Title: From Scattering in Kerr Backgrounds to Higher-Spin Amplitudes

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Abstract: We revisit the scattering of massless waves of helicity $|h|=0,\frac{1}{2},1,2$ in Schwarzschild and Kerr backgrounds, in the long-wavelength regime. The Newman-Penrose scattering amplitudes arising from the Black Hole Perturbation Theory (BHPT) framework are found in agreement with the classical limit of QFT amplitudes at finite values of the scattering angle and arbitrary spin orientation. The latter amplitudes are obtained from on-shell methods and describe the $2\to 2$ scattering of a massless particle of helicity $|h|$ with a massive particle of arbitrary spin S, where $S=0$ corresponds to the Schwarzschild case. The effect of the black hole spin in the polarization of the waves is found in agreement with previous analysis. Finally, unitarity constraints based on partial amplitudes and positive time delay are also discussed.

From scattering in Kerr background to Higher-spin Amplitudes

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2106.xxxxx

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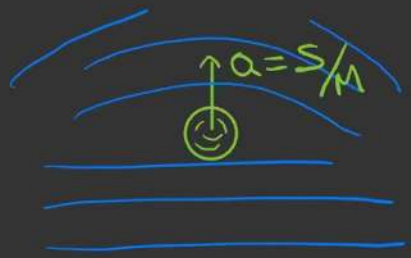
The 24th Capra Meeting on Radiation Reaction in General Relativity

June 10th, 2021

Two problems in GR and High energy Physics

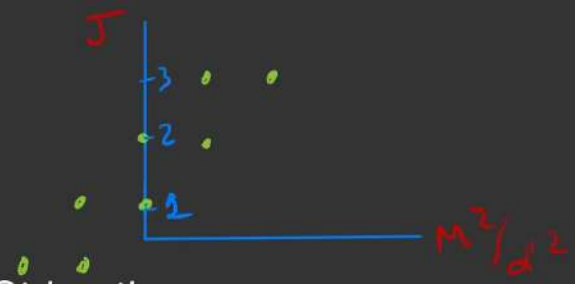
1) (Perturbations of) Kerr Metric



★ Analytic solutions for an isolated BH under certain assumptions

$GM\omega \ll 1$, $a\omega \sim G(\ldots)$ [Dolan 2008]

2) Interacting Higher-Spins



★ String theory
★ Causality in $\sim R^2$ theories [Maldacena 2014]
★ EFT description, ex. Bound states QCD

Not well defined high energy limit, m → 0

Claim: The 3pt amplitude contains essentially the same Information as the Kerr metric to order G

[Guevara, Ochirov & Vines 2018; Arkani-Hamed, O'Connell & Huang 2019]

- No radiation in (3,1) signature (stationary) . But Radiation modes appear in (2,2) signature.

On the Gravity side, the linearized Kerr metric in harmonic gauge [Vines 2017]

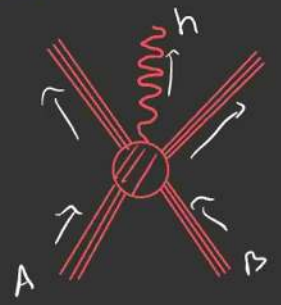
$$h^{Kerr}_{\mu\nu} \xrightarrow{(2,2)} P_\mu P_\nu P_\alpha P_\beta e^{\pm a\cdot\partial} \frac{GM}{r}$$

Same exponential structure for an infinite-spin scattering Amplitude!

However, perturbations of the Kerr metric in (3,1) indeed carry radiative modes.

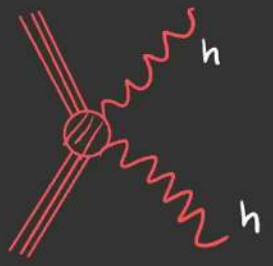
Those perturbations can be match with higher point amplitudes

Binary BH problem



[Golberger & Ridgway, Li, Rothstein, Porto, Levi, Damour , O'Connell, Luna, Nicholson, White, Guevara, Bautista, Manu, Gosh, Ladah, Parra-Martinez, Ruf, Zeng, Hermann, Jakobsen, Mogull, Plefka, Steinhoff, Di Vecchia, Heissenberg, Russo, Veneziano ...]

Wave perturbations (h=0,1/2,1,2)



What is the 4pt amplitude of Kerr?

[Dolan, Guadagnini, Peters, Logi, Kovacs,]

Hints from the BBH problem

[GOV, Huang et al, Helset &Haddad, Simeonsen &Vines...]

A clear connection between the 4 pt Amplitude and the Kerr BH is missing!

Since wave perturbations contain radiation modes, the amplitude should match to a Newman-Penrose scalar

?

The null tetrad $\{\ell^\mu, n^\mu, m^\mu, \bar{m}^\mu\}$

$\ell \cdot n = -m\bar{m} = 1$

$$\psi_{-2}(x) = \bar{m}^\mu n^\nu \bar{m}^\rho n^\sigma C_{\mu\nu\rho\sigma}$$

Weyl or Faraday radiation scalars depending on h.

★ What is the dictionary between the two ?

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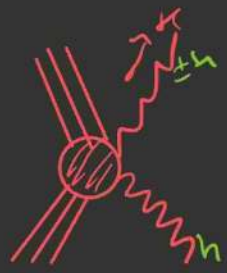

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★ What is the dictionary between the two ?

Observation: At future null infinite, only the modes with 3 momentum $\vec{k}$ aligned with the direction of observation $\frac{\vec{x}}{|\vec{x}|}$, contribute to the radiation flux (See ex. [strominger 2017])

Identify $\quad k^\mu = \omega \ell^\mu(x) \,, \quad \varepsilon^{*\mu} = -\bar{m}^\mu(x) \,, \quad \varepsilon^\mu = m^\mu(x)$

Such that: $\quad k^2 \propto \ell^2 = 0 \,, \quad k \cdot \epsilon \propto \ell \cdot m = 0 \;\wedge\; \varepsilon \cdot \varepsilon^* = -m \cdot \bar{m} = 1$

But the scattering amplitude is still a quantum object, besides, the proposed map is written in a non-perturbative fashion.

Still need to provide:

1) A prescription to take the classical limit
2) A prescription to match the two sides in the low energy limit
3) A prescription to do the match order by order in perturbation theory.

Scalar wave in the Schwarzschild background

In the assimptiticaly flat region we have to solve

$\nabla^\mu \nabla_\mu \phi = 0$ in $g_{\mu\nu} = \eta_{\mu\nu} + h^{Schw.}_{\mu\nu}$ Use the gauge $\partial^\mu h^{Sch.}_{\mu\nu} = 0$

$\Box^{flat} \phi = h^{\mu\nu}_{Sch.} \partial_\mu \partial_\nu \phi$, Set. $\phi(x) = e^{i\omega t} \tilde{\phi}(\vec{x})$

$$\boxed{[\Delta + \omega^2]\tilde{\phi} = h^{\mu\nu}_{Sch.}(x)\, \partial_\mu \partial_\nu \tilde{\phi}}$$ Familiar problem from QM

Can expand the total wave into an incoming plane wave and a scattered wave.

$$\tilde{\phi}(x) = e^{i\vec{\kappa}\cdot\vec{x}} \phi^{in} + \phi^s(x)$$

To leading order in $6mw << 1$ Born approximation Qm

$$-(\omega^2 + \Delta)\phi^s(x) = h^{\mu\nu}_{Sch.} \kappa_\mu \kappa_\nu e^{i\kappa\cdot x} \phi^{in}$$

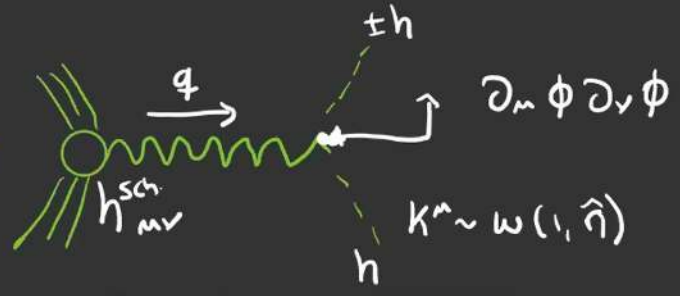
As $|\vec{x}| \to \infty$, we find

$$\text{Amplitude} = \frac{\phi^S(x)}{\phi_{in}} = \frac{e^{i\omega|\vec{x}|}}{|\vec{x}|} \int d^3y \, e^{iq\cdot y} \, h^{sch.}_{\mu\nu}(q) \, k^\mu k^\nu$$

It is just a Fourier transform

$$\text{Amplitude} = \frac{e^{i\omega|\vec{x}|}}{|\vec{x}|} \, \tilde{h}^{sch.}_{\mu\nu}(q) \, k^\mu k^\nu$$

Amplitude $\sim$



The field theory scattering amplitude naturally arises from the classical computation !

Some lessons from this simple example:

- The expansion $GM\omega \ll 1$ efficiently accounts for the low energy limit, as well as the perturbative expansion (G)
- In general, at a given order GM/r the scattering amplitude computation will include Quantum corrections of the form $\omega/m \sim 1/rm$ which can be disregard by the "classical limit "

$$\omega \to \hbar\bar{\omega} \quad , \quad r \to \bar{r}/\hbar \quad \text{taking} \quad \hbar \to 0$$

- Equivalent to a multiple soft limit where the momenta of real and virtual massless particles scales as

$$k_i = \hbar\bar{k} \to 0$$

- In more general helicity scenarios, we need to solve the Reggie-Wheeler equation, and for spinning BH the Teukolsky equation

Scattering in Kerr background and the Teukolsky equation

The NP scalar $\psi_n \propto \sum_{l=h}^{\infty} \sum_{m=-l}^{l} e^{i\omega t}\, {}_nR_{lm}(r,\omega,a)\, {}_nS_{lm}(\theta,\varphi,\omega,a)$

Solutions to the radial Teukolsky eq.

Spin-weighted Spheroidal Harmonics

- We still like to solve it in the low energy limit $\epsilon = GM\omega \ll 1$
- Additional dimensionless combinations show up $z = a\omega$, find solutions in powers of z.
- But unlike for the Schwarzschild BH, $\epsilon \ll 1$ does not take full care of the perturbative expansion (G). In particular, the rotation parameter

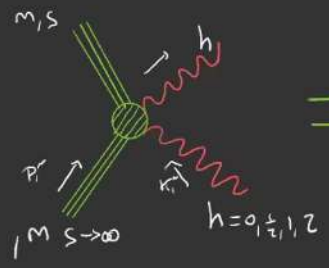
$$a^* = \frac{z}{\epsilon} = \frac{a}{GM} < 1$$ is untouched by the low energy expansion.

Analytically continue $a^* \to \infty$. This breaks down BHPT, and effectively erases the BH horizon (Naked singularity)

On the amplitudes side

For $a\omega \sim \mathcal{O}(1)$, and since $\omega \to \hbar\bar{\omega}$ the classical limit, then the spin $S = aM \to \infty$ The Kerr BH corresponds to an infinite spin particle

With this dictionary at hand, then one can extract Higher-spin amplitudes from the Kerr BH!



$$= {}_h\Psi \propto e^{i\omega t} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} {}_hR_{\ell m}(r, \omega a)\, {}_hS_{\ell m}(\theta, \varphi, a\omega)$$

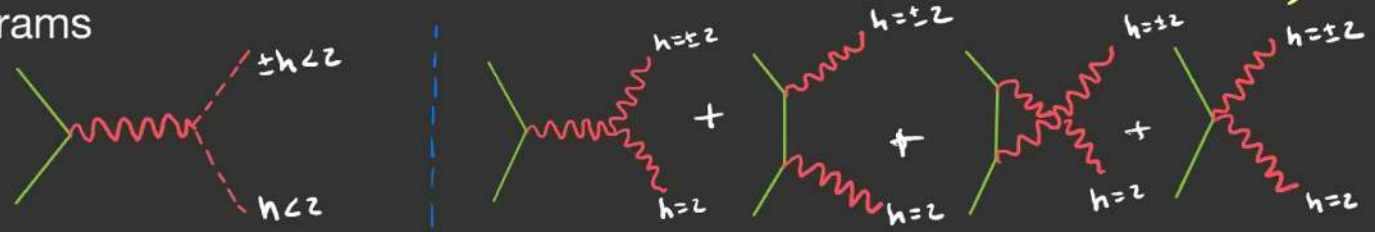
Difficulty: In many cases doing the infinite sum in the classical computation is almost impossible!

But there is light at the end of the tunnel. We can expand an exact Ansatz for the scattering amplitude, in the basis of ${}_hS_{\ell m}$ and compare mode by mode (at finite valu of spin, the sums truncate)

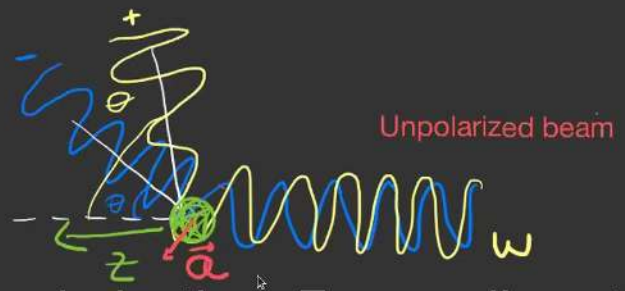
$$\sum_{\ell, m} A_{\ell m}\, {}_hS_{\ell m}(\theta, \varphi, a\omega) = \sum_{\ell m} {}_hR_{\ell m}(r, \omega a)\, {}_hS_{\ell m}(\theta, \varphi, a\omega)$$

To close: Some lessons from higher-spin amplitudes

- For $h<2$ only one diagram contribute to the amplitude, whereas for $h=2$, 4 diagrams



- Waves of different helicties scatter by different angles off the Kerr BH



Unpolarized beam

[Logi and Kovacs 1977]
$a \sim a^2$. Extend to arbitrary a^n

Spin induce polarization. For small scattering angle and @ θ^1

$$P_h \underset{\theta\to 0}{\simeq} -h\, a_z |\omega| \theta^2 \;, \quad h = 0, \tfrac{1}{2}, 1 \quad ; \quad P_{h=2} \underset{\theta\to 0}{\simeq} -a_z |\omega| \theta^2$$ [Dolan 2008]

[Logi & Kovacs 1977, Barbieri & Guadagnuni 2005] x 2

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To close: Some lessons from higher-spin amplitudes

- Consistent higher-spin amplitudes: Unitary, Causal (Positive time delay) well defined in the high energy $M \to 0$ limit!
- Perfect agreement with known lower spin amplitudes ($s \leq 2$)
- Give new building blocks to study the Binary BH problem problem (h=2, and in the weak gravity regime)
- Efficient to sum up the infinite series appearing from BHPT

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To get some flavour of what the amplitudes look like, for GW

$$A^{s_1 +-}_{h=2} = A^{(0)+-}_{h=2} \, e^{(-2w - k_2 - k_3)\cdot a} + \text{corrections } \mathcal{O}(a^5)$$

where $w^\mu = \frac{p_1 \cdot k_2}{p_1 \cdot \epsilon_2} \epsilon_2^\mu$

See Justin's talk

Thank you 😊