Title: Gauge Invariant Self-Force Calculations with a Spinning Secondary

Speakers: Josh Mathews

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Abstract: "We calculate the first order metric perturbation to a Schwarzschild background spacetime induced by a spinning secondary body in the Regge-Wheeler and Zerilli gauges. In particular we specialise to a secondary with spin (anti-)aligned to the total orbital angular momentum in a quasi-circular orbit. From the metric perturbation we can calculate gauge invariant self-force quantities such as Detweilerâ€™s redshift invariant and compare with known PN results. In doing so we present the first strong field calculation of a conservative self-force quantity with a spinning secondary and emphasise:

1) The treatment of the additional spin term to the singular field by deriving additional tensor harmonic regularisation parameters.

2) Parametrising at fixed frequency and practically extracting the linear in spin contribution to the metric perturbation."

Gauge Invariant Self-Force Calculations with a Spinning Secondary

Josh Mathews

Barry Wardell
Adam Pound

UCD DUBLIN
UCD RELATIVITY GROUP

IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

Schwarzschild Perturbations in a Nutshell

The RWZ Formalism in the Frequency Domain

Looking for first order MP induced by secondary body's $T^{\alpha\beta}$.

$$\mathbf{g}_{\alpha\beta} = g_{\alpha\beta} + \epsilon h_{\alpha\beta} + O(\epsilon^2)$$

LEFE manipulated - solve the RWZ equation for 'master functions'.

$$\left[\frac{\partial^2}{\partial r_*^2} - V_l(r) + \omega^2\right] R_{lm}(r) = Z_{lm}(r)$$

Spherical harmonic decomposition, frequency domain.

bhptoolkit.org

Reconstruct metric from master functions, sum over modes.

$$h_{aB}\left(x^\mu\right) = \sum_{\ell,m} h_a^{\ell m} X_B^{\ell m}$$

Gauge invariant self-force quantities from the metric perturbation + derivative.

(Martel + Poisson 2005)

Spinning Secondary Bodies

Specialised to circular orbit with (anti-)aligned spin. T SSC.

Fixed Radius

$$(r_0, \Omega_0 + \sigma\Omega_\sigma)$$

$$u^\alpha = u_0^\alpha + \sigma u_\sigma^\alpha + O[\sigma^2]$$

- Requires a solving an extra 4th order ODE

Fixed Frequency

$$(r_0 + \sigma r_\sigma, \Omega_0)$$

$$u^\alpha = u_0^\alpha + O[\sigma^2]$$

- Directly Comparable with PN results
- Requires an extra derivative

$$\left[\frac{\partial^2}{\partial r_*^2} - V_l(r) + \omega^2\right] R_{lm}(r) = Z_{lm}(r)$$

Helical gauge condition: $(\partial_t + \Omega\partial_\phi)\xi^\alpha = 0$

Dissipative Calculation: Flux Balance

$$\frac{D\mathcal{E}}{d\tau} = \frac{1}{2}u^\alpha u^\beta \mathcal{L}_\xi h^{\mathcal{R}}_{\alpha\beta} - \frac{1}{2\mu}S^{\gamma\delta}u^\beta \nabla_\delta \mathcal{L}_\xi h^{\mathcal{R}}_{\gamma\beta} = u^t \mathcal{F}$$

$y = (M\Omega)^{2/3}$	$u^t\mathcal{F}$	$\frac{D\mathcal{E}}{d\tau}$	Δ_{rel}
1/5	-9.6454026694126*10^-4	-9.6454026694126*10^-4	4.0*10^-22
1/10	4.2421081207*10^-6	4.2421081207*10^-6	1.1*10^-24
1/100	-8.2656071209178*10^-13	-8.2656071209178*10^-13	1.0*10^-31

(Akcay et al 2019)

*In agreement with Table II: Ackay et al 2019 - which in turn was compared with the equivalent PN results.

Detweiler-Whiting Singular field

Lorenz gauge. Not yet specialised to an SSC, orbital configuration or ST.

$$\bar{h}^{S(\mu)}_{ab}(x) = 2\left[\frac{U(x,x')_{aba'b'}\, u^{a'} u^{b'}}{|\sigma_{c'}(x,x')\, u^{c'}|}\right]\Bigg|_{x'=x_{(adv)},x_{(ret)}} + 2\int_{\tau_{(ret)}}^{\tau_{(adv)}} V(x,z(\tau'))_{aba'b'}\, u^{a'} u^{b'} d\tau'$$

$$\begin{aligned}\bar{h}^{S(\sigma)}_{ab}(x) = 2 &\left[\frac{u^{p'}\nabla_{p'}U(x,x')_{aba'b'}\,\sigma_{\rho'}u^{(a'}\tilde{S}^{b')\rho'} + U(x,x')_{aba'b'}\, u^{q'}\sigma_{\rho' q'}u^{(a'}\tilde{S}^{b')\rho'}}{|\sigma_{c'}u^{c'}|\,\sigma_{d'}u^{d'}}\right]\Bigg|_{x'=x_{(adv)},x_{(ret)}} \\ -2&\left[\frac{\left[\nabla_{\rho'}U(x,x')_{aba'b'} + V(x,x')_{aba'b'}\,\sigma_{\rho'}\right]u^{(a'}\tilde{S}^{b')\rho'}}{|\sigma_{c'}u^{c'}|}\right]_{x'=x_{(adv)},x_{(ret)}} \\ -2&\left[\frac{U(x,x')_{aba'b'}\,\sigma_{\rho'}u^{(a'}\tilde{S}^{b')\rho'}\sigma_{p'q'}u^{p'}u^{q'}}{|\sigma_{c'}u^{c'}|\left(\sigma_{d'}u^{d'}\right)^2}\right]\Bigg|_{x'=x_{(adv)},x_{(ret)}} \\ -2&\int_{\tau_{(ret)}}^{\tau_{(adv)}} \nabla_{\rho'}V(x,z(\tau'))_{aba'b'}\, u^{(a'}\tilde{S}^{b')\rho'} d\tau'\end{aligned}$$

(Heffernan et al 2013)

Tensor Harmonic Regularisation Parameters

- Regularisation parameters from suitable expansion and (Scalar/Tensor) harmonic decomposition of the singular field.
- RPs included - mode-sum converges.

$$h_{\phi\phi}^{[-1]} = \pm \frac{\mu\sigma M^{3/2}}{\sqrt{r_0 - 3M}} \pm \left[\frac{\mu\sigma M^{3/2}}{\sqrt{r_0 - 3M}}\right]_{\ell \geq 2}$$

$$h_{\phi\phi}^{R} = \sum_{\ell=0}^{\infty} \left[h_{\phi\phi}^{\ell} - h_{\phi\phi}^{[-1]}(2\ell+1) - h_{\phi\phi}^{[0]} - \ldots\right]$$

(Wardell et al 2015)

Detweiler's Redshift Invariant

Circular orbit with (anti-)aligned spin.

- Conservative gauge invariant GSF quantity. In <u>perturbed</u> spacetime:

$$z_1 = \frac{1}{u^t} \qquad z_1(y) = -\frac{1}{2\sqrt{1-3y}}\left[h_{kk}(y) + \sigma M y^{1/2}\partial_r h_{kk(0)}(y)\right] \qquad h_{kk} = h_{\alpha\beta}k^\alpha k^\beta$$

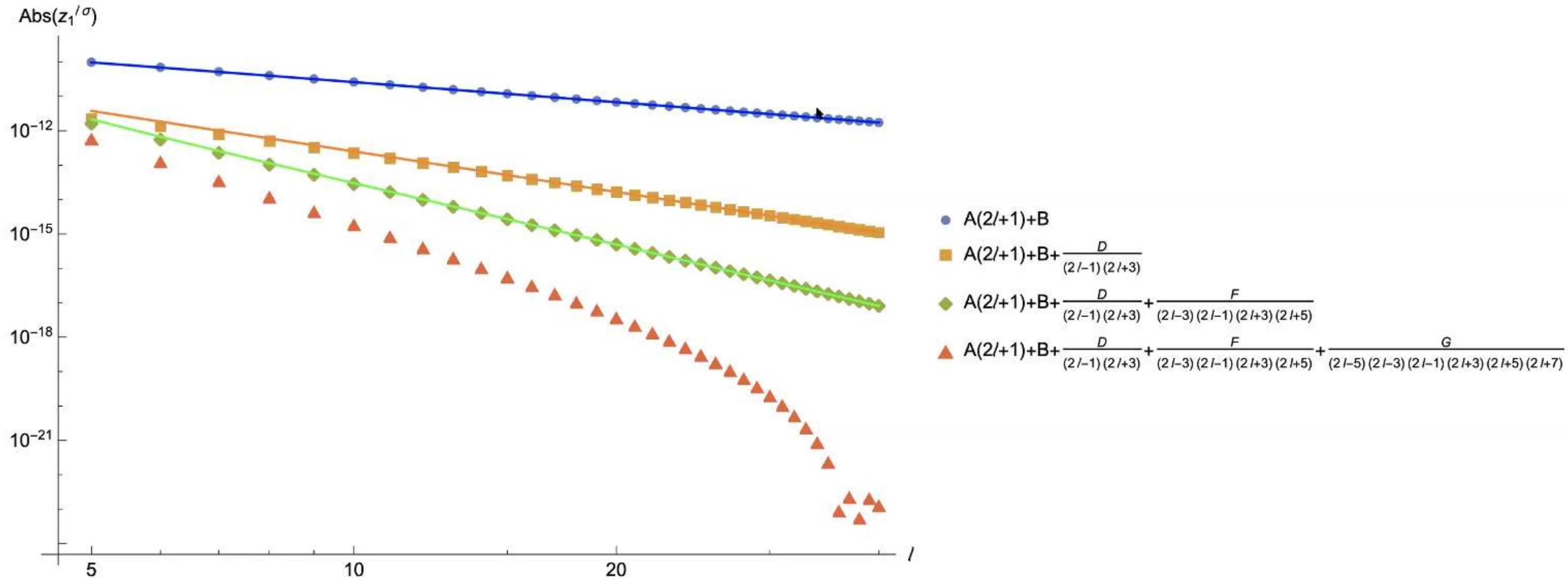
- Also gauge invariant in fixed radius parametrisation - but no longer meaningful as above.

- Extra term cancels under linearisation:

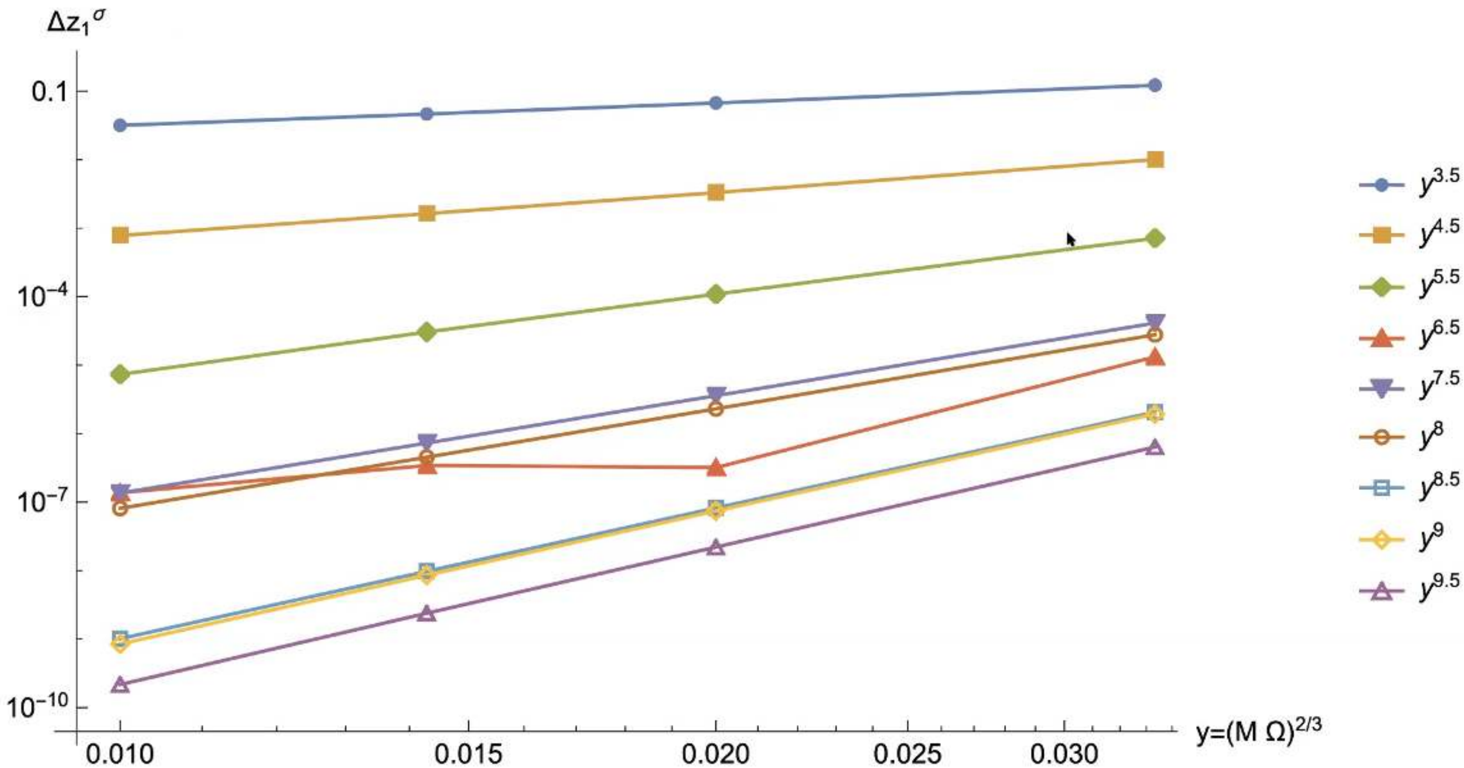
$$h_{kk}(r,y)|_{r=r_0+\sigma r_\sigma} = h_{kk}(r_0,y) + \sigma r_\sigma \partial_r h_{kk(0)}(r_0,y) + O[\sigma^2]$$

(Bini et al 2018)

Regularisation Example



Comparison with PN



(Bini et al 2018)

Concluding

- First strong field calculation of a conservative GSF quantity including the spin of the secondary.

- What was new?

  Fixed frequency calculations > fixed radius calculations.

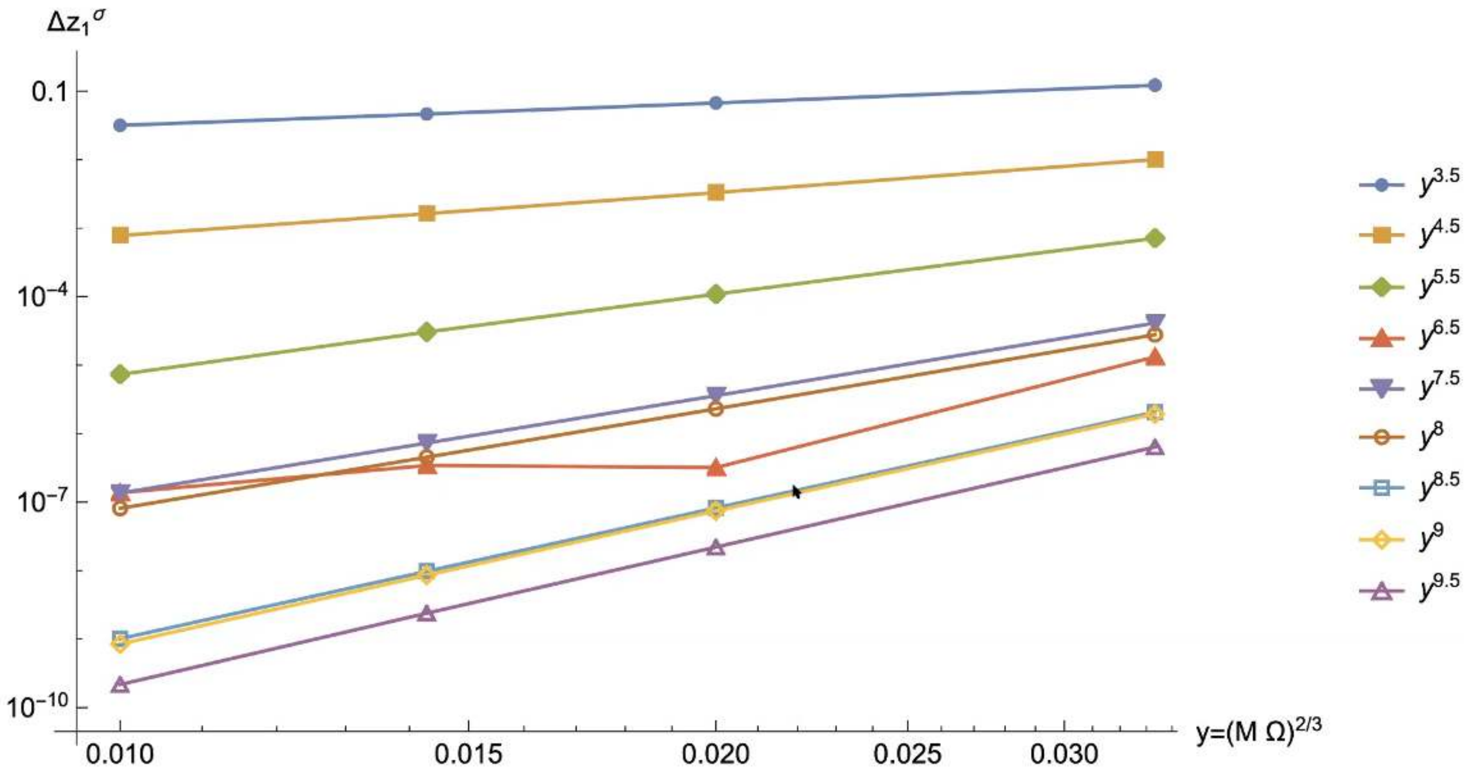
  Tensor harmonic regularisation parameters.

- What's next?

  Other gauge invariants with a spinning secondary - spin precession invariant etc.

  Fixed frequency calculations in more generic orbital configurations.

Comparison with PN



(Bini et al 2018)