Title: Discussion: Scattering Amplitudes

Speakers:

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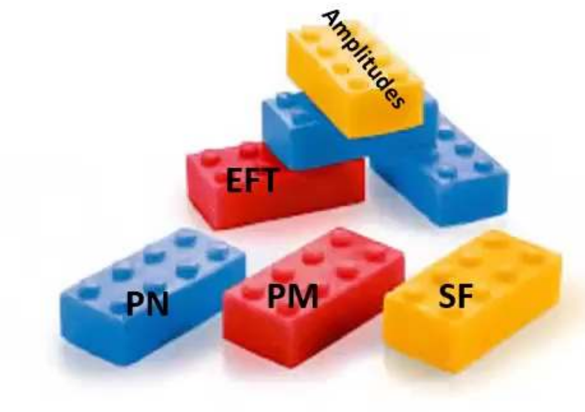
Scattering:
Discussion Session

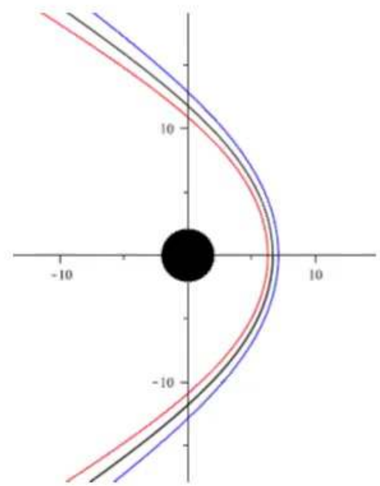
The 24° Capra meeting, June 10 , 2021

Different approaches to a two-body scattering process:



Conservative scattering process using conservative 1SF

Scattering process including radiation-reaction effects using dissipative SF



Challenges in developing analytic, PN, conservative, 1SF calculations along hyp orbits

In principle …we know what to do…that is, we need to generalize the Barack-Sago prescriptions valid for ellipticlike orbits.
But… what about subtleties?

Hyperbolic motion on the Schwarzschild background

Well known

$$\bar{u}^t = \frac{dt}{d\tau} = \frac{\bar{E}}{f}, \qquad (\bar{u}^r)^2 = \left(\frac{dr}{d\tau}\right)^2 = \bar{E}^2 - f\left(1 + \frac{\bar{L}^2}{r^2}\right), \qquad \bar{u}^\phi = \frac{d\phi}{d\tau} = \frac{\bar{L}}{r^2}$$

geos

$$f = 1 - 2M/r \qquad \bar{E} = E/m, \quad j = L/(mM) \equiv \bar{L}/M$$

$$r = \frac{Mp}{1+e\cos\chi}, \quad p = \frac{1}{u_p}$$

$$\bar{E} = \sqrt{\frac{(p-2)^2 - 4e^2}{p(p-3-e^2)}} = \sqrt{\frac{(1-2u_p)^2 - 4e^2u_p^2}{1-3u_p-u_pe^2}}$$

$$j = \frac{p}{\sqrt{(p-3-e^2)}} = \frac{1}{\sqrt{u_p(1-3u_p-u_pe^2)}},$$

$$\chi \in [-\chi_{(\max)}, \chi_{(\max)}], \text{ with } \chi_{(\max)} = \arccos(-1/e)$$

$\bar{E} > 1 \quad e > 1$

$$M\frac{d\chi}{d\tau} = u_p^2 j(1+e\cos\chi)\sqrt{1-6u_p-2u_pe\cos\chi}, \qquad \frac{d\phi}{d\chi} = \frac{1}{\sqrt{1-6u_p-2eu_p\cos\chi}}$$

Elliptic functions

$$\phi(\chi) = \frac{\kappa}{\sqrt{eu_p}}\left[K(\kappa) - F\left(\cos\frac{\chi}{2}, \kappa\right)\right] \qquad \frac{\chi_{\rm scatt}}{2} = \phi(\chi_{(\max)}) - \frac{\pi}{2}$$

When doing perturbations...

$$T^{\mu\nu}(t,r,\theta,\phi) = \mu \int_{-\infty}^{+\infty} \delta^{(4)}(x - z_p(\tau)) u^\mu(\tau) u^\nu(\tau) d\tau = \frac{\mu}{u^t r^2} \delta(r - r_p(t)) \delta\left(\theta - \frac{\pi}{2}\right) \delta(\phi - \phi_p(t)) u^\mu(t) u^\nu(t)$$

The background geos are used at the rhs of the EE

The solutions of the geos should be written as simple as possible (this is just the first step)

→ difficulty in handling ellip functions! («moderately» high PN expansion of these solutions?)

→ Is a large-eccentricity expansion always necessary? (for Fourier domain computations only?)

→ Is the χ angle still the best parametrization as it was the case for bound motion?

$$r = \frac{Mp}{1 + e\cos\chi}, \qquad p = \frac{1}{u_p}$$

Perturbation eqs. a la RWZ or a la Teukosky→ with the perturbed, regularized metric one would like to see how the geodesics modify

$$\frac{du_\alpha}{d\tau} = \frac{1}{2} u^\mu u^\nu \frac{\partial}{\partial x^\alpha} (\bar{g}_{\mu\nu} + h_{\mu\nu}) \Big|_{\rm orbit}$$

(perturbed motion still in the equatorial plane)

To start the RWZ or Teukolsky procedure one needs PN solutions and MST solutions

- → **Now ω runs over an infinite spectrum [in the bound case we had only a few ω s…]**
- → **One expects that certain frequencies will dominate the spectrum: ω =v/b**
- → **In general one should study the contributions arising from both low frequencies and high frequencies, and use then the method of regions.**

Generalizing the Barack-Sago procedure to hyp orbits

$$u = u^\alpha \partial_\alpha = \frac{dx^\alpha}{d\tau}\partial_\alpha = (\bar{u}^\alpha + \delta u^\alpha)\partial_\alpha$$

$$\delta u^t = \frac{\delta E}{f}\,, \qquad \delta u^\phi = \frac{\delta L}{r^2}\,, \qquad \delta u^r = \frac{\bar{u}^t \delta E - \bar{u}^\phi \delta L - \frac{1}{2}h_{00}}{\bar{g}_{rr}\bar{u}^r}$$

$$\delta u^\alpha = \hat{\delta} u^\alpha + \frac{1}{2}h_{00}\bar{u}^\alpha\,,$$

BS normalization: convenient

$$\hat{\delta}E = \delta E - \frac{1}{2}\bar{E}h_{00}\,,$$

$$\hat{\delta}u^r = \delta u^r - \frac{1}{2}\bar{u}^r h_{00}\,,$$

$$\hat{\delta}L = \delta L - \frac{1}{2}j h_{00}\,,$$

$$\boxed{\bar{g}_{rr}\bar{u}^r\hat{\delta}u^r = \bar{u}^t\hat{\delta}E - \bar{u}^\phi\hat{\delta}L\,.}$$

Equations for energy and angular momentum

$$\frac{d}{d\tau}\hat{\delta}E = -F_t\,, \qquad \frac{d}{d\tau}\hat{\delta}L = F_\phi \qquad F^\mu = -\frac{1}{2}(\bar{g}^{\mu\nu} + \bar{u}^\mu\bar{u}^\nu)\bar{u}^\lambda\bar{u}^\rho(2h_{\nu\lambda;\rho} - h_{\lambda\rho;\nu})$$

Interested in conservative effects…first…

$$F_\alpha^{\rm cons}(\chi) = \frac{1}{2}\left(F_\alpha(\chi) - F_\alpha(-\chi)\right)$$

$$\hat{\delta}E(\chi) = -\int_0^\chi F_t^{\rm cons}(\chi)\frac{d\tau}{d\chi}d\chi + \hat{\delta}E(0) \equiv \mathcal{E}(\chi) + \hat{\delta}E(0)\,,$$

$$\hat{\delta}L(\chi) = \int_0^\chi F_\phi^{\rm cons}(\chi)\frac{d\tau}{d\chi}d\chi + \hat{\delta}L(0) \equiv \mathcal{L}(\chi) + \hat{\delta}L(0)\,.$$

→ **How to fix the integration constants?**

Bound case reminder

- In the bound case one imposes the vanishing of $\hat{\delta} u^r$ [implying the vanishing of δu^r too] at the periastron ($\chi = 0$) and apoastron ($\chi = \pi$) and these two conditions fix both $\hat{\delta} E(0)$ and $\hat{\delta} L(0)$.

→ What about the most natural choice in the unbound case?

Minimum approach and χ max ?

$$\hat{\delta} E(\chi)\,, \qquad \hat{\delta} L(\chi)\,, \qquad \hat{\delta} u^r(\chi)\,;$$

All vanishing at -χ max ?

Necessity? And in the non-conservative case?

u^r vanishing at the minimum approach ?

Deep discussion by T. Damour already in 2009

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Gravitational self-force in a Schwarzschild background and the effective one-body formalism

Thibault Damour

Computing χ **from SF**

$$\frac{d\phi}{d\tau} = \bar{u}^\phi + \delta u^\phi$$
$$\frac{dr}{d\tau} = \bar{u}^r + \delta u^r$$

Advantages?
Local+nonlocal
High PN

$$\frac{d\phi}{d\chi} = \frac{d\phi}{dr}\frac{dr}{d\chi} = \frac{\bar{u}^\phi}{\bar{u}^r}\left(1 + \frac{\delta u^\phi}{\bar{u}^\phi} - \frac{\delta u^r}{\bar{u}^r}\right)\frac{dr}{d\chi}$$

$$\delta\phi(\bar{E}, j) = \delta\phi^{(0)}(\bar{E}, j) + \delta\phi^{(1)}(\bar{E}, j)$$

$$\delta\phi^{(1)}(\bar{E}, j) = \int_{-\chi_{\max}}^{\chi_{\max}} \frac{\hat{\delta}L}{r^2}\frac{d\tau}{d\chi}d\chi - \int_{-\chi_{\max}}^{\chi_{\max}} \frac{\bar{u}^\phi}{\bar{u}^r}\hat{\delta}u^r\frac{d\tau}{d\chi}d\chi$$

WHEELER'S FIRST MORAL PRINCIPLE. ***Never make a calculation until you know the answer.*** Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every puzzle. Courage: no one else needs to know what the guess is. Therefore make it quickly, by instinct. A right guess reinforces this instinct. A wrong guess brings the refreshment of surprise. In either case life as a spacetime expert, however long, is more fun!

Comments by L. Barack, O. Long, C. Whittal, C. Kavanagh...