Title: Transient resonances in EMRIs

Speakers: Zachary Nasipak

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Transient resonances in EMRIs:
A scalar model [arXiv:2105.15188]

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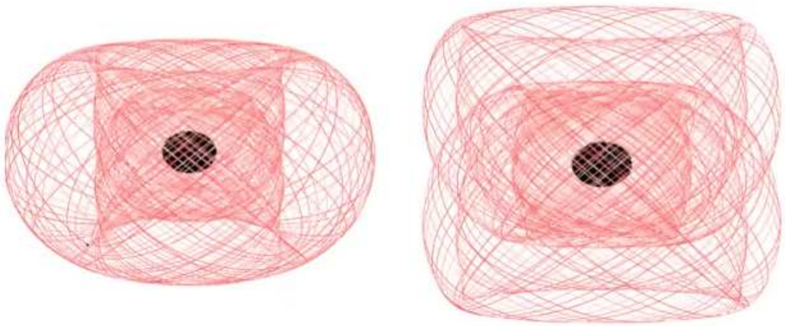

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THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL
ICERM
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24th Capra Meeting
Perimeter Institute (Virtual)
11 June 2021

Motivation

- **Transient orbital $r\theta$-resonance** $\Upsilon_r/\Upsilon_\theta \approx \beta_r/\beta_\theta \quad (\beta_i \in \mathbb{Z})$
  - Persist for resonant timescale $T_{\text{res}} \sim M\epsilon^{-1/2}$
  - Resonances affect evolution of frequencies/orbital quantities

$$\dot{\mathscr{E}} \sim F_t = \epsilon \tilde{f}_t^{00} + \epsilon \sum_{kn} \tilde{f}_t^{kn} e^{-i(kq_\theta + nq_r)}$$

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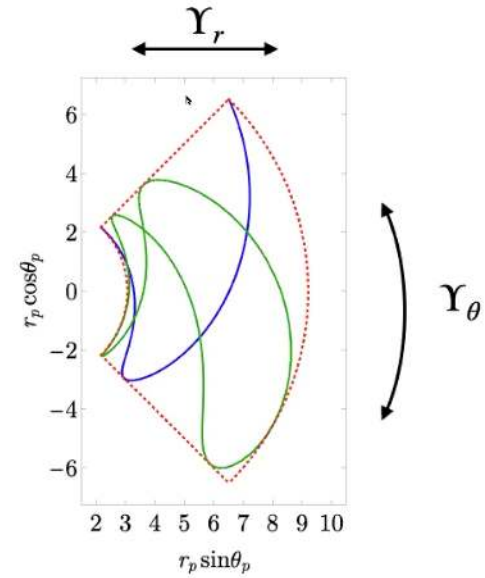

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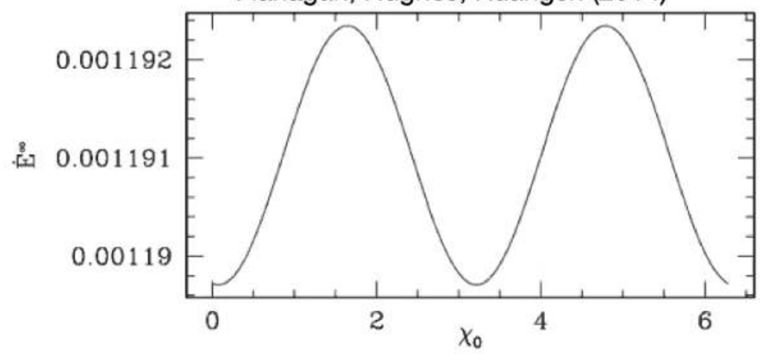
Flanagan, Hughes, Ruangsri (2014)



- **Complications due to resonances**
  - Leads to $O(\epsilon^{-1/2})$ "jumps" in actions/orbital quantities
  - Jumps depend on phase at which EMRI enters resonance
  - These effects lead to post-1/2 adiabatic effect

$$q_\alpha = \epsilon^{-1} q_\alpha^{\rm ad} + \epsilon^{-1/2} q_\alpha^{\text{post}-1/2} + q_\alpha^{\text{post}-1} + O(\epsilon)$$

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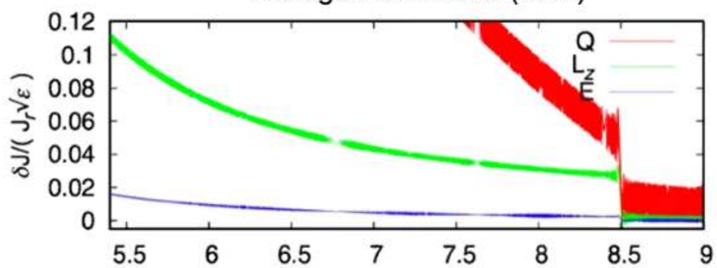

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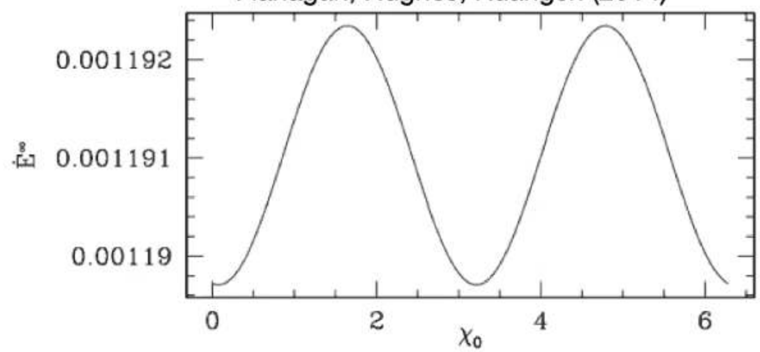

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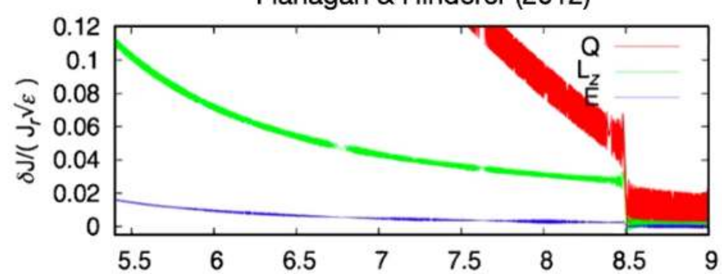

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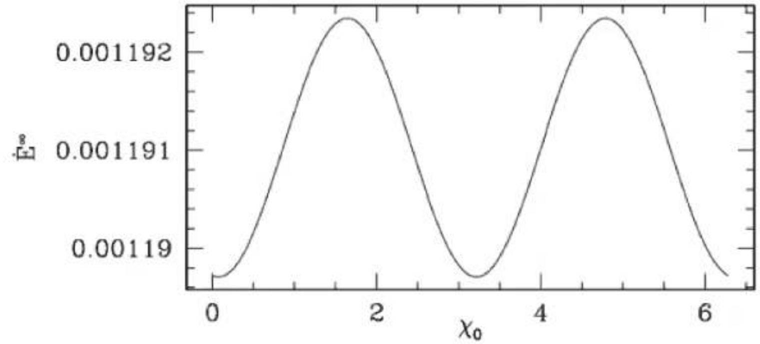
- **Focus of this work [arXiv:2105.15188]**
  - Calculate scalar self-force during resonances; examine impact on secular evolution of orbital quantities, i.e., $\langle \dot{\mathscr{E}} \rangle, \langle \dot{\mathscr{L}}_z \rangle, \langle \dot{\mathscr{Q}} \rangle$

Flanagan, Hughes, Ruangsri (2014)



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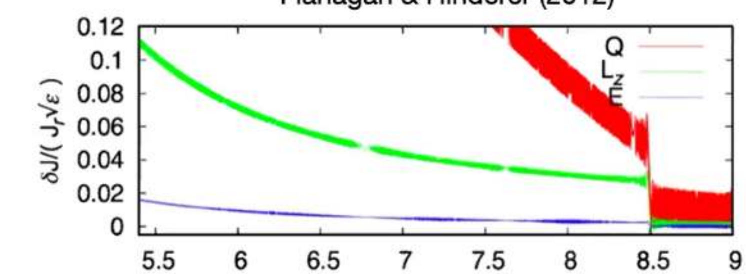
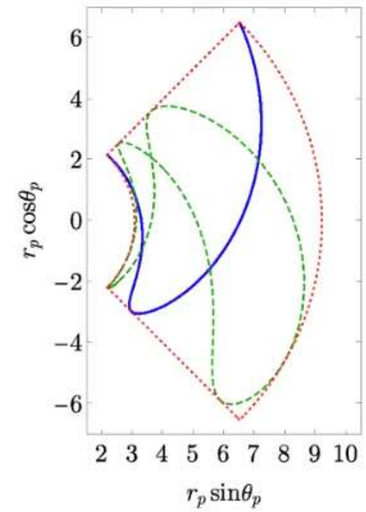


Secular evolution due to self-force

- **At resonance** $\Upsilon_r/\Upsilon_\theta = \beta_r/\beta_\theta$
  - Define resonant frequency $\Upsilon = \Upsilon_r/\beta_r = \Upsilon_\theta/\beta_\theta$, resonant angle $\bar{q} = \Upsilon\lambda$, initial phase $\bar{q}_0$
  - Secular evolution of actions expressed as single integral over resonant angle variable



$$\bar{F}_t = \bar{F}_t(\bar{q};\bar{q}_0) \qquad \langle\dot{\mathscr{E}}\rangle = -\frac{1}{\Upsilon_t}\int_0^{2\pi}\frac{d\bar{q}}{2\pi}\bar{\Sigma}_p\bar{F}_t \qquad \langle\dot{\mathscr{L}}_z\rangle = \frac{1}{\Upsilon_t}\int_0^{2\pi}\frac{d\bar{q}}{2\pi}\bar{\Sigma}_p\bar{F}_\varphi$$

$$\bar{\Sigma}_p = \bar{\Sigma}_p(\bar{q};\bar{q}_0)$$

$$\bar{K}_p^{\mu\nu} = \bar{K}_p^{\mu\nu}(\bar{q};\bar{q}_0) \qquad \langle\dot{\mathscr{Q}}\rangle = -2(\mathscr{L}_z - a\mathscr{E})\Big(\langle\dot{\mathscr{L}}_z\rangle - a\langle\dot{\mathscr{E}}\rangle\Big) + \frac{2}{\Upsilon_t}\int_0^{2\pi}\frac{d\bar{q}}{2\pi}\bar{\Sigma}_p\bar{K}_p^{\mu\nu}\bar{u}_\mu\bar{F}_\nu$$

  - Secular evolution depends on initial phase at resonance
  - Conservative contributions not guaranteed to vanish based on form of integrals
    - Based on flux-balance laws, $\langle\dot{\mathscr{E}}\rangle$ and $\langle\dot{\mathscr{L}}_z\rangle$ will not depend on $\bar{F}_\alpha^{\text{cons}}$

$$\langle\dot{\mathscr{E}}\rangle + \langle\dot{E}\rangle^\infty + \langle\dot{E}\rangle^{\mathscr{H}} = 0 \qquad \langle\dot{\mathscr{L}}_z\rangle + \langle\dot{L}_z\rangle^\infty + \langle\dot{L}_z\rangle^{\mathscr{H}} = 0$$

    - No flux-balance law for $\langle\dot{\mathscr{Q}}\rangle$, could depend on conservative perturbations
      - Consistent with observations by Isoyama et al. (2019)
      - Integrability conjecture by Flanagan and Hinderer (2012) argues conservative contributions should still vanish

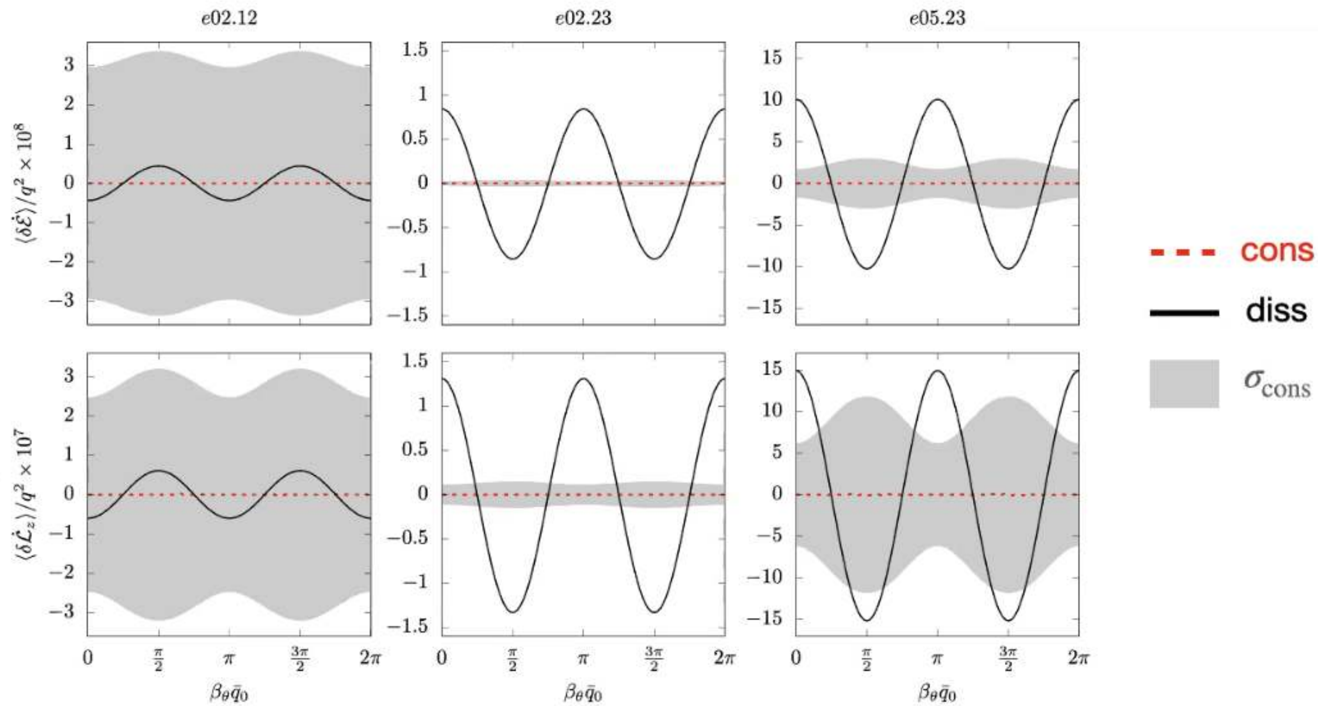
Energy & angular momentum

- **Residual variations due to initial phase**

$$\int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathscr{E}} \rangle^{\text{cons}} = 0$$

$$\int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathscr{E}} \rangle^{\text{diss}} = \text{const}$$

$$\langle \delta\dot{\mathscr{E}} \rangle = \langle \dot{\mathscr{E}} \rangle - \int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathscr{E}} \rangle$$



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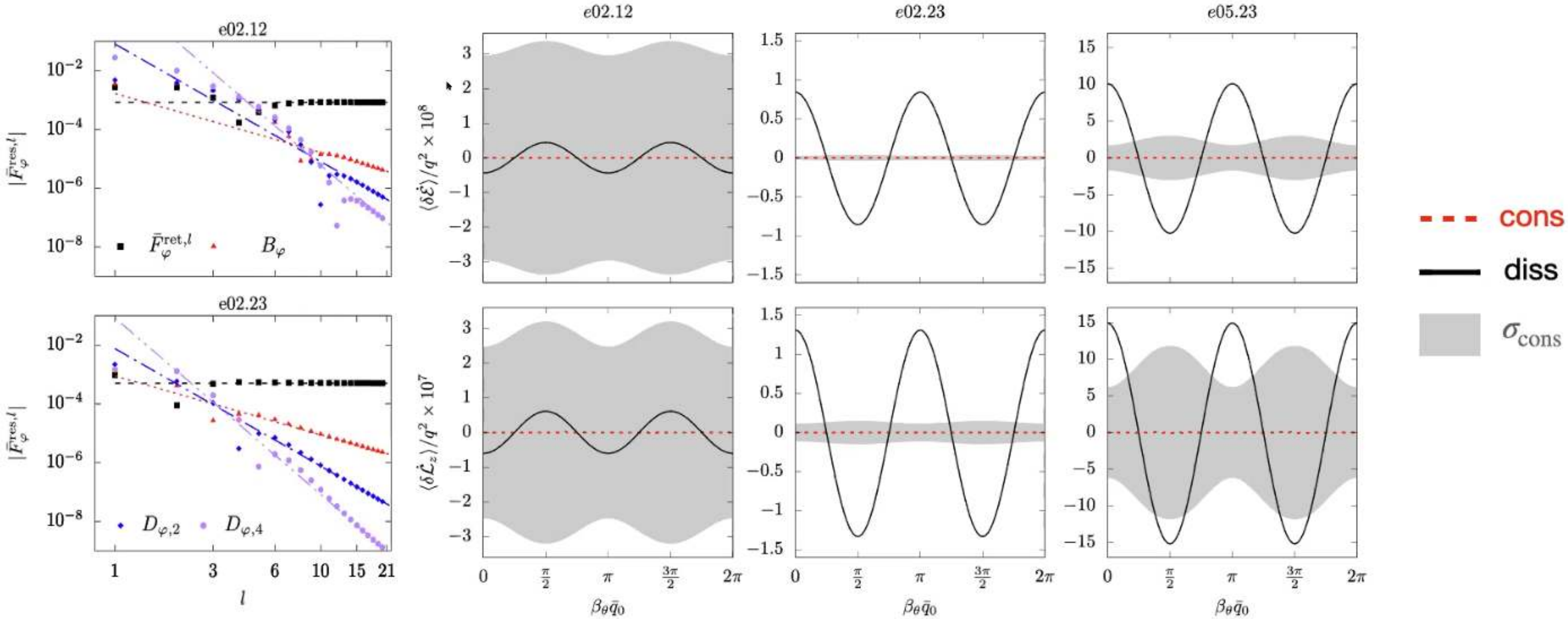

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Secular evolution of Carter constant

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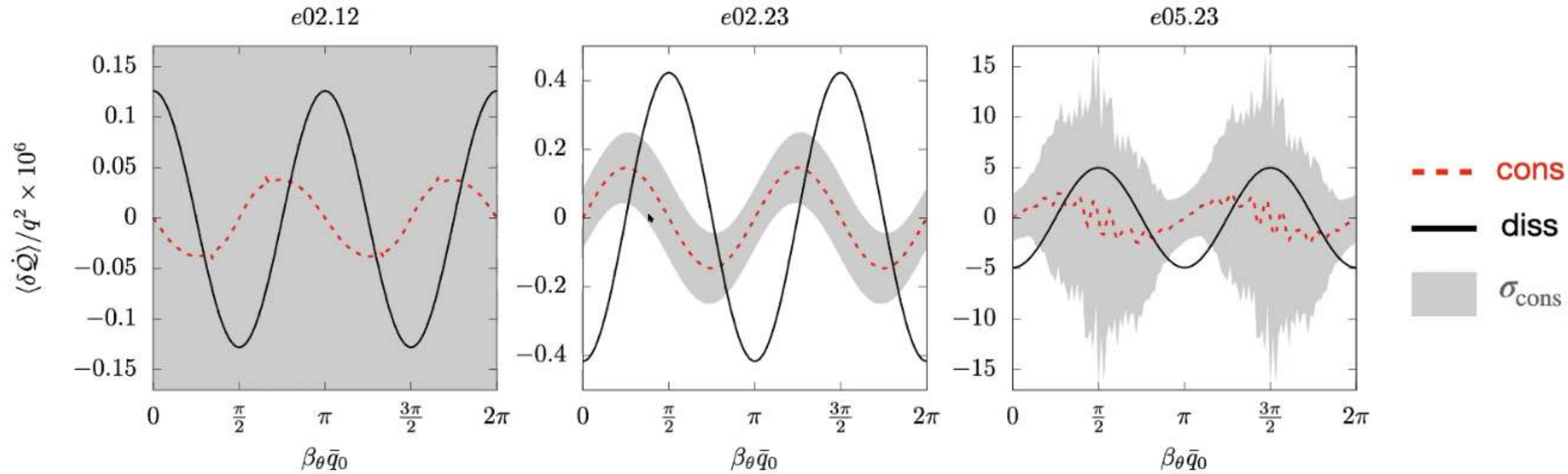

Secular evolution of Carter constant

- **Preliminary results w/ new, higher-order regularization scheme**
  - 2 new improvements
    1. Directly regularize $\langle \dot{\mathcal{Q}} \rangle$
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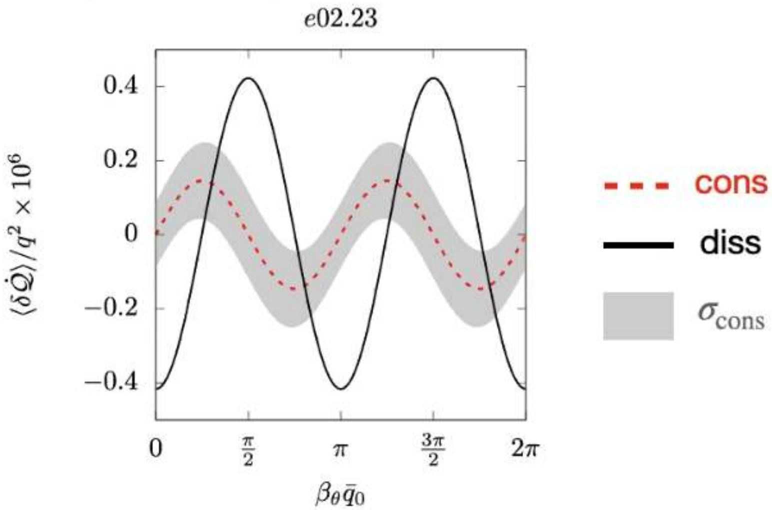

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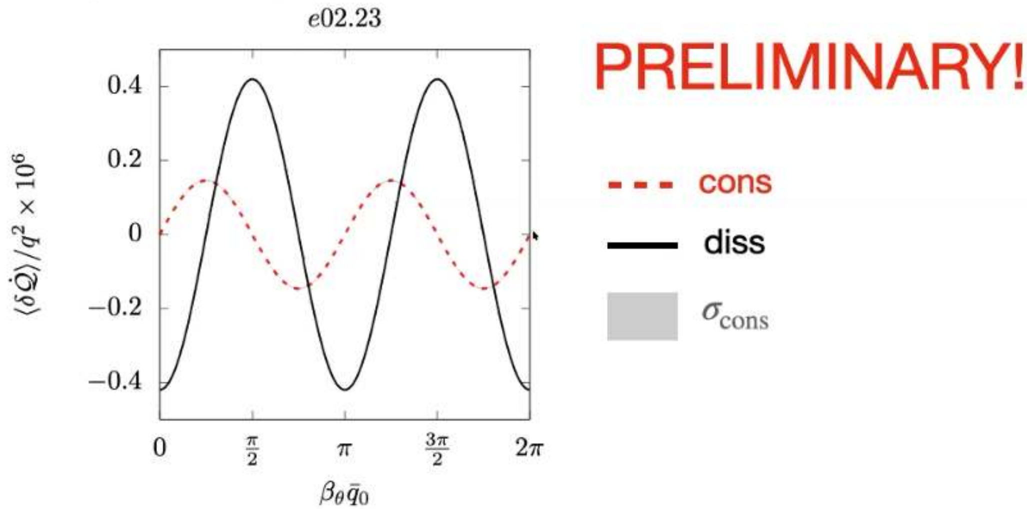

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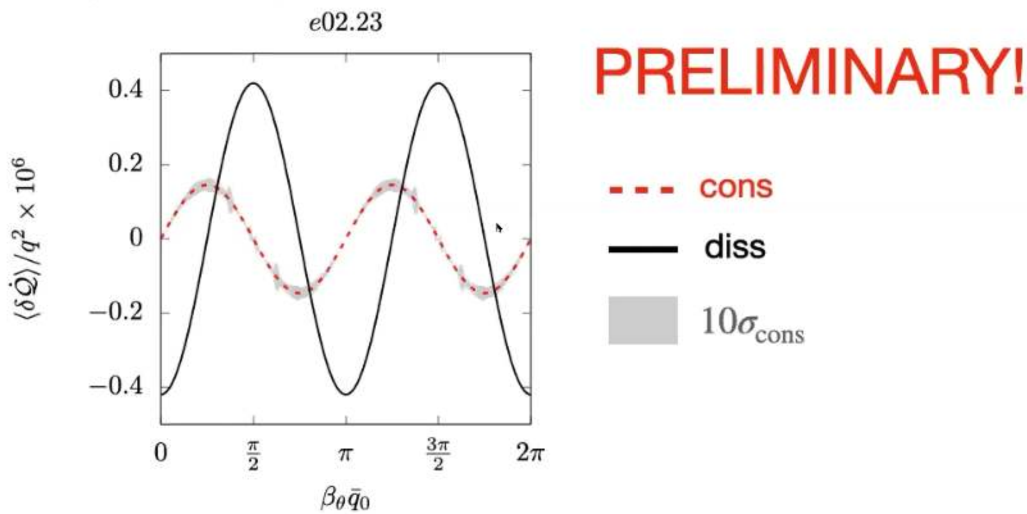

Conclusion

- **Impact of resonances on secular evolution**
  - Flux-balance laws preserved during resonances (as expected)
  - Numerical evidence that conservative perturbations contribute to secular evolution of Carter constant $\langle \dot{\mathcal{Q}} \rangle$ during $r\theta$-resonances
  - Regularization procedures required to calculate post-1/2 adiabatic corrections
  - Errors in conservative self-force will accumulate over resonant timescale $T_{\mathrm{res}} \sim M\epsilon^{-1/2}$
  - More accurate conservative calculations requires higher-order regularization schemes

- **Future work**
  - Verify preliminary results, test other resonant orbits
  - Calculate how including conservative perturbations impacts resonant "jumps" in actions/frequencies
  - Test whether these results extend to the gravitational case