Title: Tidal Love numbers of Kerr black holes clarified

Speakers: Alexandre Le Tiec

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Abstract: The open question of whether a black hole can become tidally deformed by an external gravitational field has profound implications for fundamental physics, astrophysics and gravitational-wave astronomy. Love tensors characterize the tidal deformability of compact objects such as astrophysical (Kerr) black holes under an external static tidal field. We prove that all Love tensors vanish identically for a Kerr black hole in the nonspinning limit or for an axisymmetric tidal perturbation. In contrast to this result, we show that Love tensors are generically nonzero for a spinning black hole. Specifically, to linear order in the Kerr black hole spin and the weak perturbing tidal field, we compute in closed form the Love tensors that couple the mass-type and current-type quadrupole moments to the electric-type and magnetic-type quadrupolar tidal fields. For a dimensionless spin ~ 0.1, the nonvanishing quadrupolar Love tensors are ~ 0.002, thus showing that black holes are particularly "rigid" compact objects. We also show that the induced quadrupole moments are closely related to the physical phenomenon of tidal torquing of a spinning body interacting with a tidal gravitational environment.

Tidal Love numbers of Kerr black holes clarified

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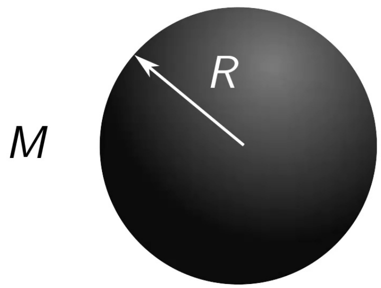
What's new since last Capra?

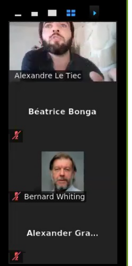
- Analytic continuation in multipolar order ℓ vindicated
  [Page 1976; Chia 2020; Charalambous et al. 2021]
- Connection to Kerr black hole tidal torquing
  [Thorne & Hartle 1980; Poisson 2004]
- Purely dissipative nature of Kerr black hole tidal deformability
  [Chia 2020; Goldberger et al. 2020; Charalambous et al. 2021]
- Do Kerr black hole tidal Love numbers vanish, yes or no?!

Agreement on maths but disagreement on **nomenclature**

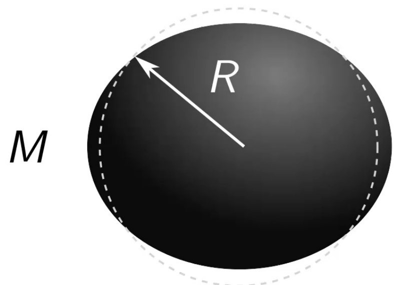
Newtonian theory of Love numbers



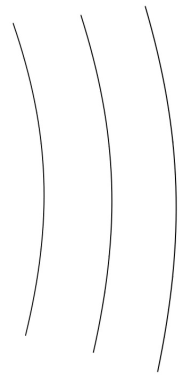
$$U = \frac{M}{r}$$



Newtonian theory of Love numbers



$$Q_{ab} = \lambda_2 \mathcal{E}_{ab}$$

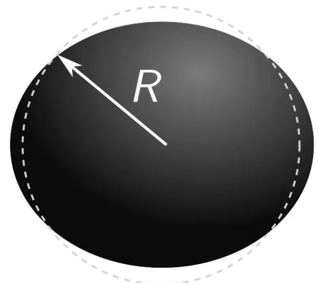


$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

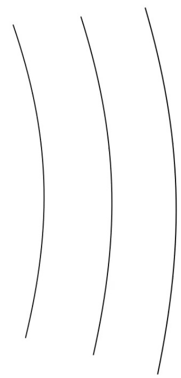
$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} + \frac{3}{2} \frac{x^a x^b Q_{ab}}{r^5}$$

Newtonian theory of Love numbers

M



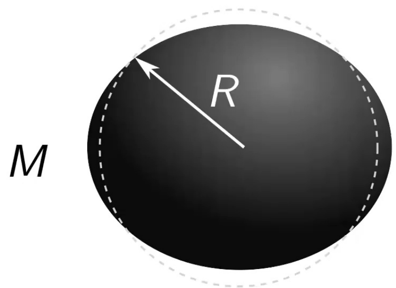
$$Q_{ab} = \lambda_2 \mathcal{E}_{ab} = -\tfrac{2}{3} k_2 R^5 \mathcal{E}_{ab}$$



$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

$$U = \frac{M}{r} - \sum_{\ell \geqslant 2} \sum_{|m| \leqslant \ell} \frac{(\ell-2)!}{\ell!} \, r^\ell \mathcal{E}_{\ell m} \left[1 + 2 k_\ell \left(\frac{R}{r} \right)^{2\ell+1} \right] Y_{\ell m}$$

Newtonian theory of Love numbers



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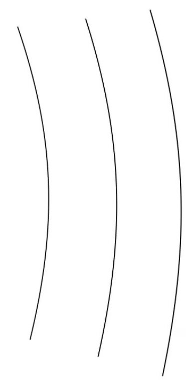
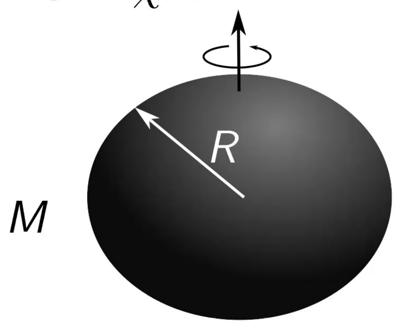


$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

$$\psi_0 = \sum_{\ell \geqslant 2} \sum_{|m| \leqslant \ell} \sqrt{\frac{(\ell+2)(\ell+1)}{\ell(\ell-1)}}\, r^{\ell-2}\, \mathcal{E}_{\ell m} \left[1 + 2 k_\ell \left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

Newtonian theory of Love numbers

$S = \chi M^2$



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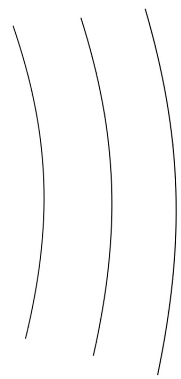

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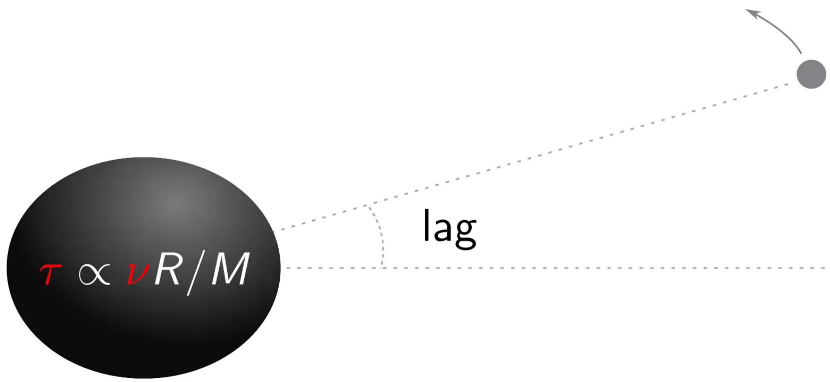
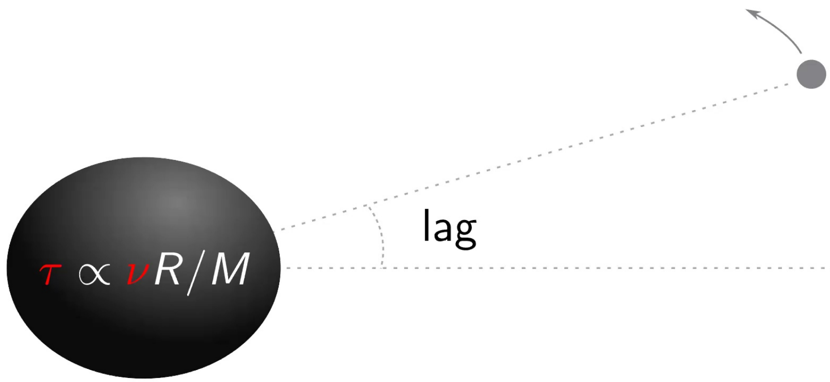
$$k_{\ell m} = k_\ell^{(0)} + i m \chi\, k_\ell^{(1)} + O(\chi^2)$$

Viscosity and tidal dissipation



Viscosity and tidal dissipation

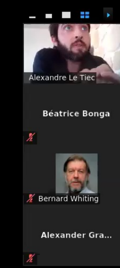


- Tidal lag:

$$Q_{ab}(t) = -\frac{2}{3}k_2 R^5 \left[\mathcal{E}_{ab}(t) - \tau\dot{\mathcal{E}}_{ab}(t) + \cdots\right]$$
$$= -\frac{2}{3}k_2 R^5 \left[\mathcal{E}_{ab}(t-\tau) + \cdots\right]$$

- Tidal torquing:

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc}\langle Q_{bd}\mathcal{E}^d{}_c\rangle = \frac{2}{3}(k_2\tau)R^5\,\varepsilon^{abc}\langle \dot{\mathcal{E}}_{bd}\mathcal{E}^d{}_c\rangle$$



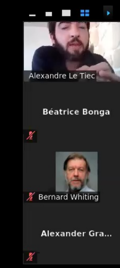
Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_{a_1\cdots a_\ell} \propto \left[C_{0a_10a_2;a_3\cdots a_\ell}\right]^{\text{STF}}, \quad \mathcal{B}_{a_1\cdots a_\ell} \propto \left[\varepsilon_{a_1bc}C_{a_20bc;a_3\cdots a_\ell}\right]^{\text{STF}}$$

- Metric and Geroch-Hansen multipole moments:

$$g_{\alpha\beta} = \mathring{g}_{\alpha\beta} + \underbrace{h^{\text{tidal}}_{\alpha\beta}}_{\sim r^\ell} + \underbrace{h^{\text{resp}}_{\alpha\beta}}_{\sim r^{-(\ell+1)}} \longrightarrow \begin{cases} M_{\ell m} = \mathring{M}_{\ell m} + \delta M_{\ell m} \\ S_{\ell m} = \mathring{S}_{\ell m} + \delta S_{\ell m} \end{cases}$$



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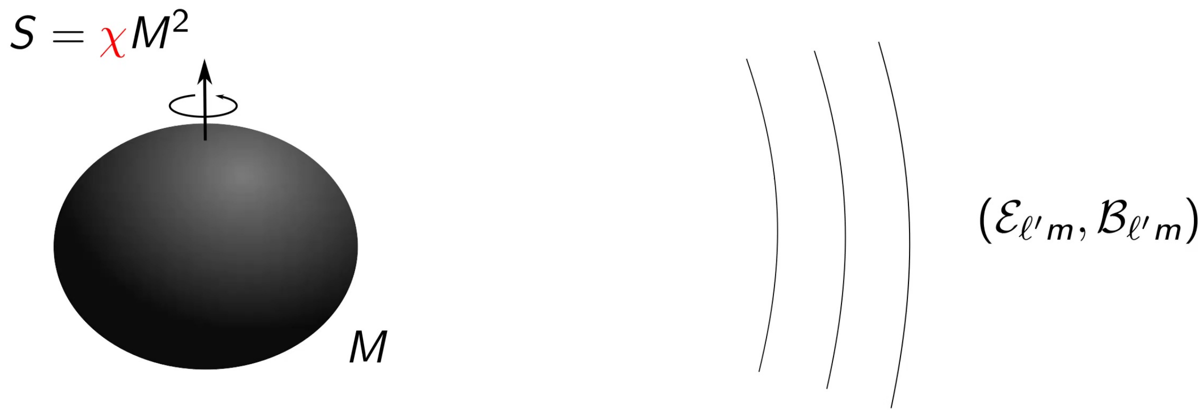
- *Four* families of tidal deformability parameters:

$$\lambda^{M\mathcal{E}}_{\ell\ell' m} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell' m}} \qquad \lambda^{S\mathcal{B}}_{\ell\ell' m} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell' m}}$$

$$\lambda^{S\mathcal{E}}_{\ell\ell' m} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{E}_{\ell' m}} \qquad \lambda^{M\mathcal{B}}_{\ell\ell' m} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{B}_{\ell' m}}$$

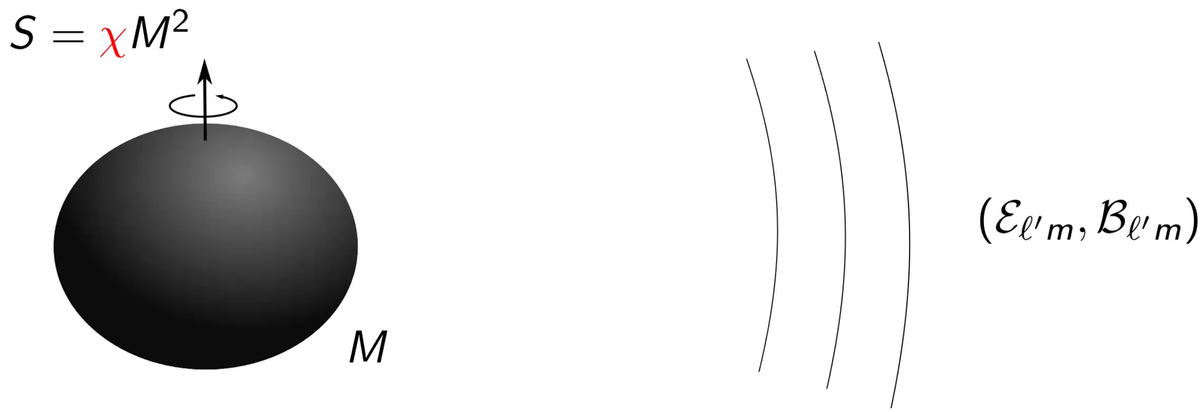
Investigating Kerr's Love



- Metric reconstruction through the Hertz potential Ψ:

$$(\mathcal{E}_{\ell' m}, \mathcal{B}_{\ell' m}) \to \psi_0 \to \Psi \to h_{\alpha\beta} \to (M_{\ell m}, S_{\ell m}) \to \lambda_{\ell\ell' m}^{M/S,\mathcal{E}/\mathcal{B}}$$

Investigating Kerr's Love



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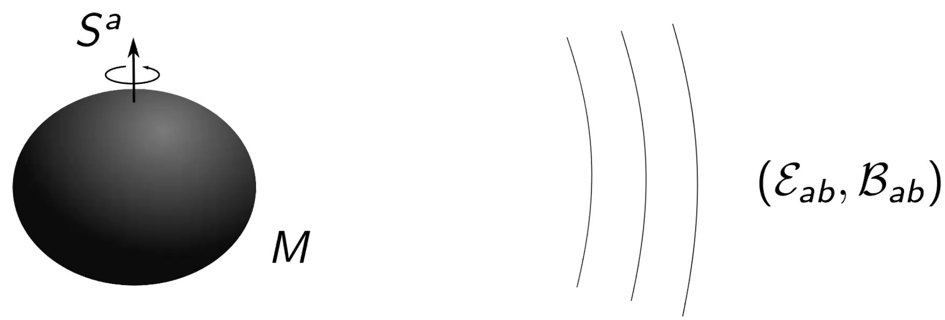
- Quadrupolar tidal Love numbers of a Kerr black hole:

$$\lambda^{M\mathcal{E}}_{2\ell' m} = \lambda^{S\mathcal{B}}_{2\ell' m} \doteq \frac{im\chi}{180} (2M)^5 \, \delta_{\ell' 2} \,, \quad \lambda^{M\mathcal{B}}_{2\ell' m} = \lambda^{S\mathcal{E}}_{2\ell' m} = 0$$

Tidal torquing of a spinning black hole



- Any spinning body interacting with a tidal environment suffers a tidal torquing [Thorne & Hartle 1980]

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc} \langle M_{bd} \mathcal{E}^d{}_c + S_{bd} \mathcal{B}^d{}_c \rangle$$

- Applied to a spinning black hole this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi \left[2 \langle \mathcal{E}^{ab} \mathcal{E}_{ab} \rangle - 3 \langle \mathcal{E}_{ab} s^b \mathcal{E}^{ac} s_c \rangle + (\mathcal{E} \to \mathcal{B}) \right]$$

- Full agreement with independant calculation by [Poisson 2004]

Tidal deformability and horizon viscosity

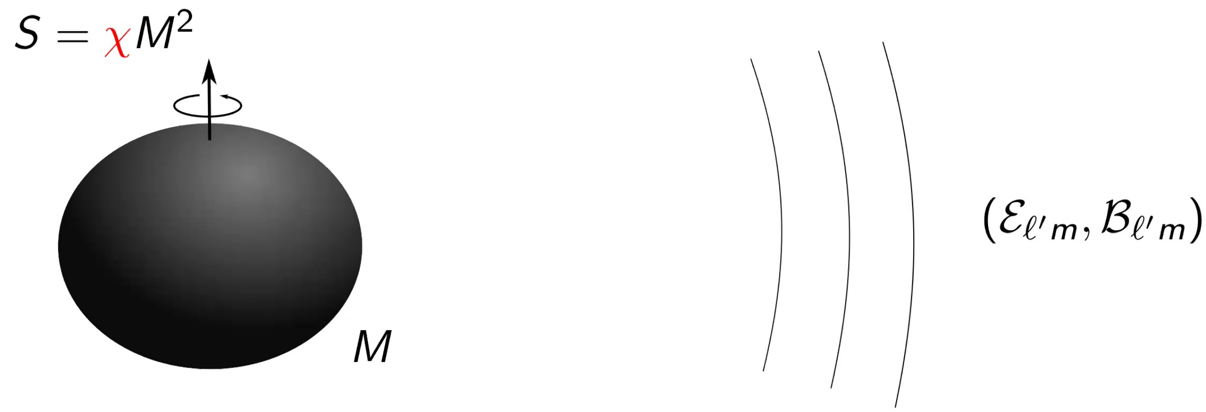
- A black hole has surface shear and bulk viscosity [Damour 1982]

$$\eta_S = \frac{1}{16\pi} \quad \text{and} \quad \zeta_S = -\frac{1}{16\pi}$$

- Black hole tidal heating and torquing imply remarkable analogy with Newtonian viscous fluid [Poisson 2009]

$$(k_2 \tau) R^5 = \frac{16}{15} M^6 \quad \Longrightarrow \quad k_2 \nu \sim M$$

Investigating Kerr's Love



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$$(k_2\tau)R^5 = \frac{16}{15}M^6 \quad \Longrightarrow \quad k_2\nu \sim M$$

- However $k_2 = 0$ for a nonspinning black hole so τ and ν are formally infinite

- Revisit analogy with nonzero Kerr black hole tidal Love numbers and horizon surface viscosity

Summary

- Love numbers of Kerr black holes do not vanish in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- Kerr black holes deform like any other self-gravitating body, despite being particularly "rigid" compact objects
- This is closely related to the phenomenon of tidal torquing
- Tidal deformability ↔ horizon viscosity ↔ tidal torquing

Spinning black holes fall in Love!