Title: Relativistic scattering of a fast spinning neutron star by a massive black hole

Speakers: Kaye Li

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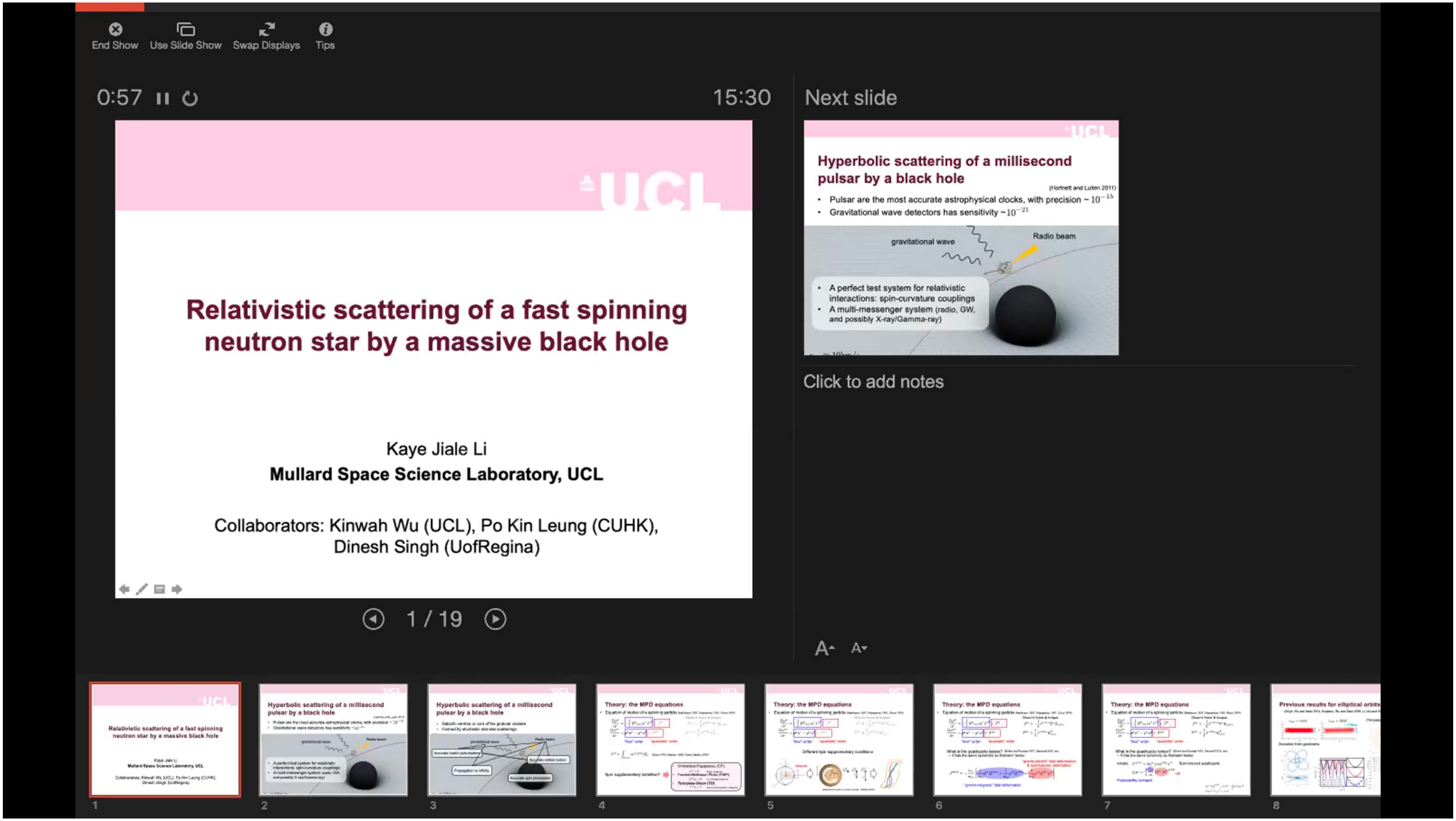
Abstract: We consider the motion of a spinning neutron star with astrophysically relevant speed in the gravity field of a massive black hole. The orbital dynamic is described by the MPD equations which include up to quadrupole interaction. We compare the orbits of the neutron star under geodesic motion and under MPD equations and show that the difference in the orbital motion can translate into a variation of pulse-arrival-time. Such a difference is within the observational limit of the radio telescopes like SKA and FAST.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UCL
Relativistic scattering of a fast spinning neutron star by a massive black hole
Kaye Jiale Li
Mullard Space Science Laboratory, UCL
Collaborators: Kinwah Wu (UCL), Po Kin Leung (CUHK), Dinesh Singh (UofRegina)
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Hyperbolic scattering of a millisecond pulsar by a black hole
(Hartnett and Luiten 2011)
Pulsar are the most accurate astrophysical clocks, with precision ~ 10^{-15}
Gravitational wave detectors has sensitivity ~10^{-21}
gravitational wave
Radio beam
A perfect test system for relativistic interactions: spin-curvature couplings
A multi-messenger system (radio, GW, and possibly X-ray/Gamma-ray)
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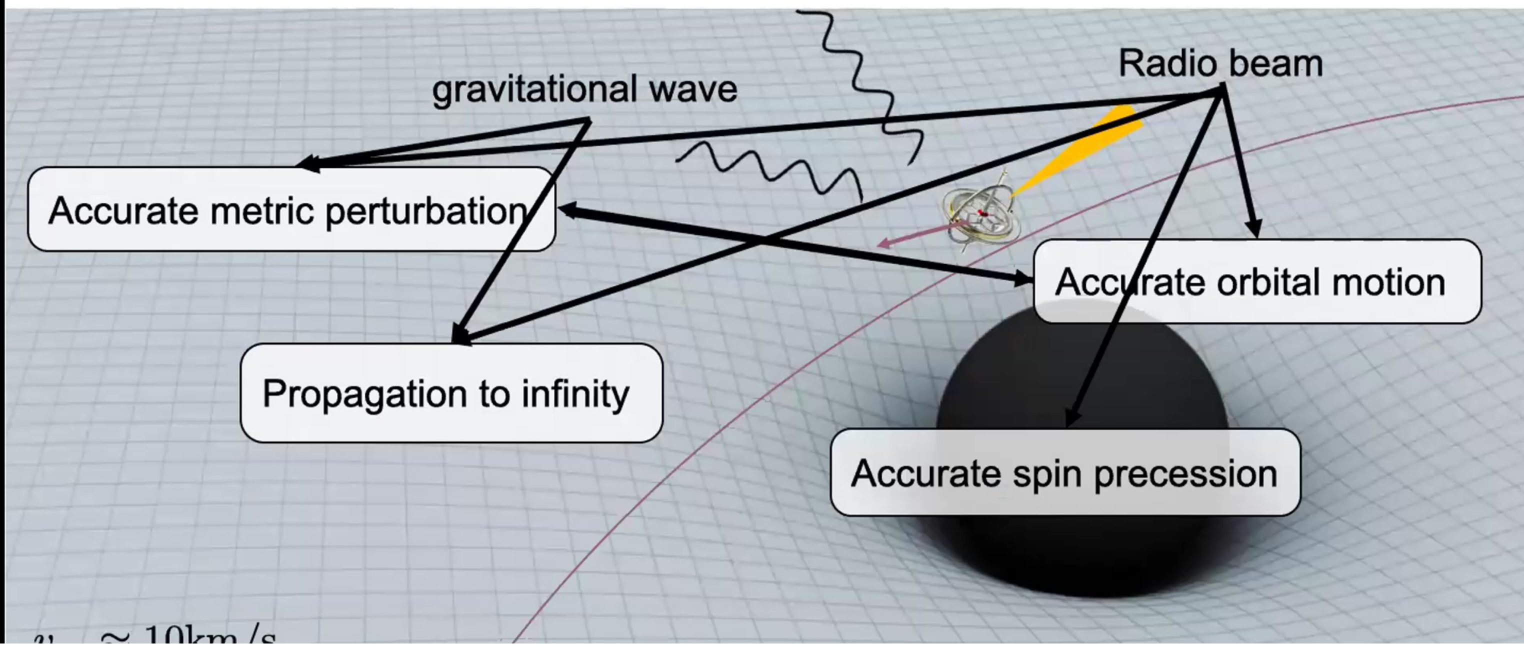
Hyperbolic scattering of a millisecond pulsar by a black hole

- Galactic centres or core of the globular clusters
- Formed by stochastic star-star scatterings



Theory: the MPD equations

- Equation of motion of a spinning particle (Mathisson 1937, Papapetrou 1951, Dixon 1974)

$$\frac{\mathrm{D}p^\mu}{\mathrm{d}\tau} = \boxed{-\frac{1}{2}R^\mu{}_{\nu\alpha\beta}u^\nu s^{\alpha\beta}} \boxed{+\mathcal{F}^\mu\ ,}$$

$$\frac{\mathrm{D}s^{\mu\nu}}{\mathrm{d}\tau} = \boxed{p^\mu u^\nu - p^\nu u^\mu} \boxed{+\mathcal{T}^{\mu\nu}}$$

"liner" order "quadratic" order

Dixon's force & torque

$$\mathcal{F}^\mu \equiv -\frac{1}{6}J^{\alpha\beta\gamma\sigma}\nabla^\mu R_{\alpha\beta\gamma\sigma}$$

$$\mathcal{T}^{\mu\nu} \equiv \frac{4}{3}J^{\alpha\beta\gamma[\mu}R^{v]}_{\gamma\alpha\beta}$$

$$S^{\alpha\beta} \equiv \int_{\Sigma(x,u)} \mathrm{d}x^{[\alpha}T^{\beta]\gamma}\mathrm{d}\Sigma_\gamma$$ (Dixon 1974, Madore 1969, Costa, Natário 2015)

Spin supplementary condition?

- Corinaldesi-Papapetrou (CP):
  $S^{\mu 0} = 0$ Static observer
- Frenkel-Mathisson-Pirani (FMP):
  $S^{\alpha\beta}u_\beta = 0$ Co-moving observer
- ***Tulczyjew-Dixon (TD):***
  $S^{\mu\nu}P_\nu = 0$ "Zero-3-momentum" observer

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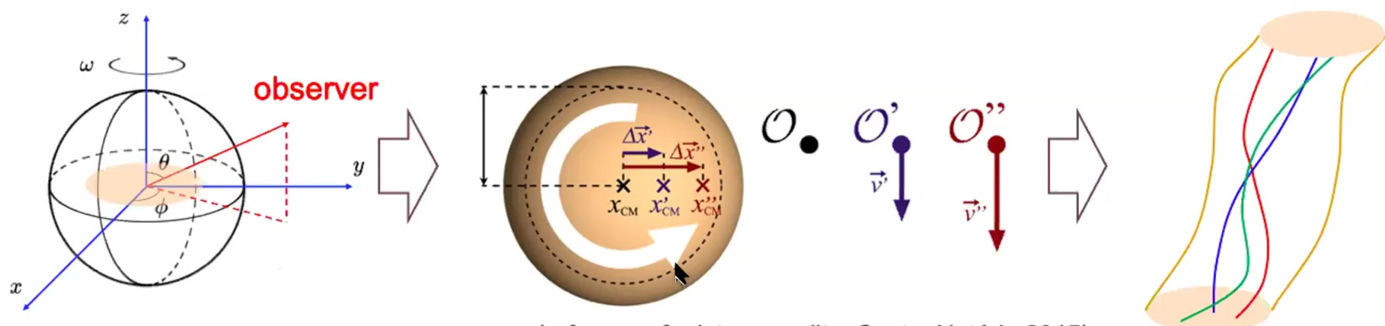
Different spin supplementary conditions



(reference & picture credits: Costa, Natário 2015)

Theory: the MPD equations

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What is the quadrupole tensor? (Ehlers and Rudolph 1977, Steinhoff 2011, etc)
-- It has the same symmetry as Riemann tensor

"gravito-electric" tidal deformation
& spin-induced deformation

$$J^{abcd} = -\frac{m_u}{m_p}\left[\frac{1}{m_u}p^{[a}Q^{b]cd} + \frac{1}{m_p}p^{[d}Q^{c]ba} + \frac{3}{m_p^2}p^{[a}Q^{b][c}p^{d]}\right]$$

"gravito-magnetic" tidal deformation

Theory: the MPD equations

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What is the quadrupole tensor? (Ehlers and Rudolph 1977, Steinhoff 2011, etc)

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Ansatz: $J^{\alpha\beta\gamma\delta} = 4u^{[\alpha}\chi(u)^{\beta][\gamma}u^{\delta]}$ Spin-induced quadrupole

$$\chi(u) = \frac{3}{4}\frac{C_Q}{m}[S^{\alpha\gamma}S^\beta_\gamma]^{\mathrm{STF}}$$ ~S^2

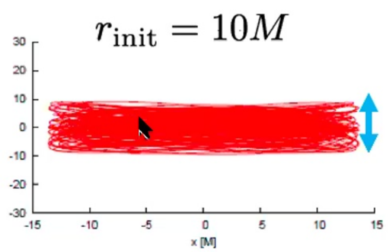
Polarizability constant

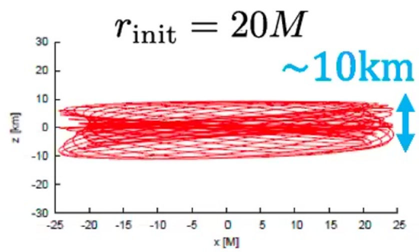
$$[S^{\alpha\gamma}S^\beta_\gamma]^{\mathrm{STF}} = S^\alpha S^\beta - \frac{1}{3}S^2 P(u)^{\alpha\beta}$$

$$P(u)^{\alpha\beta} \equiv g^{\alpha\beta} + u^\alpha u^\beta$$

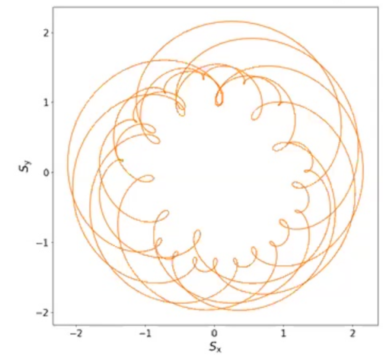
Previous results for elliptical orbits

(Singh, Wu and Sarty 2014, Kimpson, Wu and Zane 2018, Li, Wu and Singh 2019)

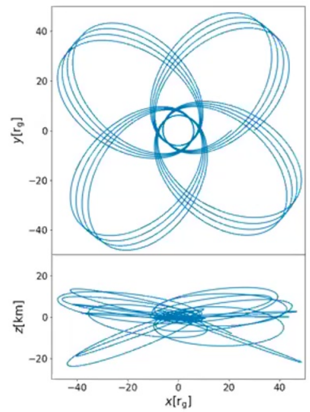


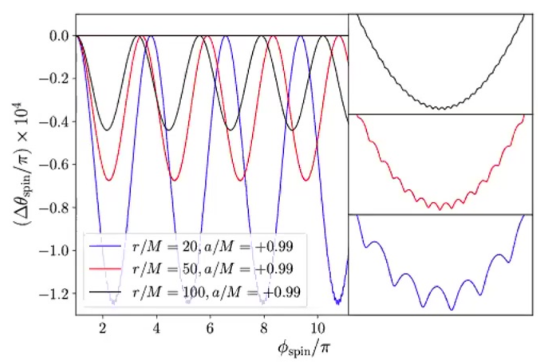


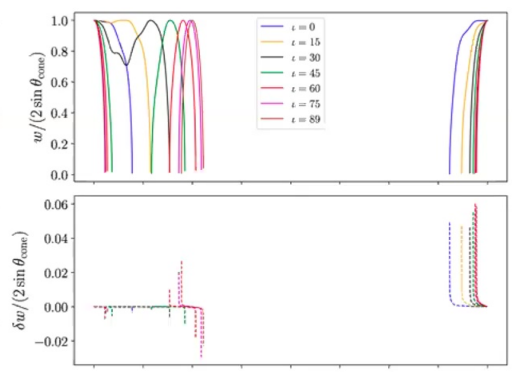
Precession of the spin axis



Deviation from geodesics





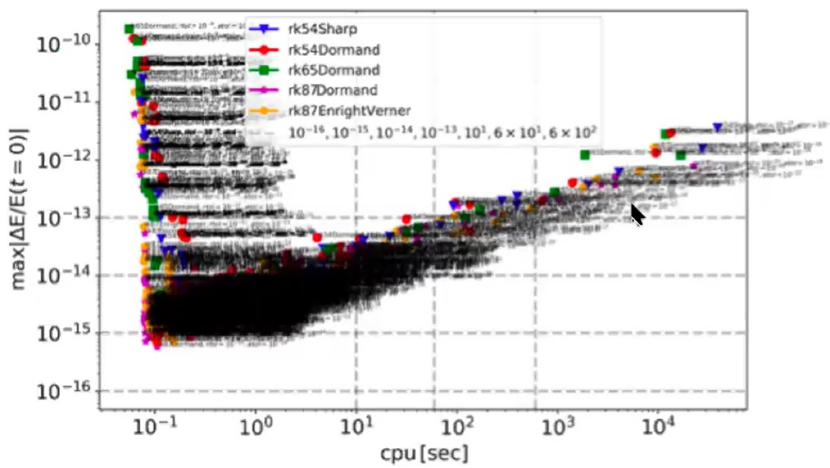


Solving the MPD equations

- Local truncation error & round-off error

Truncation error $$y_{n+1} = y_n + h \sum_{i=1}^{s} b_i k_i \;\Rightarrow\; y_{n+1} - y(t_{n+1}) \rightarrow \text{error}$$

Round-off error $$\text{decimal} : 0.1 \rightarrow \text{binary} : 0.00011001100110011001101\ldots$$

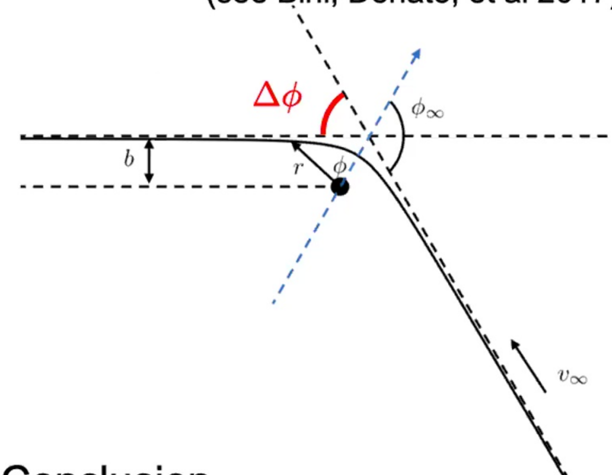


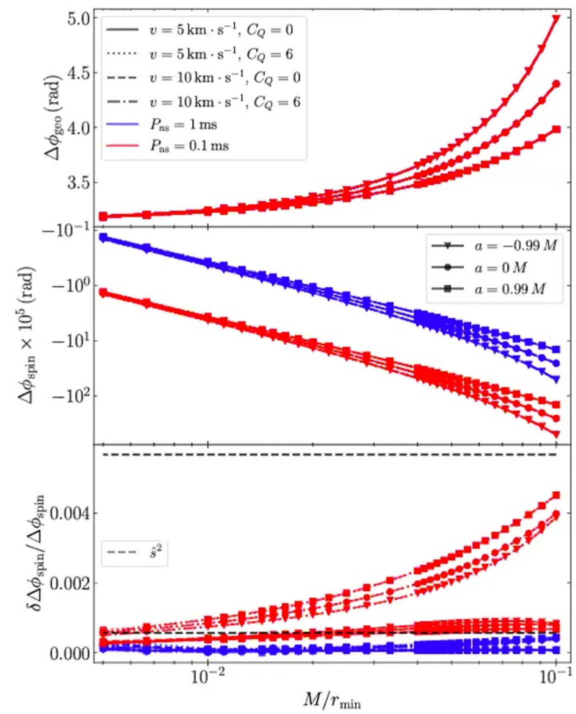
Solution to the MPD equations

- Comparing with analytical solution at linear order (equatorial motion)

$$\Delta\phi = 2\phi_0(\chi) + 2\hat{s}\phi_{\hat{s}}(\chi) - \pi$$

(see Bini, Donato, et al 2017)





- Conclusion

$$\mathrm{err}\,(\Delta\phi) \leq 10^{-9}\mathrm{rad}$$

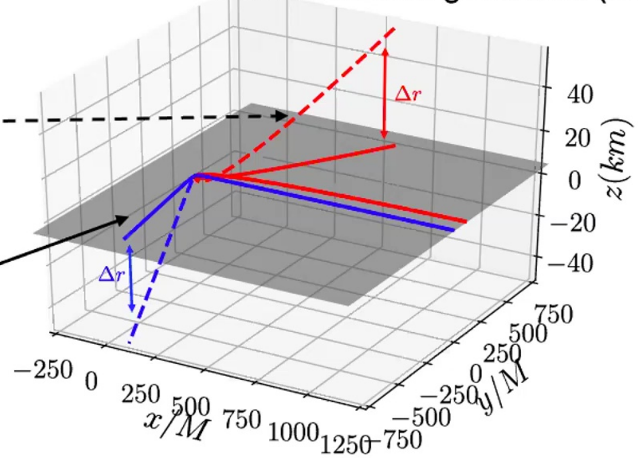
Deviation from the geodesics

- The MPD equation

$$\frac{\mathrm{D}p^\mu}{\mathrm{d}\tau} = -\frac{1}{2} R^\mu{}_{\nu\alpha\beta} u^\nu s^{\alpha\beta} + \mathcal{F}^\mu \ ,$$
$$\frac{\mathrm{D}s^{\mu\nu}}{\mathrm{d}\tau} = p^\mu u^\nu - p^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

- The geodesics equation

$$\frac{\mathrm{D}p^\mu}{\mathrm{d}\tau} = 0 \ ,$$
$$\frac{\mathrm{D}s^{\mu\nu}}{\mathrm{d}\tau} = 0$$



- When will this deviation becomes detectable?

  Take $\Delta r = 30\,\mathrm{km} \approx 100\,\mu\mathrm{s}$

- We cannot observer the MSP for infinity long time.

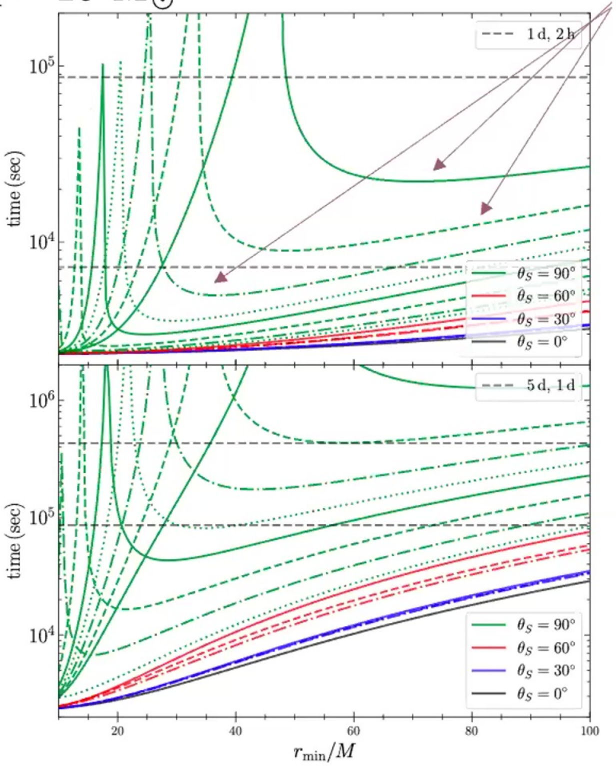
  Suppose the pulsar is observed for 30min-24h after the close flyby

When will this deviation becomes detectable?

$M_{\rm BH} = 10^3 M_\odot$

different orientations of spin

$P_{\rm ns} = 1\,{\rm ms}$



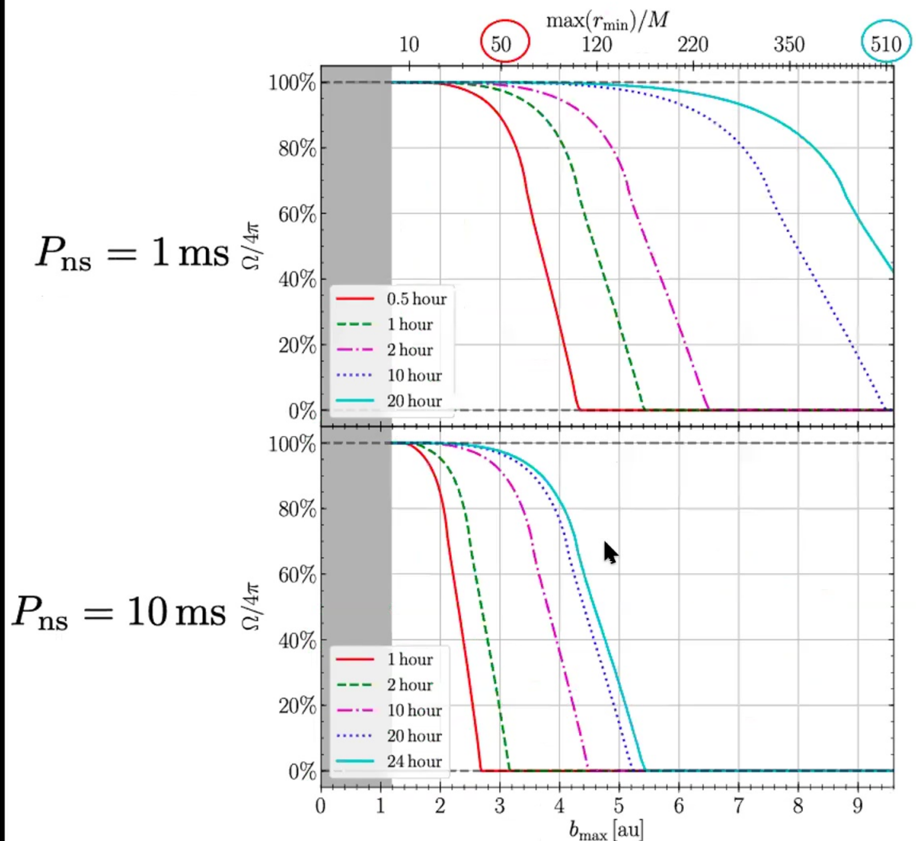
$P_{\rm ns} = 10\,{\rm ms}$

Mostly within 2 hours

$$\text{time} \propto M_{\rm BH} P_{\rm ns}$$

Mostly within 1 day

What is the probability of observing a deviation of 30km?



- The spin's effects of an MSP on a wider orbit is still observable given sufficient long observation time

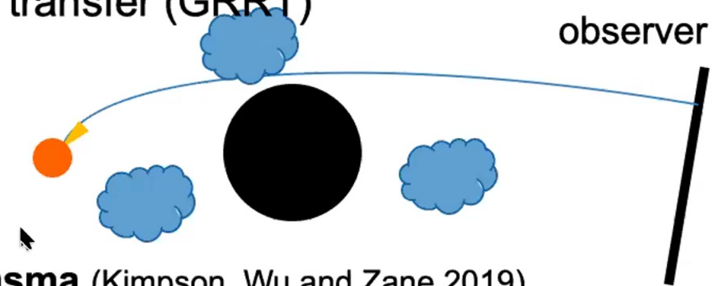
- A pulsar with smaller spin can also be used as probes of MPD dynamics

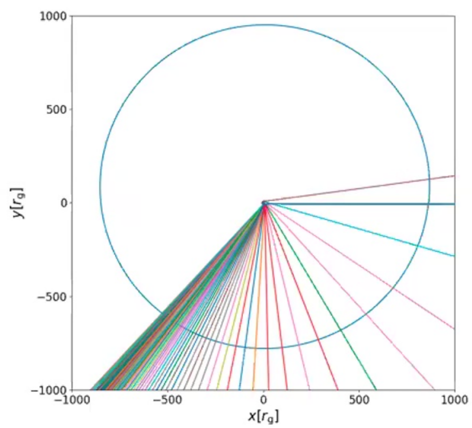
$b \propto v_\infty^{-1}$

$v_\infty = 10\,\mathrm{km/s}$ in this figure

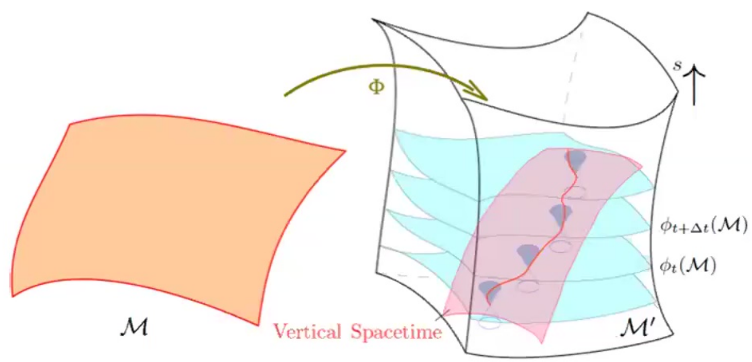
Step towards radio observations

- Gravitational relativity radiation transfer (GRRT)
  - Precession of the spin's axis
  - Gravitational redshift
  - Gravitational abbreviation
  - ….
  - **Spatial dispersion due to plasma** (Kimpson, Wu and Zane 2019)
  - **Dynamical space time** (Hu, Wu, Martin Li, Lin and Younsi 2021)





(Kimpson, Wu and Zane 2019)



(Hu, Wu, Martin Li, Lin and Younsi 2021)

Summary

- For millisecond pulsars with $1\mathrm{ms} \leq P_{\mathrm{ns}} \leq 10\mathrm{ms}$, the equation of motion is described with MPD equations.
- The forces due to spin-curvature and quadrupole-curvature couplings lead to deviation from the geodesics, which is observable within hours for 1000 solar mass black hole, longer observing time for more massive black holes.
- Modelling the radio emission requires solving the null geodesic on a dynamical space-time manifold with the presence of plasma on the line-of-sight.