

Title: Progress and prospects in the mingling of quantum-scattering-amplitudes, post-Minkowskian, post-Newtonian, and self-force calculations

Speakers: Justin Vines

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Abstract: Recent years have seen a surge of progress in post-Minkowskian (PM, weak-field but arbitrary-speed) approximation methods for the gravitational two-body problem, complementing and reorganizing the still much further developed post-Newtonian (PN, weak-field and slow-motion) approximation. This has been driven by simplifying insights, powerful computational tools, and new results coming from the study of on-shell scattering amplitudes in quantum field theories and their classical limits. We will review some of these developments, focusing on the particularly impactful observation (ultimately also understandable from a purely classical perspective but born of the dialog with quantum amplitudes) that certain PM and PN results for arbitrary mass ratios can be determined from surprisingly low orders in the extreme-mass-ratio/self-force expansion.

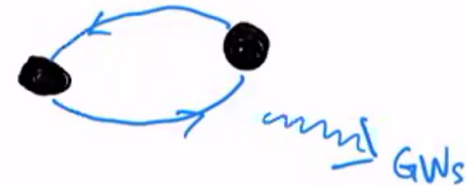
Progress and prospects in the mingling of  
quantum-scattering-amplitudes, post-Minkowskian,  
post-Newtonian, and self-force calculations

Justin Vines

AEI Potsdam

24th Capra Meeting – Perimeter Institute, Waterloo, Canada – 10 June 2021

To solve the binary black hole problem,  
for inspiraling bound orbits,  $v^2 \sim \frac{GM}{r} (\ll c^2: \text{PN})$   
...

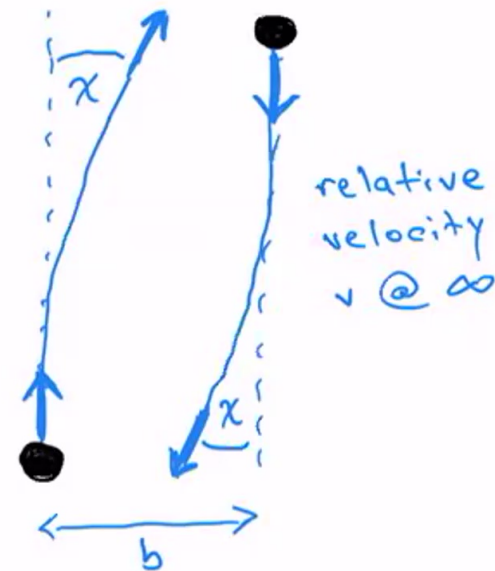


### Why scatter?

- cleaner than bound orbits,  
clearer gauge-invariant info.,  
~ free-particle states at  $\infty$
- scattering data also has info.  
on bound-orbit dynamics  
e.g., for (local) (aligned-spin) conservative dynamics,  
scattering angle  $\chi(v, b)$  determines Hamiltonian  $H(r, p)$   
governing both bound and unbound orbits

**Why post-Minkowskian?**  $\frac{GM}{r} \ll c^2 \sim v^2$  (PM)

- natural and relevant for scattering
- complement/resum/reorganize PN
- access to ultrarelativistic (UR) limit
- keep *special relativity* intact!



### Why quantum? (or not?)

- exploit powerful computational methods developed for QCD (SYM, SUGRA,...)  
{ generalized unitary; spinor and twistor variables; double copy:  $\text{GR} = (\text{QCD})^2$ ; ... }  
and associated rich structures of *on-shell scattering Amplitudes* in QFTs

... S. Weinberg's talk

Done

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for inspiraling bound orbits,  $v^2 \sim \frac{GM}{r}$  ( $\ll c^2$ : PN)

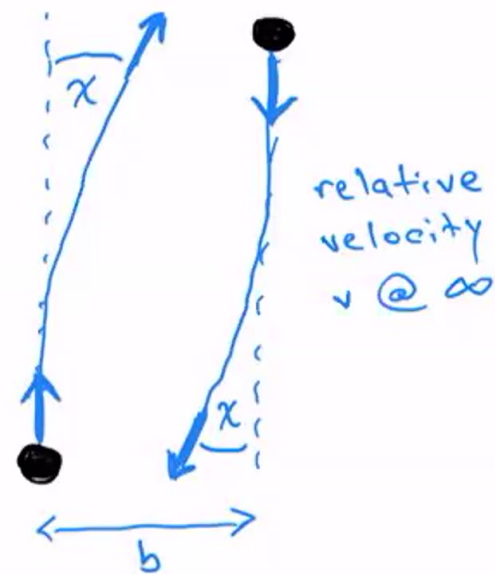
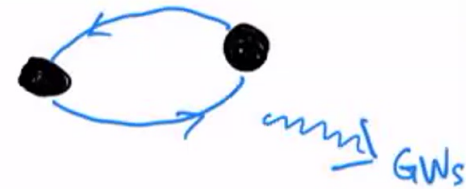
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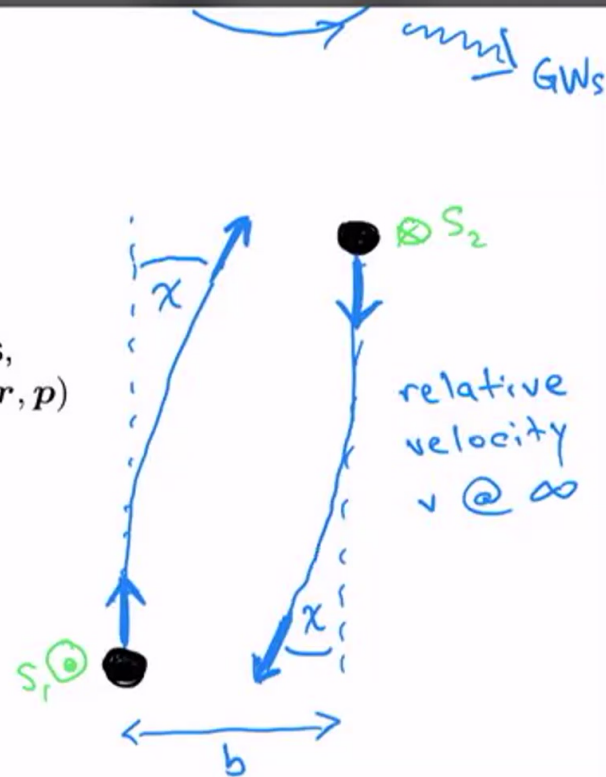
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Done

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$$\begin{aligned}
 \frac{H}{\mu} &= \frac{Mc^2}{\mu} + \frac{p^2}{2\mu^2} - \frac{1-3\nu}{8c^2} \frac{p^4}{\mu^4} + \frac{1-5\nu+5\nu^2}{16c^4} \frac{p^6}{\mu^6} + \dots \rightarrow \sum_a \frac{\sqrt{m_a^2 + p^2}}{\mu} \\
 M &= m_1 + m_2 - \frac{GM}{r} - \frac{3+2\nu}{2c^2} \frac{p^2}{\mu^2} \frac{GM}{r} + \frac{5-20\nu-8\nu^2}{8c^4} \frac{p^4}{\mu^2} \frac{GM}{r} + \dots \quad 1PM \\
 \mu &= \nu M = \frac{m_1 m_2}{M} + \frac{1+\nu}{2c^2} \frac{G^2 M^2}{r^2} + \frac{10+27\nu+3\nu^2}{4c^4} \frac{p^2}{\mu^2} \frac{G^2 M^2}{r^2} + \dots \quad 2PM \\
 &\quad - \frac{1+6\nu+0\nu^2}{4c^4} \frac{G^3 M^3}{r^3} + \dots \quad 3PM
 \end{aligned}$$

(isotropic gauge)

Westpfahl '85 - 'brute force'  $\rightarrow \chi_{2PM}$ Bicek+ '08 - ADM  $\rightarrow$  (complicated)  $H_{1PM}$ Damour '16 -  $\chi_{1PM} \rightarrow$  (simple)  $H_{1PM(EOB)}$ Damour '17 - Amp./ $\chi_{2PM} \rightarrow H_{2PM}$ Bjerrum-Bohr+ '18 - Amp.  $\rightarrow \chi_{2PM}$ Cheung+ '18 - Amp.  $\rightarrow H_{2PM}$ Bern+ '19 - Amp.  $\rightarrow H_{3PM}, \chi_{3PM}^{cons.}$ 

Olin-Porto+ '19, '20 - classical PM EFT,

boundary-to-bound, confirm  $\chi_{3PM}^{cons.}$ Damour '19 - questions about the high-energy (UR) limit of  $\chi_{cons.}$ Kosower+ '18 - KMOC: Amp.  $\rightarrow$  observables

Mogull+ '20, '21 - WQFT, waveform

Parra-Martinez+ '20 - diff.eqs. for integrals

Di Vecchia+ '20, '21 - Eikonal approach, importance of radiation in UR limit, 3PM

Damour '20 -  $\Delta J_{2PM} \rightarrow \chi_{3PM}^{rad.}$ , added to  $\chi_{3PM}^{cons.}$ , resolves UR limitHerrmann+ '21 - KMOC  $\rightarrow \Delta E_{3PM}$ , complete  $\Delta p_{3PM}^\mu$  (cons. + rad.)Bern+ '21 - Amp.  $\rightarrow \chi_{4PM}^{cons.-local}$

Done

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$$\frac{1}{\mu} = \frac{Mc}{\mu} + \frac{p}{2\mu^2} - \frac{1-3\nu}{8c^2} \frac{p^2}{\mu^4} + \frac{1-3\nu+3\nu^2}{16c^4} \frac{p^3}{\mu^6} + \dots \rightarrow \sum_a \frac{\sqrt{m_a^2 + p^2}}{\mu}$$

$$M = m_1 + m_2 - \frac{GM}{r} - \frac{3+2\nu}{2c^2} \frac{p^2}{\mu^2} \frac{GM}{r} + \frac{5-20\nu-8\nu^2}{8c^4} \frac{p^4}{\mu^2} \frac{GM}{r} + \dots \quad 1PM$$

$$\mu = \nu M = \frac{m_1 m_2}{M} + \frac{1+\nu}{2c^2} \frac{G^2 M^2}{r^2} + \frac{10+27\nu+3\nu^2}{4c^4} \frac{p^2}{\mu^2} \frac{G^2 M^2}{r^2} + \dots \quad 2PM$$

$$(isotropic gauge) \quad - \frac{1+6\nu+0\nu^2}{4c^4} \frac{G^3 M^3}{r^3} + \dots \quad 3PM$$

Westpfahl '85 - 'brute force'  $\rightarrow \chi_{2PM}$ Bicek+ '08 - ADM  $\rightarrow$  (complicated)  $H_{1PM}$ Damour '16 -  $\chi_{1PM} \rightarrow$  (simple)  $H_{1PM}(EOB)$ Select All ur '17 - Amp.  $\rightarrow \chi_{2PM}$ Bjerrum-Bohr+ '18 - Amp.  $\rightarrow \chi_{2PM}$ Cheung+ '18 - Amp.  $\rightarrow H_{2PM}$ Bern+ '19 - Amp.  $\rightarrow H_{3PM}, \chi_{3PM}^{cons.}$   $G^3$ Kälin-Porto+ '19, '20 - classical PM EFT, boundary-to-bound, confirm  $\chi_{3PM}^{cons.}$ Damour '19 - questions about the high-energy (UR) limit of  $\chi_{3PM}^{cons.}$ ;argued mass dependence of  $\chi$  (0SF  $\rightarrow$  2PM; 1SF  $\rightarrow$  4PM; 2SF  $\rightarrow$  6PM; ...)  $\rightarrow$  "Tutti Frutti"Kosower+ '18 - KMOC: Amp.  $\rightarrow$  observables

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Done

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$$\begin{aligned}
 M &= m_1 + m_2 \\
 \mu &= \nu M = \frac{m_1 m_2}{M} \\
 (\text{isotropic gauge})
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{GM}{r} - \frac{3+2\nu}{2c^2} \frac{p^2}{\mu^2} \frac{GM}{r} + \frac{5-20\nu-8\nu^2}{8c^4} \frac{p^4}{\mu^2} \frac{GM}{r} + \dots \quad \text{1PM} \\
 & + \frac{1+\nu}{2c^2} \frac{G^2 M^2}{r^2} + \frac{10+27\nu+3\nu^2}{4c^4} \frac{p^2}{\mu^2} \frac{G^2 M^2}{r^2} + \dots \quad \text{2PM} \\
 & - \frac{1+6\nu+0\nu^2}{4c^4} \frac{G^3 M^3}{r^3} + \dots \quad \text{3PM}
 \end{aligned}$$

Westpfahl '85 - 'brute force'  $\rightarrow \chi_{2\text{PM}}$ Bicek+ '08 - ADM  $\rightarrow$  (complicated)  $H_{1\text{PM}}$ Damour '16 -  $\chi_{1\text{PM}} \rightarrow$  (simple)  $H_{1\text{PM}}(\text{EOB})$ Damour '17 - Amp.  $\chi_{2\text{PM}} \rightarrow H_{2\text{PM}}$ Bierrum-Bohr+ '18 - Amp.  $\rightarrow \chi_{2\text{PM}}$ Onceng+ '18 - Amp.  $\rightarrow H_{2\text{PM}}$ Bern+ '19 - Amp.  $\rightarrow H_{3\text{PM}}, \chi_{3\text{PM}}^{\text{cons.}}$ Kälin-Porto+ '19, '20 - classical PM EFT, boundary-to-bound, confirm  $\chi_{3\text{PM}}^{\text{cons.}}$ Damour '19 - questions about the high-energy (UR) limit of  $\chi_{3\text{PM}}^{\text{cons.}}$ ;argued mass dependence of  $\chi$  (0SF  $\rightarrow$  2PM; 1SF  $\rightarrow$  4PM; 2SF  $\rightarrow$  6PM; ...)  $\rightarrow$  "Tutti Frutti"Kosower+ '18 - KMOC: Amp.  $\rightarrow$  observables

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SUGRAG<sup>3</sup>



[Damour 1912.] [see also JV+ 1812., Bern+ 1908., Kälin+ 1910., Antonelli+ 2010.]

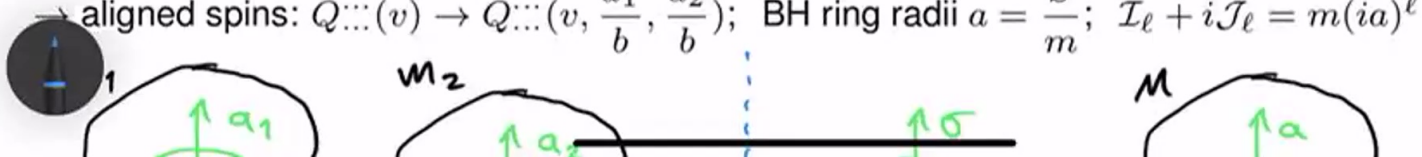
structure of PM expansion, Poincaré symmetry, dimensional analysis, ...  $\Rightarrow c=1$

$$|\Delta p| = \frac{Gm_1m_2}{b} \left[ Q^{1\text{PM}}(v) + \frac{G}{b} \left( m_1 Q_1^{2\text{PM}}(v) + m_2 Q_2^{2\text{PM}}(v) \right) + \frac{G^2}{b^2} \left( m_1^2 Q_{11}^{3\text{PM}}(v) + m_2^2 Q_{22}^{3\text{PM}}(v) + m_1 m_2 Q_{12}^{3\text{PM}}(v) \right) + \frac{G^3}{b^3} \left( m_1^3 Q_{111}^{4\text{PM}}(v) + m_2^3 Q_{222}^{4\text{PM}}(v) + m_1^2 m_2 Q_{112}^{4\text{PM}}(v) + m_1 m_2^2 Q_{122}^{4\text{PM}}(v) \right) \right] + \mathcal{O}(G^5)$$

$$E^2 = m_1^2 + m_2^2 + \frac{2m_1 m_2}{\sqrt{1-v^2}}$$

$$\chi = \frac{GE}{b} \left[ X^{1\text{PM}}(v) + \frac{GM}{b} X_1^{2\text{PM}}(v) + \frac{G^2 M^2}{b^2} \left( X_{11}^{3\text{PM}}(v) + \nu X_{12}^{3\text{PM}}(v) \right) + \frac{G^3 M^3}{b^3} \left( X_{111}^{4\text{PM}}(v) + \nu X_{112}^{4\text{PM}}(v) \right) \right] + \mathcal{O}(G^5),$$

aligned spins:  $Q_{\ell\ell\ell}(v) \rightarrow Q_{\ell\ell\ell}(v, \frac{a_1}{b}, \frac{a_2}{b})$ ; BH ring radii  $a = \frac{S}{m}$ ;  $\mathcal{I}_\ell + i\mathcal{J}_\ell = m(ia)^\ell$



argued mass dependence of  $\chi$  (0SF  $\rightarrow$  2PM; 1SF  $\rightarrow$  4PM; 2SF  $\rightarrow$  6PM; ...)  $\rightarrow$  "Tutti Frutti"

[Damour 1912.] [see also JV+ 1812., Bern+ 1908., Kälin+ 1910., Antonelli+ 2010.]

structure of PM expansion, Poincaré symmetry, dimensional analysis, ...  $\Rightarrow$

$c=1$

$$\begin{aligned}
 |\Delta p| &= \frac{Gm_1m_2}{b} \left[ Q^{1\text{PM}}(v) \right. && \text{1} \leftrightarrow \text{2 symm.} \\
 &+ \frac{G}{b} \left( \underline{m_1 Q_1^{2\text{PM}}(v)} + \underline{m_2 Q_2^{2\text{PM}}(v)} \right) && E^2 = m_1^2 + m_2^2 + \frac{2m_1m_2}{\sqrt{1-v^2}} \\
 &+ \frac{G^2}{b^2} \left( m_1^2 Q_{11}^{3\text{PM}}(v) + m_2^2 Q_{22}^{3\text{PM}}(v) + m_1m_2 Q_{12}^{3\text{PM}}(v) \right) \\
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 \chi &= \frac{GE}{b} \left[ X^{1\text{PM}}(v) + \frac{GM}{b} X_1^{2\text{PM}}(v) + \frac{G^2M^2}{b^2} \left( X_{11}^{3\text{PM}}(v) + \nu X_{12}^{3\text{PM}}(v) \right) \right. \\
 &\quad \left. + \frac{G^3M^3}{b^3} \left( X_{111}^{4\text{PM}}(v) + \nu X_{112}^{4\text{PM}}(v) \right) \right] + \mathcal{O}(G^5),
 \end{aligned}$$

$\rightarrow$  aligned spins:  $Q_{\dots}(v) \rightarrow Q_{\dots}(v, \frac{a_1}{b}, \frac{a_2}{b})$ ; ~~BH ring radii  $a$~~   $= \frac{S}{m}$ ;  $\mathcal{I}_\ell + i\mathcal{J}_\ell = m(ia)^\ell$

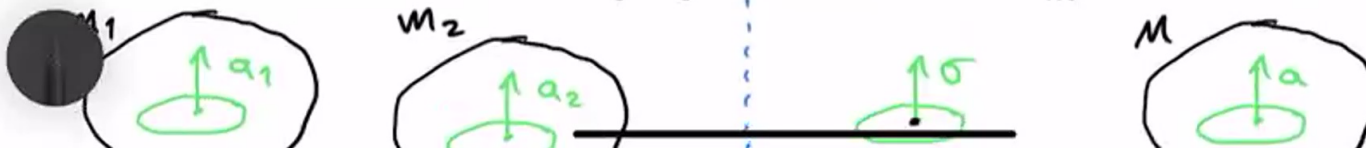
4 of 12 [mour 1912.] [see also JV+ 1812., Bern+ 1908., Kälín+ 1910., Antonelli+ 2010.]

structure of PM expansion, Poincaré symmetry, dimensional analysis, ...  $\Rightarrow c=1$

$$\begin{aligned}
 |\Delta p| &= \frac{Gm_1m_2}{b} \left[ Q^{1\text{PM}}(v) \quad 1 \leftrightarrow 2 \text{ symm.} \right. \\
 &\quad + \frac{G}{b} \left( m_1 Q_1^{2\text{PM}}(v) + m_2 Q_2^{2\text{PM}}(v) \right) \\
 &\quad + \frac{G^2}{b^2} \left( m_1^2 Q_{11}^{3\text{PM}}(v) + m_2^2 Q_{22}^{3\text{PM}}(v) + m_1 m_2 Q_{12}^{3\text{PM}}(v) \right) \\
 &\quad \left. + \frac{G^3}{b^3} \left( m_1^3 Q_{111}^{4\text{PM}}(v) + m_2^3 Q_{222}^{4\text{PM}}(v) + m_1^2 m_2 Q_{112}^{4\text{PM}}(v) + m_1 m_2^2 Q_{122}^{4\text{PM}}(v) \right) \right] + \mathcal{O}(G^5) \\
 \chi &= \frac{GE}{b} \left[ X^{1\text{PM}}(v) + \frac{GM}{b} X_1^{2\text{PM}}(v) + \frac{G^2 M^2}{b^2} \left( X_{11}^{3\text{PM}}(v) + \nu X_{12}^{3\text{PM}}(v) \right) \right. \\
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 \end{aligned}$$

$E^2 = m_1^2 + m_2^2 + \frac{2m_1 m_2}{\sqrt{1-v^2}}$

$\rightarrow$  aligned spins:  $Q^{\dots}(v) \rightarrow Q^{\dots}(v, \frac{a_1}{b}, \frac{a_2}{b})$ ; BH ring radii  $a = \frac{S}{m}$ ;  $\mathcal{I}_\ell + i\mathcal{J}_\ell = m(ia)^\ell$



Done

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$$\begin{aligned}
 & + \frac{G}{b} \left( \underbrace{m_1 Q_1^{2\text{PM}}(v)} + \underbrace{m_2 Q_2^{2\text{PM}}(v)} \right) \\
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 & \left. + \frac{G^3 M^3}{b^3} \left( X_{111}^{4\text{PM}}(v) + \nu X_{112}^{4\text{PM}}(v) \right) \right] + \mathcal{O}(G^5),
 \end{aligned}$$

$2 \frac{1+\nu^2}{\nu^2}$

$3\pi \frac{4+\nu^2}{\nu^2}$

→ aligned spins:  $Q_{\ell}^{\text{PM}}(v) \rightarrow Q_{\ell}^{\text{PM}}(v, \frac{a_1}{b}, \frac{a_2}{b})$ ; BH ring radii  $a = \frac{S}{m}$ ;  $\mathcal{I}_\ell + i\mathcal{J}_\ell = m(ia)^\ell$



real BBH



MPD

"test BH" in a Kerr background



Done

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spin <sup>0</sup>	1PM (0PN)	2PM (1PN)	3PM (2PN)	4PM (3PN)	5PM (4PN)	6PM (5PN)	7PM (6PN)
spin <sup>1</sup>		LO S <sup>1</sup> (1.5PN)	NLO S <sup>1</sup> (2.5PN)	N <sup>2</sup> LO S <sup>1</sup> (3.5PN)	N <sup>3</sup> LO S <sup>1</sup> (4.5PN)	N <sup>4</sup> LO S <sup>1</sup> (5.5PN)	N <sup>5</sup> LO S <sup>1</sup> (6.5PN)
spin <sup>2</sup>			LO S <sup>2</sup> (2PN)	NLO S <sup>2</sup> (3PN)	N <sup>2</sup> LO S <sup>2</sup> (4PN)	N <sup>3</sup> LO S <sup>2</sup> (5PN)	N <sup>4</sup> LO S <sup>2</sup> (6PN)
spin <sup>3</sup>				LO S <sup>3</sup> (3.5PN)	NLO S <sup>3</sup> (4.5PN)	N <sup>2</sup> LO S <sup>3</sup> (5.5PN)	N <sup>3</sup> LO S <sup>3</sup> (6.5PN)
...					LO S <sup>4</sup> (4PN)	NLO S <sup>4</sup> (5PN)	N <sup>2</sup> LO S <sup>4</sup> (6PN)
						LO S <sup>5</sup> (5.5PN)	NLO S <sup>5</sup> (6.5PN)
							LO S <sup>6</sup> (6PN)

Select All

**Traditional PN** - (Blanchet+ completing 4PN S<sup>0</sup> radiation)

**EFT of PN** - Foffa+, Bluemlein+ toward 5PN+

- Levi+ toward 4.5 and 5PN spin<sup>1,2,3,4</sup>

**Tutti Fruitti** - Bini+ 5PN complete modulo 2SF terms; ... 6PN ...

- Antonelli+ complete 4.5PN S<sup>1</sup> and 5PN aligned S<sub>1</sub>S<sub>2</sub> from 1SF

- Khalil's talk 5.5PN S<sup>1</sup> mod. 2SF and 5PN aligned S<sup>2</sup> from 1SF

- Siemonsen+ aligned 4.5PN S<sup>3</sup> and 5PN S<sup>4</sup> from 0SF – check w/ 1SF

**Traditional PM** - Bini+ '17 LO S<sup>1</sup>, '18 NLO S<sup>1</sup>; JV '17 LO S<sup>∞</sup>

**EFT of PM** - Liu+ NLO S<sup>1</sup> and S<sup>2</sup>



real BBH

"test BH" in a Kerr background

	0SF		1SF		2SF		3SF
spin <sup>0</sup>	1PM (0PN)	2PM (1PN)	3PM (2PN)	4PM (3PN)	5PM (4PN)	6PM (5PN)	7PM (6PN)
spin <sup>1</sup>		LO S <sup>1</sup> (1.5PN)	NLO S <sup>1</sup> (2.5PN)	N <sup>2</sup> LO S <sup>1</sup> (3.5PN)	N <sup>3</sup> LO S <sup>1</sup> (4.5PN)	N <sup>4</sup> LO S <sup>1</sup> (5.5PN)	N <sup>5</sup> LO S <sup>1</sup> (6.5PN)
spin <sup>2</sup>			LO S <sup>2</sup> (2PN)	NLO S <sup>2</sup> (3PN)	N <sup>2</sup> LO S <sup>2</sup> (4PN)	N <sup>3</sup> LO S <sup>2</sup> (5PN)	N <sup>4</sup> LO S <sup>2</sup> (6PN)
spin <sup>3</sup>				LO S <sup>3</sup> (3.5PN)	NLO S <sup>3</sup> (4.5PN)	N <sup>2</sup> LO S <sup>3</sup> (5.5PN)	N <sup>3</sup> LO S <sup>3</sup> (6.5PN)
...					LO S <sup>4</sup> (4PN)	NLO S <sup>4</sup> (5PN)	N <sup>2</sup> LO S <sup>4</sup> (6PN)
						LO S <sup>5</sup> (5.5PN)	NLO S <sup>5</sup> (6.5PN)
							LO S <sup>6</sup> (6PN)

**Traditional PN** - (Blanchet+ completing 4PN S<sup>0</sup> radiation)

**EFT of PN** - Foffa+, Bluemlein+ toward 5PN+

- Levi+ toward 4.5 and 5PN spin<sup>1,2,3,4</sup>

**Tutti Frutti** - Rini+ 5PN complete modulo 2SF terms — 6PN

### PN Tutti Frutti calculations require

- assumption  $\exists$  Hamiltonian for (local) conservative dynamics (beyond  $N^3LO$ , special treatment of tail terms)
- the mass dependence of the scattering data
- analytic PN expansions of bound-orbit SF gauge-invariants (redshift, spin precession, tidal invariants, ...) [Bini+, Kavanagh+]
  - 1SF spin prec. in Schw. *with linear-in-spin* would complete aligned 5PN  $S^2$
  - 1SF redshift and spin prec. in equatorial Kerr *with eccentricity* would resolve aligned 4.5PN  $S^3$  and 5PN  $S^4$
- the First Law of binary mechanics [Le Tiec+]

The last three ingredients are  $\sim$ understood only for aligned spins.

— *How to generalize to precession?* —

Is there a single scalar gauge-invariant which characterizes spin-precessing dynamics?

- see spinning eikonal phase of Bern+ '20, Kosmopoulos+ '21

With precession, it's nontrivial to relate scattering data to Hamiltonian.

— Make use of conserved quantities? Are spinning BBHs 'integrable'?

- 2PN integrability: Tanay+ '20
- pole-dipole in Kerr: Carter-like constant (+1) Rüdiger '85, Witzany '19

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### Why scatter?

- cleaner than bound orbits,  
clearer gauge-invariant info.,  
~ free-particle states at  $\infty$
- scattering data also has info.  
on bound-orbit dynamics

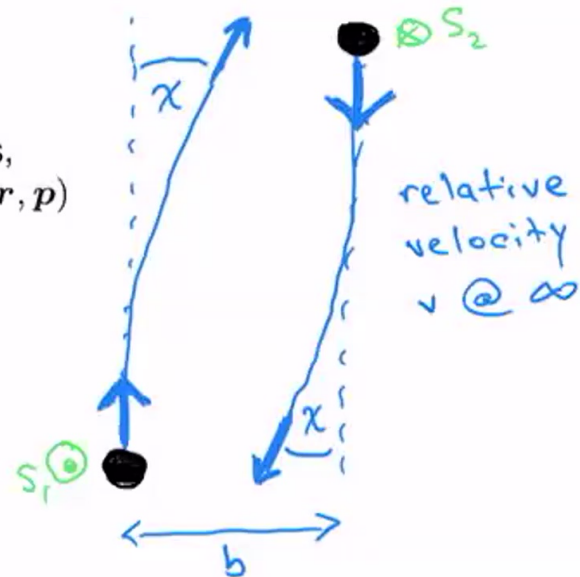
e.g., for (local) (aligned-spin) conservative dynamics,  
scattering angle  $\chi(v, b)$  determines Hamiltonian  $H(r, p)$   
governing both bound and unbound orbits

### Why post-Minkowskian? $\frac{GM}{r} \ll c^2 \sim v^2$ (PM)

- natural and relevant for scattering
- complement/resum/reorganize PN
- access to ultrarelativistic (UR) limit
- keep *special relativity* intact!

### Why quantum? (or not?)

- exploit powerful computational methods developed for QCD (SYM, SUGRA,...)  
{ generalized unitary; spinor and twistor variables; double copy:  $GR = (QCD)^2$ ; ... }
- and associated rich structures of *on-shell scattering Amplitudes* in QFTs



see Solon's talk