Title: Tidal Deformation and Dissipation of Rotating Black Holes

Speakers: Horng Sheng Chia

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 11, 2021 - 11:30 AM

URL: http://pirsa.org/21060078

Abstract: Black holes are never isolated in realistic astrophysical environments; instead, they are often perturbed by complicated external tidal fields. How does a black hole respond to these tidal perturbations? In this talk, I will discuss both the conservative and dissipative responses of the Kerr black hole to a weak and adiabatic gravitational field. The former describes how the black hole would change its shape due to these tidal interactions, and is quantified by the so-called â€œLove numbersâ€•. On the other hand, the latter describes how energy and angular momentum are exchanged between the black hole and its tidal environment due to the absorptive nature of the event horizon. In this talk, I will describe how the Love numbers of the Kerr black hole in a static tidal field vanish identically. I will also describe how the Kerr black hole's dissipative response implies that energy and angular momentum can either be lost to or extracted from the black hole, with the latter process commonly known as the black hole superradiance. I will end by discussing how these tidal responses leave distinct imprints on the gravitational waves emitted by binary black holes.

Tidal Deformation and Dissipation of Rotating Black Holes

Horng Sheng Chia

Institute for Advanced Study

arXiv: 2010.07300

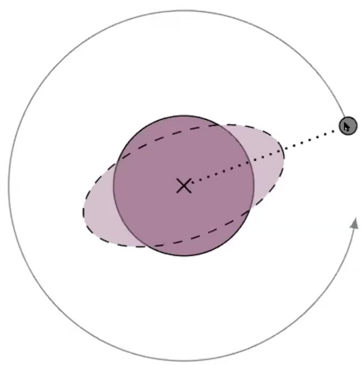
24th CAPRA Meeting, June 2021

Static Tidal Response

In Newtonian gravity, a body would respond to a **static** external tidal field by acquiring the induced mass moments:

$$\delta Q_{\ell m} \propto 2k_{\ell m}\mathcal{E}_{\ell m}$$

where the proportionality constants $k_{\ell m}$ are called the Love numbers.



Love (1912)
Poisson, Will (Gravity textbook)

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Time-Dependent Tidal Response

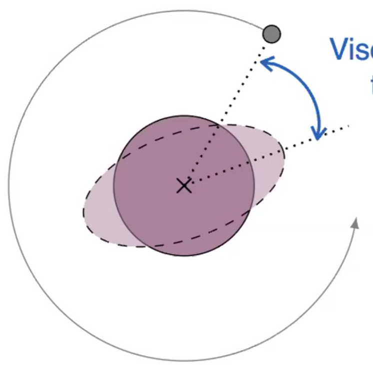
For a **slowly-varying** external tidal field, the induced response is

$$\delta Q_{\ell m}(t) = \int_0^\infty \mathrm{d}\tau \, F_{\ell m}(\tau)\mathcal{E}_{\ell m}(t-\tau)\,,$$

$$\propto 2k_{\ell m}\,\mathcal{E}_{\ell m}(t) - \tau_0 \nu_{\ell m}\,\dot{\mathcal{E}}_{\ell m}(t) + \cdots$$

static tides | tidal dissipation* | dynamical tides

where $\nu_{\ell m}$ are the dissipation numbers associated to the object's viscosity.



Poisson, Will (Gravity textbook)

*also called tidal heating, tidal torquing, tidal acceleration etc.

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Tidal Response of a Newtonian Body

In Fourier space, the tidal response of a Newtonian body is

$$\delta Q_{\ell m}(\omega) = F_{\ell m}(\omega)\mathcal{E}_{\ell m}(\omega)\,,$$

where ω is the frequency of the external tidal field, and

$$F_{\ell m}(\omega) = 2k_{\ell m} + i\omega\tau_0\nu_{\ell m} + \cdots$$

real part = **conservative** effect

imaginary part = **dissipative** effect

Goldberger, Rothstein [0409156, 0511133]
Chakrabarti, Delsate, Steinhoff [1306.5820]

Black Hole Perturbation Theory

The black hole's tidal response can be solved with the **Teukolsky equation**:

$$\rho^4\psi_4 = \sum_{\ell m} e^{-i\omega t + im\phi} R_{\ell m}(r)\,{}_{-2}S_{\ell m}(\theta)$$

The radial equation can be written as*

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r^2} + \left(\frac{2iP_+ - 1}{r - r_+} - \frac{2iP_- + 1}{r - r_-} - 2i\omega\right)\frac{\mathrm{d}R}{\mathrm{d}r} + \left(\frac{4iP_-}{(r-r_-)^2}\right.$$
$$\left. \boxed{-\frac{4iP_+}{(r-r_+)^2}} + \frac{A_- + iB_-}{(r-r_-)(r_+ - r_-)} - \frac{A_+ + iB_+}{(r-r_+)(r_+ - r_-)}\right)R = \frac{T}{\Delta}\,,$$

Dominates as $r \to r_+$

where we have organised the radial equation in terms of its poles at $r_\pm, \infty$. The "superradiance factor" $P_+ \propto m\Omega_H - \omega$ captures all of the physics of the event horizon.

Press and Teukolsky (1974)
HSC [2010.07300]
*In the the ingoing-Kerr coordinates and the Kinnersley null tetrad

Measuring the Tidal Response

In Newtonian gravity, the response function of a perturbed body can be measured through its **potential**:

$$U = -\frac{M}{r} + \sum_{\ell m} \frac{\mathcal{E}_{\ell m} r^{\ell}}{\ell(\ell-1)} \left[1 + F_{\ell m}(\omega) \left(\frac{r_0}{r} \right)^{2\ell+1} \right] Y_{\ell m}(\theta, \phi)$$

"growing term"
(applied external tidal field)

"decaying term"
(object's response)

In GR, the response function can be inferred through the decaying terms of the **metric** or the **Weyl scalars**.

Spherical Body: Love Numbers

It is instructive to compute ψ_4 of a **perturbed Schwarzschild spacetime** for a spherical body (Birkhoff's theorem). In the static limit, $\omega = 0$ (no information about dissipation),

$$\psi_4^{\rm Sph}(\omega=0) = \sum_I \sum_{\ell m} \mathcal{M}^I_{\ell m}\, {}_{-2}Y_{\ell m}(\theta,\phi) \times \left[r^{\ell-2} G_\ell(r) + 2\, k^I_{\ell m} r^{\ell-2} \left(\frac{r_0}{r}\right)^{2\ell+1} D_\ell(r) \right],$$

$\sim r^{\ell-2}$ "growing term" (applied external tidal field)

$\sim r^{-\ell-3}$ "decaying term" (object's response)

where $I = \{E, B\}$ label quantities of electric- and magnetic-type characters:

$$\mathcal{M}^E_{\ell m} \propto \mathcal{E}_{\ell m}\,, \qquad \mathcal{M}^B_{\ell m} \propto i\mathcal{B}_{\ell m}\,,$$

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096]
HSC [2010.07300]

*Recall the analogous $\sim r^{\ell}$ and $\sim r^{-\ell-1}$ scalings in the Newtonian potential

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Schwarzschild Black Hole: Love Numbers

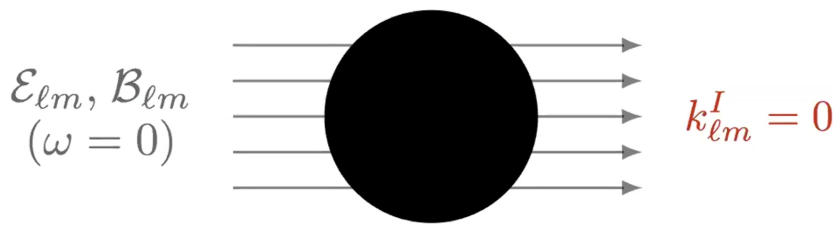
For a Schwarzschild black hole, $r_0 = 2M$.

$$\psi_4^{\rm Sph}(\omega = 0) \propto r^{\ell-2} G_\ell(r) + 2 k^I_{\ell m} r^{\ell-2} \left(\frac{r_0}{r}\right)^{2\ell+1} D_\ell(r)\,.$$

Furthermore, as we approach the event horizon,

$$r^{\ell-2} G_\ell(r) \sim (r - 2M)^2\,, \qquad r \to 2M\,,$$

while D_ℓ diverges logarithmically. The boundary condition at the event horizon* forces the decaying terms to vanish identically, which is only possible if $k^I_{\ell m} = 0$.



No deformation

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096], Kol, Smolkin [1110.3764]

*The purely-ingoing wave scales as $\psi_4 \sim (r - r_+)^2$

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Schwarzschild Black Hole: Tidal Response

To obtain both the conservative and dissipative responses of black holes, we solve the Teukolsky equation. For the Schwarzschild black hole,

$$\psi_4^{\mathrm{Schw}} \propto r^{\ell-2}\left[(1+\cdots)+F_{\ell m}^{I,\mathrm{Schw}}\left(\frac{2M}{r}\right)^{2\ell+1}(1+\cdots)\right], r \gg 2M$$

$$F_{\ell m}^{I,\mathrm{Schw}}(\omega) = 0 + i\omega(2M)\nu_{\ell m}^{\mathrm{Schw}} + \cdots$$

Well-known result in the literature

We derive the **general expression** for the dissipation numbers $\nu_{\ell m}^{\mathrm{Schw}}$, which recovers known results for the first few orders of ℓ in the literature.

HSC [2010.07300], Poisson [2012.10184]

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096], Kol, Smolkin [1110.3764]

Goldberger, Rothstein [0511133]

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Kerr Black Hole: Tidal Response

We also solve the response function of the Kerr black hole:

$$F^{I,\mathrm{Kerr}}_{\ell m}(\omega, \Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu^{\mathrm{Kerr}}_{\ell m} + \cdots$$

- **Love numbers of rotating black holes for static tides are zero**
  - true for all spins, all $\{\ell, m\}$, and both electric-type and magnetic-type tides
  - generalizes partial results known in the literature
- Tidal dissipation is proportional to the **superradiance** factor

$$\propto m\Omega_H - \omega$$

which can either be negative (energy loss) or positive (energy extraction)

HSC [2010.07300]
Poisson [1411.4711], Pani, Gualtieri, Maselli, Ferrari [1503.07365]
Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917]

Claims of “Spinning Black Holes Fall in Love”

Le Tiec, Casals [2007.00214] + Franzin [2010.15795]:

Featured in Physics | Editors' Suggestion

Spinning Black Holes Fall in Love

Alexandre Le Tiec and Marc Casals
Phys. Rev. Lett. **126**, 131102 – Published 30 March 2021

$$k_{\ell m} = -\frac{\mathrm{i}}{4\pi}\sinh(2\pi m\gamma)\,|\Gamma(\ell+1+2\mathrm{i}m\gamma)|^2\,\frac{(\ell-2)!(\ell+2)!}{(2\ell)!(2\ell+1)!}. \quad (22)$$

$\omega = 0$

Version 1 of arXiv paper:

be able to detect the quadrupolar tidal deformability of spinning black holes which we have here uncovered. One of the main sources for LISA is the radiation-reaction driven inspiral

Version 2 of arXiv paper (PRL):

Speculation.—As suggested by this tidal lag and as argued in Ref. [66], the purely imaginary TLNs (8) may give rise to dissipative effects *only*, such as the Kerr tidal torquing discussed in Ref. [41]. However, under the assumption that the induced quadrupole moments (13) also give rise to conservative effects, there is the exciting

$$F^{I,\mathrm{Kerr}}_{\ell m}(\omega,\Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu^{\mathrm{Kerr}}_{\ell m} + \cdots$$

HSC [2010.07300]

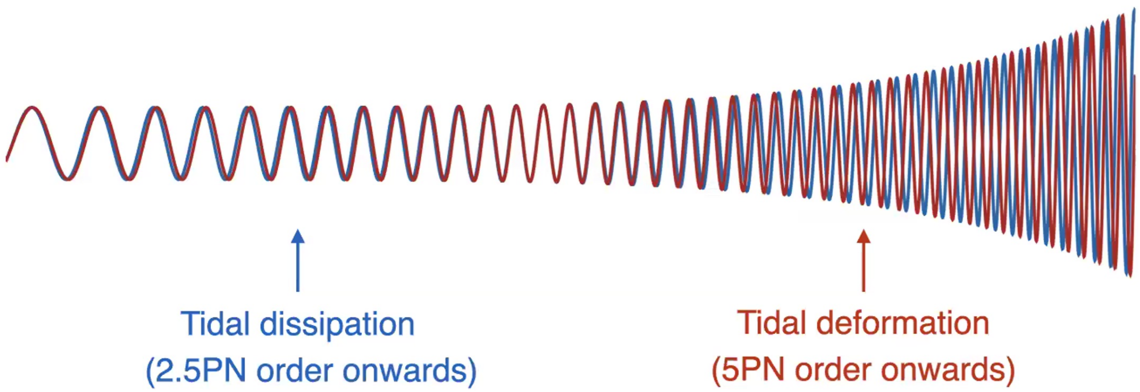
Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917]

The Many Definitions of Love Numbers?

The GW community commonly refers to $k_{\ell m}$, not $F_{\ell m}$, as Love numbers

$$F_{\ell m}^{I,\mathrm{Kerr}}(\omega, \Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu_{\ell m}^{\mathrm{Kerr}} + \cdots$$

Regardless of difference in definitions/nomenclatures, the **physical imprints** of these tidal effects on waveforms are **unambiguous**.



Flanagan, Hinderer [0709.1915], Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096]
Hartle (1973), Poisson, Sasaki [9412027], Tagoshi, Mano, Takasugi [9711072]

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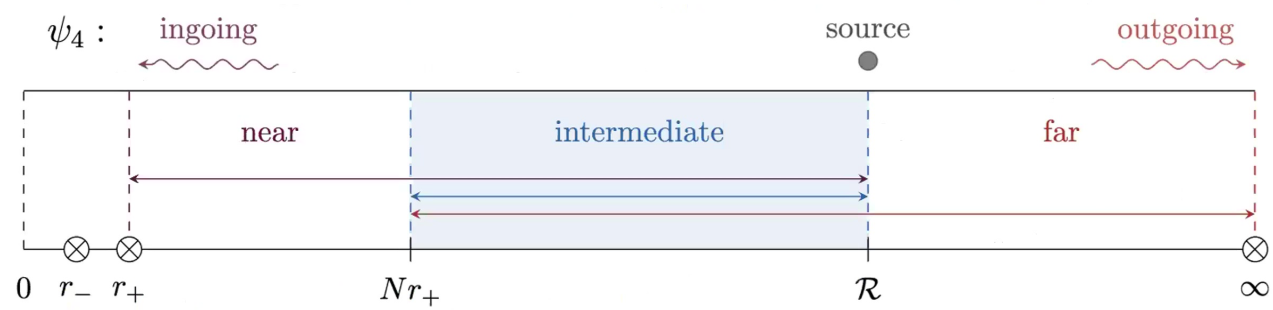
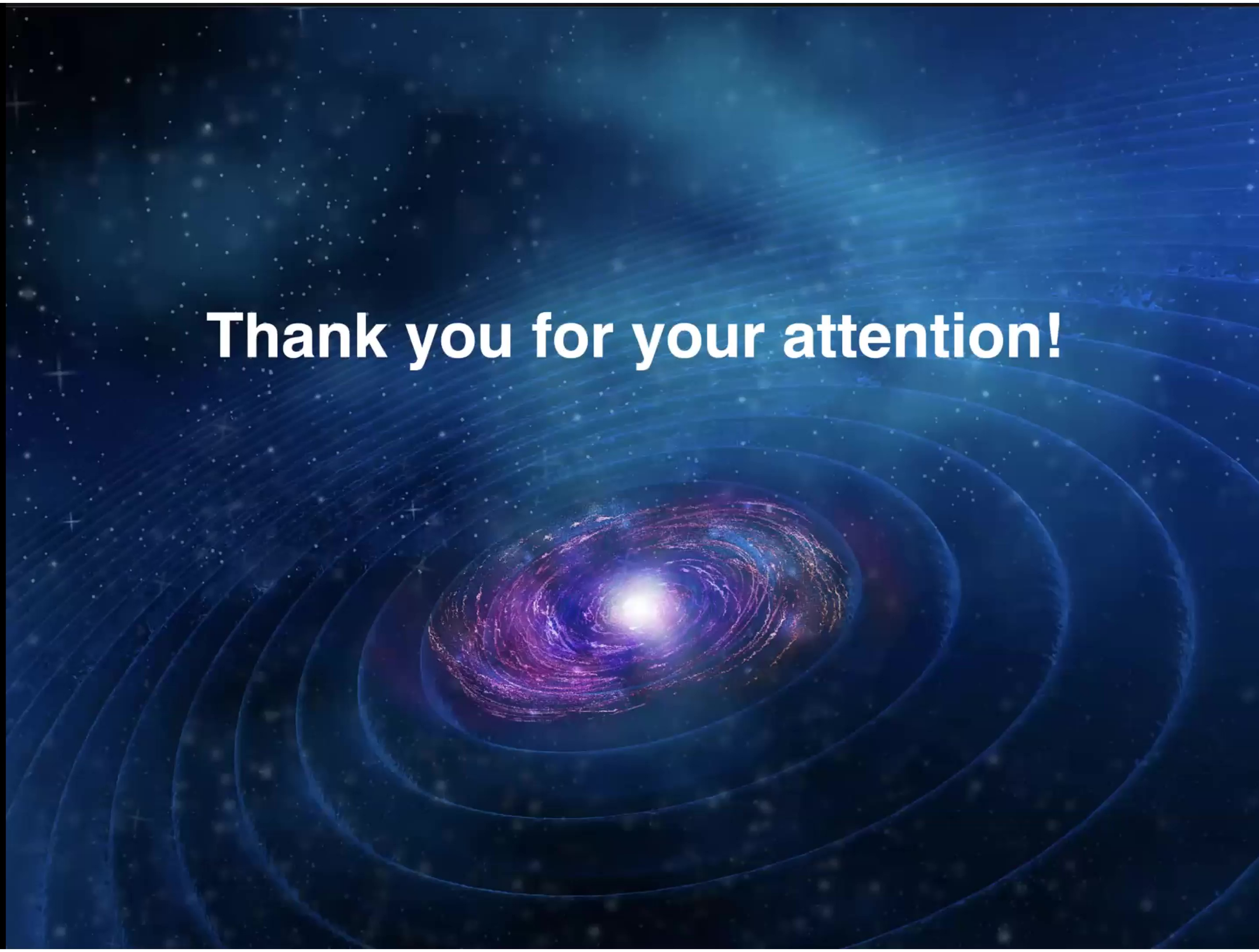
Love is Unambiguous

There are also claims that Love numbers in General Relativity are ambiguous (due to arbitrary coordinate transformation)



However, this ambiguity can be resolved by performing a **matched asymptotic expansion** between the solutions in different spacetime regions.

Fang, Lovelace [0505156], Pani, Gualtieri, Maselli, Ferrari [1503.07365], Gralla [1710.11096]
Sasaki, Tagoshi (Living Review), HSC [2010.07300] , Charalambous, Dubovsky, Ivanov [2102.08917]


Thank you for your attention!