Title: Frequency domain approach to self-force in hyperbolic scattering

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Abstract: Hyperbolic scattering orbits, able to penetrate deep into the sub-ISCO region even at relatively low energies, provide an excellent probe of the strong-field regime outside black holes. Self-force calculations of the scatter angle can greatly advance the development of post-Minkowskian theory and of the EOB model of binary dynamics. We develop a frequency-domain method for calculating the 1st order scalar self-force acting on a charge moving along a hyperbolic Schwarzschild geodesic, outlining the formulation of the problem, the challenges faced and our attempted solutions. Particular attention will be paid to issues faced by the usual method of extended homogeneous solutions (EHS) used to circumvent the Gibbs phenomenon.

Frequency Domain Approach to Self-Force in Hyperbolic Scattering

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Equations of Motion

The scalar field equation of motion is given by

$$\nabla_\mu \nabla^\mu \Phi = -4\pi T \tag{1}$$

and the scalar charge density T is that of a point particle. We separate into spherical and Fourier harmonics:

$$\Phi = \int d\omega \sum_{\ell,m} \frac{1}{r} \psi_{\ell m\omega} Y_{\ell m}(\theta, \varphi) e^{-i\omega t}, \tag{2}$$

and the equation of motion becomes

$$\frac{d^2 \psi_{\ell m\omega}}{dr_*^2} - (V_\ell(r) - \omega^2)\psi_{\ell m\omega} = S_{\ell m\omega}(r). \tag{3}$$

Extended Homogeneous Solutions

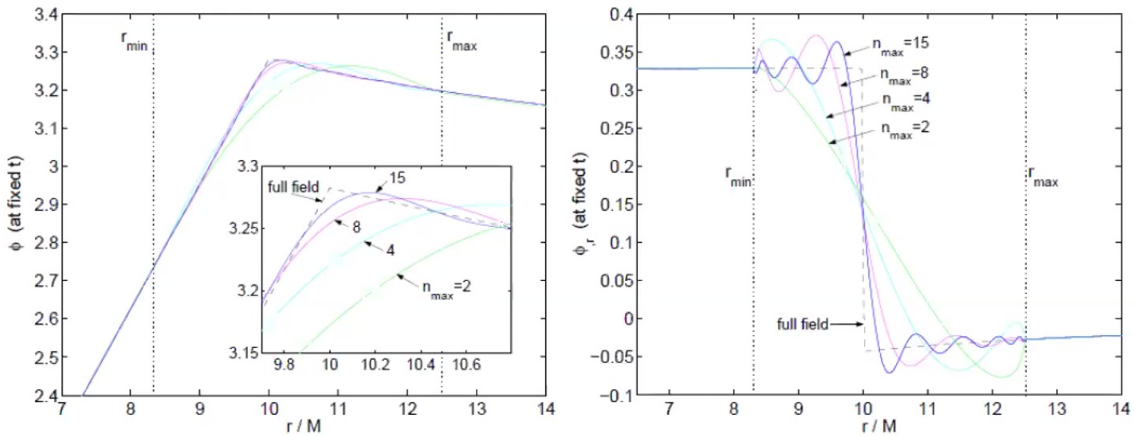


Figure: Convergence of Fourier series for scalar monopole (Schwarzschild eccentric orbit) [Barack, Ori, Sago 2008]

MEHS: express time domain field $\Phi_{lm}(t,r)$ in terms of analytic functions on either side of the worldline.

$$r\Phi_{\ell m}(t,r) = \tilde{\psi}^{+}_{\ell m}(t,r)\Theta(r - r_p(t)) + \tilde{\psi}^{-}_{\ell m}(t,r)\Theta(r_p(t) - r) \quad (6)$$

Extended Homogeneous Solutions (2)

First define the extended homogeneous solutions on $r > 2M$

$$\tilde{\psi}^{\pm}_{\ell m\omega}(r) := \psi^{\pm}_{\ell m\omega}(r) \int_{r_{min}}^{r_{max}} \frac{\psi^{\mp}_{\ell m\omega}(r') S_{\ell m\omega}(r') dr'}{W_{\ell m\omega} f(r')} \qquad (7)$$

and construct the corresponding time domain functions $\tilde{\psi}^{\pm}(t, r)$.

Key ideas:

1. In the source free region $r \geq r_{max}$, $\psi_{\ell m}(t, r) = \tilde{\psi}^{+}_{\ell m}(t, r)$.
2. $\psi_{\ell m}(t, r)$ and $\tilde{\psi}^{+}_{\ell m}(t, r)$ are analytic throughout $r > r_p(t)$.
3. Hence they must agree throughout $r \geq r_p(t)$.

Make a similar argument for $r \leq r_p(t)$.

[Barack, Ori, Sago 2008]

Inhomogeneous Solutions in the Unbound Case

For $\omega \neq 0$ variation of parameters again gives us the inhomogeneous field

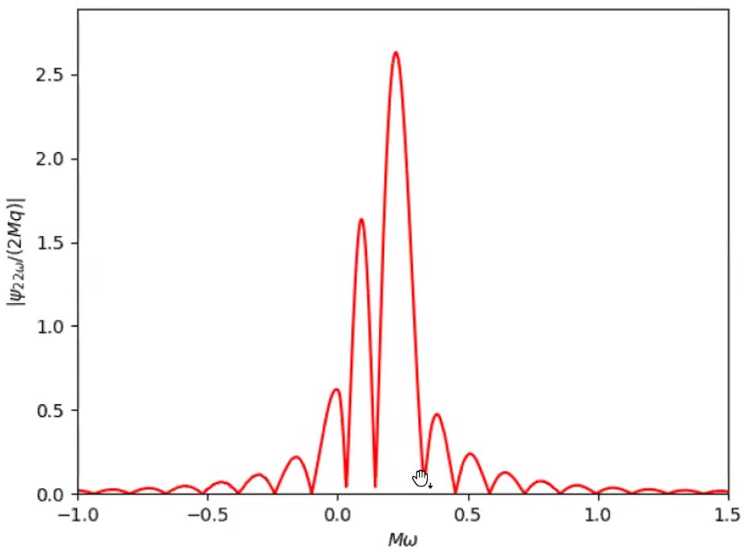
$$\psi_{\ell m\omega}(r) = \psi^{+}_{\ell m\omega}(r)\int_{r_{min}}^{r}\frac{\psi^{-}_{\ell m\omega}(r')S_{\ell m\omega}(r')}{W_{\ell m\omega}}\frac{dr'}{f(r')} + \psi^{-}_{\ell m\omega}(r)\int_{r}^{\infty}\frac{\psi^{+}_{\ell m\omega}(r')S_{\ell m\omega}(r')}{W_{\ell m\omega}}\frac{dr'}{f(r')} \tag{8}$$

Marginal convergence of this integral, integrand

$$\sim \frac{\text{oscillations}}{r}$$

as $r \longrightarrow \infty$.

Sample Spectrum



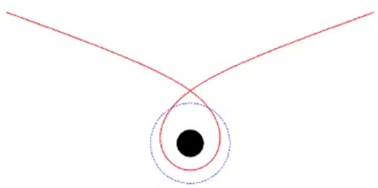


Figure: $\psi_{22\omega}(r = 6M)$ vs frequency for the geodesic $E = 1.1, r_{min} = 4M$ (illustrated).

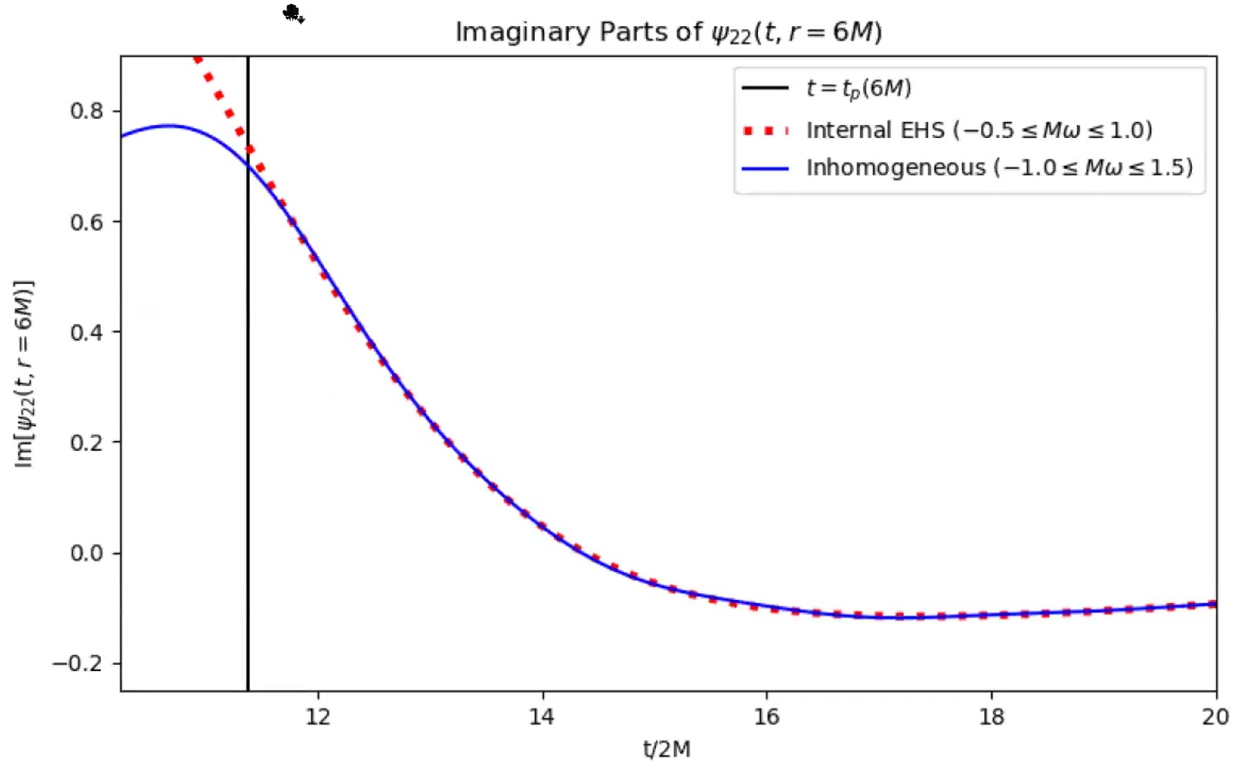
EHS in unbound case: possible solutions

- Could some form of EHS still apply?

- Extension into $u = 1/r < 0$:
  1. Scattering orbit extends to orbit in $u < 0$ region, periodic in Mino time.
  2. Need to find a global time coordinate which allows field equation to be separated into frequency modes before this is tractable.

- One-sided regularisation using only lower EHS.

Reconstructing Time Domain Field (Internal)

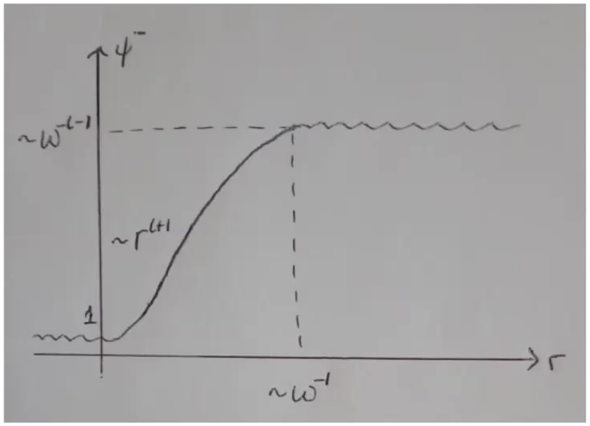


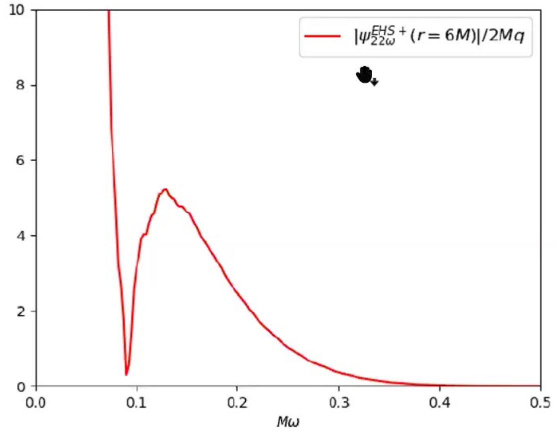
Slowly Converging Radial Integrals

- Need to compute radial integrals extending out to ∞, want to truncate at finite radius.
- Slow oscillations$/r$ behaviour of integrand.
- In wave zone integrand can be expanded in $1/r$ and resulting integrals known analytically.
  1. Puncture integrand to get higher rate of convergence
  2. OR Analytical correction to truncated integral
- Particularly acute for external normalisation integral.

$$C^{+}_{\ell m\omega} = \int_{r_{min}}^{+\infty} \frac{\psi^{-}_{\ell m\omega}(r')S_{\ell m\omega}(r')dr'}{W_{\ell m\omega}f(r')} \tag{9}$$

The IR Problem

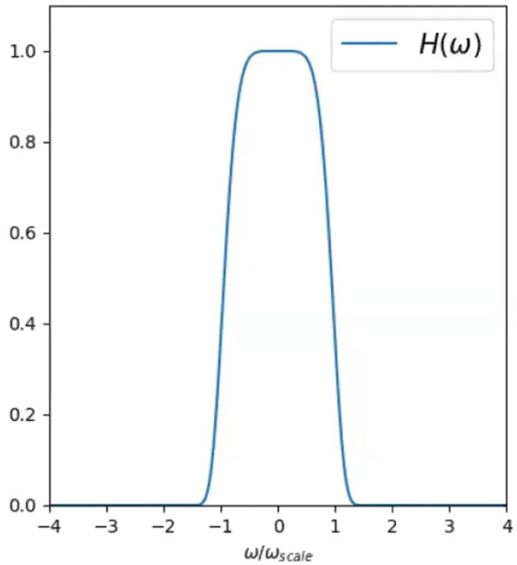




$$\psi^{EHS+}_{\ell m\omega}(r) = \psi^{+}_{\ell m\omega}(r)\int_{r_{min}}^{+\infty}\frac{\psi^{-}_{\ell m\omega}(r')S_{\ell m\omega}(r')dr'}{W_{\ell m\omega}f(r')} \tag{10}$$

Heuristics suggest genuine power law divergence...

Resolving the IR problem: Windowed EHS



Introduce a suitable window function, e.g.

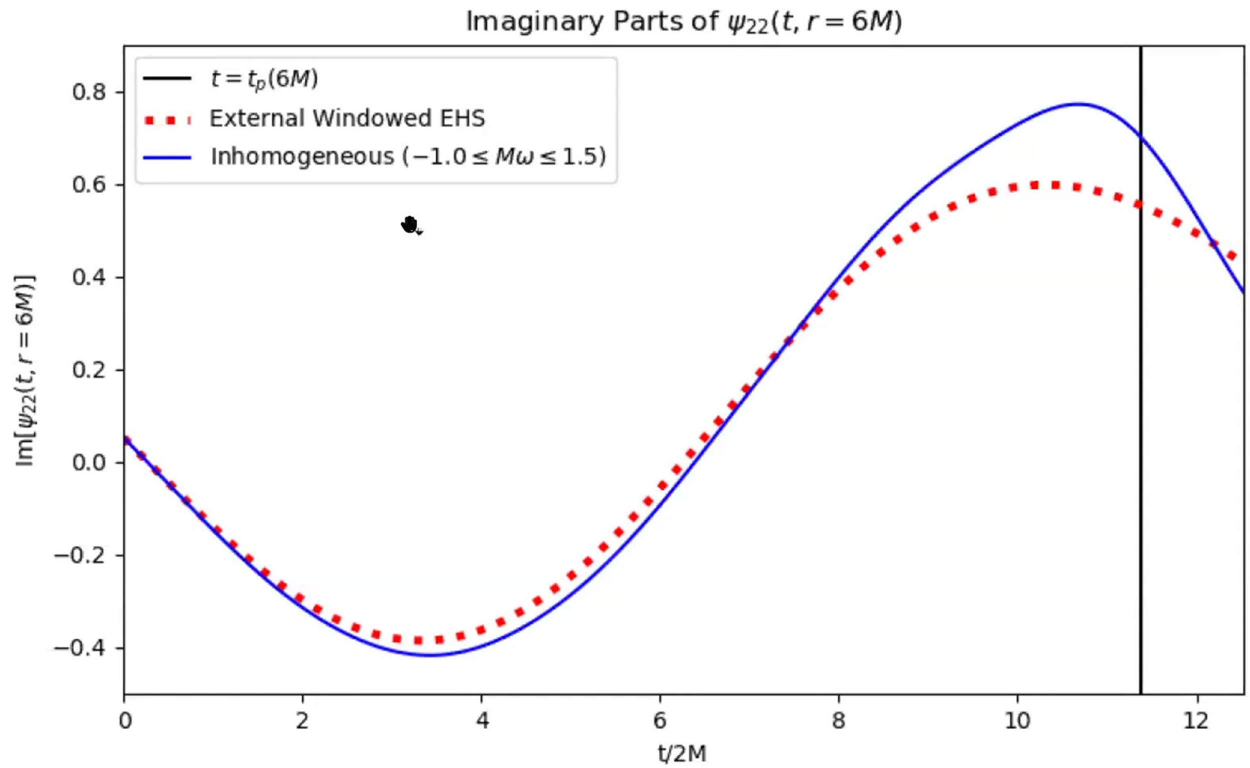
$$H(\omega) = \exp\left[-(\omega/\omega_{scale})^{2n}\right] \tag{11}$$

to split solution into high and low frequency parts.

Assuming EHS can be applied with usual form, outside the orbit:

$$\psi_{\ell m}(t,r) = \int_{-\infty}^{+\infty} \left[H(\omega)\psi^{inh}_{\ell m\omega}(r) + (1 - H(\omega))\psi^{EHS+}_{\ell m\omega}(r)\right] e^{-i\omega t} d\omega. \tag{12}$$

Reconstructing Time Domain Field (External)



Next Steps

- Work to understand how, if at all, we can apply EHS in external region.
- Dealing with numerical issues.
- Time domain reconstruction.
- SSF calculations and effect on scatter angle and time delay.
- Comparison with time-domain calculations (Oliver Long).