Title: Hybrid waveform for neutron star binaries

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Abstract: "We consider the motion of nonspinning, compact objects orbiting around a Kerr black hole with tidal couplings. The tide-indcued quadrupole moment modifies both the orbital energy and out-going fluxes, so that over the inspiral timescale there is an accumulative shift in the orbital and gravitational wave phase. Previous studies on compact object tidal effects have been carried out in the Post-Newtonian (PN) and Effetive-One-Body (EOB) formalisms. In this work, within the black hole perturbation framework, we propose to characterize the tidal influence in the expansion
of mass ratios, while higher-order PN corrections are naturally included. For the equatorial and circular orbit, we derive the leading order, frequency depedent tidal phase shift which agrees with the Post-Newtonian result at low frequencies but deviates at high frequencies. We also find that such phase shift has weak dependence (â‰¤ 10%) on the spin of the primary black hole. Combining this black hole perturbation waveform with the Post-Newtonian waveform, we propose a frequency-domain, hybrid waveform that shows comparable accuracy as the EOB waveform in characterizing
the tidal effects, as calibrated by numerical relativity simulations. Further improvement is expected as the next-leading order in mass ratio and the 2PN tidal corections are included. This hybrid approach is also applicable for generating binary black hole waveforms."

Black-Hole Perturbation Plus Post-Newtonian Theory: Hybrid Waveform for Neutron Star Binaries

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Black hole perturbation approach

1. Constructing GW waveform models are crucial for efficiently detecting compact binary systems.
2. Black hole perturbation approach is the leading solution to EMRI. But recent studies show that if the GW phase is written as the post-adiabatic expansion [Maarten, 2020; Le Tiec, 2011]

$$\psi(\omega) = \frac{\psi_0(\omega)}{\eta} + \psi_1(\omega) + \eta\psi_2(\omega) + \dots, \tag{1}$$

where ω is the orbital frequency and $\eta = m_1 m_2/(m_1 + m_2)^2$ is the symmetric mass ratio, Eq. (1) may be a fast-converging series even for equal-mass binaries,

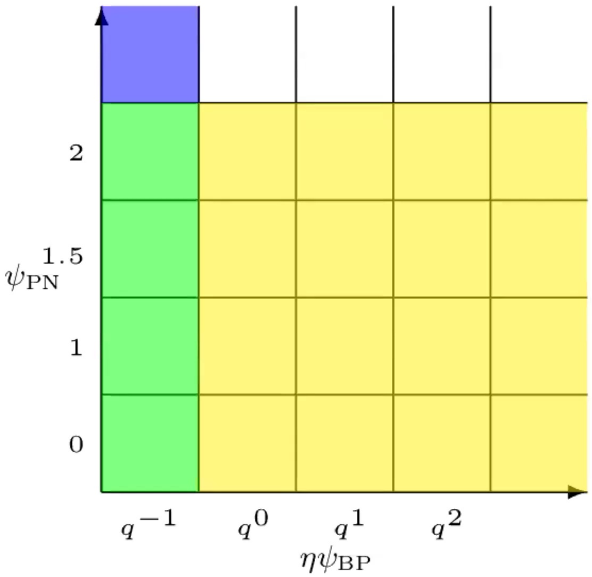
Hybrid Waveform

Tide-induced phase shift can be expanded in the velocity[Vines, 2011; Damour, 2012] and mass ratios

$$\eta\psi_{\rm BP} = \lambda(q^{-1}\psi_{\rm BP}^{-1} + \psi_{\rm BP}^{0} + \sum_{n\geq 1} q^n \psi_{\rm BP}^{n}) .$$

$$\eta\psi_{\rm PN} = v^5(\psi_{\rm 0PN} + v^2\psi_{\rm 1PN} + v^3\psi_{\rm 1.5PN} + v^4\psi_{\rm 2PN} + ...)$$

where $q = m_1/m_2 \leq 1$, $v = \sqrt{M/r}$, M is the total mass.



hybrid waveform:

$$\psi_{\rm hyd} = \psi_{\rm PN} + \psi_{\rm BP} - \psi_{\rm ovp} \qquad (2)$$

MPD equations

Considering the influence of quadrupole moment curvature coupling, the equations of motion of an extended body are [Steinhoff, 2010]

$$\frac{Dp_a}{d\tau} = -\frac{1}{6}\nabla_a R_{bcde} J^{bcde}, \tag{3}$$

$$p^a = m_0 u^a + \frac{4}{3} u_b R^{[a}{}_{bcd} J^{b]cde} \tag{4}$$

where tidal quadrupole moment Q^{ab} and tidal quadrupole deformations J^{abcd} are

$$Q^{ab} = -\lambda E^{ab},$$

$$J^{abcd} = -\frac{3m_0}{m_1^3} p^{[a} Q^{b][c} p^{d]},$$

with $m_1^2 = -p^a p_a$, $m_0 = -p_a u^a$.

Dynamical tide

The dimensionless dynamical tidal Love number $k_2 = \frac{2\lambda}{3R^5}$ [Hinderer, 2016] is

$$k_l^{\rm dyn} = k_l \left[a_l + \frac{b_l}{2} \left(\frac{Q^{\rm DT}_{m=l}}{Q^{\rm AT}_{m=l}} + \frac{Q^{\rm DT}_{m=-l}}{Q^{\rm AT}_{m=-l}} \right) \right] \tag{5}$$

where

$$\begin{aligned} \frac{Q_m^{\rm DT}}{Q_m^{\rm AT}} &= \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} + \frac{\omega_f^2}{2(m\Omega)^2 \epsilon_f \Omega'_f (\phi - \phi_f)} \\ &\pm \frac{i\omega_f^2}{(m\Omega)^2 \sqrt{\epsilon_f}} e^{\pm i\Omega'_f \epsilon_f (\phi-\phi_f)^2} \int_{-\infty}^{\sqrt{\epsilon_f}(\phi-\phi_f)} e^{\mp i\Omega'_f s^2}\, ds\,. \end{aligned} \tag{6}$$

where $a_2 = 1/4$, $b_2 = 3/4$, $\Omega^2 = M/r^3$ and ϵ_f is the ratio between the orbital timescales and the gravitational radiation reaction timescales, ϕ is the orbital phase and ϕ_f denotes the orbital phase evaluation at $\omega = \omega_f$.

Conserved quantities

When the internal quadrupole moment is included, we can construct conserved quantities for extended bodies in the Kerr spacetime based on the Killing vector fields. The quantity [Steinhoff, 2010]

$$Q_\xi = p_a \xi^a \tag{7}$$

is conserved if ξ^a is a Killing vector, and a mass-like quantity

$$\mu = m_0 + \frac{\lambda}{4} E_{ab} E^{ab} + \mathcal{O}(\lambda^2) \tag{8}$$

is approximately constant if we neglect the higher order tidal effects.

equations of motion on the equatorial plane

From these three conserved quantities, we can obtain the explicit equations of motion on the equatorial plane:

$$\left(\frac{dt}{d\tau}\right) = \frac{E}{\mu\Sigma}\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\right] + \frac{aJ}{\mu}\left(1 - \frac{r^2+a^2}{\Delta}\right) + F_1(r, a, E, J)$$

$$\left(\frac{d\phi}{d\tau}\right) = \frac{J}{\mu\Sigma} + \frac{aE}{\mu}\left(\frac{r^2+a^2}{\Delta} - 1\right) - \frac{a^2 J}{\mu\Delta} + F_2(r, a, E, J)$$

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{\left[E(r^2+a^2) - aJ\right]^2}{\Sigma^2} - \frac{\Delta\left[r^2 + (J - aE)^2\right]}{\Sigma^2} + F_3(r, a, E, J)$$

Binding Energy and Energy Flux

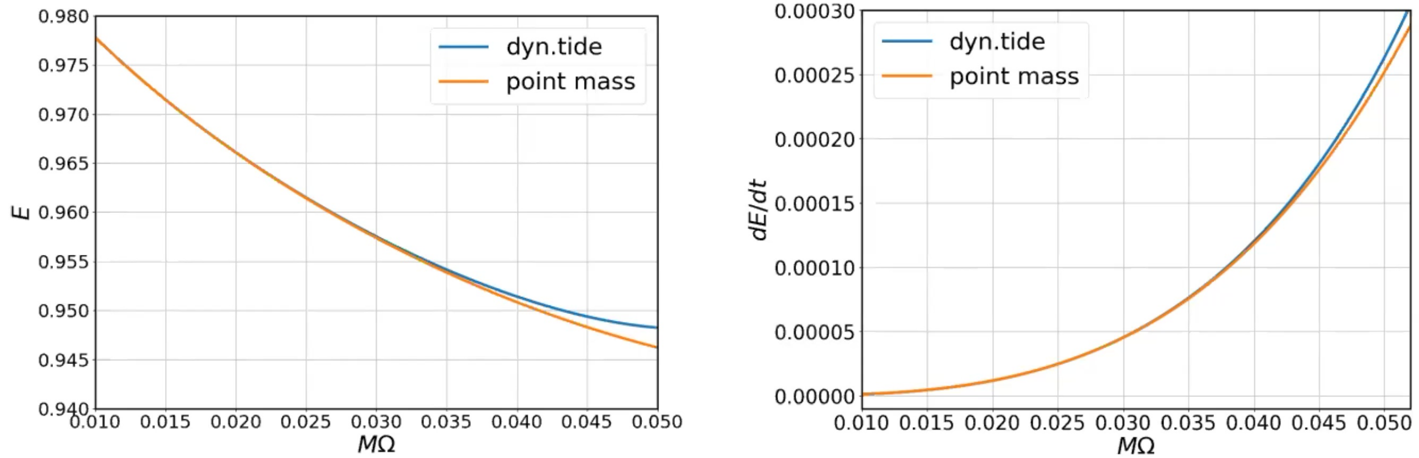


Fig. 1: The energy and energy flux computed for an equal-mass, black hole-neutron star binary. The neutron stars have a polytropic equation of state $P = K\rho^{\Gamma}$, with $\Gamma = 2$, $K = 101.45$. The neutron star mass is $m = 1.4M_{\odot}$ and the radius is $R = 14.4\mathrm{km}$.

Gravitational wave Phase

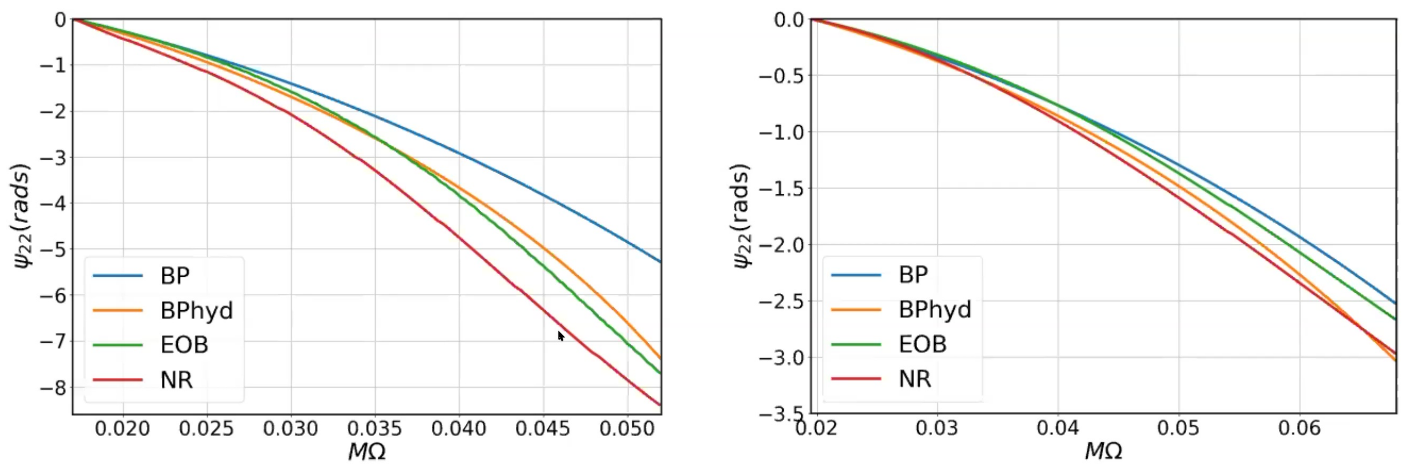


Fig. 2: Tidal phases of Perturbation theory, the hybrid method, the EOB framework and numerical relativity simulation for an equal-mass, black hole-neutrons star system. Left panel: Equal mass ratio. Right panel: BHNS mass ratio 2:1.

Influence of Spin Parameter

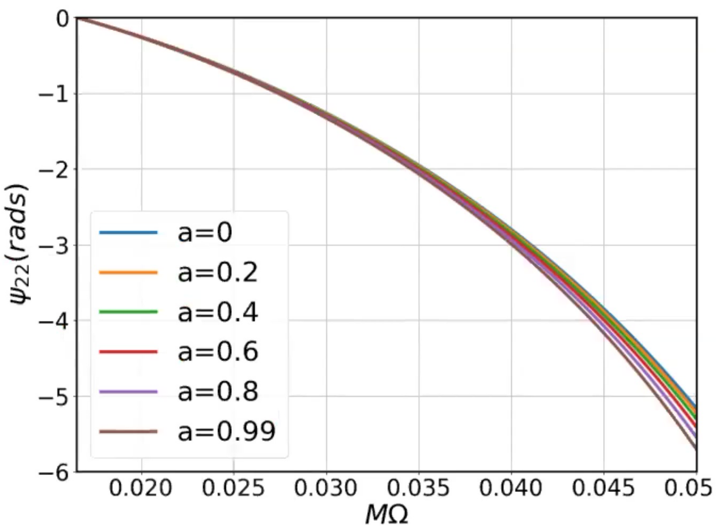


Fig. 3: Tidal phases in the black hole perturbation waveform with spin ranging from $a = 0$ to $a = 0.99$ for six equal-mass, black hole-neutron star systems.

Future works

* Apart from the above two scenarios, we still need more detailed comparison and characterization. For example, consider other equations of state of neutron star.
* It is necessary to work out the $\psi_{\rm BP}^{(0)}$ and beyond-2PN corrections to achieve better accuracy.

Thank you!