

Title: Transient resonances in EMRIs

Speakers: Zachary Nasipak

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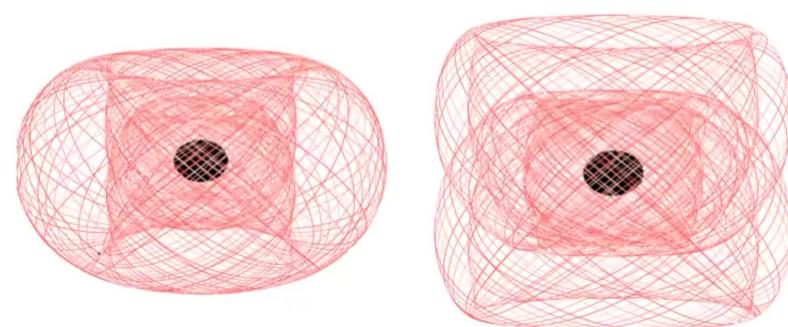
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Transient resonances in EMRIs: A scalar model [arXiv:2105.15188]

Zachary Nasipak (+ Charles R. Evans)

NASA Postdoctoral Fellow - Goddard Space Flight Center



Additional affiliations/support:



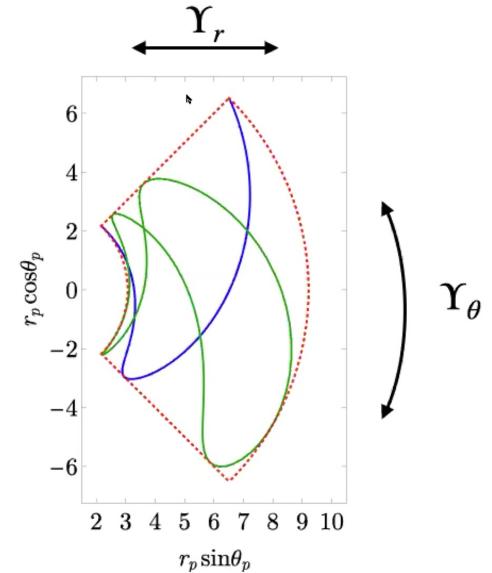
24th Capra Meeting
Perimeter Institute (Virtual)
11 June 2021

Motivation

- **Transient orbital $r\theta$ -resonance** $\Upsilon_r/\Upsilon_\theta \approx \beta_r/\beta_\theta$ ($\beta_i \in \mathbb{Z}$)
 - Persist for resonant timescale $T_{\text{res}} \sim M\epsilon^{-1/2}$
 - Resonances affect evolution of frequencies/orbital quantities

$$\dot{\mathcal{E}} \sim F_t = \epsilon \tilde{f}_t^{00} + \epsilon \sum_{kn} \tilde{f}_t^{kn} e^{-i(kq_\theta + nq_r)}$$

$$\langle \dot{\mathcal{E}} \rangle \sim \epsilon \tilde{f}_t^{00} + \epsilon \sum_{k\beta_\theta - n\beta_r = 0} \tilde{f}_t^{kn} e^{-i(kq_{\theta 0} + nq_{r0})}$$



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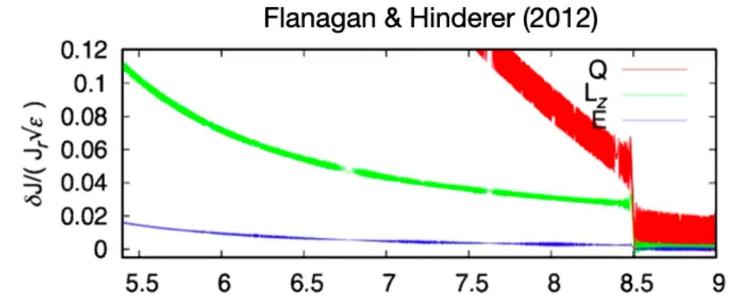
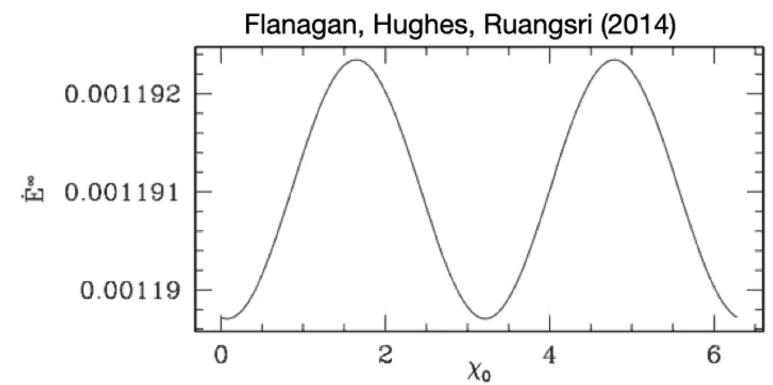
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- **Complications due to resonances**

- Leads to $O(\epsilon^{-1/2})$ “jumps” in actions/orbital quantities
- Jumps depend on phase at which EMRI enters resonance
- These effects lead to post-1/2 adiabatic effect

$$q_\alpha = \epsilon^{-1} q_\alpha^{\text{ad}} + \epsilon^{-1/2} q_\alpha^{\text{post}-1/2} + q_\alpha^{\text{post}-1} + O(\epsilon)$$



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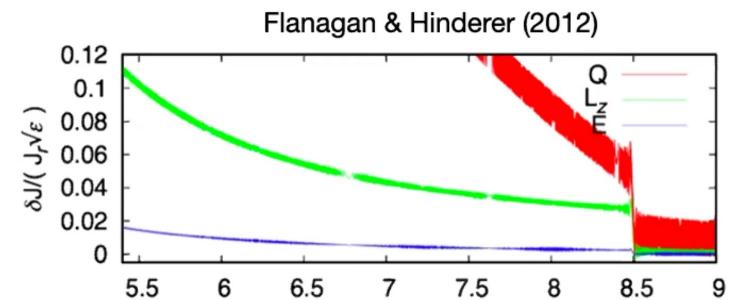
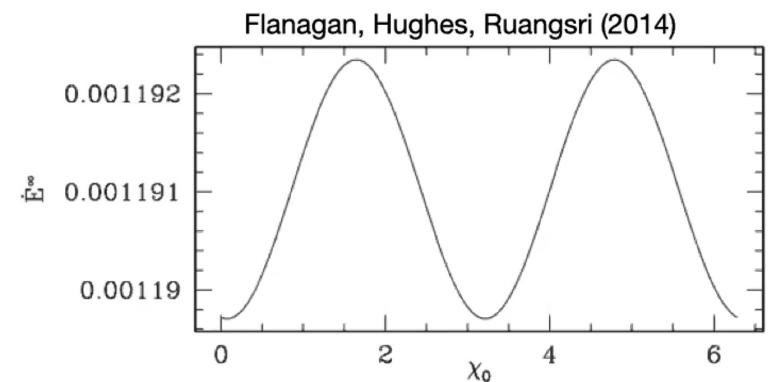
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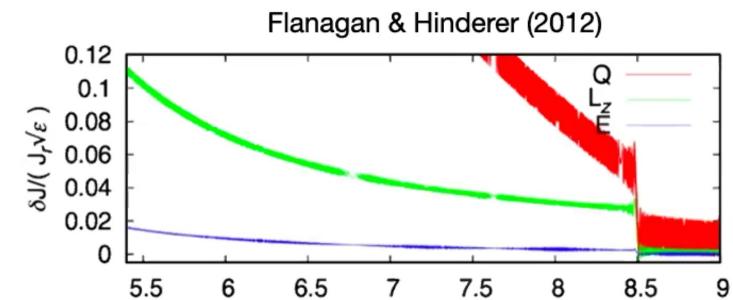
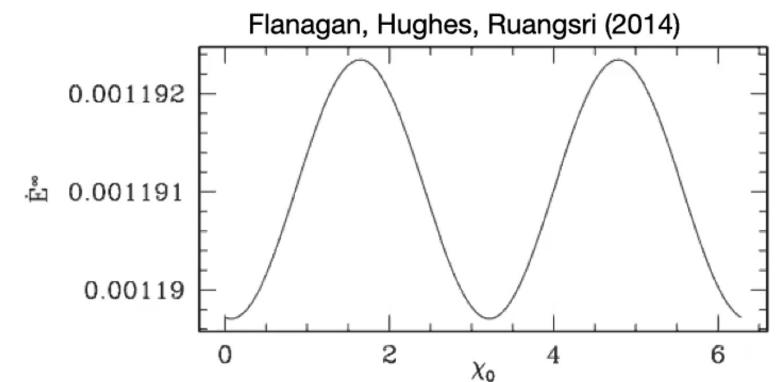
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- **Focus of this work [arXiv:2105.15188]**

- Calculate scalar self-force during resonances; examine impact on secular evolution of orbital quantities, i.e., $\langle \dot{\mathcal{E}} \rangle, \langle \dot{\mathcal{L}}_z \rangle, \langle \dot{\mathcal{Q}} \rangle$



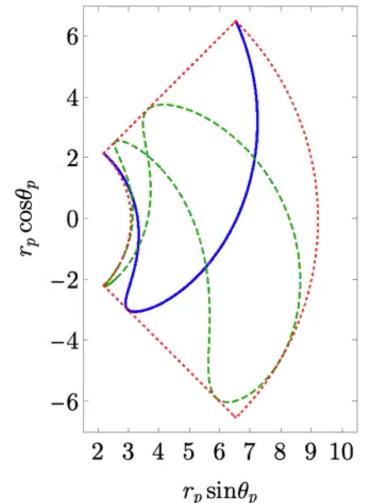
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Secular evolution due to self-force

- **At resonance** $\Upsilon_r/\Upsilon_\theta = \beta_r/\beta_\theta$
 - Define resonant frequency $\Upsilon = \Upsilon_r/\beta_r = \Upsilon_\theta/\beta_\theta$, resonant angle $\bar{q} = \Upsilon\lambda$, initial phase \bar{q}_0
 - Secular evolution of actions expressed as single integral over resonant angle variable

$$\begin{aligned}\bar{F}_t &= \bar{F}_t(\bar{q}; \bar{q}_0) & \langle \dot{\mathcal{E}} \rangle &= -\frac{1}{\Upsilon_t} \int_0^{2\pi} \frac{d\bar{q}}{2\pi} \bar{\Sigma}_p \bar{F}_t & \langle \dot{\mathcal{L}}_z \rangle &= \frac{1}{\Upsilon_t} \int_0^{2\pi} \frac{d\bar{q}}{2\pi} \bar{\Sigma}_p \bar{F}_\varphi \\ \bar{\Sigma}_p &= \bar{\Sigma}_p(\bar{q}; \bar{q}_0) & & & & \\ \bar{K}_p^{\mu\nu} &= \bar{K}_p^{\mu\nu}(\bar{q}; \bar{q}_0) & \langle \dot{\mathcal{Q}} \rangle &= -2(\mathcal{L}_z - a\mathcal{E}) \left(\langle \dot{\mathcal{L}}_z \rangle - a\langle \dot{\mathcal{E}} \rangle \right) + \frac{2}{\Upsilon_t} \int_0^{2\pi} \frac{d\bar{q}}{2\pi} \bar{\Sigma}_p \bar{K}_p^{\mu\nu} \bar{u}_\mu \bar{F}_\nu\end{aligned}$$



- Secular evolution depends on initial phase at resonance
- Conservative contributions not guaranteed to vanish based on form of integrals
 - Based on flux-balance laws, $\langle \dot{\mathcal{E}} \rangle$ and $\langle \dot{\mathcal{L}}_z \rangle$ will not depend on $\bar{F}_\alpha^{\text{cons}}$

$$\langle \dot{\mathcal{E}} \rangle + \langle \dot{E} \rangle^\infty + \langle \dot{E} \rangle^\mathcal{H} = 0 \quad \langle \dot{\mathcal{L}}_z \rangle + \langle \dot{L}_z \rangle^\infty + \langle \dot{L}_z \rangle^\mathcal{H} = 0$$

- No flux-balance law for $\langle \dot{\mathcal{Q}} \rangle$, could depend on conservative perturbations
 - Consistent with observations by Isoyama et al. (2019)
 - Integrability conjecture by Flanagan and Hinderer (2012) argues conservative contributions should still vanish



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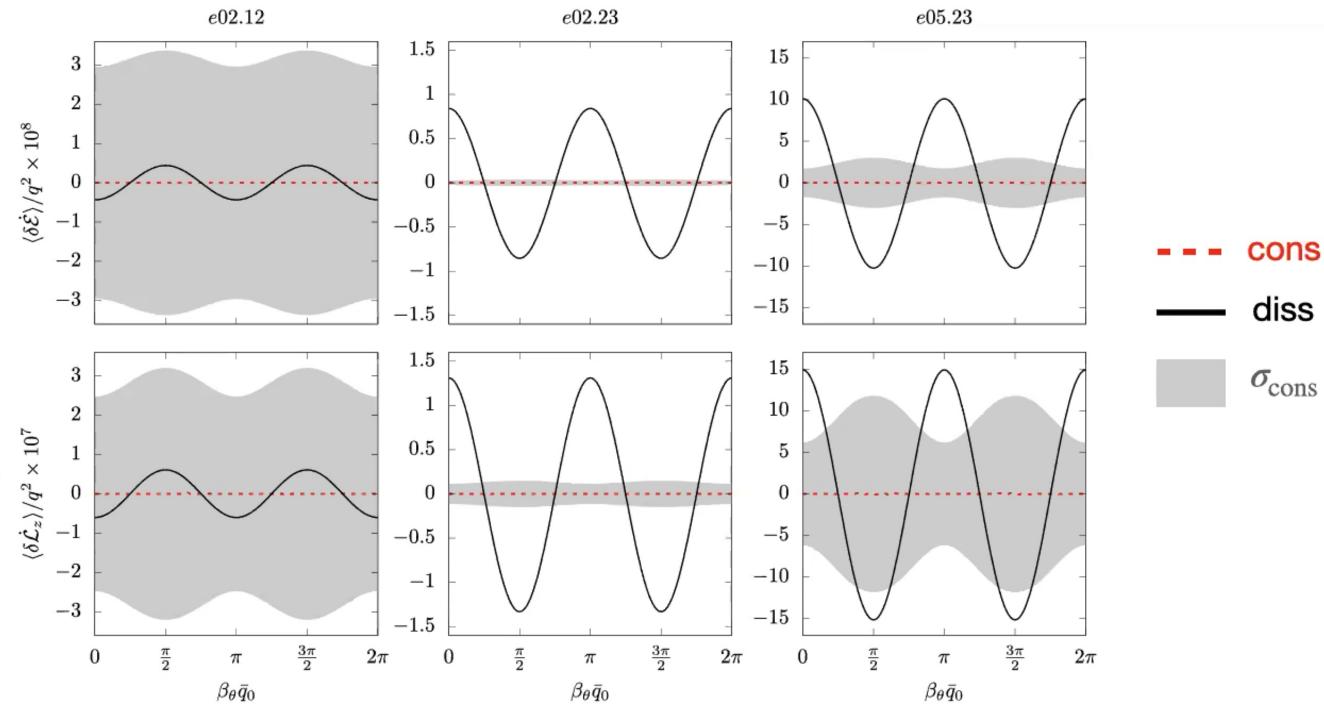
Energy & angular momentum

- Residual variations due to initial phase

$$\int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathcal{E}} \rangle^{\text{cons}} = 0$$

$$\int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathcal{E}} \rangle^{\text{diss}} = \text{const}$$

$$\langle \delta \dot{\mathcal{E}} \rangle = \langle \dot{\mathcal{E}} \rangle - \int_0^{2\pi} \frac{d\bar{q}_0}{2\pi} \langle \dot{\mathcal{E}} \rangle$$

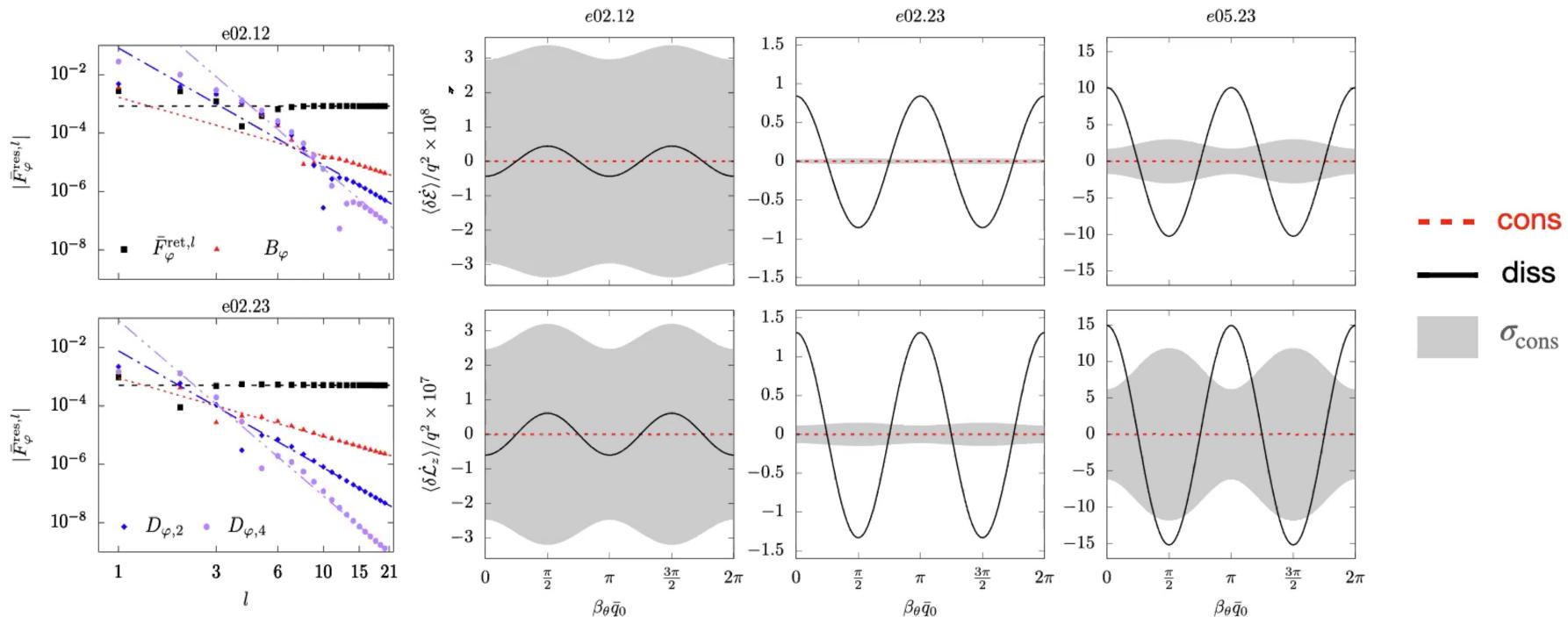


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Energy & angular momentum

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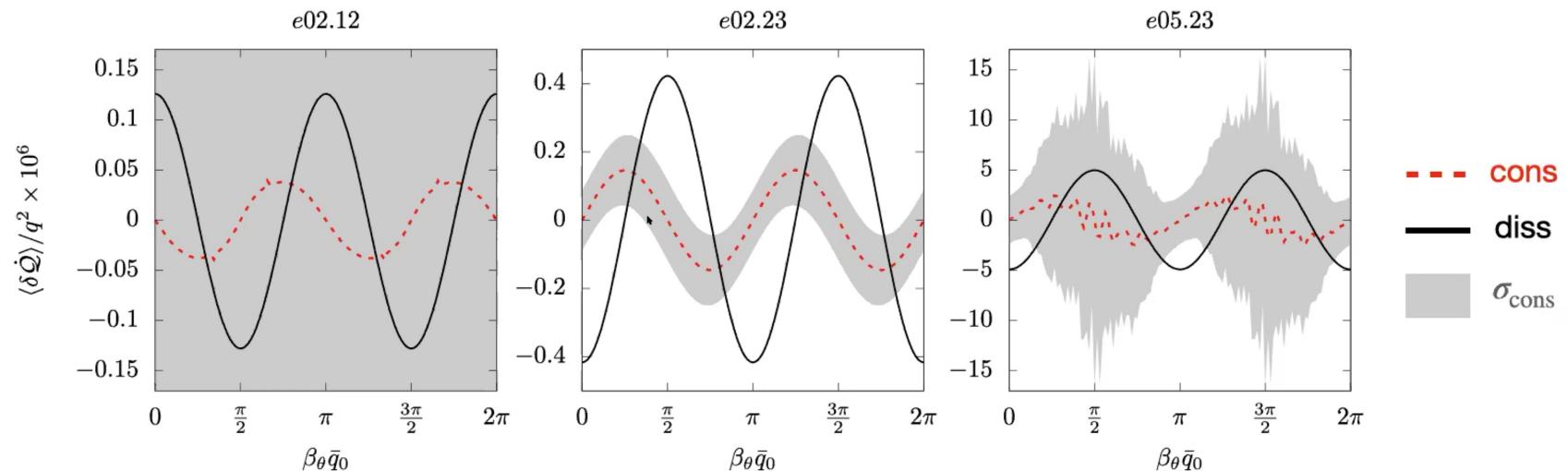


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Secular evolution of Carter constant

- Residual variations due to initial phase



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Secular evolution of Carter constant

- Preliminary results w/ new, higher-order regularization scheme
 - 2 new improvements
 1. Directly regularize $\langle \dot{Q} \rangle$
 2. Incorporate higher-order regularization parameters obtained by Anna Heffernan

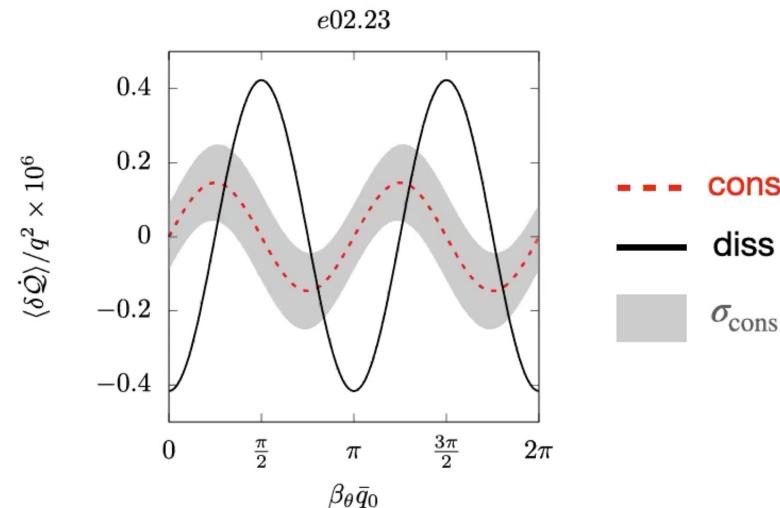


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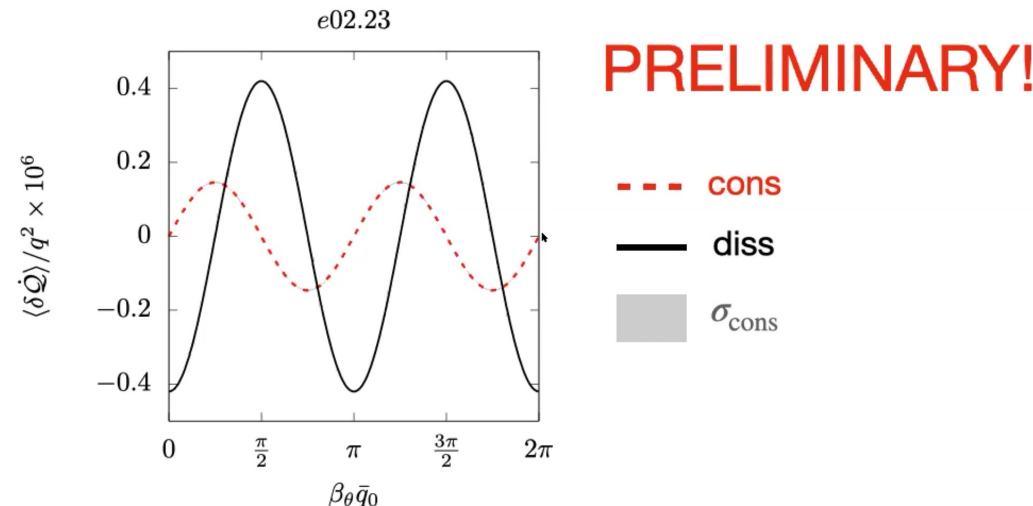


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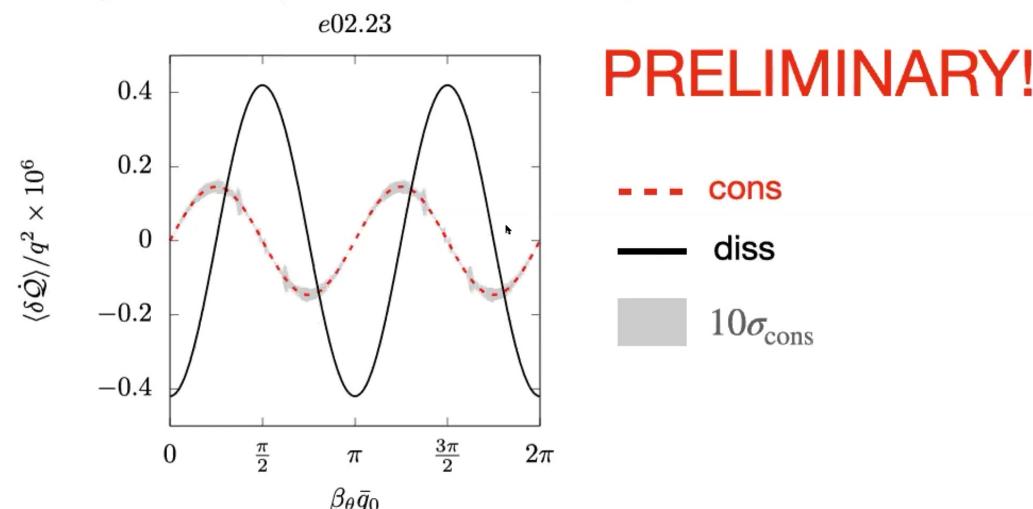


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Conclusion

- **Impact of resonances on secular evolution**
 - Flux-balance laws preserved during resonances (as expected)
 - Numerical evidence that conservative perturbations contribute to secular evolution of Carter constant $\langle \dot{\mathcal{Q}} \rangle$ during $r\theta$ -resonances
 - Regularization procedures required to calculate post-1/2 adiabatic corrections
 - Errors in conservative self-force will accumulate over resonant timescale $T_{\text{res}} \sim M\epsilon^{-1/2}$
 - More accurate conservative calculations requires higher-order regularization schemes
- **Future work**
 - Verify preliminary results, test other resonant orbits
 - Calculate how including conservative perturbations impacts resonant “jumps” in actions/frequencies
 - Test whether these results extend to the gravitational case



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