Title: Importance of tidal resonances in EMRIs

Speakers: Priti Gupta

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 11, 2021 - 10:45 AM

URL: http://pirsa.org/21060075

Abstract: In recent work, tidal resonances induced by the tidal field of nearby stars or black holes have been identified as potentially significant in the context of extreme mass-ratio inspirals (EMRIs). These resonances occur when the three orbital frequencies describing the orbit are commensurate. During the resonance, the orbital parameters of the small body experience a â€˜jumpâ€™ leading to a shift in the phase of the gravitational waveform. We study how common and important such resonances are over the entire orbital parameter space. We find that a large proportion of inspirals encounter a low-order tidal resonance in the observationally important regime.

Importance of tidal resonances in EMRIs

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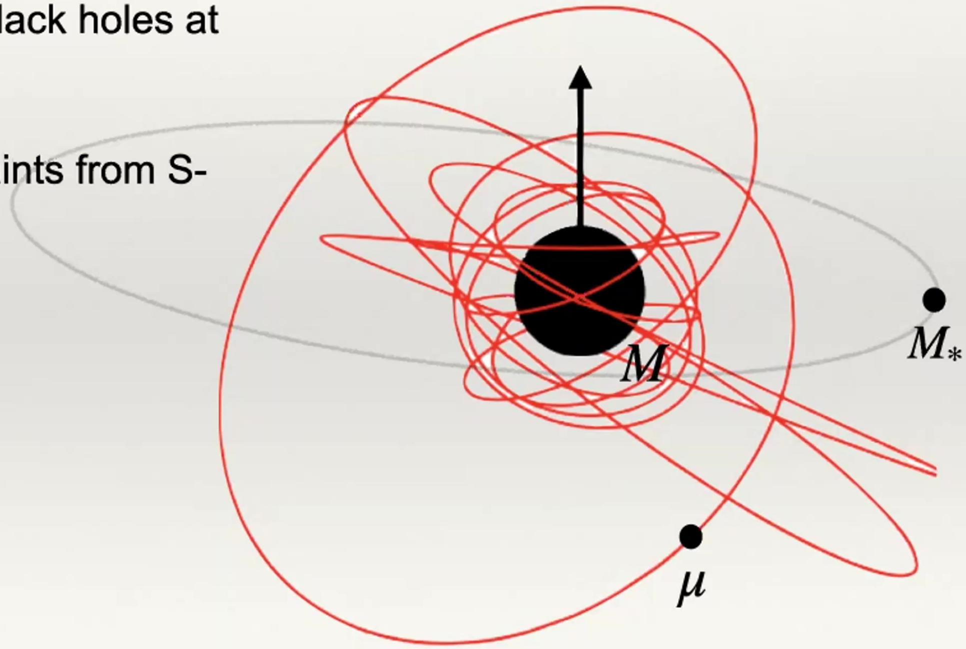
24th CAPRA meeting - 7-11 June 2021

Tidal perturbers near EMRIs

- Mass segregation + dynamical friction so that more massive black holes sink to the centre [Emami & Loeb `19]: $20 - 30\,M_\odot$ black holes at distance ~ 2 - 5 AU from Sgr A* .

- Observational signature: Constraints from S-stars.

arXiv: 2002.02341, 0810.4674



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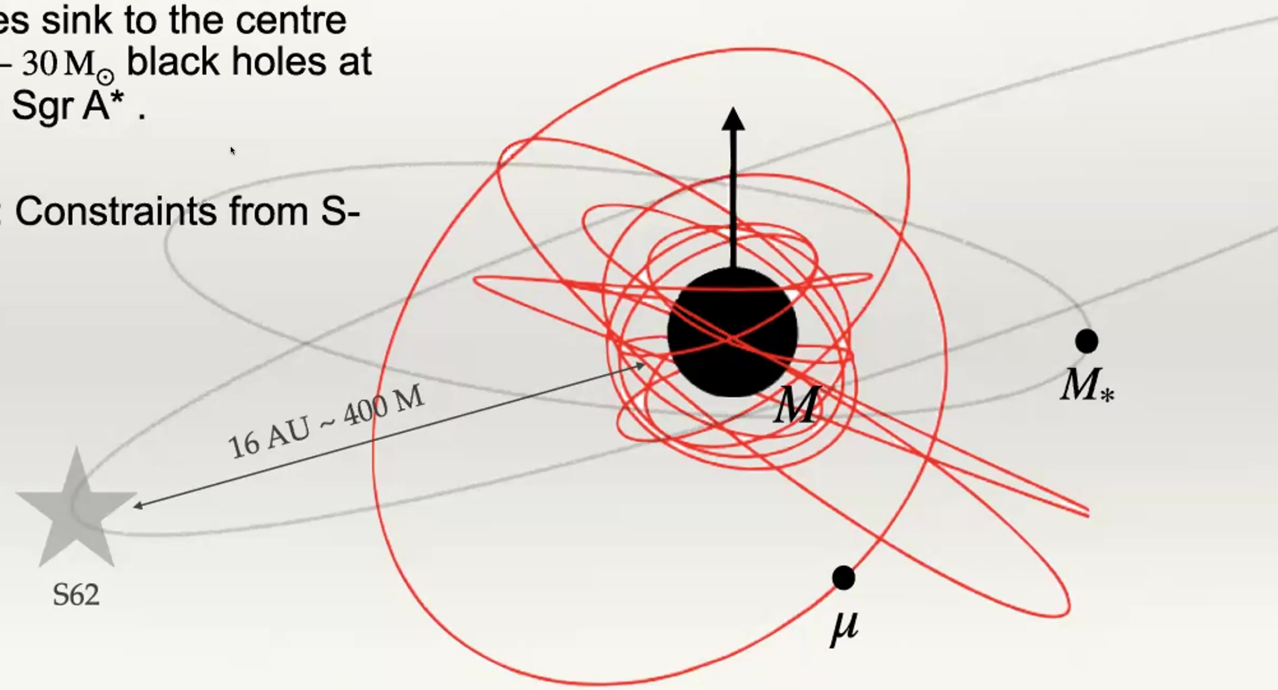

Introducing a perturber...

An EMRI orbit deviates due to the gravitational self-force and the tidal field from nearby stars and BHs.

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \epsilon\, g^{(1)}_{i,\mathrm{td}}(\underline{q_\phi}, q_\theta, q_r, \mathbf{J}) + \eta\, g^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)\,,$$

$$\frac{dJ_i}{d\tau} = \epsilon\, G^{(1)}_{i,\mathrm{td}}(\underline{q_\phi}, q_\theta, q_r, \mathbf{J}) + \eta\, G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)\,.$$

$\eta = \mu/M$

$\epsilon = \dfrac{M^2 M_*}{R^3}$

Characterizes the strength of the tidal field produced by the pertuber M_*

Tidal Resonance

- Adiabatic approximation → Fourier Domain + Averaging → Tidal Resonance condition

$$\frac{dJ_i}{d\tau} \approx \eta \langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) \rangle + \epsilon \langle G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) \rangle$$

$$G^{(1)}_i(q_\phi, q_\theta, q_r, \mathbf{J}) = \sum_{m,k,n} G^{(1)}_{i,mkn}(\mathbf{J}) e^{i\underbrace{(mq_\phi + kq_\theta + nq_r)}}$$

Rapidly oscillating for most index pairs m,*k*,*n*

$$\langle G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) \rangle = G^{(1)}_{i,\mathrm{tide},000}(\mathbf{J})$$

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$$n\omega_r + k\omega_\theta + m\omega_\phi = 0$$

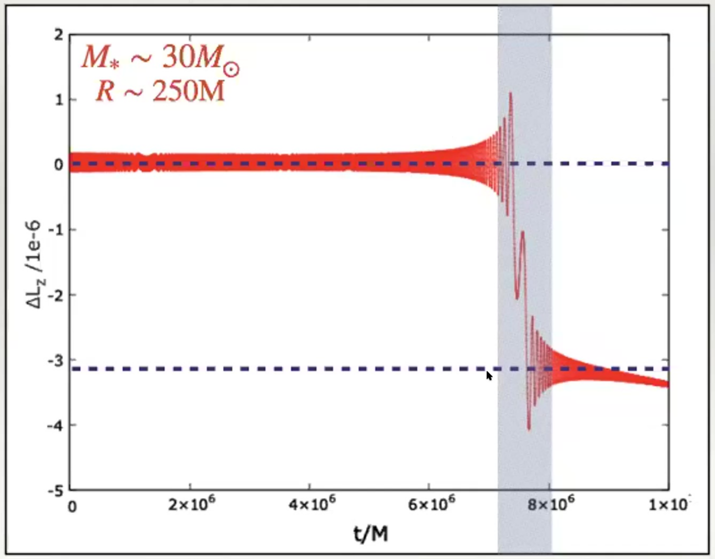
Tidal resonance condition

Why worry about resonances?

$$\langle G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J})\rangle = G^{(1)}_{i,\mathrm{tide},000}(\mathbf{J}) \;+\; G^{(1)}_{\mathrm{tide,nkm}}(\mathrm{J})$$

- Kick size is typically small $\mathcal{O}(\epsilon\eta^{-1/2})$ but if encountered early in the inspiral → significant dephasing $\mathcal{O}(\epsilon\eta^{-3/2})$.

Sample Resonance (n:k:m) = (3:0:-2)



Treatment: Tidally perturbed Kerr

- We need perturbation to the central BH's spacetime due to the tidal field.

> Metric of tidally perturbed Kerr from [Gonzales + Yunes, 2005]

In our work,

> We choose tidal perturber on the equatorial plane and consider its quadrupolar nature.

>Assumes tidal field is stationary $T_{td} >> T_{Res}$

Given the metric, we can compute the induced acceleration and corresponding changes in L_z & Q.

$$a^{\alpha} = -\frac{1}{2}(g^{\alpha\beta}_{\rm Kerr} + u^{\alpha}u^{\beta})(2h_{\beta\lambda;\rho} - h_{\lambda\rho;\beta})u^{\lambda}u^{\rho}$$

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$T_{td} < T_{Res}$

$m(\omega_\phi \pm \Omega_{\phi,td}) + k\omega_\theta + n\omega_r = 0$

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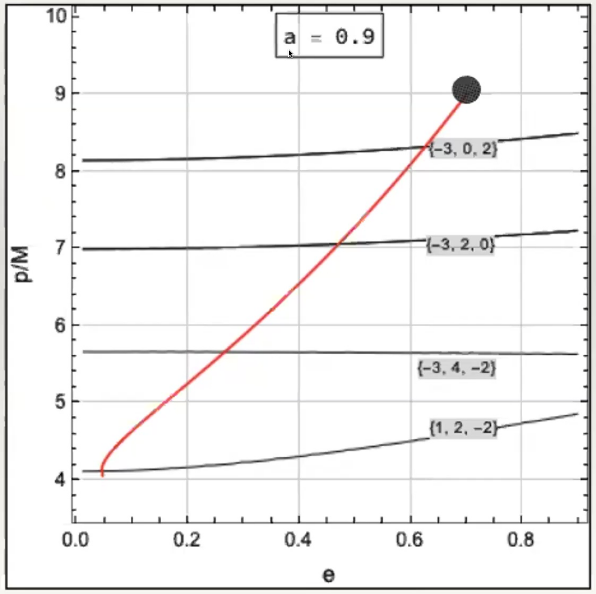
Resonances during inspiral

- Every inspiral encounters at least one of these resonances during final year of inspiral.
- The time of resonance during the inspiral depends strongly on binary parameters.

$$n\omega_r + k\omega_\theta + m\omega_\phi = 0$$

$$\frac{a_{\rm semi}}{M} < 20 \times \left(\frac{M}{4\times10^6 M_\odot}\right)^{-2/3} \left(\frac{f_{\rm LISA}}{10^{-4}{\rm Hz}}\right)^{-2/3}.$$



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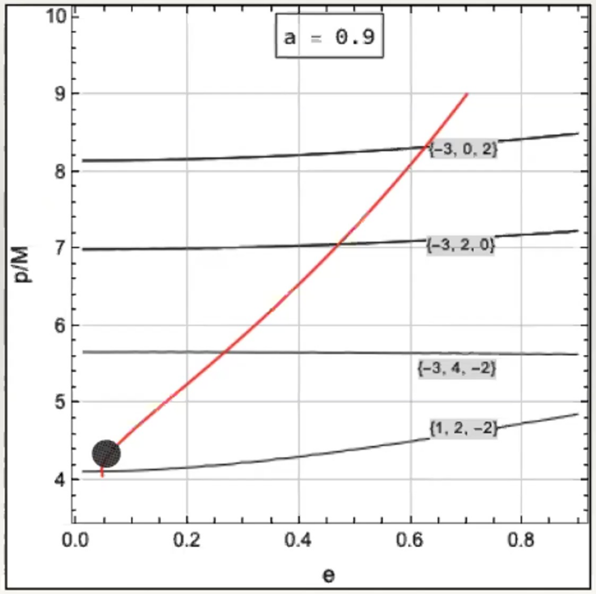
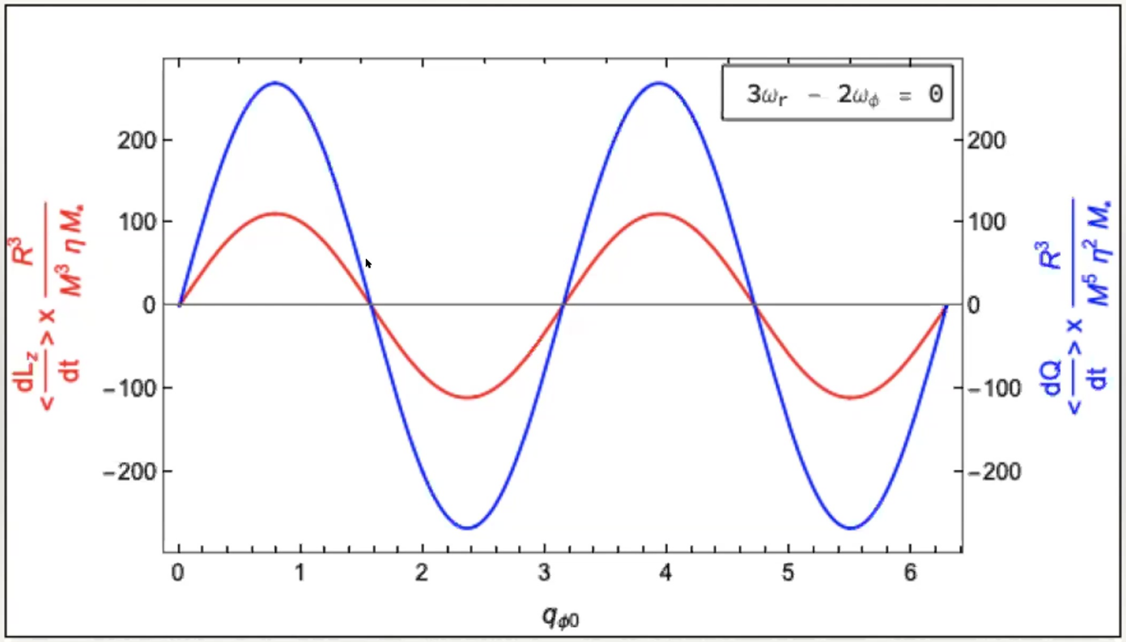


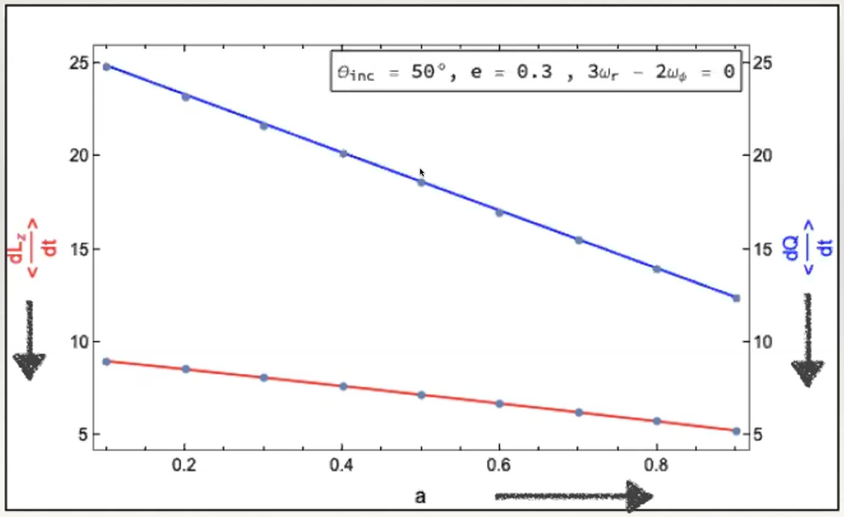
Sensitive dependence on phase



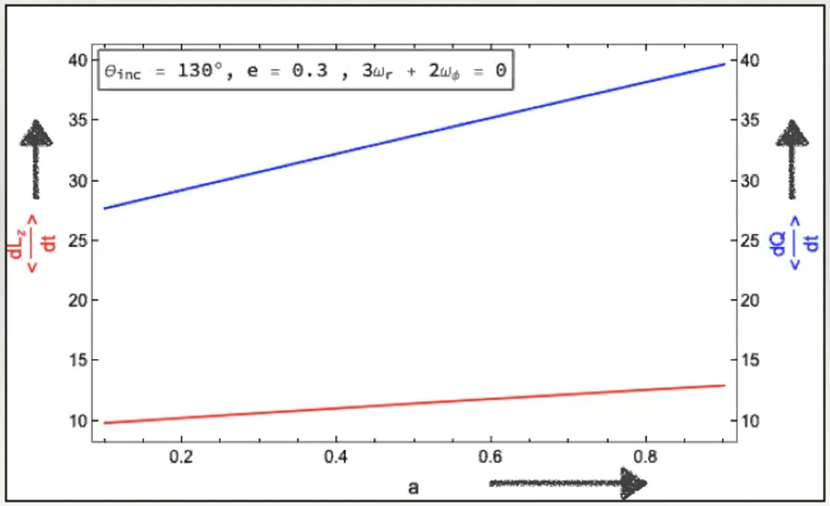
Trends followed by tidal resonances

We can see how resonance strength depends on the orbital parameters : {**a**,**p**,e,x}

Prograde

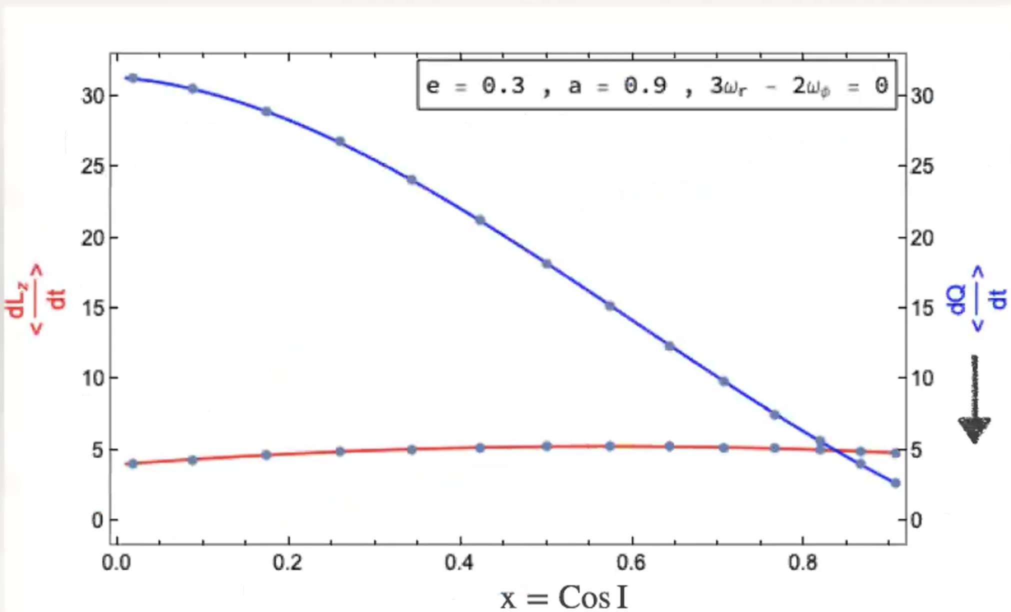


Retrograde



Trends followed by tidal resonances

Dependence on the orbital inclination: **x**



Inclination decreasing

Impact on orbital phase of GWs

To estimate the effect, two orbits are evolved and compared

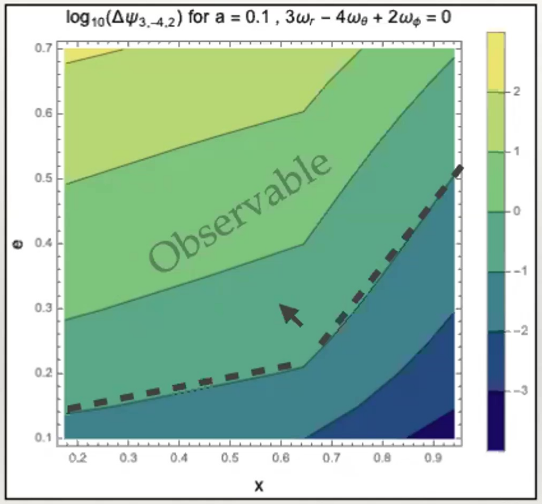
$$\{E, Q, L_z\} \to \omega_\phi^{(1)} \quad \text{versus} \quad \{E, Q + \Delta Q, L_z + \Delta L_z\} \to \omega_\phi^{(2)}$$

$$\Delta\Psi := \int_0^{T_{\mathrm{plunge}}} 2\Delta\omega_\phi dt$$

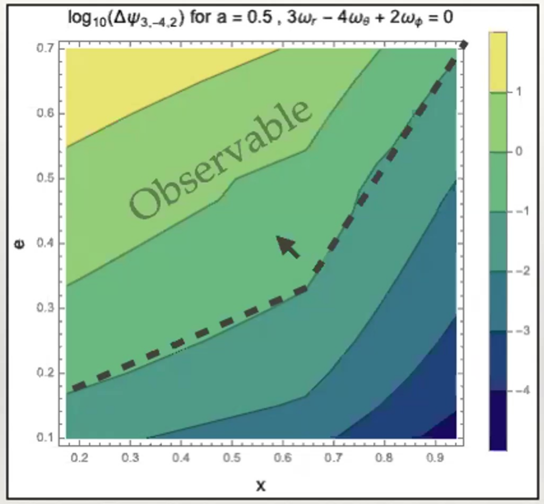
Phase resolution of LISA. $\Delta\psi \sim \mathcal{O}(1)$

Parameter Survey - Prograde Orbits

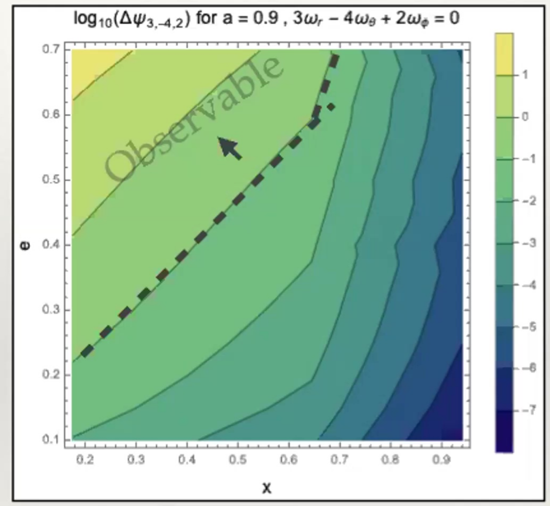
a = 0.1



a = 0.5



a = 0.9



$$\Delta\Psi'_{nkm} = \Delta\Psi_{nkm}\left(\frac{M'}{M}\right)^{7/2}\left(\frac{\mu'}{\mu}\right)^{-3/2}\left(\frac{M'_\star}{M_\star}\right)\left(\frac{R'}{R}\right)^{-3}.$$

$\mu = 30\,M_\odot, M = 4 * 10^6\,M_\odot$

$M_* \sim 30 M_\odot$ $R \sim 250\mathrm{M}$

$3\omega_r - 4\omega_\theta + 2\omega_\phi = 0$

Summary and Ongoing Work

arXiv: 2104.03422

- Tidal field can change EMRI waveforms significantly depending on the distance and mass of the tidal perturbers ----- hamper detection rate.
- Important to understand such environmental effects when constraining deviations from GR.
- Opportunity to learn about distribution of stellar mass objects that are close to SMBHs.

- Generalise position of tidal perturber and include dynamical tidal field.
- Study mismatching and parameter estimation bias from tidal resonances.