simple classical situation



We have collapsed out the physical systems.

M is a classical box - though with physics (maybe quantum) hidden inside.

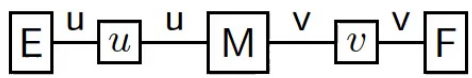
We can write this as



where u = xxxy and v = yxww.

This simple classical situation holds the key to understanding time symmetry in operational theories.

Double distribution uniqueness



Forwards gives

$$p_{\text{forward}}(u, v) = p_{\text{E}}(u) p_{\text{M}}(v|u)$$

Backwards gives

$$p_{\text{backward}}(u, v) = p_{\text{F}}(v) p_{\text{M}}(u|v)$$

These probabilities must be equal

$$p_{\text{E}}(u) p_{\text{M}}(v|u) = p_{\text{F}}(v) p_{\text{M}}(u|v)$$

Double distribution uniqueness

We have

$$p_{\mathsf{E}}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}}(v)p_{\mathsf{M}}(u|v)$$

$$p_{\mathsf{E}'}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}'}(v)p_{\mathsf{M}}(u|v)$$

Dividing gives

$$\frac{p_{\mathsf{E}}(u)}{p_{\mathsf{E}'}(u)} = \frac{p_{\mathsf{F}}(v)}{p_{\mathsf{F}'}(v)}$$

Since probabilities add to 1, we obtain

$$p_{\mathsf{E}}(u) = p_{\mathsf{E}'}(u) \quad \text{and} \quad p_{\mathsf{F}}(v) = p_{\mathsf{F}'}(v)$$

Double distribution uniqueness

We have

$$p_{\mathsf{E}}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}}(v)p_{\mathsf{M}}(u|v)$$

$$p_{\mathsf{E}'}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}'}(v)p_{\mathsf{M}}(u|v)$$

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$$\frac{p_{\mathsf{E}}(u)}{p_{\mathsf{E}'}(u)} = \frac{p_{\mathsf{F}}(v)}{p_{\mathsf{F}'}(v)}$$

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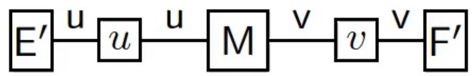
This is true unless $p_{\mathsf{M}}(u|v) = 0$ or $p_{\mathsf{M}}(v|u) = 0$

Double distribution uniqueness



Forwards gives

$$p'_{\mathsf{forward}}(u,v) = p_{\mathsf{E}'}(u)p_{\mathsf{M}}(v|u)$$

Backwards gives

$$p'_{\mathsf{backward}}(u,v) = p_{\mathsf{F}'}(v)p_{\mathsf{M}}(u|v)$$

By the circuit probability assumption they must be equal

$$p_{\mathsf{E}'}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}'}(v)p_{\mathsf{M}}(u|v)$$

Double distribution uniqueness

We have

$$p_{\mathsf{E}}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}}(v)p_{\mathsf{M}}(u|v)$$

$$p_{\mathsf{E}'}(u)p_{\mathsf{M}}(v|u) = p_{\mathsf{F}'}(v)p_{\mathsf{M}}(u|v)$$

Dividing gives

$$\frac{p_{\mathsf{E}}(u)}{p_{\mathsf{E}'}(u)} = \frac{p_{\mathsf{F}}(v)}{p_{\mathsf{F}'}(v)}$$

Since probabilities add to 1, we obtain

$$p_{\mathsf{E}}(u) = p_{\mathsf{E}'}(u) \quad \text{and} \quad p_{\mathsf{F}}(v) = p_{\mathsf{F}'}(v)$$

This is true unless $p_{\mathsf{M}}(u|v) = 0$ or $p_{\mathsf{M}}(v|u) = 0$

Double flatness

One way to achieve double distribution uniqueness is

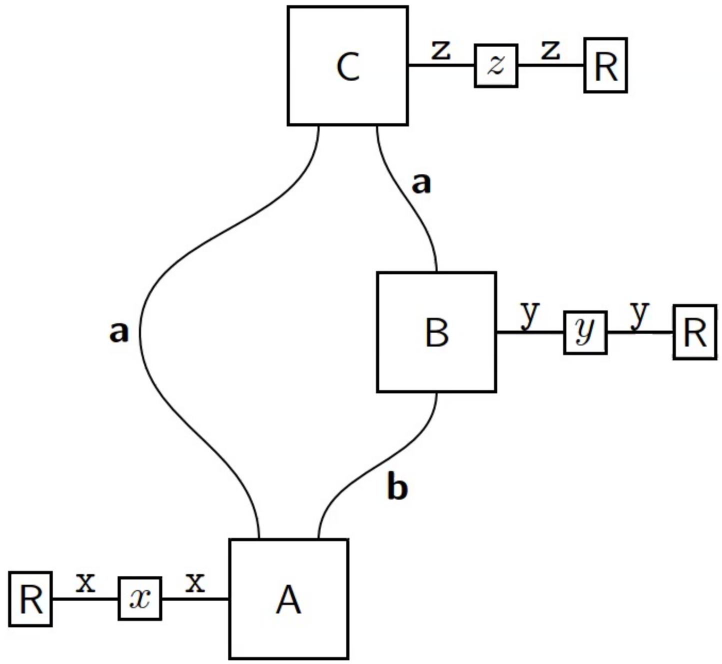
- consider only flat distributions, and

- impose *double flatness*.

Double flatness holds in Quantum Theory.

Double flatness is deeply related to double causality.

The first point means that, in the time symmetric point of view, we consider circuits of the form



This is necessary so that we get the same probability whether we do the calculation forward or backward in time.

Double summation

Forward summation

$$\sum_{u} \mathsf{prob}\left(\boxed{\mathsf{R}} \overset{\mathtt{u}}{\text{---}} \boxed{u} \overset{\mathtt{u}}{\text{------}} \boxed{\mathsf{M}} \overset{\mathtt{v}}{\text{------}} \boxed{v} \overset{\mathtt{v}}{\text{---}} \boxed{\mathsf{R}} \right) = \frac{1}{N_{\mathsf{v}}} \qquad (2)$$

Backward summation

$$\sum_{v} \mathsf{prob}\left(\boxed{\mathsf{R}} \overset{\mathtt{u}}{\text{---}} \boxed{u} \overset{\mathtt{u}}{\text{------}} \boxed{\mathsf{M}} \overset{\mathtt{v}}{\text{------}} \boxed{v} \overset{\mathtt{v}}{\text{---}} \boxed{\mathsf{R}} \right) = \frac{1}{N_{\mathsf{u}}} \qquad (3)$$

Double summation is equivalent to double flatness.

Physicality

We wish to impose *physicality conditions* which guarantee that

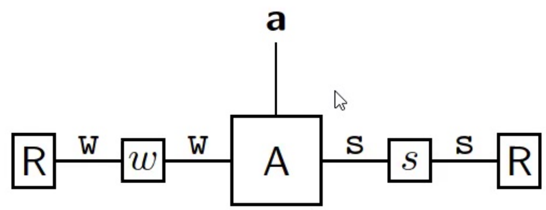
- Complete positivity that probabilities for circuits are non-negative.
- Double summation (equivalently double flatness). Motivated by double distribution uniqueness and causality considerations.

We have

- Complete positivity $\Longleftarrow$ T-positivity
- Double summation $\Longleftarrow$ double causality

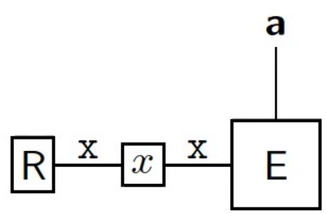
Forward spanning property

A theory has the *forward spanning property* if an arbitrary system preparation



(4)

is equivalent to a weighted sum of income only preparations

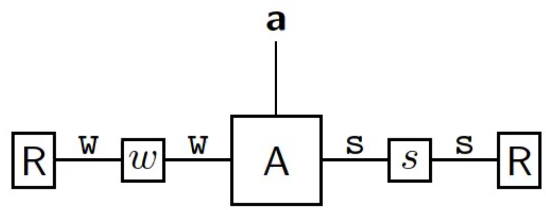


(5)

where the weights are real numbers (can be negative).

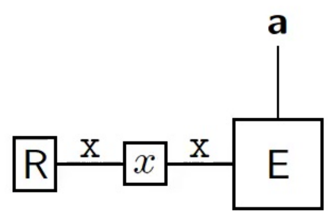
Forward purity property

A theory has the *forward purity property* if an arbitrary system preparation



(6)

is equivalent to a weighted sum of income only preparations

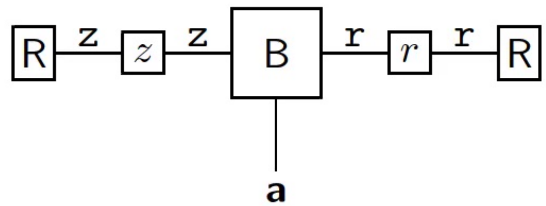


(7)

where the weights are non-negative real numbers.

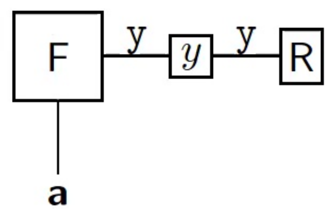
Backward spanning property

A theory has the *backward spanning property* if an arbitrary system result



(8)

is equivalent to a weighted sum of outcome only results



(9)

where the weights are real numbers (can be negative).

A theory has the *double spanning property* if it has both the forward and backward spanning properties. We use it to obtain double causality.

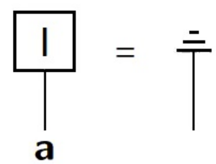
A theory has the *double purity property* if it has both the forward and backward purity properties. We use it to obtain T-positivity.

$$\text{double purity} \Longrightarrow \text{double spanning}$$

Quantum Theory has these double properties.

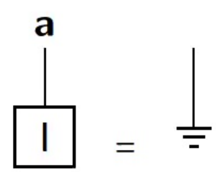
Ignore preparation and results

An operation having only an input (no output, no incomes, no outcomes).



(13)

is called an ignore result.
Similarly, an operation having only an output

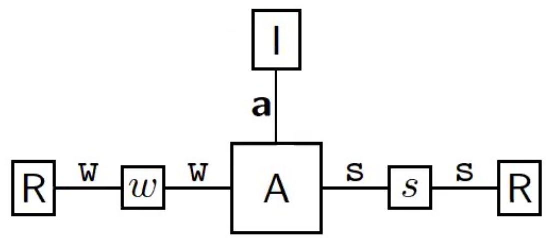


(14)

is called an ignore preparation.
Can prove the ignore preparation and result are unique from double spanning.

Proof that ignore preparations is unique

The most general circuit containing the ignore result is where we send in a general system preparation.



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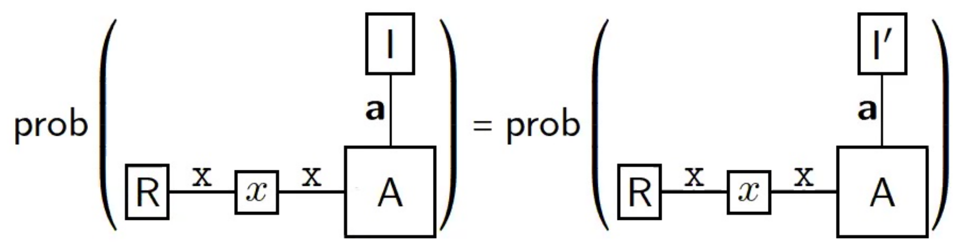
By double spanning have equivalence of I and I′ if



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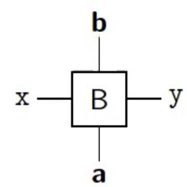
This is true by backwards flatness.

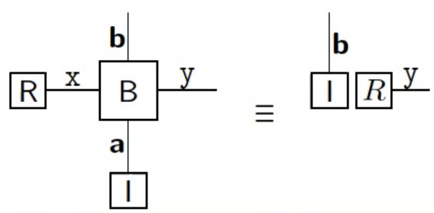
Double causality

Double causality theorem. It follows from double flatness and double spanning that a general operation,

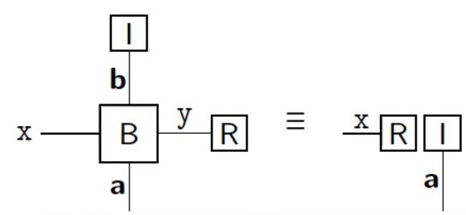


satisfies the following two conditions

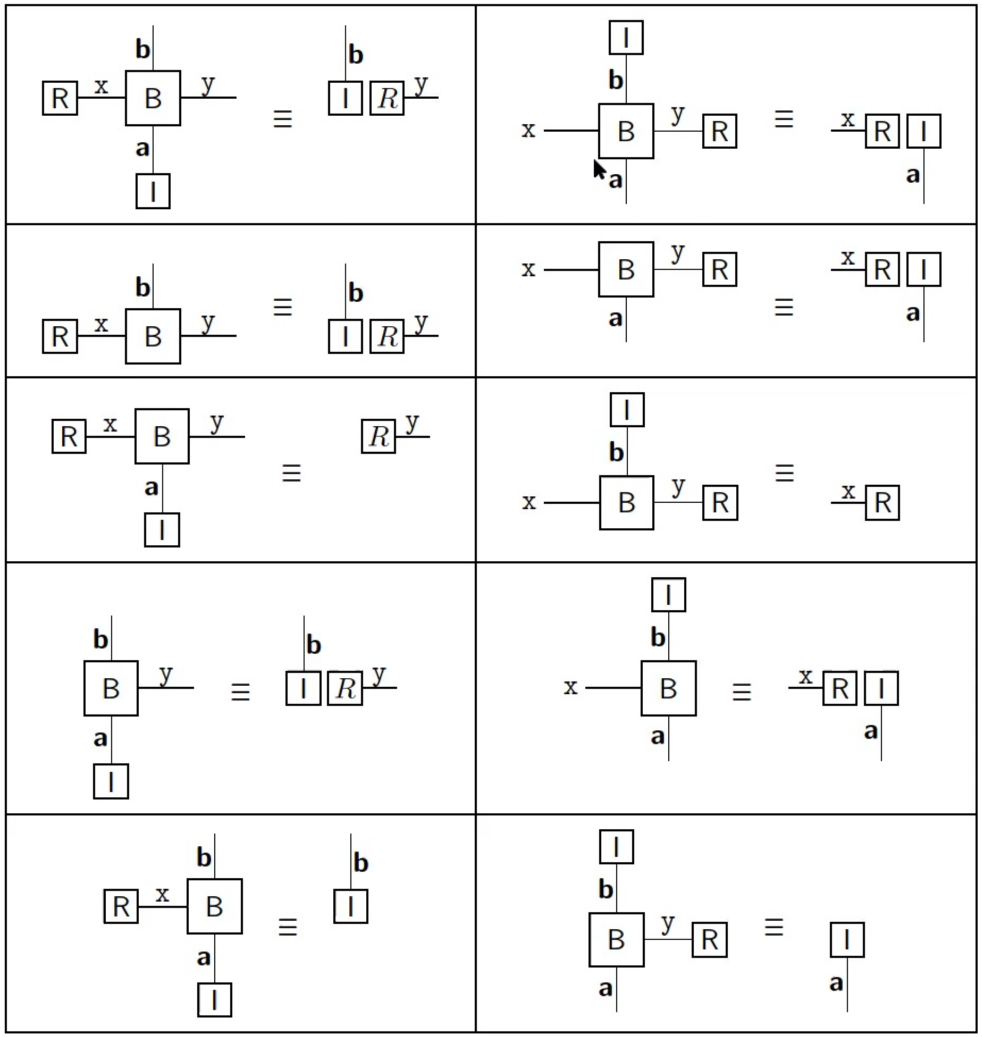
Forward causality

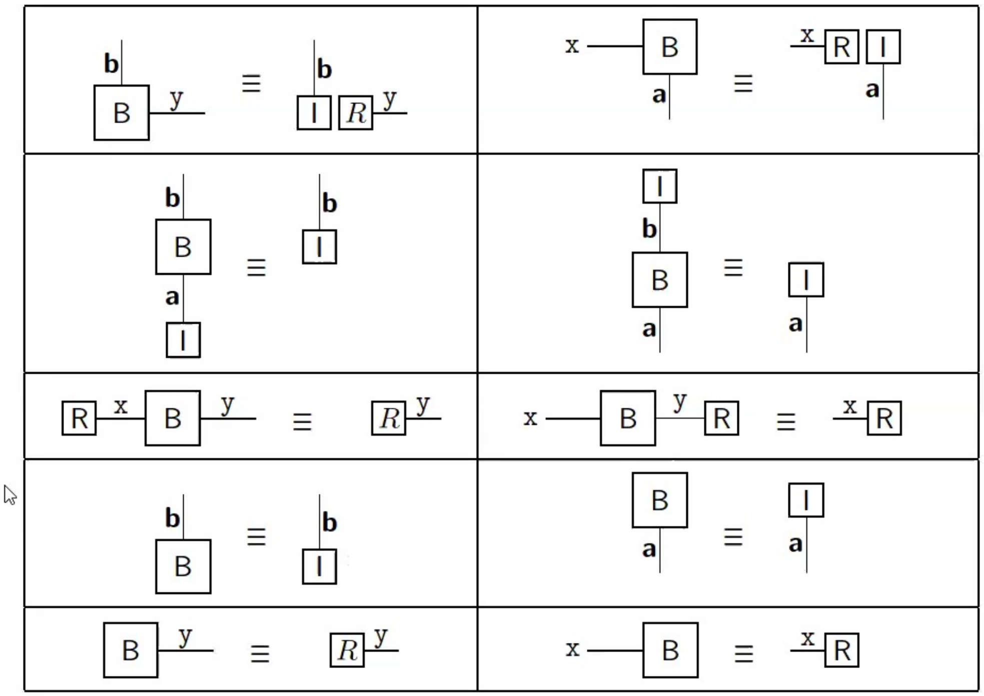


Backwards causality

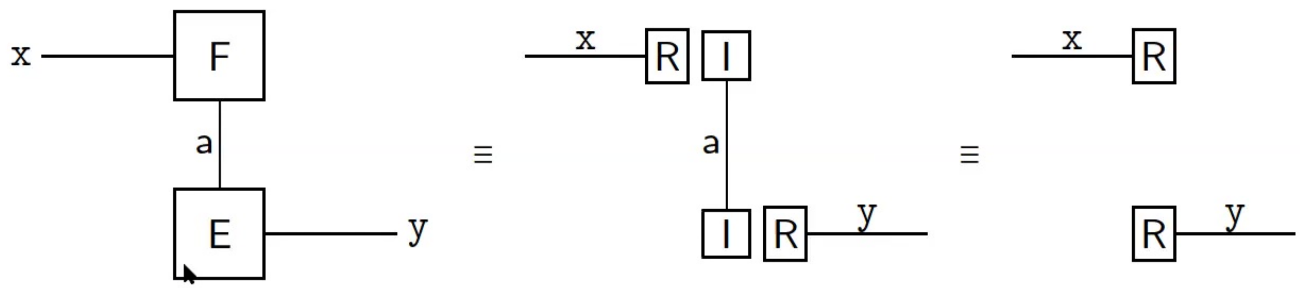


and, furthermore, if these conditions hold for all the operations comprising a circuit, then the double summation conditions hold for that circuit.

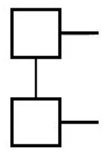
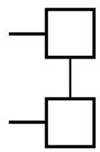
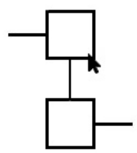
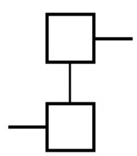




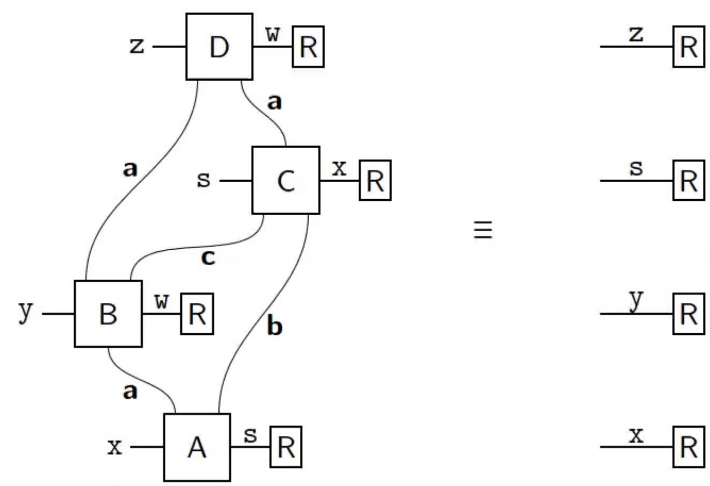
Double Z flatness



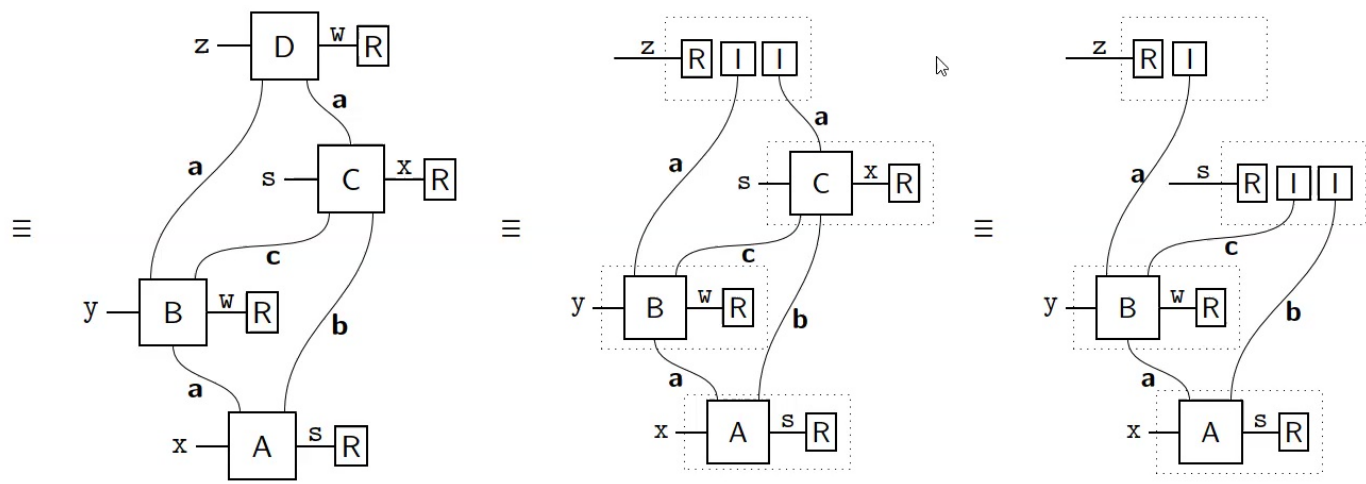
"signalling" possibilities

	no	By application of forward flatness.
	no	By application of backward flatness.
	no	By application of double Z flatness.
	yes	Double maximality is example.

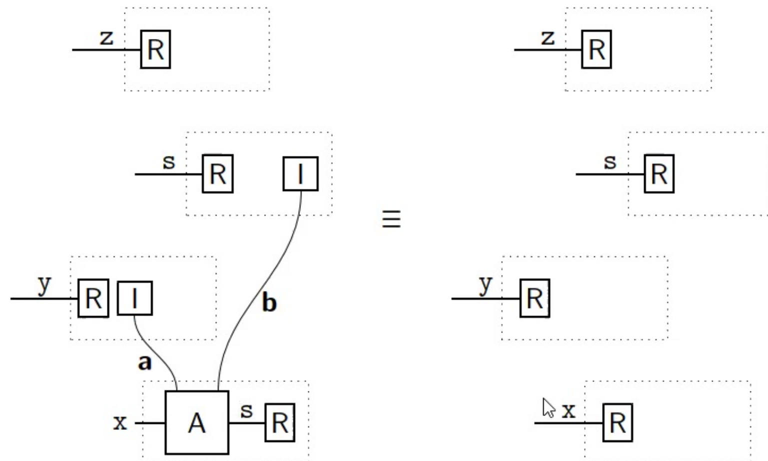
Double causality in action



Double causality in action

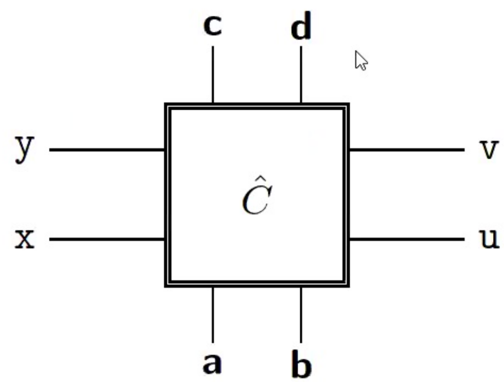


Double causality in action



Operator Tensors

Operator tensors can be represented diagrammatically



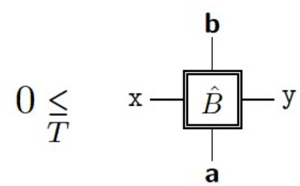
(17)

we can set up a *correspondence* from operations to operator tensors so we get the correct probabilities for circuits.

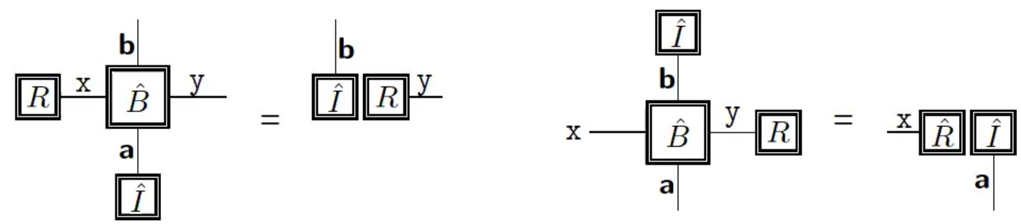
Physicality in Quantum Theory

The physicality conditions on operator tensors become

T-positivity.



Double causality.



These conditions are time symmetric.

Gauge parameters

A curious issue crops up with how we represent operator tensors. Consider the ignore operators

$$\boxed{\hat{I}}^{\,\mathbf{x}} = \frac{\alpha_{\mathbf{x}}}{\sqrt{N_{\mathbf{x}}}} \hat{\mathbb{1}}^{\mathbf{x}_1} \qquad \boxed{\hat{I}}_{\,\mathbf{x}} = \frac{1}{\alpha_{\mathbf{x}}\sqrt{N_{\mathbf{x}}}} \hat{\mathbb{1}}_{\mathbf{x}_1}$$

We are free to choose any real value for $\alpha_{\mathbf{x}}$.

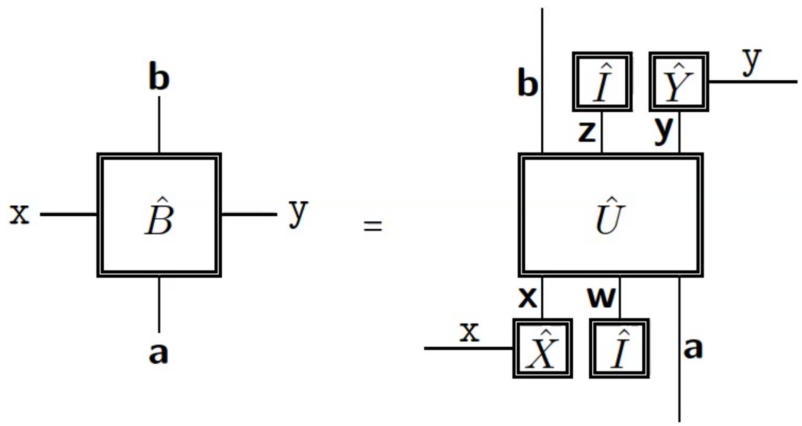
In Standard QT we write $\alpha_{\mathbf{a}} = \frac{1}{\sqrt{N_{\mathbf{a}}}}$.

A time symmetric choice is $\alpha_{\mathbf{a}} = 1$.

There is a similar gauge parameter, β_{x}, associated with the flat distribution operations R.

Time symmetric extension theorem

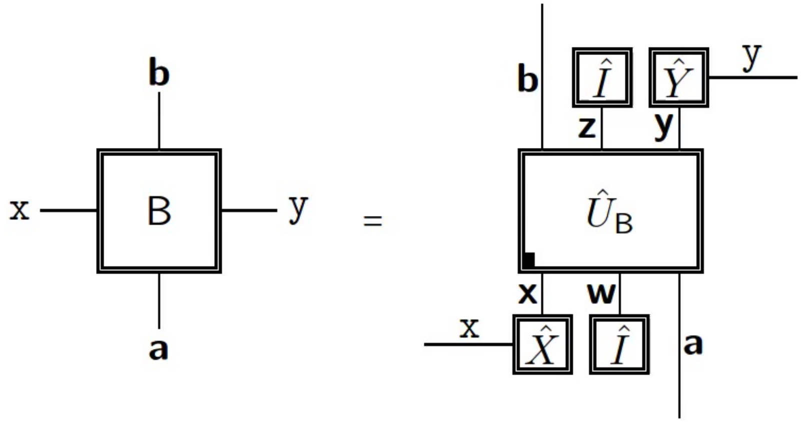
Extension theorem. *All physical operators can be written in the (extended) form*



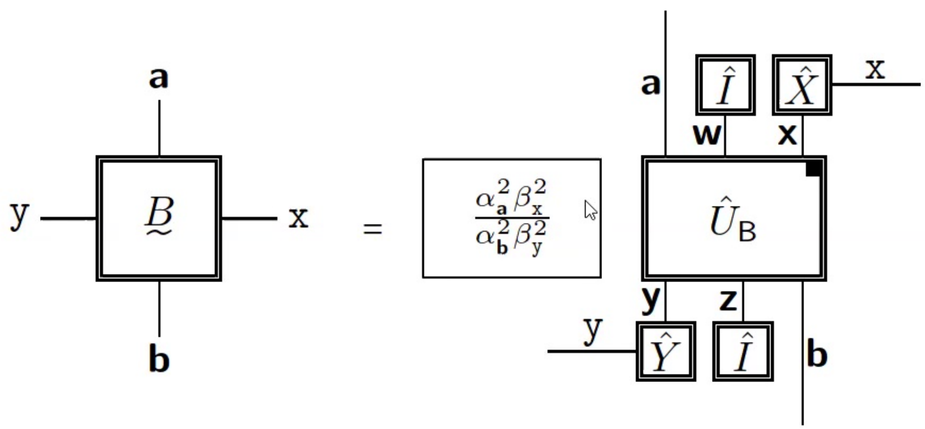
where $\hat{U}$ *is unitary, and* $\hat{X}$ *and* $\hat{Y}$ *are maximal.*

Time reverse of operator tensor

We can show that time reverse of



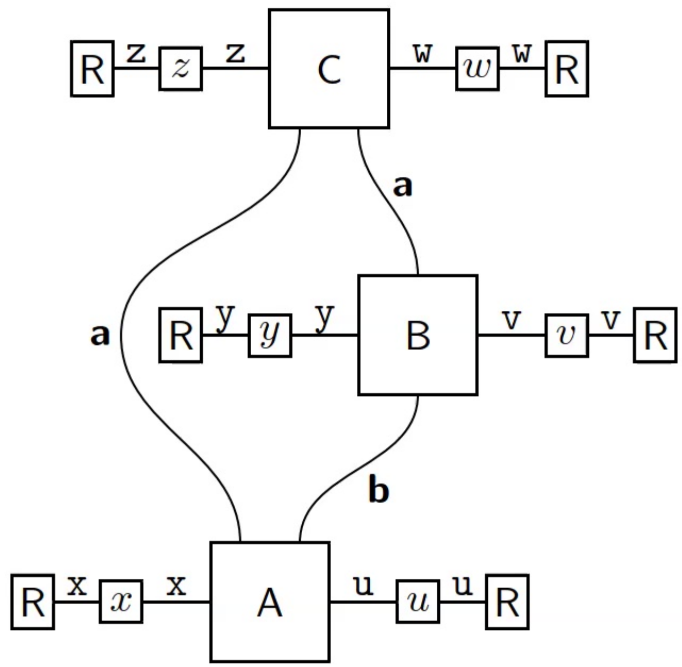
is



The black dot position indicates the adjoint.

Conditional frames of reference

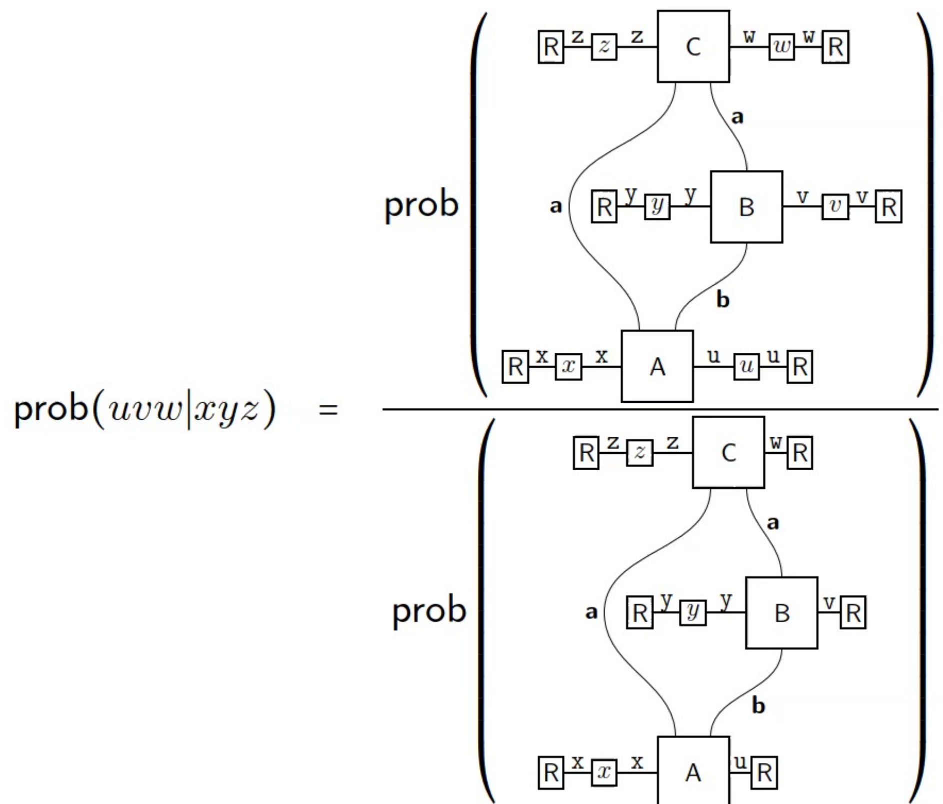
Consider the circuit



The probability is $p(x, y, z, u, v, w)$.

Conditional frames of reference

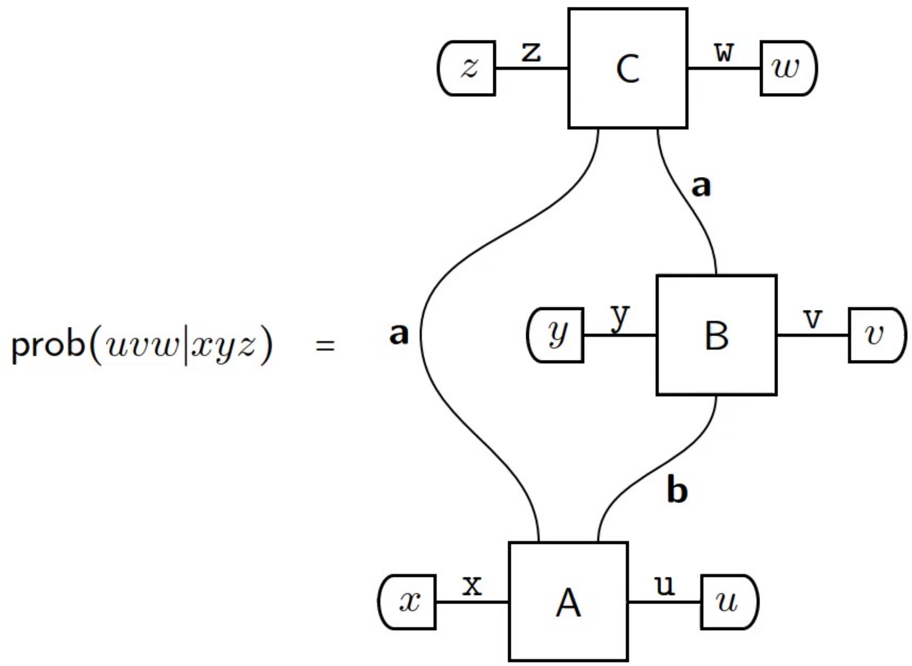
In the forward point of view we want



We know from backwards summation (or backwards flatness) that the denominator is equal to $\frac{1}{N_x N_y N_z}$.

Conditional frame of reference

This means we can write

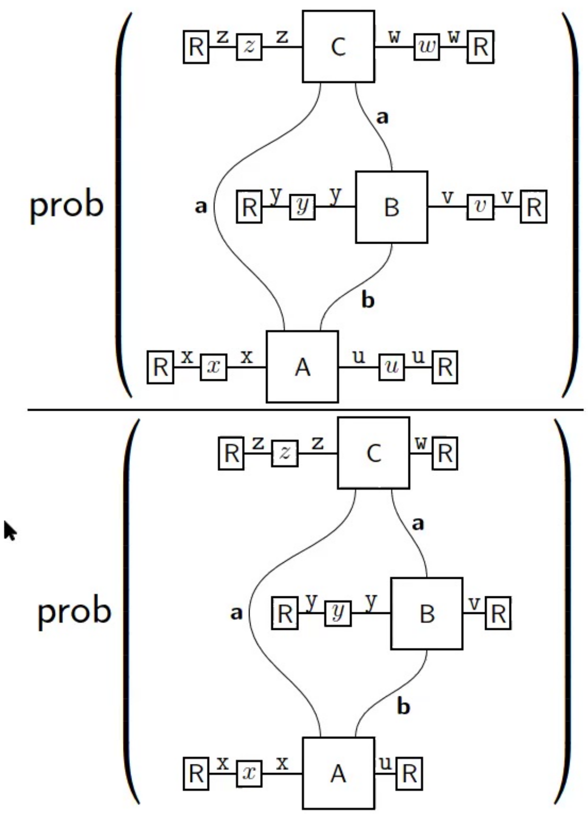


where

Conditional frames of reference

In the forward point of view we want

$\text{prob}(uvw|xyz) =$



We know from backwards summation (or backwards flatness) that the denominator is equal to $\frac{1}{N_x N_y N_z}$.

Conditional frames of reference

- We can say that the forward conditional frame of reference is given by

which we use to calculate $p(\text{outcomes}|\text{incomes})$ directly.

- Similarly, the backward conditional frame of reference is given by

Which we use to calculate $p(\text{incomes}|\text{outcomes})$ directly.

- The time symmetric conditional frame of reference is given by

We can use this to calculate $p(\text{incomes}, \text{outcomes})$ directly.

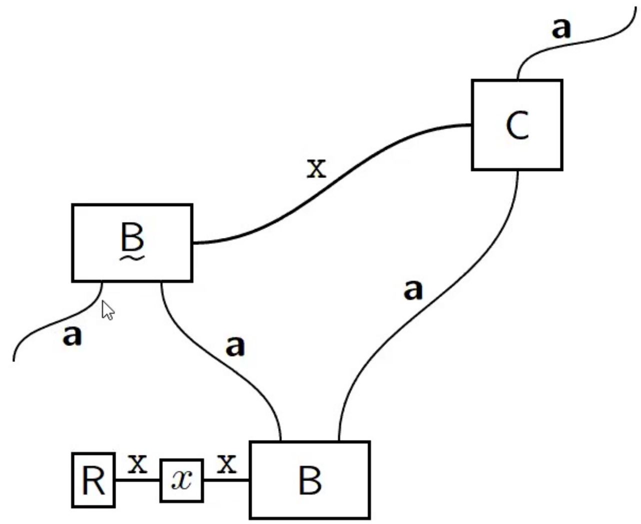
Equivalence to time forward Quantum Theory

If we transform to the forward conditional frame of reference the physicality constraints become the standard physicality constraints of (forward) operational quantum theory.

We can also transform to the backward conditional frame of reference and get physicality constraints corresponding to backwards operational quantum theory.

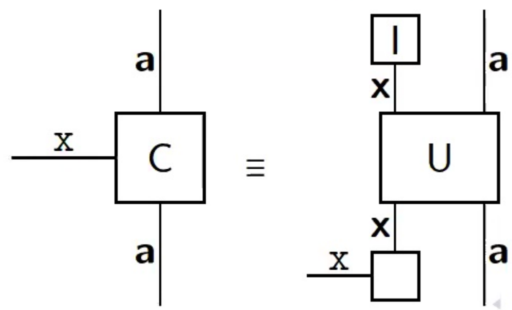
Time reverse experiments

It is fun to consider the time reverse of standard experiments. For example the standard quantum teleportation protocol looks like this



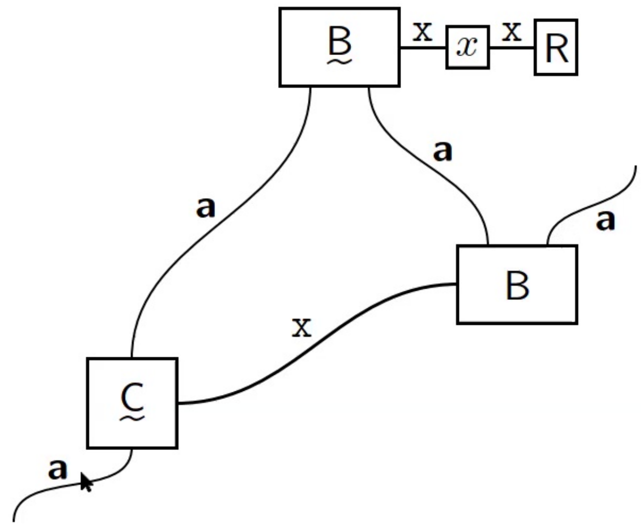
(18)

We can write x = aa. We can model C as



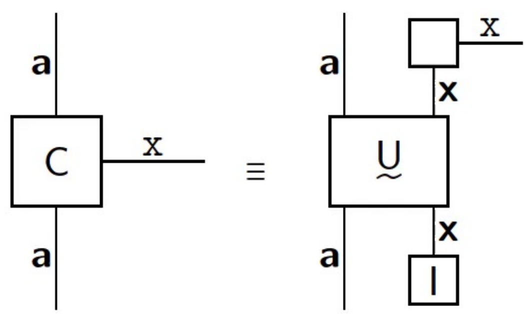
(19)

Time reverse of experiments



(20)

where



(21)

Conclusions

1. How should we interpret these results?
2. Porting classical information around.
3. General Theory of Conditional Frames.
4. Duotensor and Operator Tensor formulations.
5. Better axioms for Quantum Theory?
6. Interpretation of no-signalling result.
7. Will this help with Quantum Gravity?