Title: Self-force effects in weak-field scattering

Speakers: Kunal Lobo

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 10, 2021 - 9:30 AM

URL: http://pirsa.org/21060056

Abstract: We revisit the old problem of the self-force on a particle moving in a weak-field spacetime in the context of renewed interest in gravitational two-body scattering. We calculate the scalar, electromagnetic, and gravitational self-force on a particle moving on a straight-line trajectory in the spacetime of a Newtonian star and use these results to find the associated correction to the scattering angle in each case. In the gravitational case we must also take into account the motion of the star via a ``matter-mediated" force on the particle, which acts at the same perturbative order as the gravitational self-force.

# Self Force Effects in Weak-Field Scattering

Kunal Lobo
Advisor: Sam Gralla
University of Arizona

# Early Self Force Calculations (EM)

- 1960 – Dewitt and Brehme derived the tail integral formula for the electro-magnetic self force.
- 1964 – Dewitt and Dewitt calculated this integral for a slowly moving bound particle in the weak-field limit of a star (Post Newtonian approximation).
- 1980 – Westpfahl and Goller calculated this integral for a particle moving in a straight line with arbitrary speed in the weak-field limit of a star (Post Minkowskian approximation).
- 1985 – Westpfahl derived the EM self-force correction to small-angle scattering. Also, he calculated the gravitational scattering angle to 2PM by a different method.

# Scalar and Gravitational Self Force

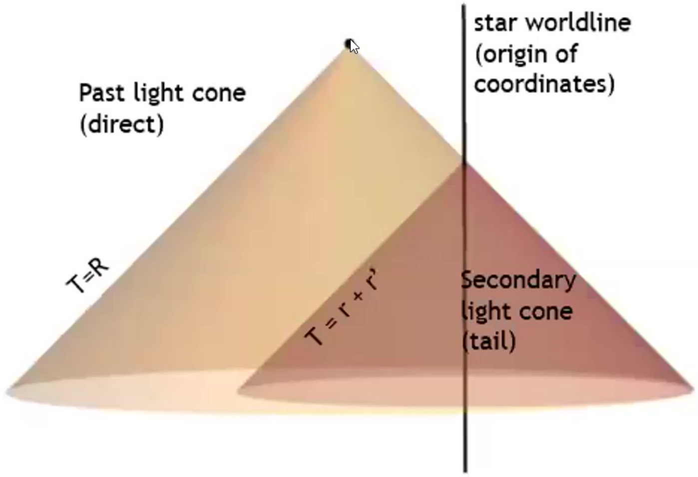
- 1997,2000 – MiSaTaQuWa (gravitational self force), Quinn (scalar self force)
- 2002 – Pfenning and Poisson revisited the Dewitt and Dewitt calculation, clarifying the EM self-force, and deriving analogous scalar and gravitational results
  - Showed the need for matter mediated forces in the gravitational case
  - Connected to the standard PN theory of binary systems
  - Provided context for the more ambitious goal of LISA waveform modeling

We will similarly revisit the Westpfahl/Goller weak-field scattering calculations

- Calculate the scalar, EM, and gravitational self force
- Calculate the matter mediated force
- Integrate the equations in closed form
- Calculate the deflection angles in each case
- Reproduce known 2PM results in gravity

# Green Functions far from Newtonian Star

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)(dx^2+dy^2+dz^2) + O(\Phi^2)$$




Tail portion has support only on and within the second light cone

- Scalar Green Function for $|\mathbf{x}|, |\mathbf{x}'| \gg \mathcal{R}.$

$$G^{\text{direct}} = \frac{\delta(T-R)}{R} - \frac{2M}{R}\log\frac{r+r'+R}{r+r'-R}\delta'(T-R)$$

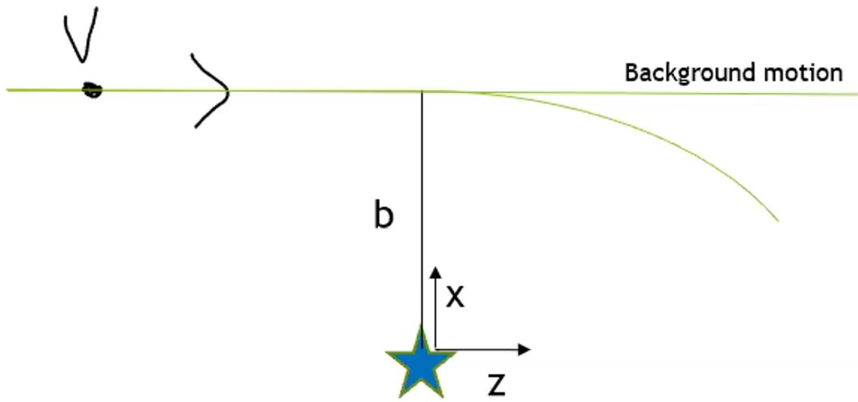
$$G^{\text{tail}} = \left(\frac{4M}{T^2-R^2} - \xi\frac{2M}{rr'}\right)\delta(T-r-r') - \frac{8MT}{(T^2-R^2)^2}\Theta(T-r-r').$$

Scalar self-force

$$f^{\alpha}_{(q)}(\tau) = q^2\int_{-\infty}^{\tau^-}\left(\nabla^{\alpha}G + u^{\alpha}u^{\beta}\nabla_{\beta}G\right)d\tau'$$

- EM and Gravitational Green's Functions are analogous and easy to obtain using Pfenning and Poisson's results

# Self Force Results




Computing the tail integral along the background straight line motion, we find

$$
\begin{aligned}
f^{z}_{(q)} &= -2q^2\gamma^{-1}\left(\gamma^2 v\mathcal{A}_1 + \mathcal{A}_2 + \xi\gamma^2 v\mathcal{B}_1 + \xi\mathcal{B}_2\right)\\
f^{x}_{(q)} &= -2q^2\gamma^{-1}\left(\mathcal{A}_3 + \xi\mathcal{B}_3\right)\\
f^{z}_{(e)} &= -e^2\gamma\left(\mathcal{C}_3 - 2v\mathcal{A}_1 - 2\gamma^{-2}\mathcal{A}_2 - v\mathcal{B}_1 + (1+v^2)\mathcal{B}_2\right)\\
f^{x}_{(e)} &= -e^2\gamma\left(\mathcal{C}_2 + v\mathcal{C}_1 - 2\gamma^{-2}\mathcal{A}_3 + (1+v^2)\mathcal{B}_3\right)\\
f^{z}_{(m)} &= -2m^2\gamma\left(\gamma^{-2}\mathcal{A}_2 - 2\mathcal{C}_3 + v\mathcal{A}_1 + 2v(2+\gamma^2 v^2)\mathcal{B}_1 - 2v^2\mathcal{B}_2\right)\\
f^{x}_{(m)} &= -2m^2\gamma\left(\gamma^{-2}\mathcal{A}_3 - 2\mathcal{C}_2 - 2v\mathcal{C}_1 - 2v^2\mathcal{B}_3\right).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_1 &= \int_{-\infty}^{t^-} \frac{d}{dt} A_{,tt'} dt' = \frac{Mv^2\left(r^2\left(1-3v^2\right) + 4rvz + z^2\left(v^2-3\right)\right)}{2r(r-vz)^4}\\
\mathcal{A}_2 &= \int_{-\infty}^{t^-} A_{,tt'z} dt' = \frac{Mv\left(r-3vz\right)\left(r^2(1-3v^2) + 4rzv + z^2(v^2-3)\right)}{6r\left(r-vz\right)^5}\\
\mathcal{A}_3 &= \int_{-\infty}^{t^-} A_{,tt'x} dt' = \frac{Mvb\left(2r^2v^3 - rz\left(1+3v^2\right) - \left(v^2-3\right)vz^2\right)}{2r(r-vz)^5}\\
\mathcal{B}_1 &= \int_{-\infty}^{t^-} \frac{d}{dt} B dt' = \frac{Mv\left(r^2v - 2rz + vz^2\right)}{r^3(r-vz)^2}\\
\mathcal{B}_2 &= \int_{-\infty}^{t^-} B_{,z} dt' = -\frac{Mz\left(r^2\left(v^2+1\right) - 3rvz + v^2z^2\right)}{r^3(r-vz)^3}\\
\mathcal{B}_3 &= \int_{-\infty}^{t^-} B_{,x} dt' = -\frac{Mb\left(r^2\left(v^2+1\right) - 3rvz + v^2z^2\right)}{r^3(r-vz)^3}\\
\mathcal{C}_1 &= \int_{-\infty}^{t^-} \frac{d}{dt}(A_{,zx'} - A_{,xz'}) dt' = \frac{Mv^2b\left(2r^2v - 3rz + vz^2\right)}{r^3(r-vz)^3}\\
\mathcal{C}_2 &= \int_{-\infty}^{t^-} \frac{d}{dt}(A_{,xt'} - A_{,tx'}) dt' = \frac{Mvb\left(2r^2v - 3rz + vz^2\right)}{r^3(r-vz)^3}\\
\mathcal{C}_3 &= \int_{-\infty}^{t^-} \frac{d}{dt}(A_{,zt'} - A_{,tz'}) dt' = \frac{Mv\left(r^3\left(1-2v^2\right) + 3r^2vz - 3rz^2 + vz^3\right)}{r^3(r-vz)^3}.
\end{aligned}
$$

$$r = \sqrt{b^2 + z^2}$$

# Matter Mediated Force

- The derivation of gravitational self-force assumes a vacuum spacetime
- Our spacetime has matter (the star), but only away from the particle, and perturbatively in M. Under these conditions, we can add a "matter-mediated" force (following Pfenning and Poisson)
- This matter-mediated force term will be the exact same order as the gravitational self-force

$$f^\mu = f^\mu_{(m)} + f^\mu_{\mathrm{mm}}.$$

## Calculating the matter mediated force

- (1) Solve for the motion of the star (treated as a point particle) in response to the particle's field
- (2) Find the perturbed metric of the perturbed star
- (3) add in the additional "gravitational force" (perturbed geodesic equation) on the particle due to this metric

# Deflection Angles

- The deflection angle is given by integrating the equations of motion

$$m\frac{d^2x^\mu}{d\tau^2} = -m\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} + f^\mu$$

This is either the SSF, EMSF, or the sum of GSF and Matter Mediated Force

$$\delta_{\text{scalar}} = \frac{M}{bv^2}\left(2(1+v^2) - \frac{q^2}{mb}\frac{\pi}{4}(v^2 + 4\xi(1-v^2)) + \dots\right) + O\left(\frac{M^2}{b^2}\right)$$

$$\delta_{\text{em}} = \frac{M}{bv^2}\left(2(1+v^2) - \frac{e^2}{mb}\frac{\pi}{4}(v^2 + 2) + \dots\right) + O\left(\frac{M^2}{b^2}\right)$$

$$\delta_{\text{grav}} = \frac{M}{bv^2}\left(2(1+v^2) + \frac{m}{b}\frac{3\pi}{4}(v^2 + 4) + \dots\right) + O\left(\frac{M^2}{b^2}\right),$$

Q is either q, e, or m

1PM Geodesic terms

$$\frac{M}{bv^2} \ll 1, \qquad \frac{Q^2}{mb} \ll 1,$$

# Derivation of 2PM Scattering Angle

Self Force + Matter Mediated Force $\delta_{\text{grav}} = \frac{M}{bv^2}\left(2(1+v^2) + \frac{m}{b}\frac{3\pi}{4}(v^2+4) + \ldots\right) + O\left(\frac{M^2}{b^2}\right)$

# Derivation of 2PM Scattering Angle

Self Force + Matter Mediated Force $\delta_{\text{grav}} = \frac{M}{bv^2}\left(2(1+v^2) + \frac{m}{b}\frac{3\pi}{4}(v^2+4) + \dots\right) + O\left(\frac{M^2}{b^2}\right)$

(1) Add in order M^2 geodesic terms $\delta_{\text{grav}} = \frac{M}{bv^2}\left(2(1+v^2) + \frac{m}{b}\frac{3\pi}{4}(v^2+4) + O\left(\frac{m^2}{b^2}\right)\right) + \frac{M^2}{b^2v^2}\frac{3\pi}{4}(v^2+4) + O\left(\frac{M^3}{b^3}\right)$

(2) Boost to CM frame

$\chi(CM) = \delta * E/M$

$E = \sqrt{M^2 + m^2 + 2Mm\gamma}$

$$\boxed{\chi = \frac{E}{bv^2}\left(2(1+v^2) + \left(\frac{m}{b} + \frac{M}{b}\right)\frac{3\pi}{4}(v^2+4)\right)}$$

(3) Notice, the symmetry of this implies that the deflection angle will be the same for both masses!

This is the 2PM Scattering Angle originally derived by Westpfahl using a different approach

# Summary

- Calculated analytic formulas for the scalar, EM, and gravitational self-force in weak-field scattering

Q is either q, e, or m

$$\frac{M}{bv^2} \ll 1, \qquad \frac{Q^2}{mb} \ll 1,$$

- For gravity, we also find the matter-mediated force
- Integrated the equations of motion exactly to get parameterized trajectories
- Calculated the deflection angle resulting from each force

- Results may be useful for code comparison
- Method shows how self-force fits into PM scattering calculations
- Pushing to higher in M or m can give new 3PM and 4PM results