Title: Exotic MOTS, the stability operator and their role in black hole mergers

Speakers: Ivan Booth

Series: Strong Gravity

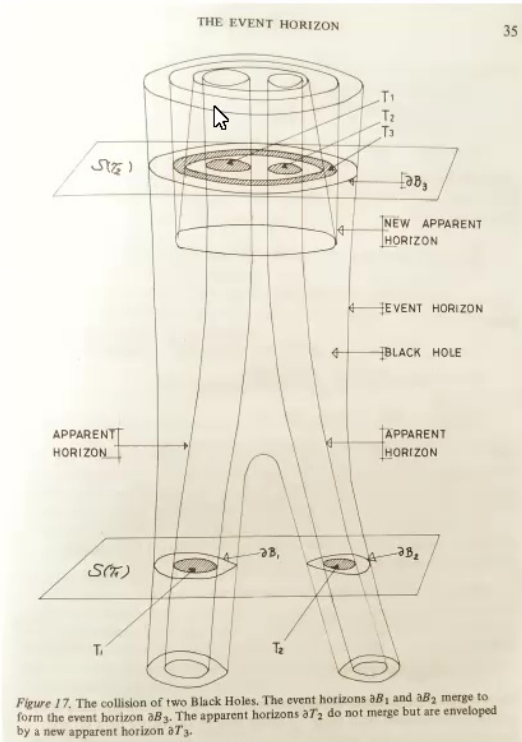
Date: June 03, 2021 - 1:00 PM

URL: http://pirsa.org/21060051

Abstract: In the last couple of years it has been demonstrated that black hole spacetimes contain many more marginally outer trapped surfaces (MOTS) than had been previously recognized. For example, there is an infinite family of axially symmetric MOTS even in the Schwarzschild solution, of which the apparent horizon is only the first element. In a recent series of papers (arXiv: 2104.10265, 2104.11343, 2104.11343) we demonstrated that these exotic new MOTS play a key role in black hole mergers and, in fact, are the missing pieces needed to complete the apparent horizon â€œpair of pantsâ€• diagram. This merger turns out to be far richer than that of event horizons and, staying with clothing analogies, is better represented by a rococo ball gown than a pair of pants.

In this talk, I will overview the techniques that we used to find these surfaces, explain the role played by the exotic MOTS in resolving the merger of apparent horizons during a (head-on, non-rotating) black hole merger and also show how the stability operator brings order to what initially appears to be a chaotic melee of (often self-intersecting!) MOTS as they create, annihilate and weave through time. The stability operator was initially introduced by Andersson, Mars and Simon to characterize when a MOTS will smoothly evolve in time, but I will show that it can also be fruitfully understood as the Jacobi operator for MOTS.

Exotic MOTSs: finding unexpected "apparent horizons"



Hawking, Les Houches 1972

Ivan Booth (Memorial)

Robie Hennigar (Memorial, Waterloo, Laurier)

Daniel Pook-Kolb (Max Planck, Hanover)

arXiv: 2104.10265, 2104.11344, 2005.05350

+

IB, RH, Saikat Mondal

arXiv: 2005:05350 Phys. Rev. D 102, 044031 (2020)

+

IB, RH, Hari Kunduri, Sarah Muth, Billy Chan

(upcoming)

MEMORIAL UNIVERSITY

UNIVERSITY OF WATERLOO

LAURIER Inspiring Lives.

MAX PLANCK INSTITUTE FOR GRAVITATIONAL PHYSICS (Albert Einstein Institute)

Bourses postdoctorales Banting Postdoctoral Fellowships

NSERC CRSNG

We want to know:

1) What is the full apparent horizon "pair of pants" diagram?

2) What is a "typical" marginally outer trapped surface?

3) Does this all have physical implications?

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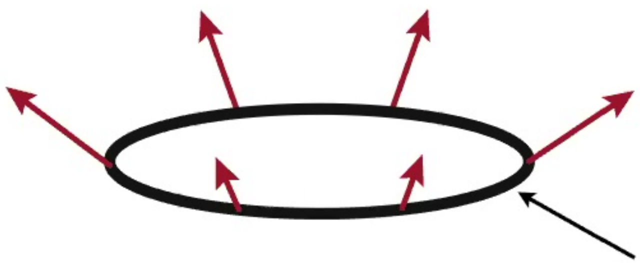
Marginally Outer Trapped Surfaces (MOTS)



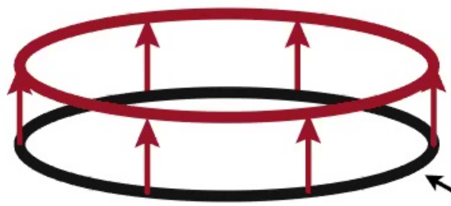
ℓ^a- outward null normal

closed spacelike two-surface

Outward null expansion: $\delta_\ell \sqrt{q} = \sqrt{q}\,\theta_{(\ell)}$

3

Marginally Outer Trapped Surfaces (MOTS)



ℓ^a- outward null normal

closed spacelike two-surface

Outward null expansion: $\delta_\ell \sqrt{q} = \sqrt{q}\theta_{(\ell)}$

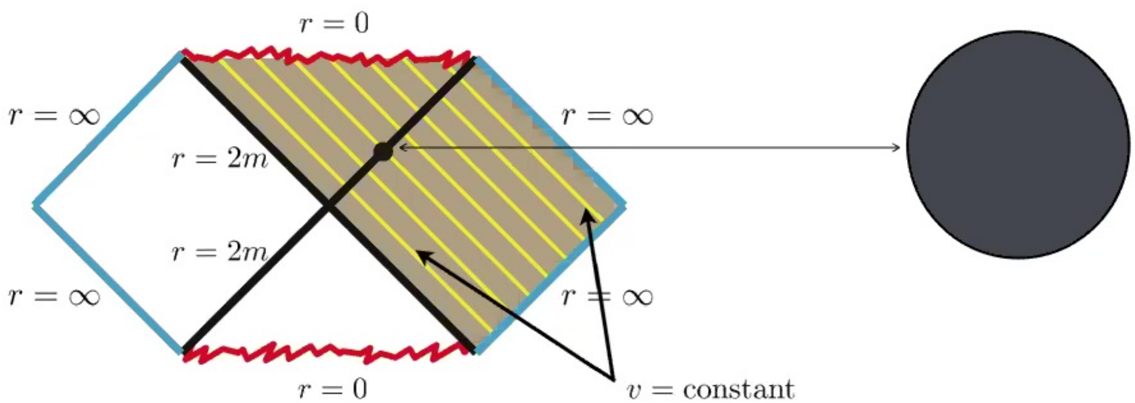
- Regular (convex) surface: $\theta_{(\ell)} > 0$
- Outer trapped surface: $\theta_{(\ell)} < 0$
- Marginally outer trapped surface (MOTS): $\theta_{(\ell)} = 0$

Apparent horizon => MOTS

3

Simplest MOTS: Schwarzschild Black Holes

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$



$$\theta_{(\ell)} = 0$$

Spherical Symmetry

Simplest MOTS: Schwarzschild Black Holes

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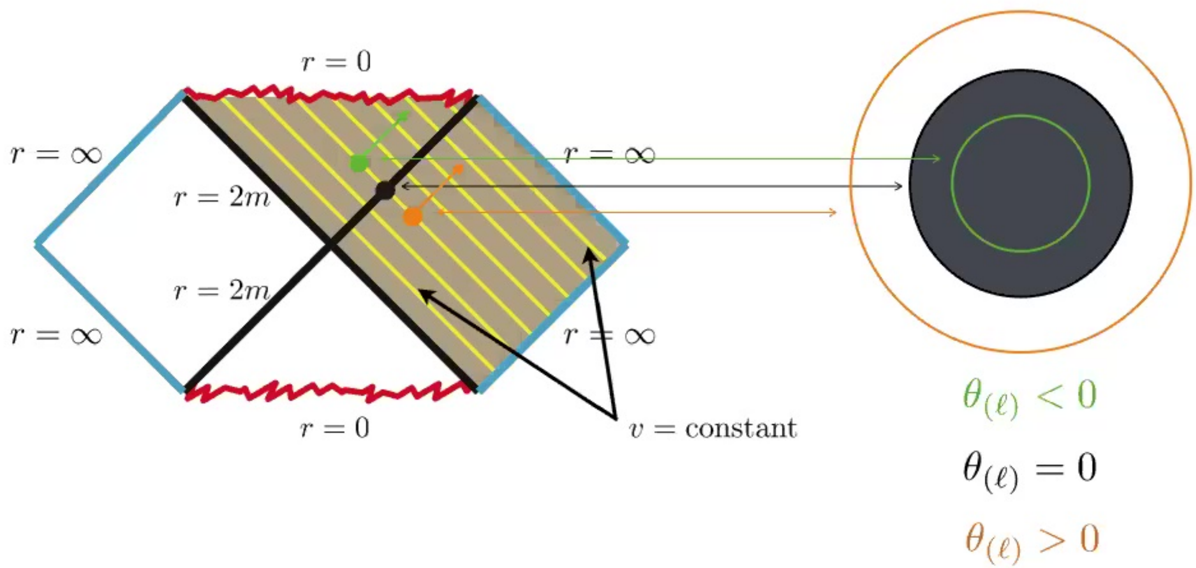

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Spherical Symmetry

Apparent horizons can "jump"

- Lemaitre-Tolman-Bondi describes the collapse of timelike dust

$$ds^2 = -dt^2 + \left(\frac{B(r_o,t)}{A(r_o,t)^{1/3}}\right)^2 dr_o^2 + R(r_o,t)^2 d\Omega^2$$ (Yodzis 70s)

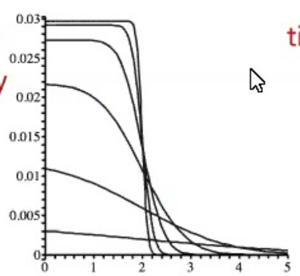
where $A(r_o,t)$, $B(r_o,t)$ and $R(r_o,t)$ are explicit functions of (r_o,t)

and $m(r_o) = \int_0^{r_o}\int_0^{\pi}\int_0^{2\pi} \sqrt{h}\rho \, dr \, d\theta \, d\phi$ (initial mass distribution)

- MOTS at $\theta_{(\ell)} = 0 \iff R(r_o,t) = 2m(r_o)$

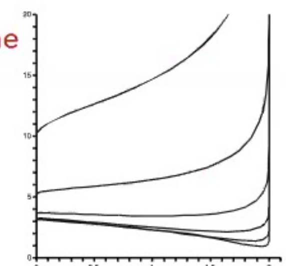
- Allows us to evolve dust/spacetime and track MOTS

initial matter density



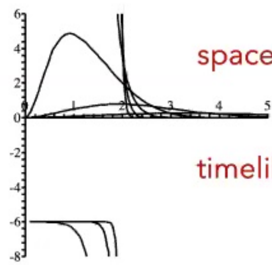
Mass distribution

time



MTT evolution

5



spacelike

timelike

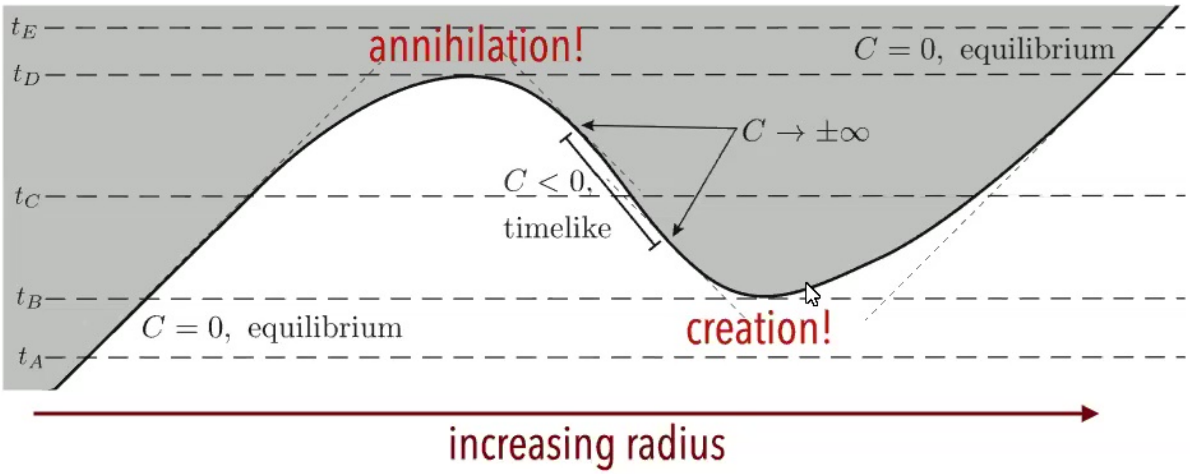
MTT signature

I Bendov (FLRW + Schwarz) PRD 2004

IB, Brits, Gonzalez, Van Den Broeck CQG 2006

Spherical Symmetry

MOTS "creation" and "annihilation"



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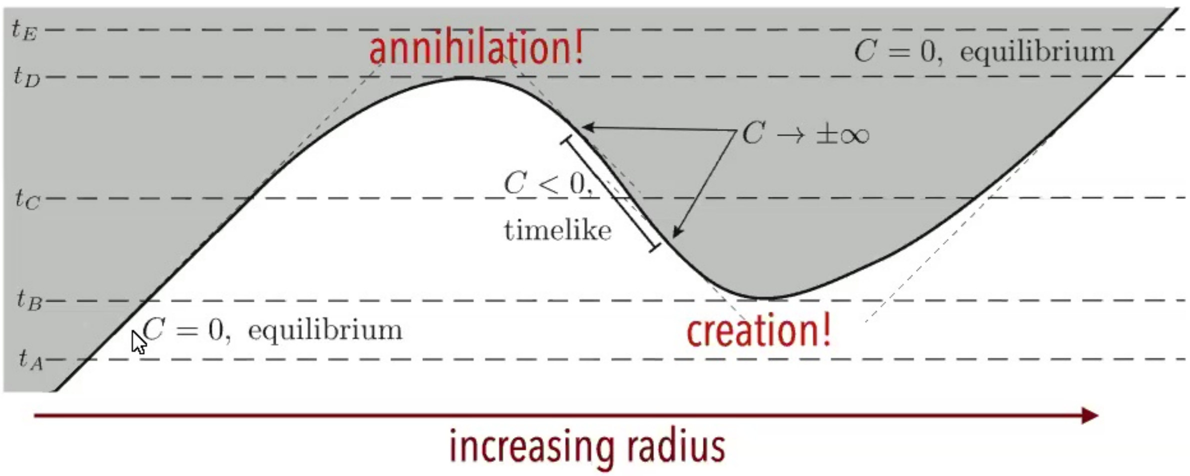
Spherical Symmetry

MOTS "creation" and "annihilation"



or weaving through time…

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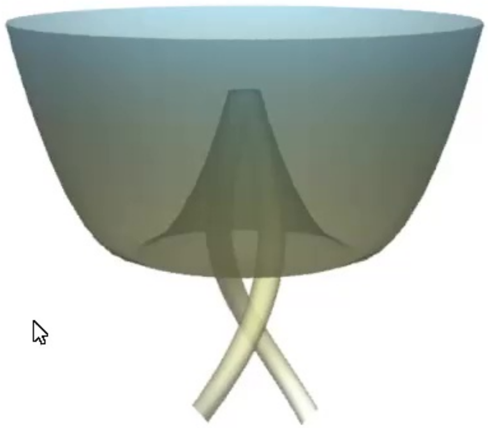
A pair of pants for apparent horizons? (pre 2018)

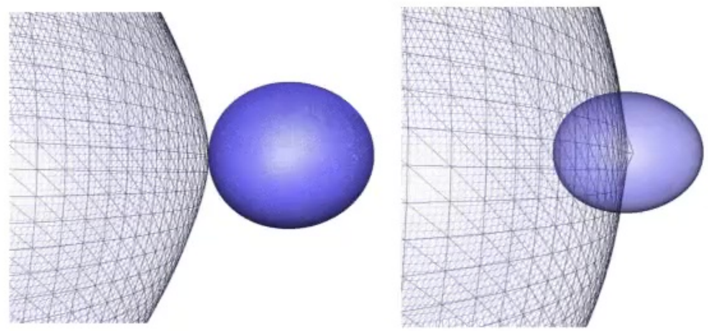
Since at least 2000: one continuous ``horizon''

(S. Hayward, Proceedings MG9, gr-qc/0008071)

Saggy pair of pants



(Gupta, Krishnan, Nielsen, Schnetter, 2018, PRD97, 084028)



(Mösta, Andersson, Metzger, Szilágyi, Winicour, 2015, CQG 32, 235003)

AH finders lost track of the original (and inner) horizons

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MOTS Finders

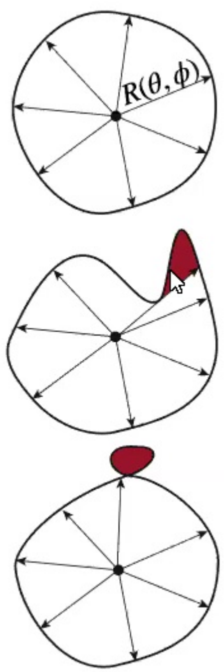
1) Standard $r = R(\theta, \phi)$

Second order elliptic PDE for $R(\theta, \phi)$

star-shaped

Not star-shaped

Not star-shaped



MOTS Finders

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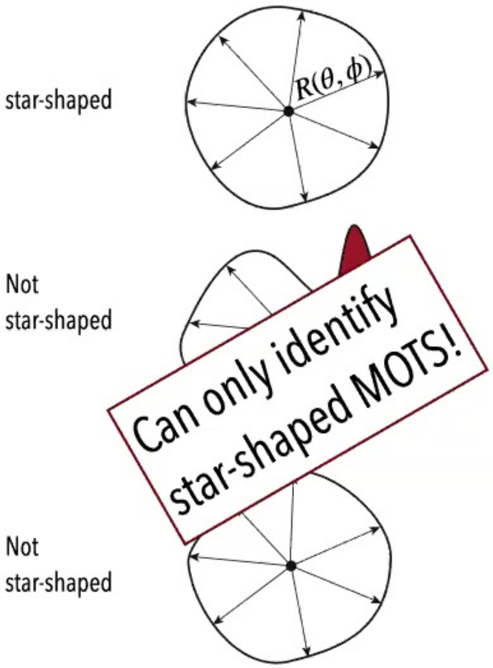
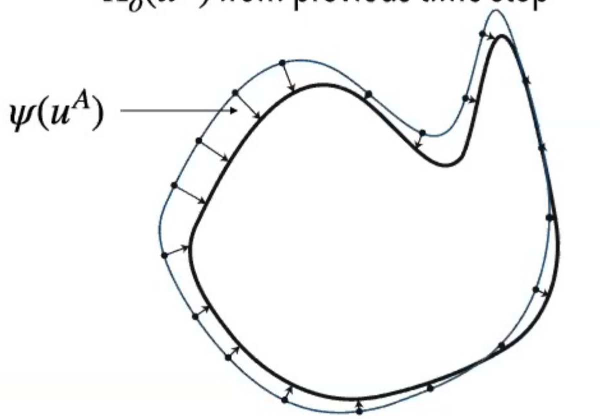


2) Reference Surface:

$$x^i = X^i_o(u^A) + \psi(u^A) N_i(u^a)$$

Second order elliptic PDE for $\psi(u^A)$

$X^i_o(u^A)$ from previous time step



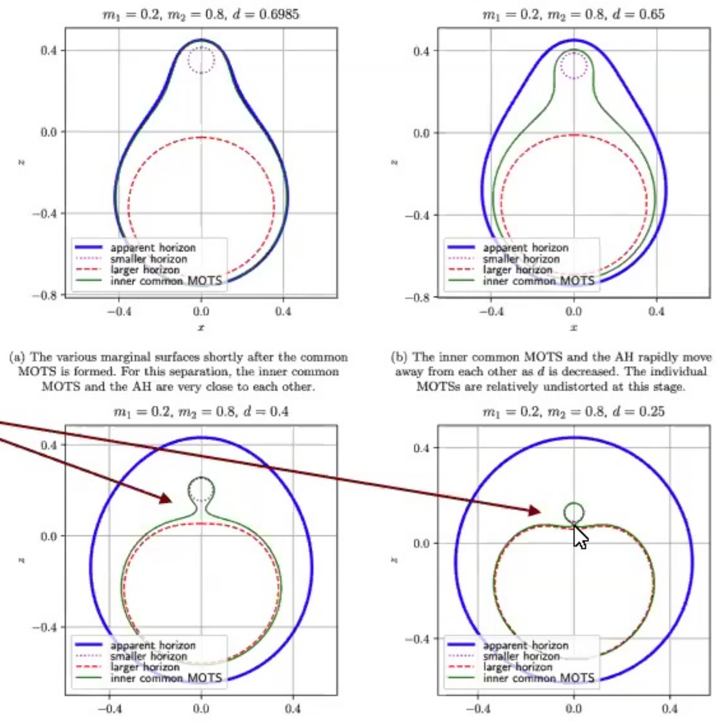
Pook-Kolb, Birnholtz, Krishnan, Schnetter
PRD 100, 084044 (2019)

2018

The existence and stability of marginally trapped surfaces

Daniel Pook-Kolb,[1,2] Ofek Birnholtz,[3] Badri Krishnan,[1,2] and Erik Schnetter[4,5,6]

Phys. Rev. D 99, 064005 (2019)



(a) The various marginal surfaces shortly after the common MOTS is formed. For this separation, the inner common MOTS and the AH are very close to each other.

(b) The inner common MOTS and the AH rapidly move away from each other as d is decreased. The individual MOTSs are relatively undistorted at this stage.

NOT star-shaped

Boyer-Lindquist (time-symmetric) initial data

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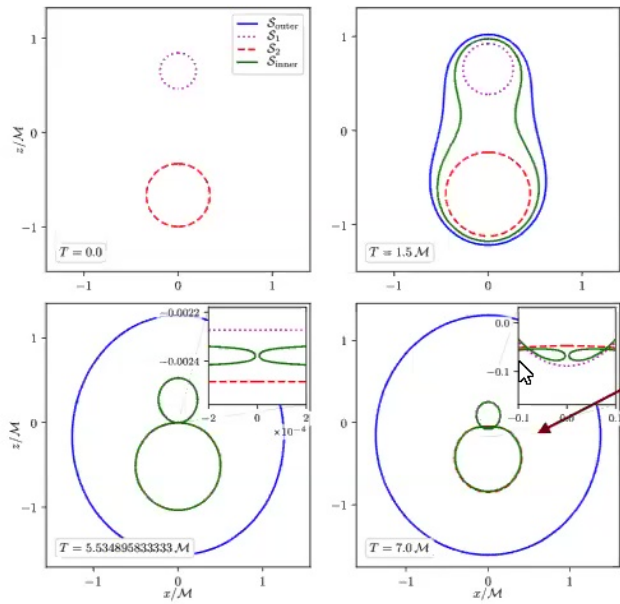
2019

Self-intersecting marginally outer trapped surfaces

Daniel Pook-Kolb,[1,2] Ofek Birnholtz,[3] Badri Krishnan,[1,2] and Erik Schnetter[4,5,6]

Phys. Rev. D 100, 084044 (2019)



DEFINITELY NOT star-shaped

+ needed good guess for reference surface

Axisymmetric black hole merger - NOT initial data

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What happens next?

Are there more MOTS in heaven and earth than were dreamt of by our horizon finders?

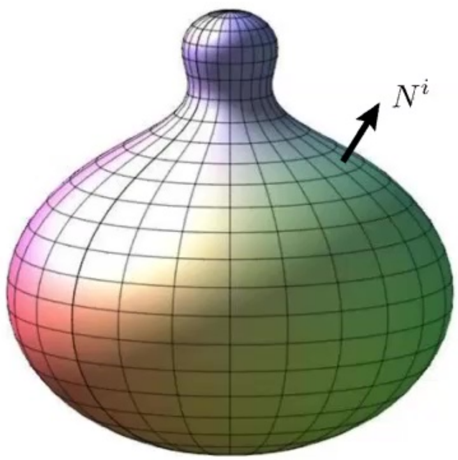
Need an agnostic MOTS finder

Axisymmetric MOT(O)S

(originally IB, R. Hennigar, S. Mondal, improved in IB, R. Hennigar, D. Pook-Kolb)

- Spacetime slice: $(\Sigma, h_{ij}, D_i, K_{ij})$ with future unit timelike normal u_α

- Surface in slice: $(S, q_{AB}, d_A, k^{(N)}_{AB})$ with outward unit spacelike normal N_i



- Timelike expansion: $k_{(u)} = q^{ij}K_{ij}$

- Spacelike expansion: $k_{(N)} = q^{AB}k^{(N)}_{AB}$

- Outward-null expansion: $\theta_+ = k_{(u)} + k_{(N)}$

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Axisymmetric MOT(O)S

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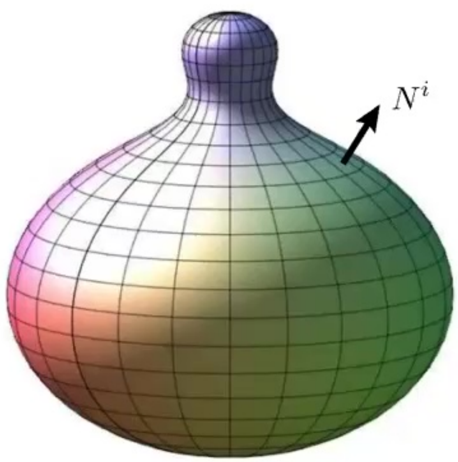

- Timelike expansion: $k_{(u)} = q^{ij} K_{ij}$
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- Outward-null expansion: $\theta_+ = k_{(u)} + k_{(N)}$

MOTS: $\theta_+ = 0$ (a 2nd order PDE)

$$\Rightarrow \quad q^{ij} D_i N_j + q^{ij} K_{ij} = 0 \Rightarrow \quad q^{ij} \partial_i N_j = -q^{ij} (\Gamma^k_{ij} N_k + K_{ij})$$

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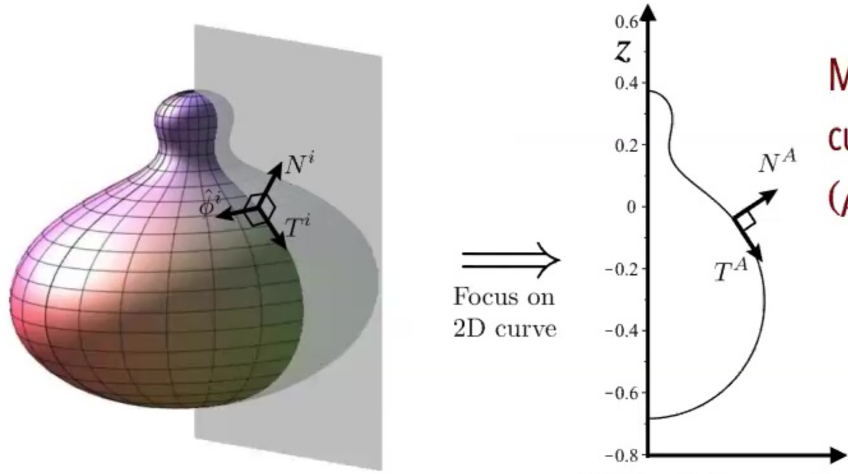
Axisymmetric alternative: MOTSodesics

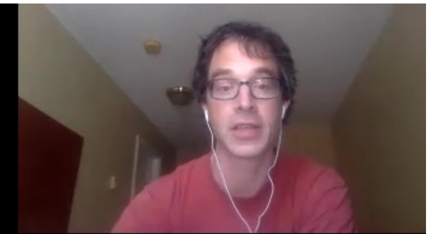


$\mathcal{S} : (P(s), Z(s), \phi)$ $\gamma : (P(s), Z(s))$

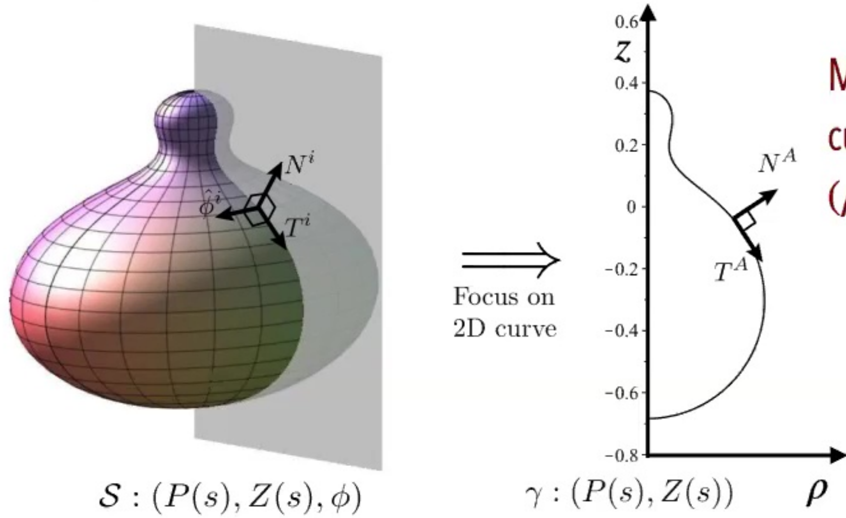
MOTSodesic: curve in the (ρ, z) plane

$$k_N = \left(T^i T^j + \hat{\phi}^i \hat{\phi}^j\right) D_i N_j = -N_b (T^a \bar{D}_a T^b) + \hat{\phi}^i \hat{\phi}^j D_i N_j$$

$$k_u = q^{ij} K_{ij} = K_{ab} T^a T^b + \frac{K_{\phi\phi}}{R^2}$$



Axisymmetric alternative: MOTSodesics



MOTSodesic: curve in the (ρ, z) plane

$$T^A \bar{D}_A T^B = \kappa N^A$$

$$\kappa = N^a \bar{D}_a \ln R + \left(K_{ab} T^a T^b + \frac{K_{\phi\phi}}{R^2} \right)$$

Coupled, 2nd order DEs

For any ICs there IS a solution (though usually not closed)

(similar to: Apparent horizons in the two black hole problem, Cadez, 1974)

Example: MOT(O)S in Schwarzschild

- Schwarzschild in Painlevé-Gullstand (spacelike, horizon penetrating slices)

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)\mathrm{d}T^2 + 2\mathrm{d}T\mathrm{d}r + \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

$$K_{ij}\mathrm{d}x^i\mathrm{d}x^j = \sqrt{\frac{M}{2r^3}}\mathrm{d}r^2 - \sqrt{2Mr}\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

$$\Downarrow$$

$$\ddot{R} = R\dot{\Theta}\left(\dot{\Theta} - \kappa\right) \quad \text{for} \quad \kappa = -\dot{\Theta} + \frac{\cot\Theta}{R}\dot{R}$$

$$\ddot{\Theta} = -\frac{\dot{R}}{R}\left(2\dot{\Theta} - \kappa\right) \qquad \pm\sqrt{\frac{m}{2R^3}}\left(3R^2\dot{\Theta}^2 + 1\right)$$

Easily solved using Maple, Mathematica or whatever

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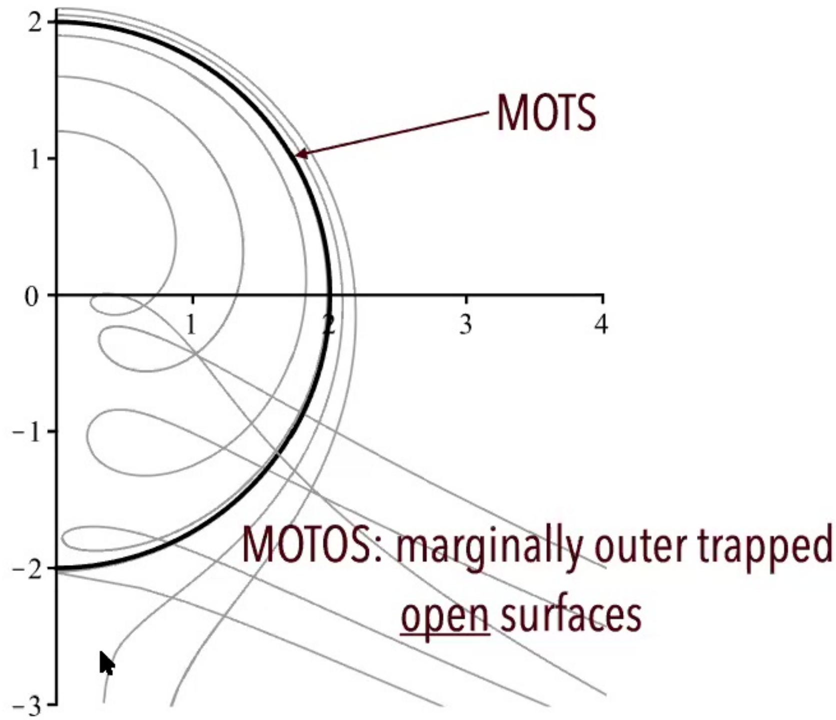
MOTSodesics of Schwarzschild I



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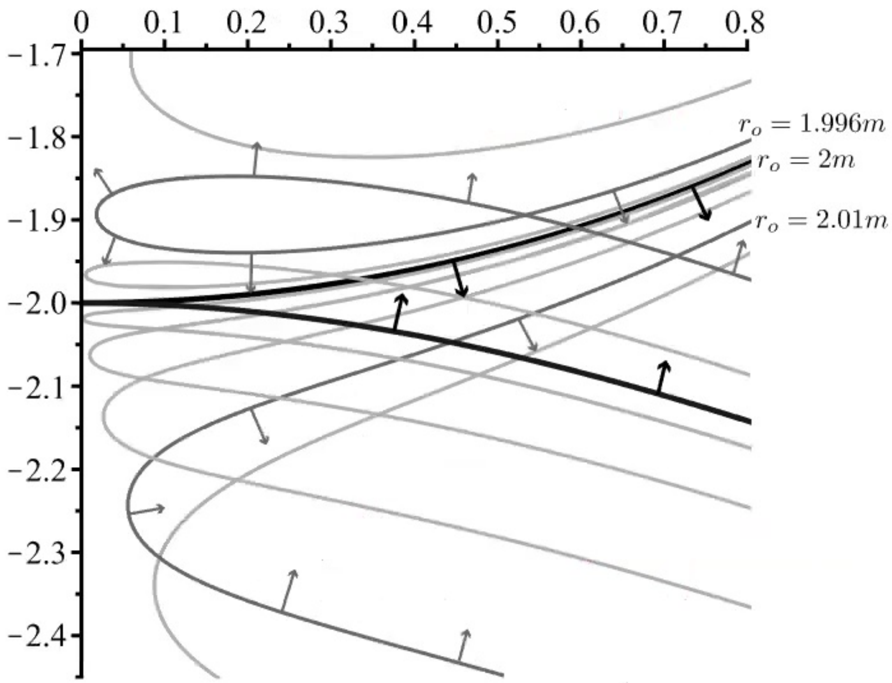
MOTSodesics of Schwarzschild I



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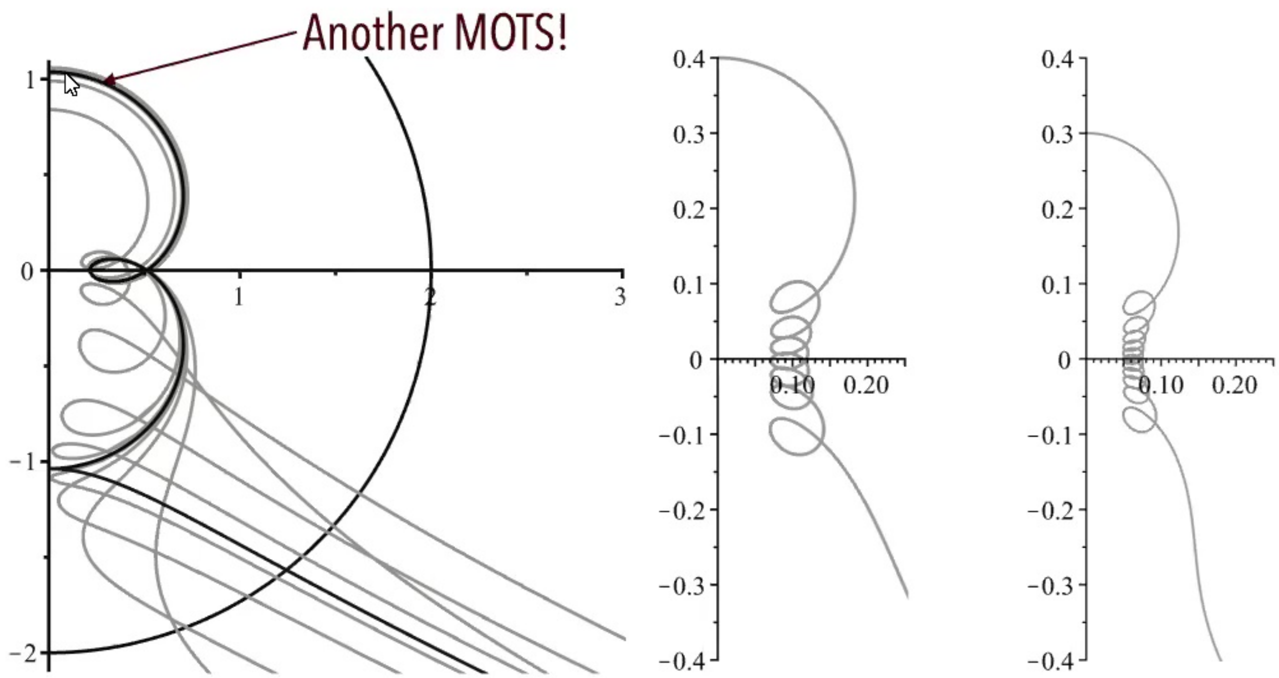
MOTSodesics of Schwarzschild I



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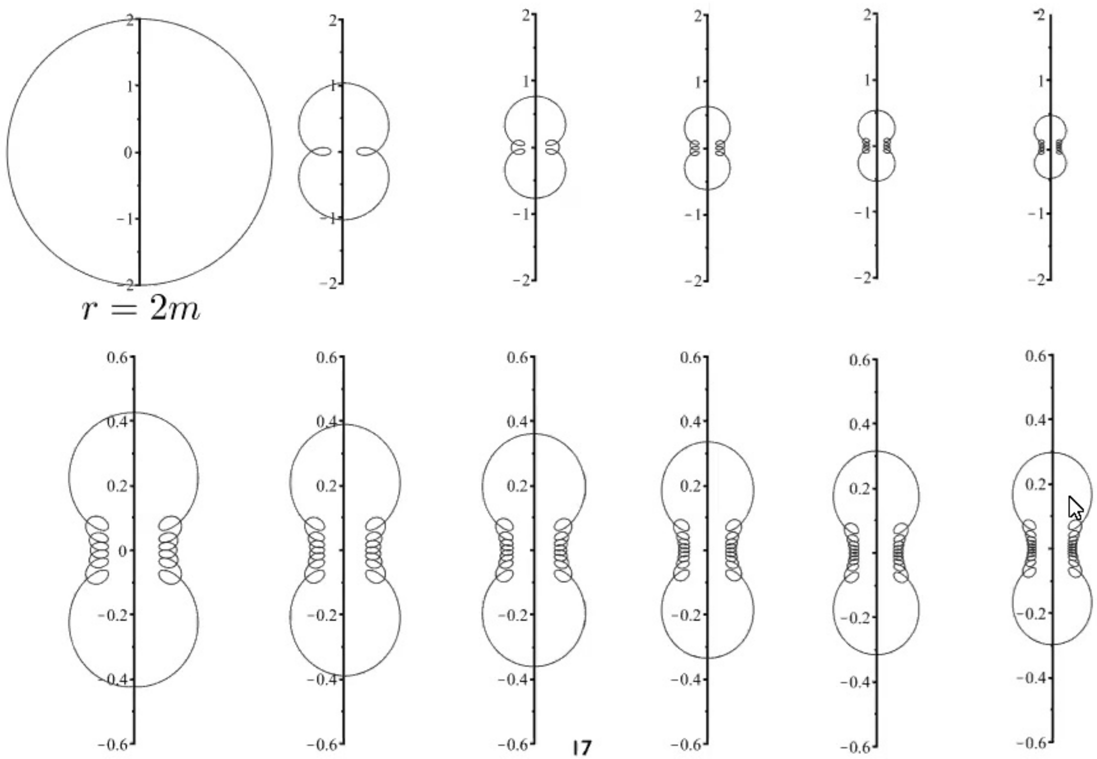
Exotic MOTS in Schwarzschild!

Marginally outer trapped surfaces in the Schwarzschild spacetime: Multiple self-intersections and extreme mass ratio mergers

PHYSICAL REVIEW D **102,** 044031 (2020)

IB, Robie Hennigar, Saikat Mondal



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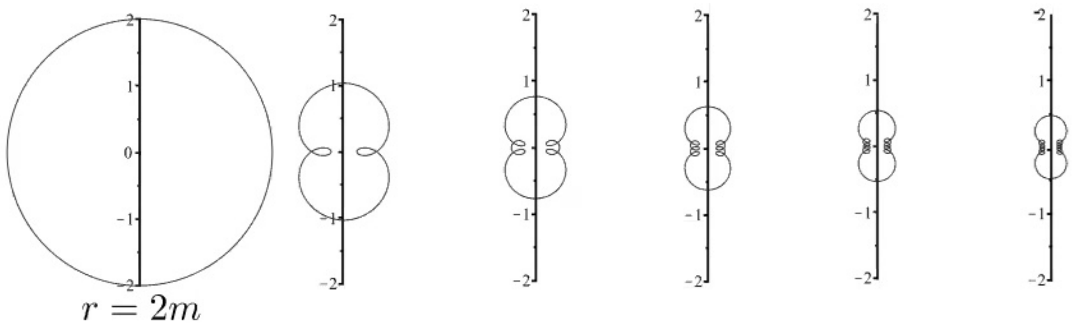

How generic are these?

Very!

Observed in (IB, RH, Hari Kunduri, Sarah Muth, Billy Chan, in preparation):
assorted Schwarzschild coordinates, Reissner-Nordström, Kerr, Gauss-Bonnet, LQG-inspired, 5D (Schwarzschild + Myers Perry)

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Should we be surprised?

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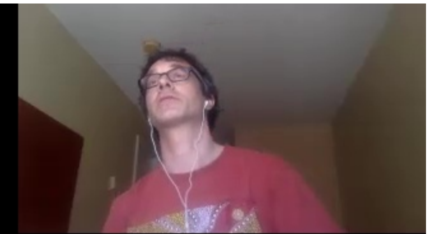


Simplest Case (Riemannian geometry)

3D spacetime = 2D slice = 1D ``surface'', $K_{ij} = 0$

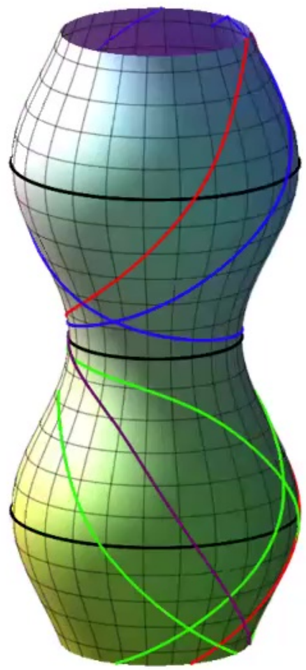
Then $\theta_{(\ell)} = \cancel{k_u} + k_N = 0$

$\Longrightarrow q^{ij} D_i N_j = 0$

$\Longrightarrow T^i T^j D_i N_j = 0$

$\Longrightarrow N_i T^j D_j T^i = 0$

$\Longrightarrow$ MOTSodesics = geodesics on (Riemannian) two-surface!



Closed geodesics (MOTS)

+ ``open'' geodesics = infinite number of marginally outer trapped **open** surfaces (MOT**O**S)

Simplest Case (Riemannian geometry)

3D spacetime = 2D slice = 1D ``surface'', $K_{ij} = 0$

Then $\theta_{(\ell)} = \cancel{k_u} + k_N = 0$

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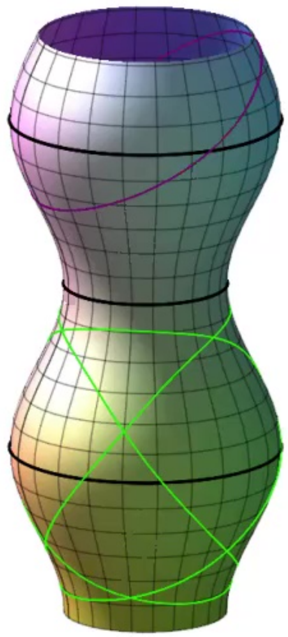

Closed geodesics (MOTS)

+ ``exotic'' closed

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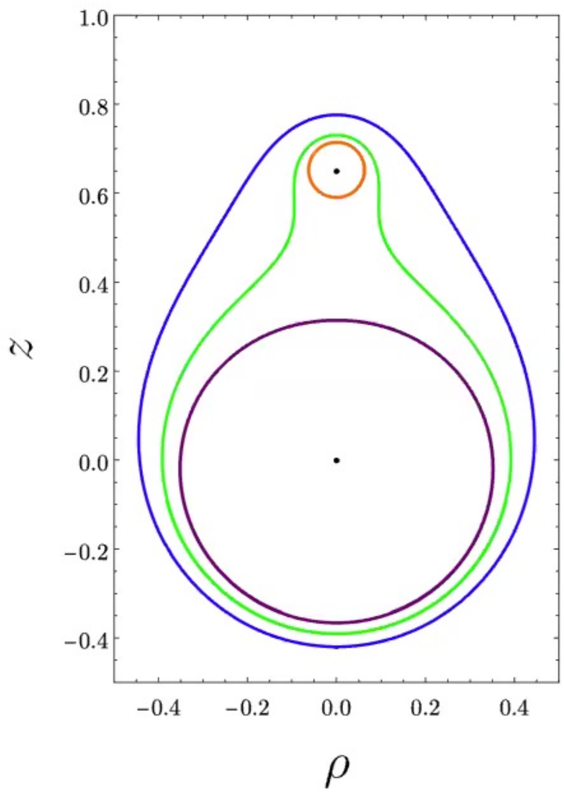
Are there more MOTS in Brill-Lindquist?



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MOTS in Brill-Lindquist Initial Data

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = \psi(r,z)^4\left(\mathrm{d}\rho^2 + \mathrm{d}z^2 + \rho^2\mathrm{d}\phi^2\right)$$

$K_{ij} = 0$ (time symmetric)

Constraints reduce to: $\nabla^2\psi = 0$

EOMS: $T^A\bar{D}_A T^B = \kappa N^B$

$$\ddot{P} = \frac{\dot{Z}^2}{P} + \frac{4\psi_r}{\psi^5} - \frac{6\dot{P}(\dot{P}\psi_r + \dot{Z}\psi_z)}{\psi}$$

$$\ddot{Z} = -\frac{\dot{Z}\dot{P}}{P} + \frac{4\psi_z}{\psi^5} - \frac{6\dot{Z}(\dot{P}\psi_r + \dot{Z}\psi_z)}{\psi}$$

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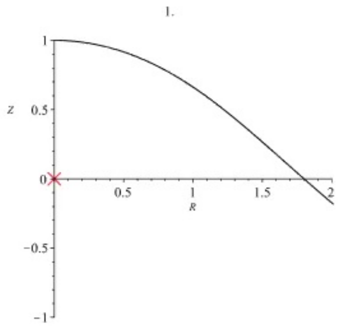
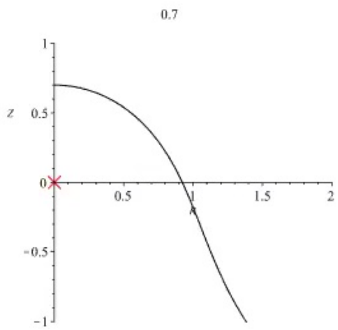
MOTOS in Brill-Lindquist (Time Symmetric) Initial Data

Schwarzschild: $\psi_{\rm Schw} = 1 + \dfrac{m}{2\sqrt{\rho^2 + z^2}}$





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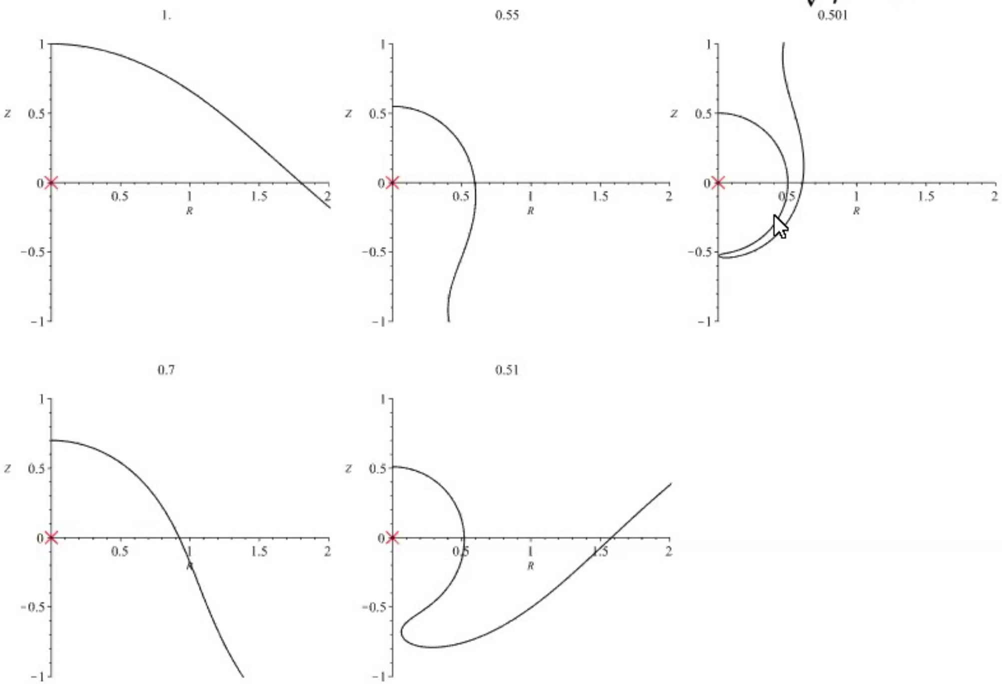

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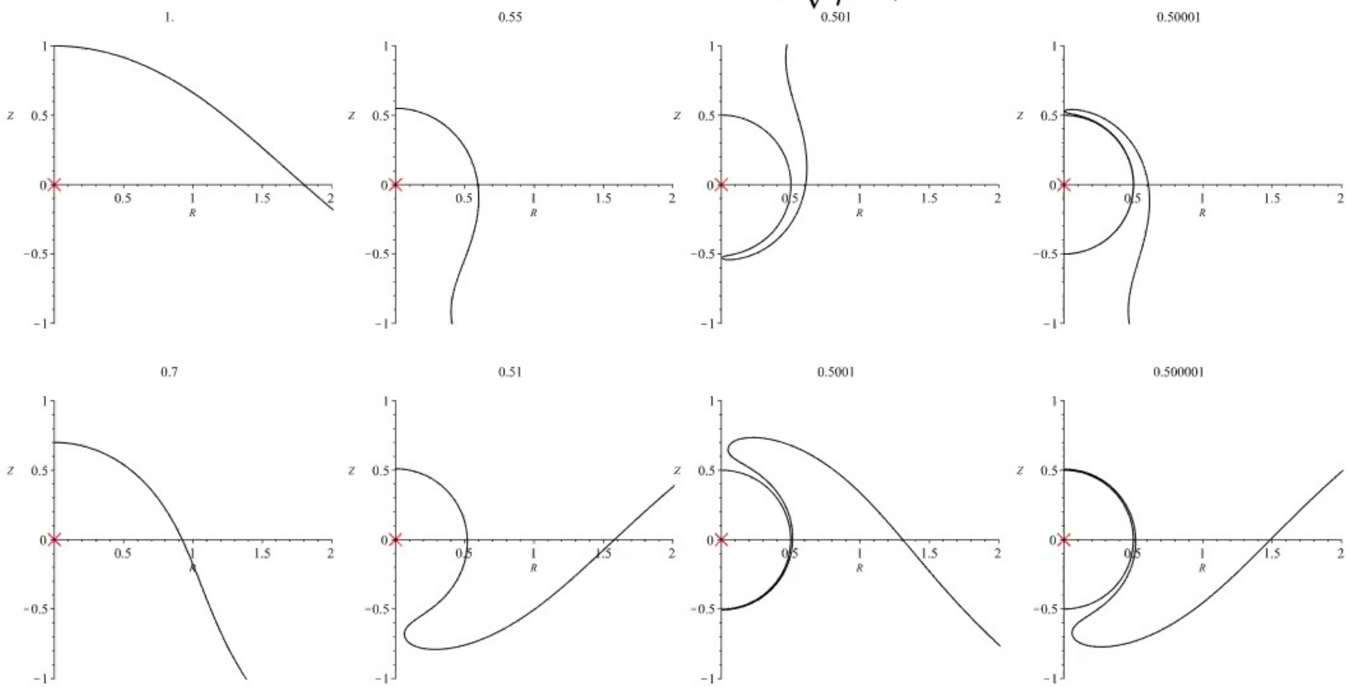

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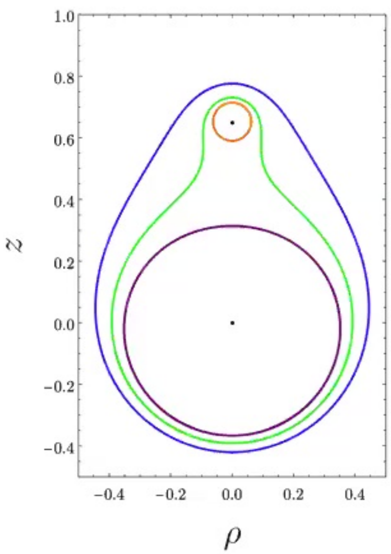
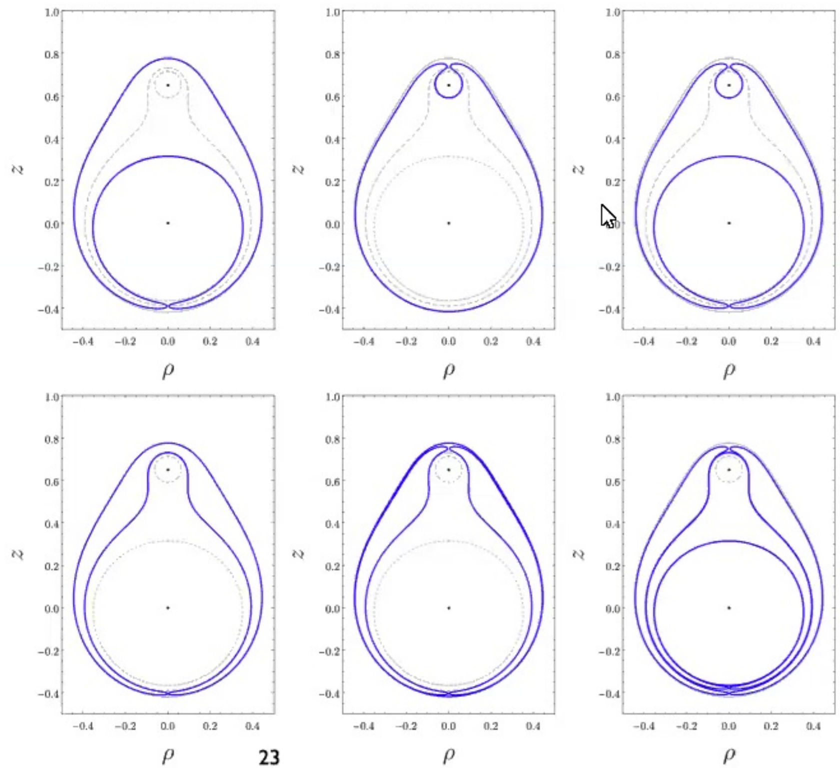
MOTS in Brill-Lindquist Initial Data

$$\psi = 1 + \frac{m_1}{2\sqrt{\rho^2 + (z - a/2)^2}} + \frac{m_2}{2\sqrt{\rho^2 + (z + a/2)^2}}$$





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What happens if this data is evolved?

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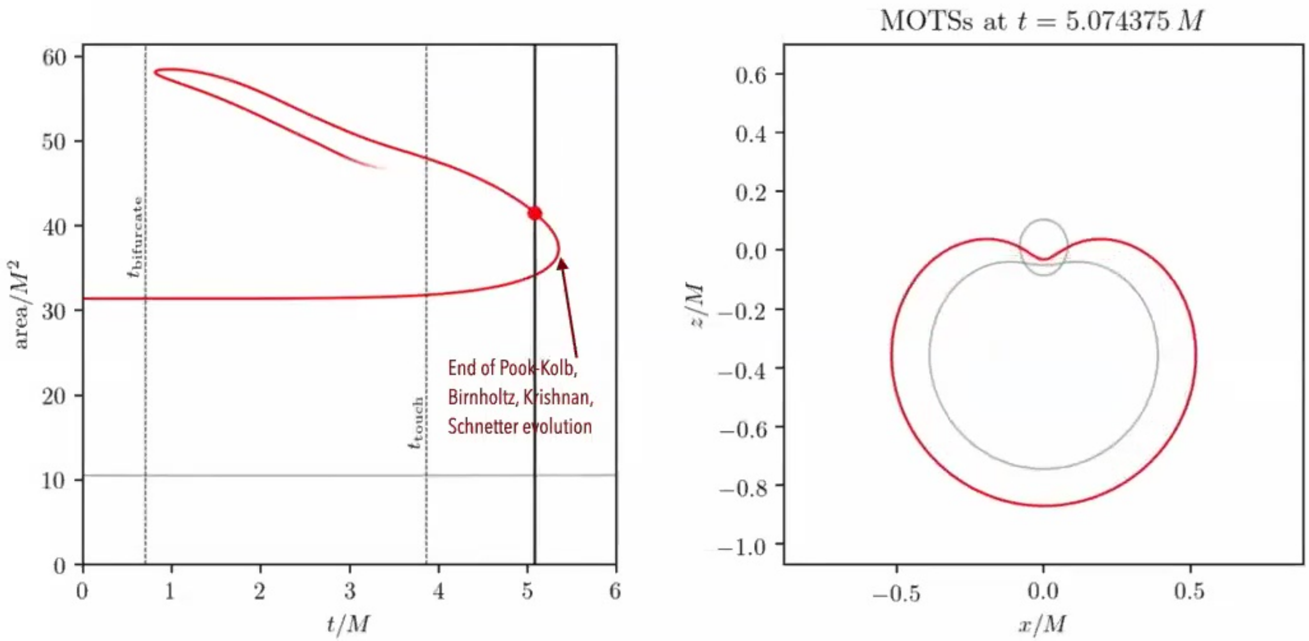
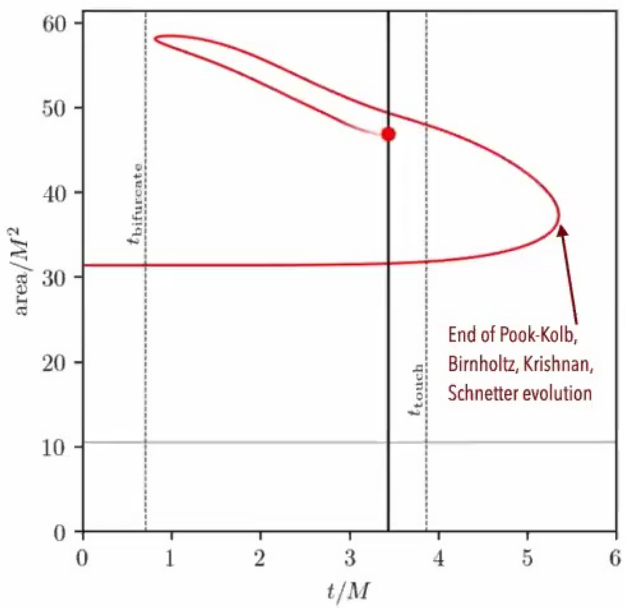
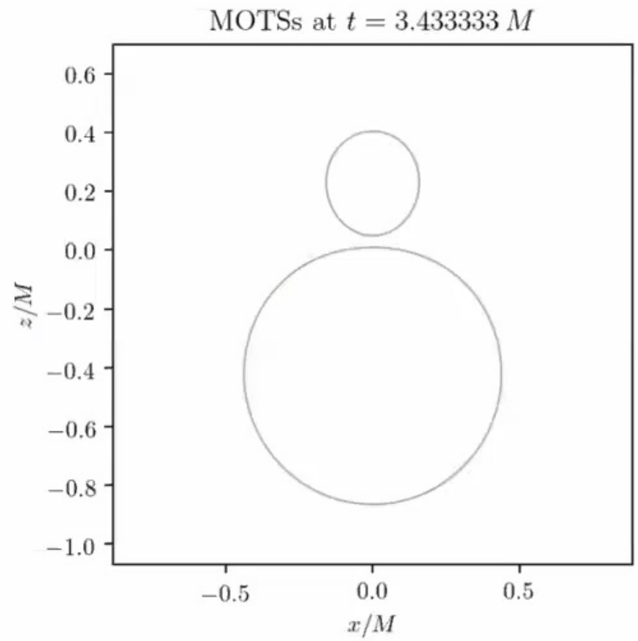
Exotic MOTS annihilate original apparent horizons



25

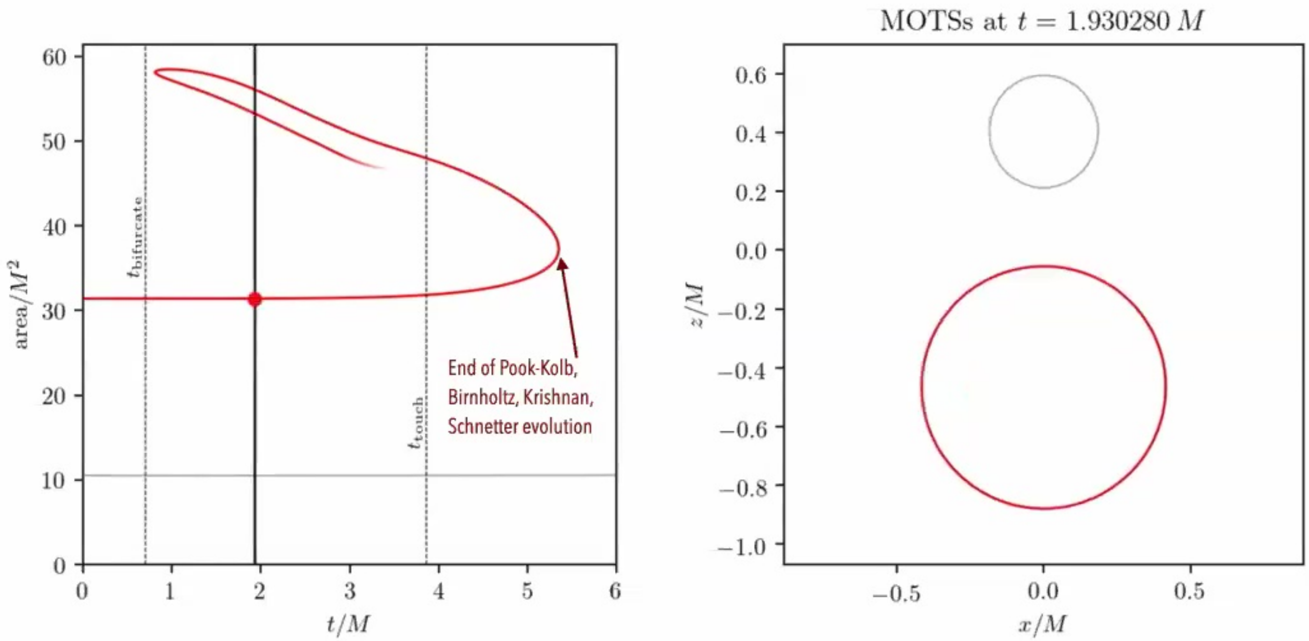
Exotic MOTS annihilate original apparent horizons





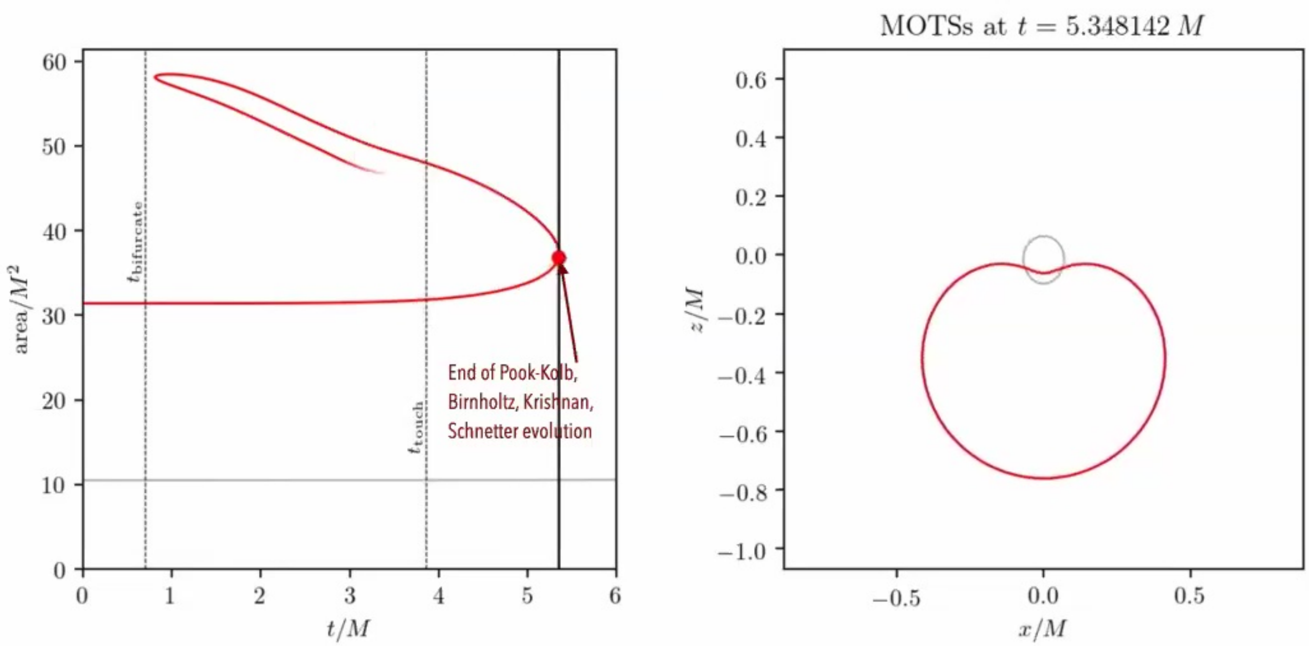
25

Exotic MOTS annihilate original apparent horizons



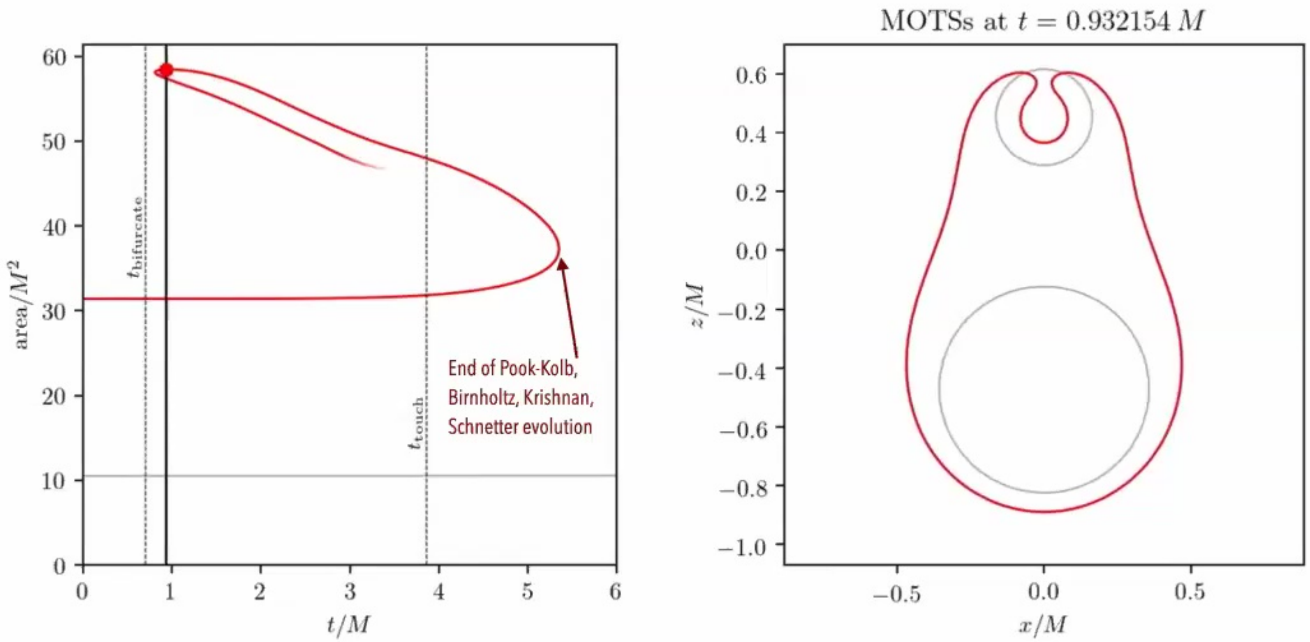
25

Exotic MOTS annihilate original apparent horizons



25

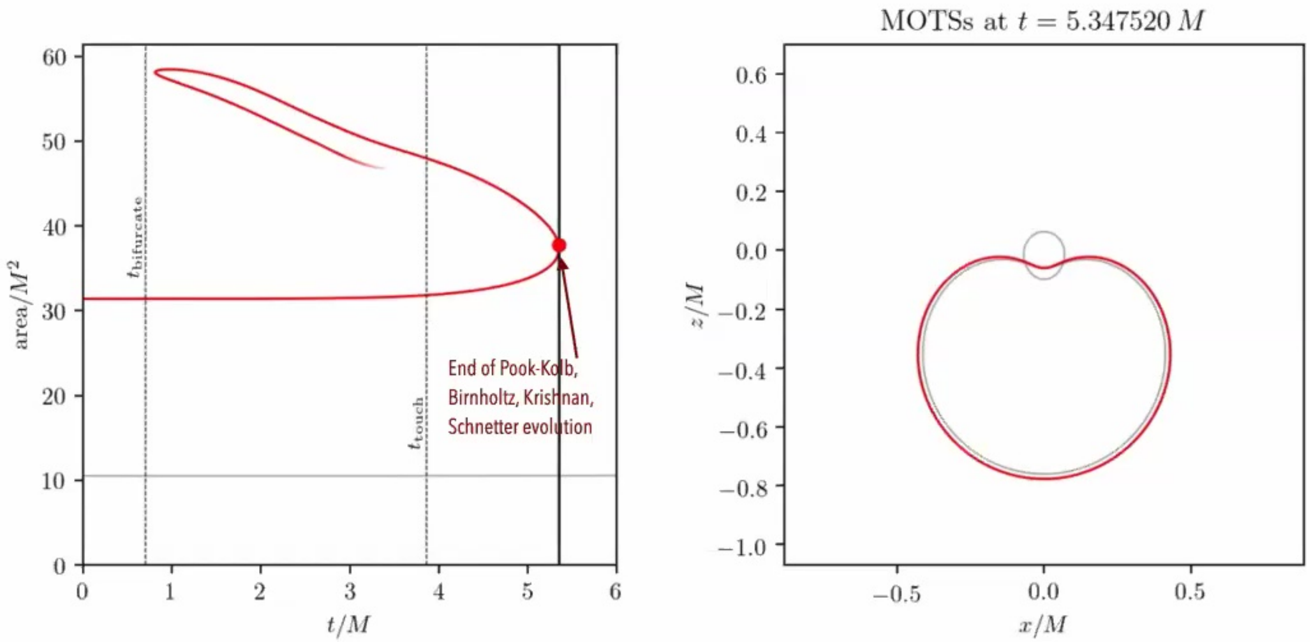
Exotic MOTS annihilate original apparent horizons



00:09 -00:03

25

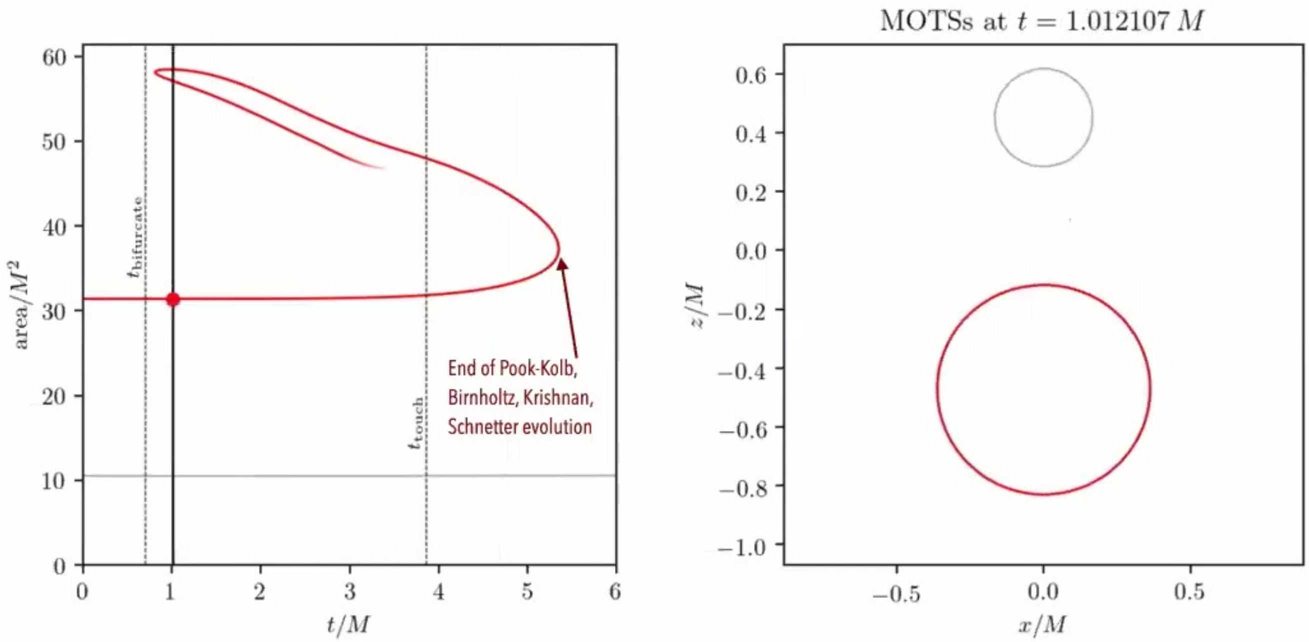
Exotic MOTS annihilate original apparent horizons



00:05 -00:07

25

Exotic MOTS annihilate original apparent horizons



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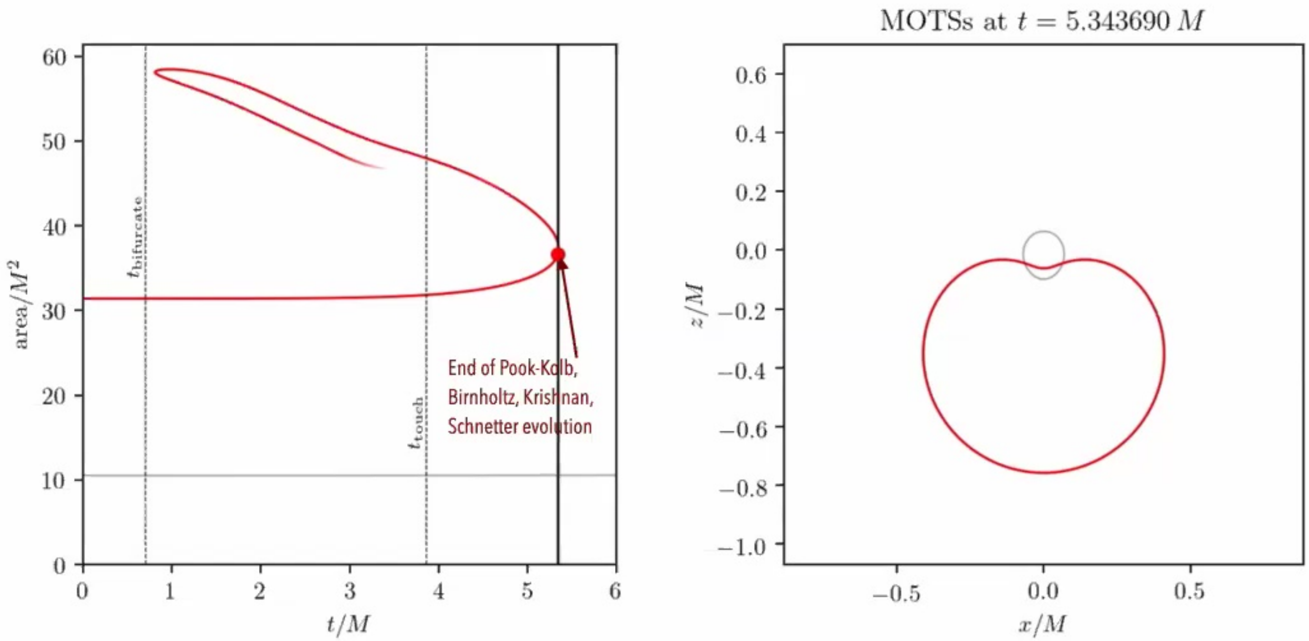
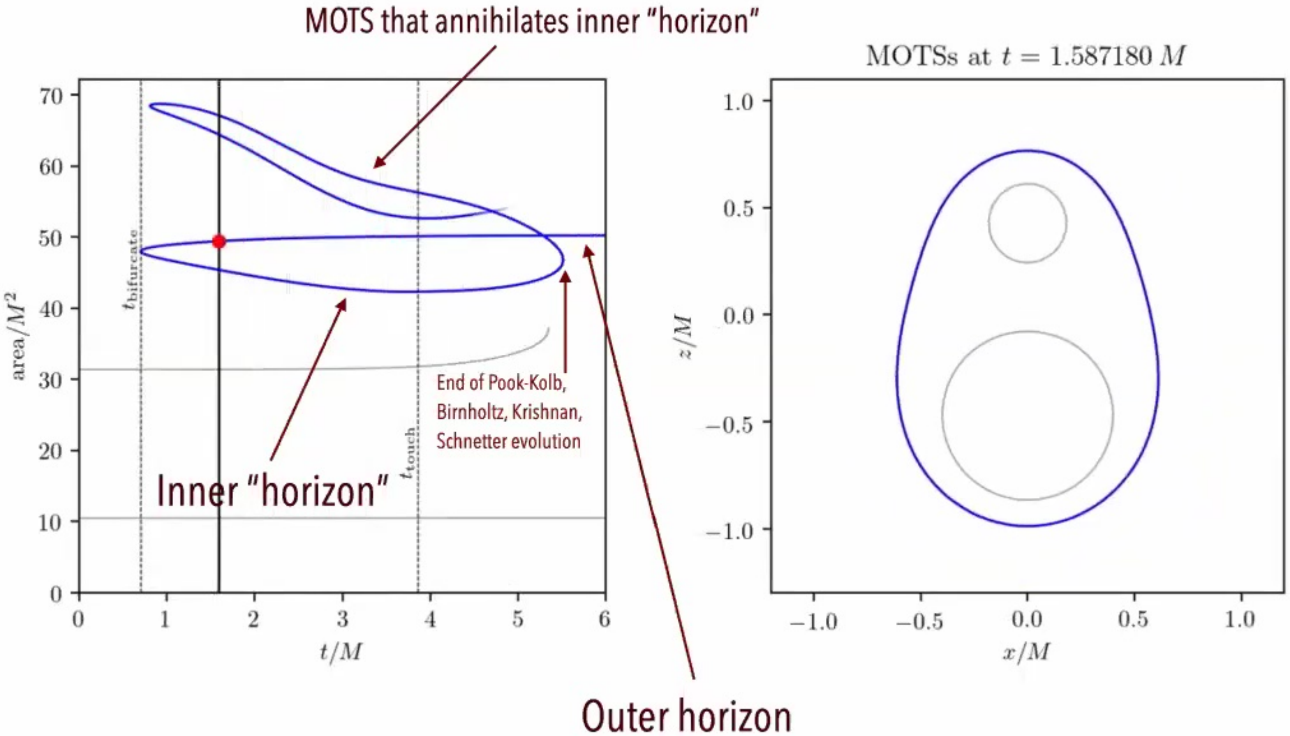
Exotic MOTS annihilate original apparent horizons



(Smaller hole: proximity to puncture prevents full resolution)

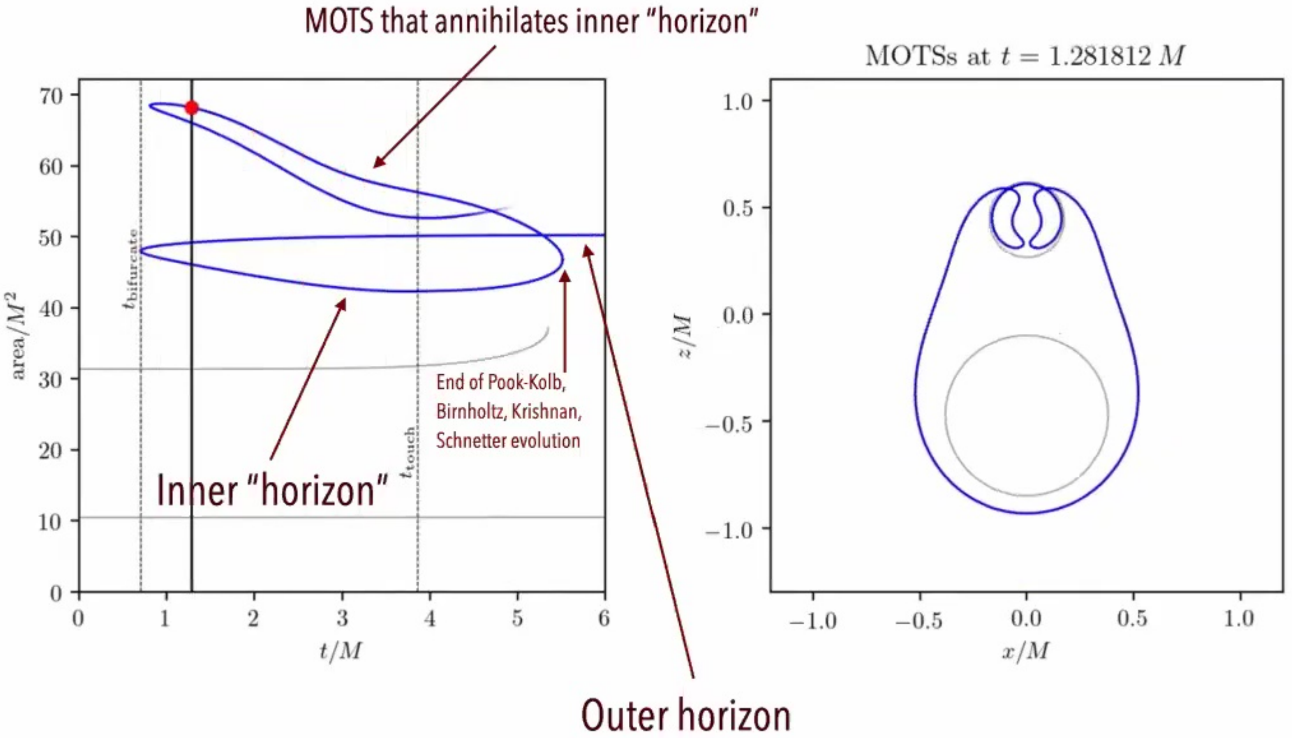
25

Evolution of Inner and Outer "Horizons"



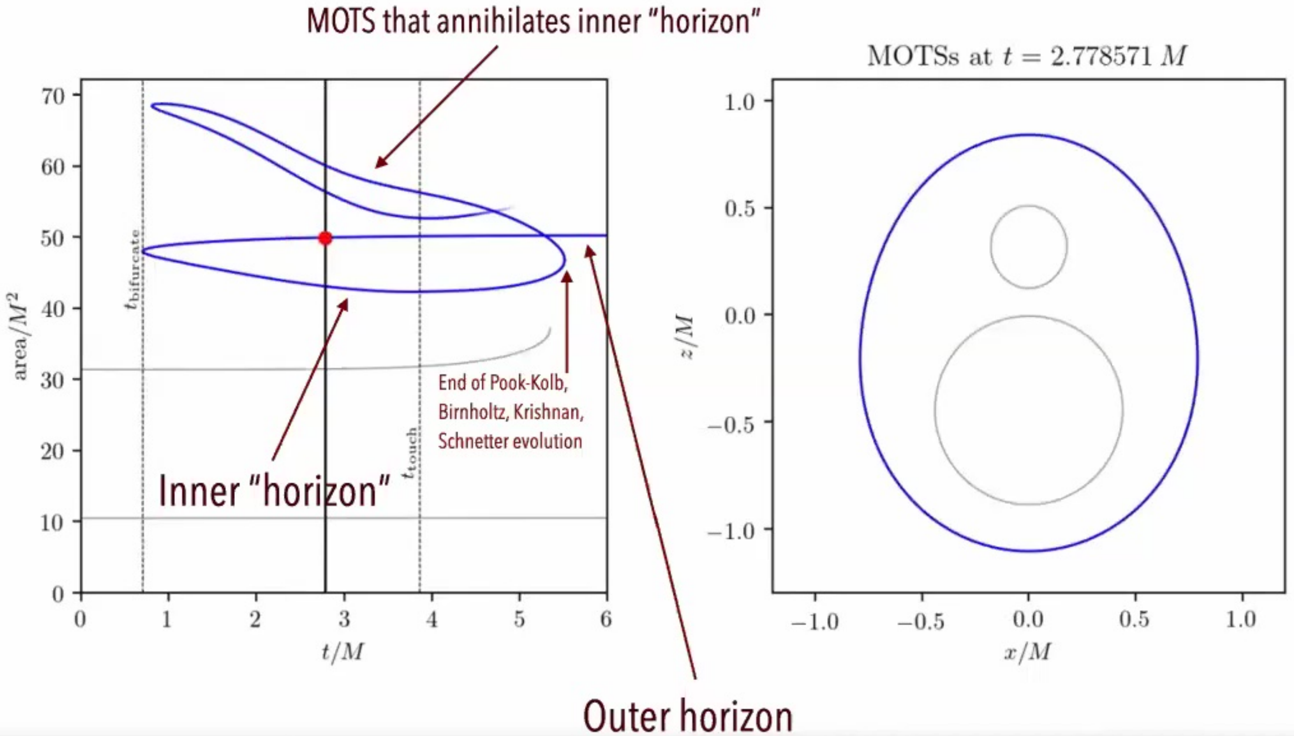
27

Evolution of Inner and Outer "Horizons"



27

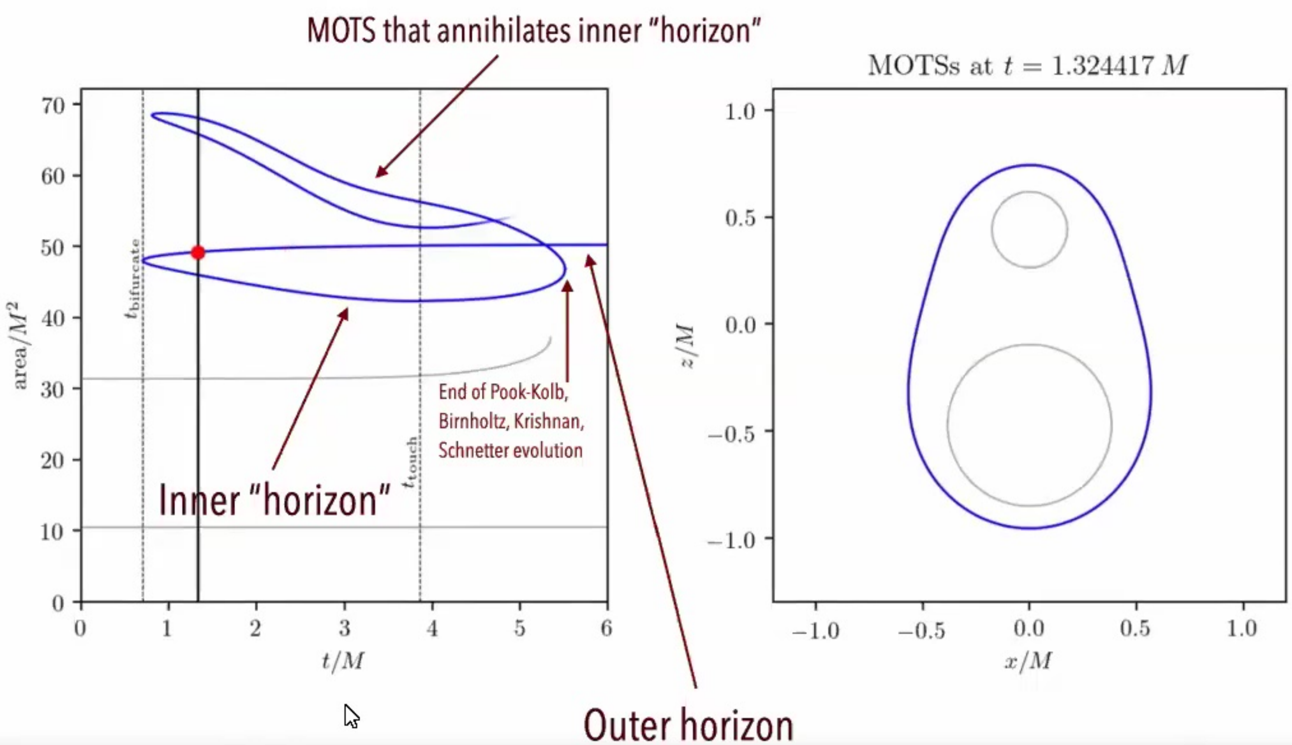
Evolution of Inner and Outer "Horizons"



00:03 -00:16

27

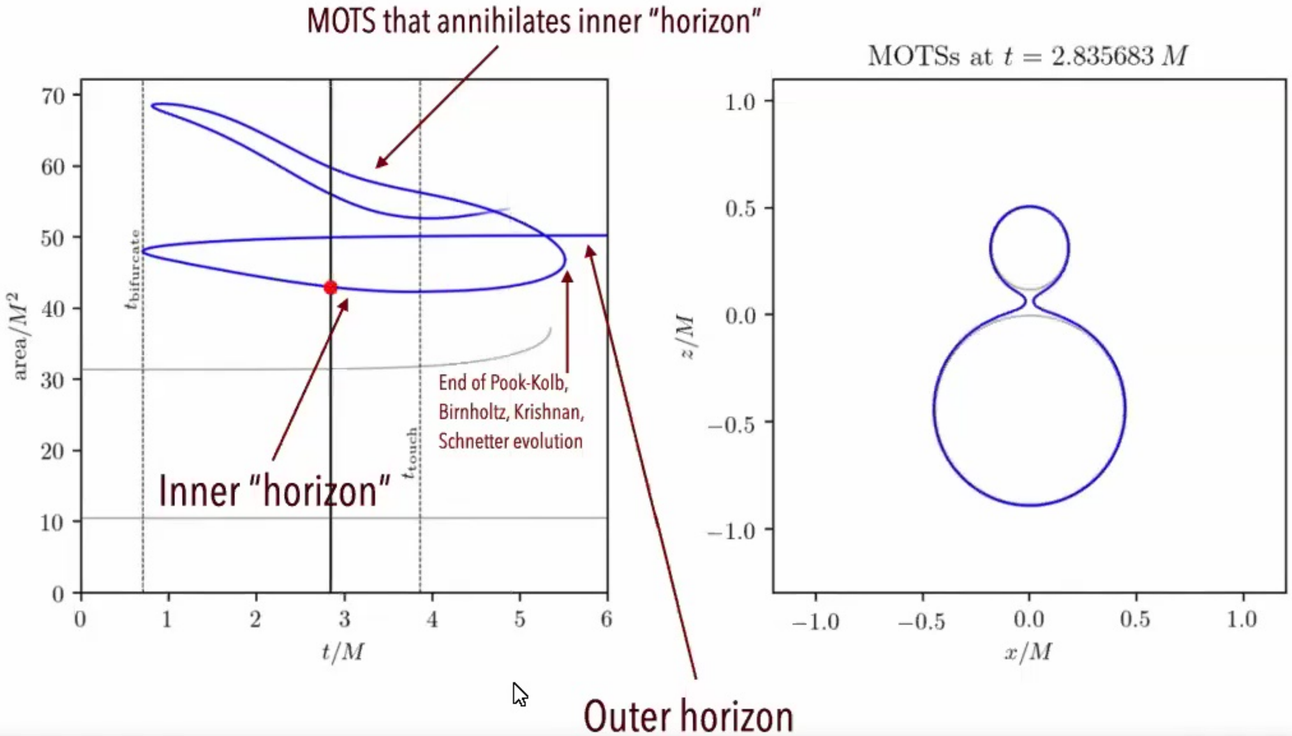
Evolution of Inner and Outer "Horizons"



00:04 -00:15

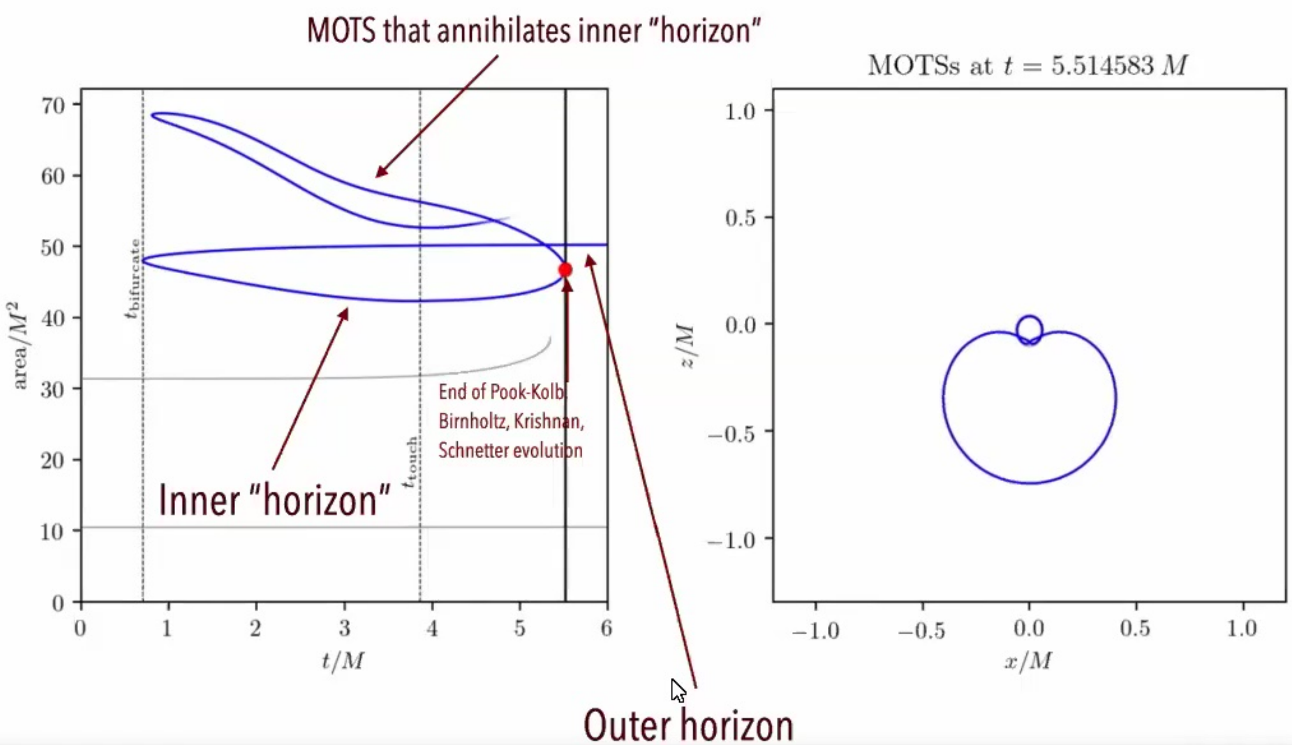
27

Evolution of Inner and Outer "Horizons"



27

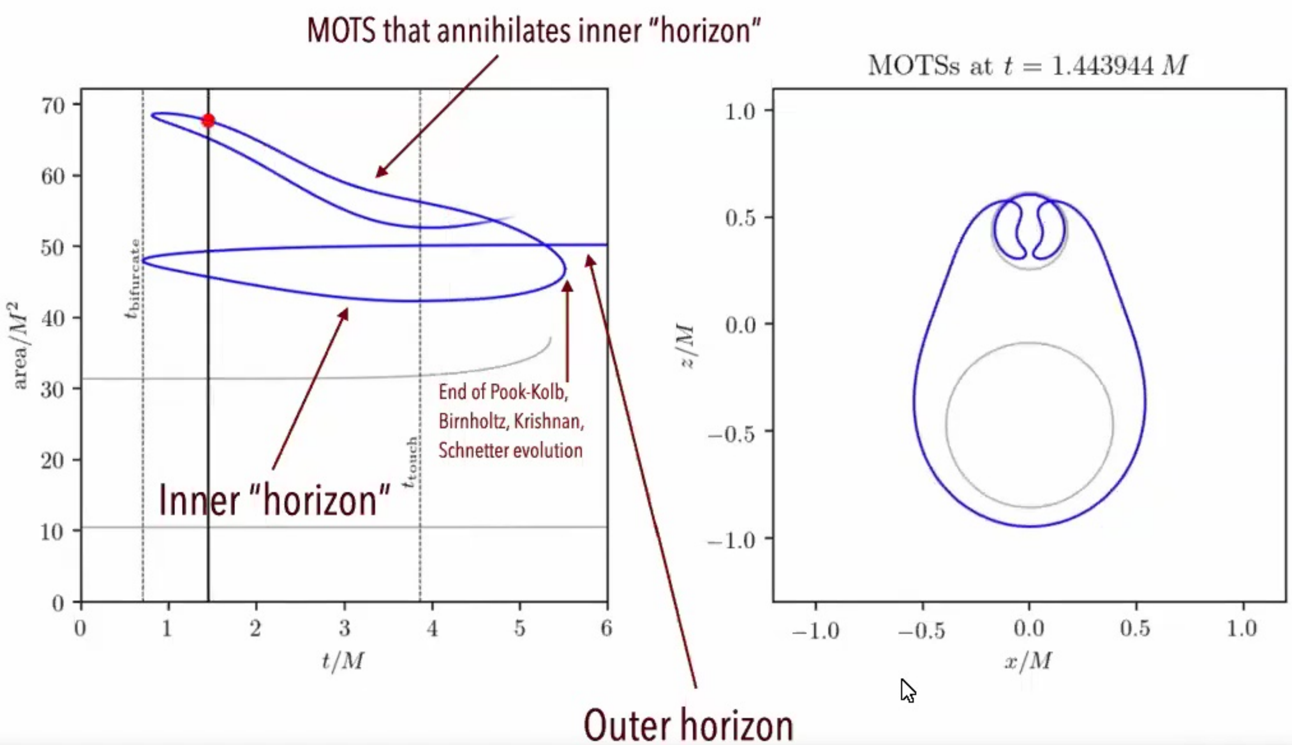
Evolution of Inner and Outer "Horizons"



00:10 -00:09

27

Evolution of Inner and Outer "Horizons"



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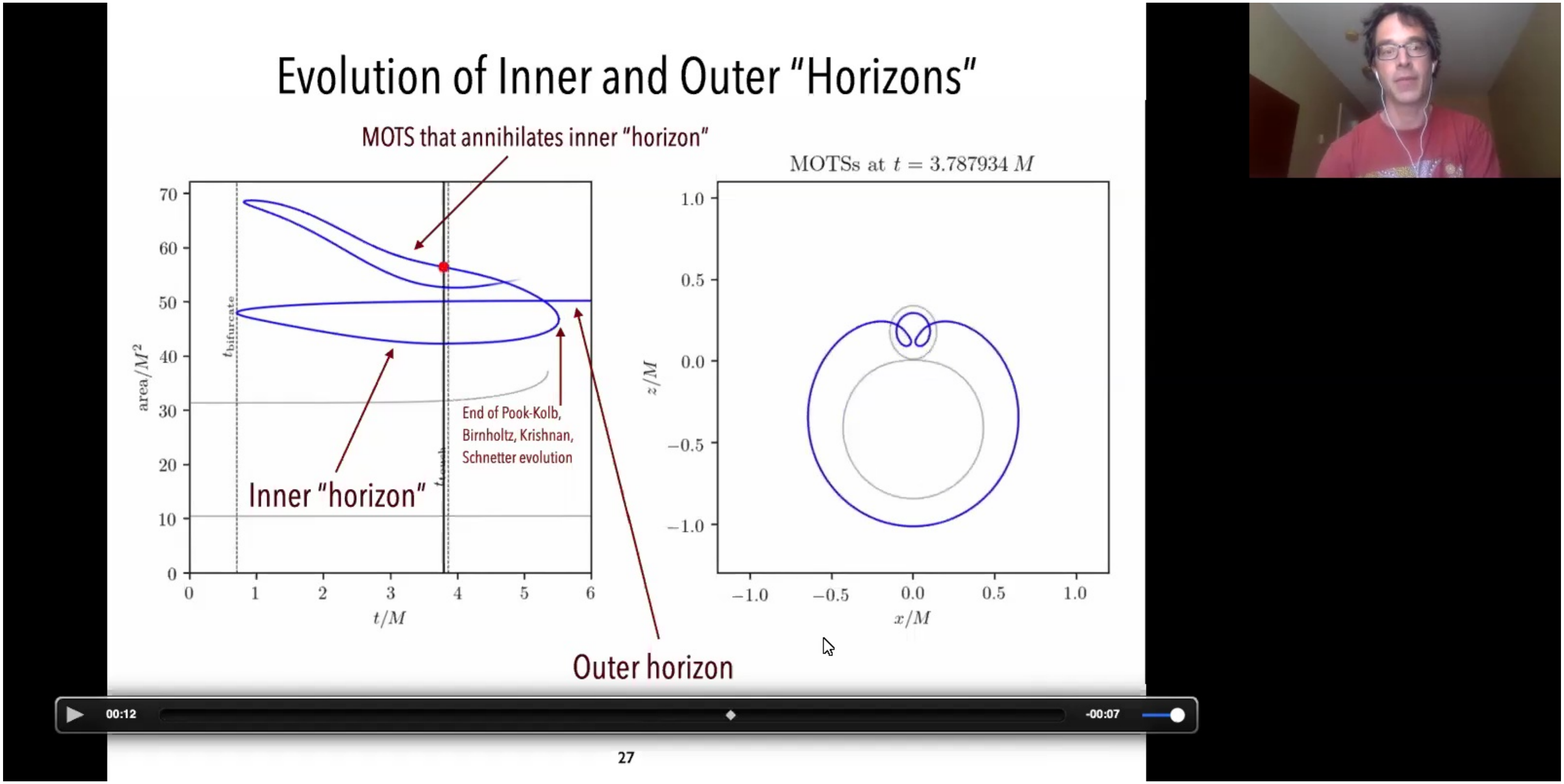
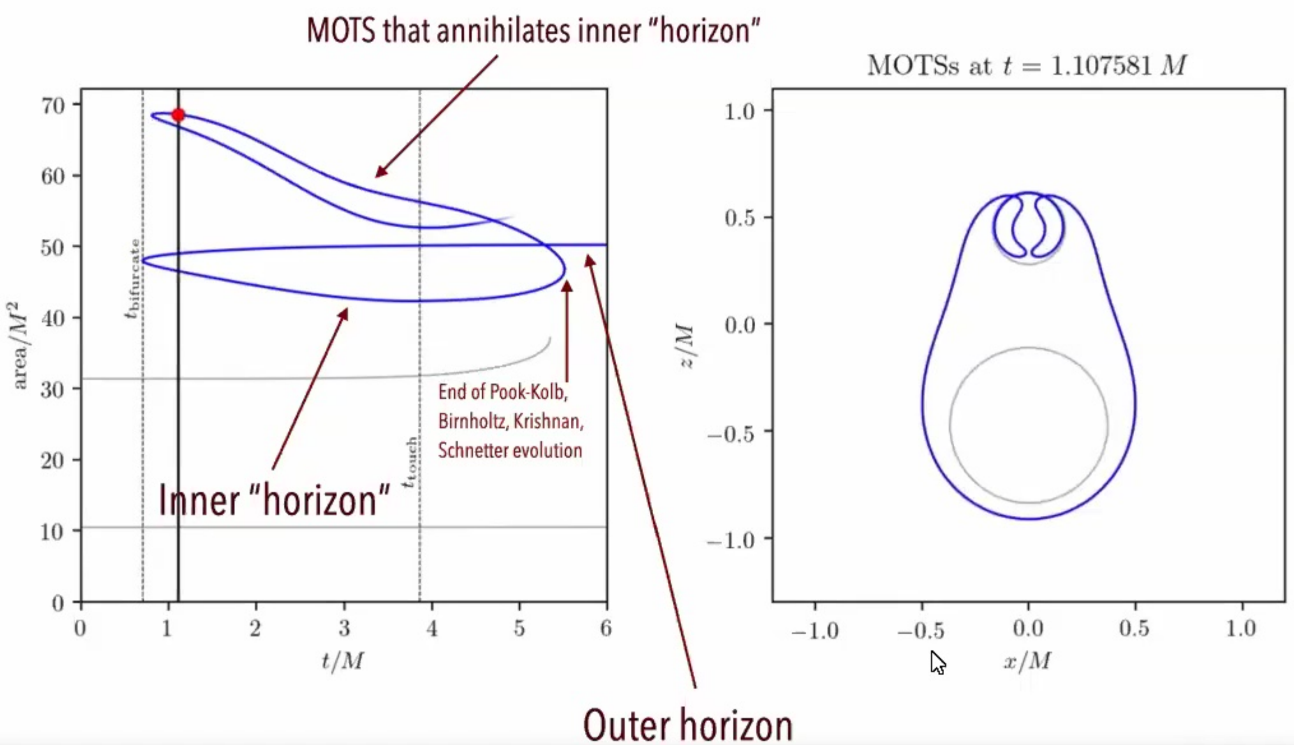
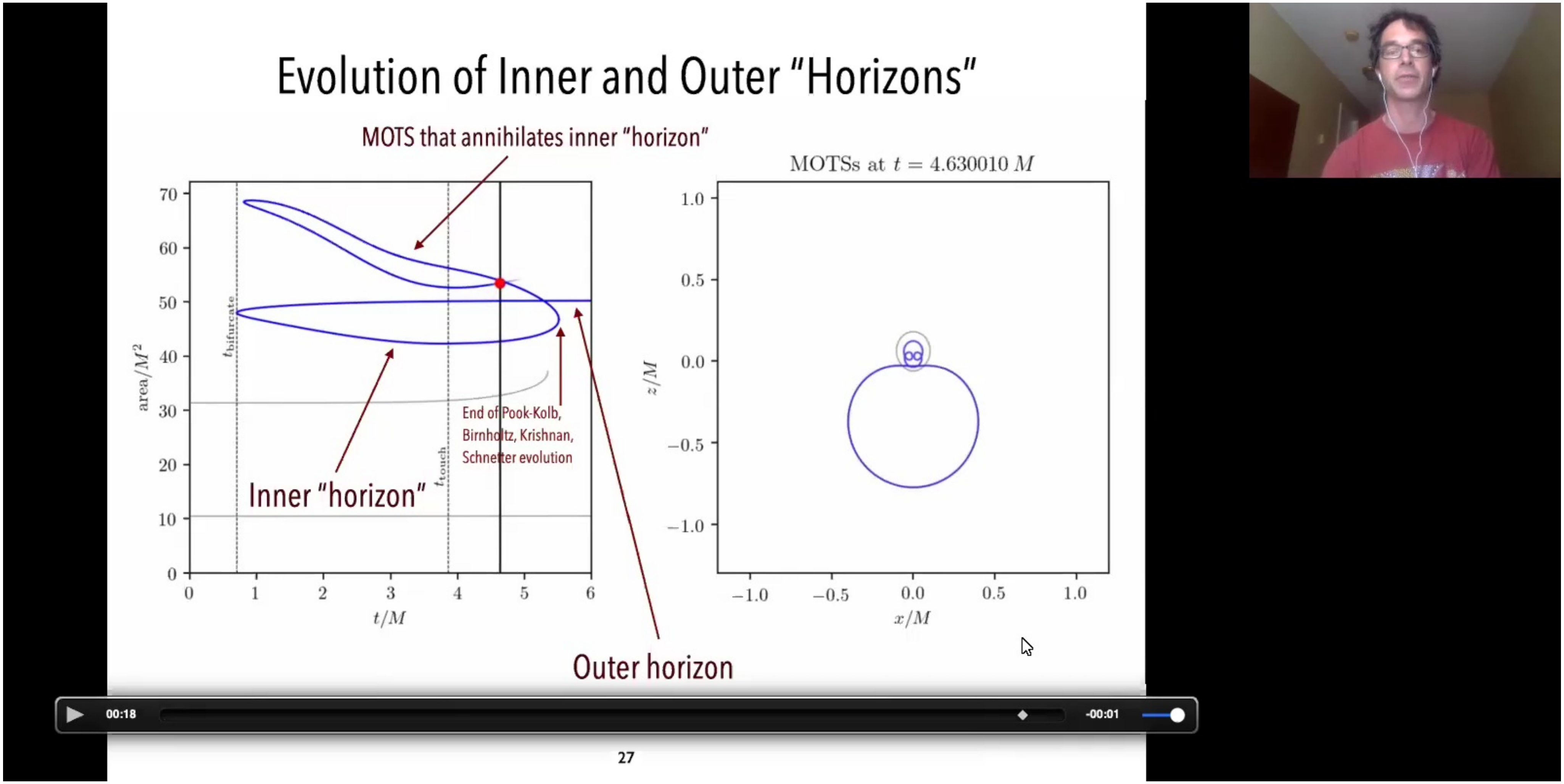

Evolution of Inner and Outer "Horizons"
MOTS that annihilates inner "horizon"
MOTSs at $t = 3.787934\ M$
area/M^2
t/M
$t_{\text{bifurcate}}$
End of Pook-Kolb, Birnholtz, Krishnan, Schnetter evolution
Inner "horizon"
z/M
x/M
Outer horizon
00:12
-00:07
27

Evolution of Inner and Outer "Horizons"

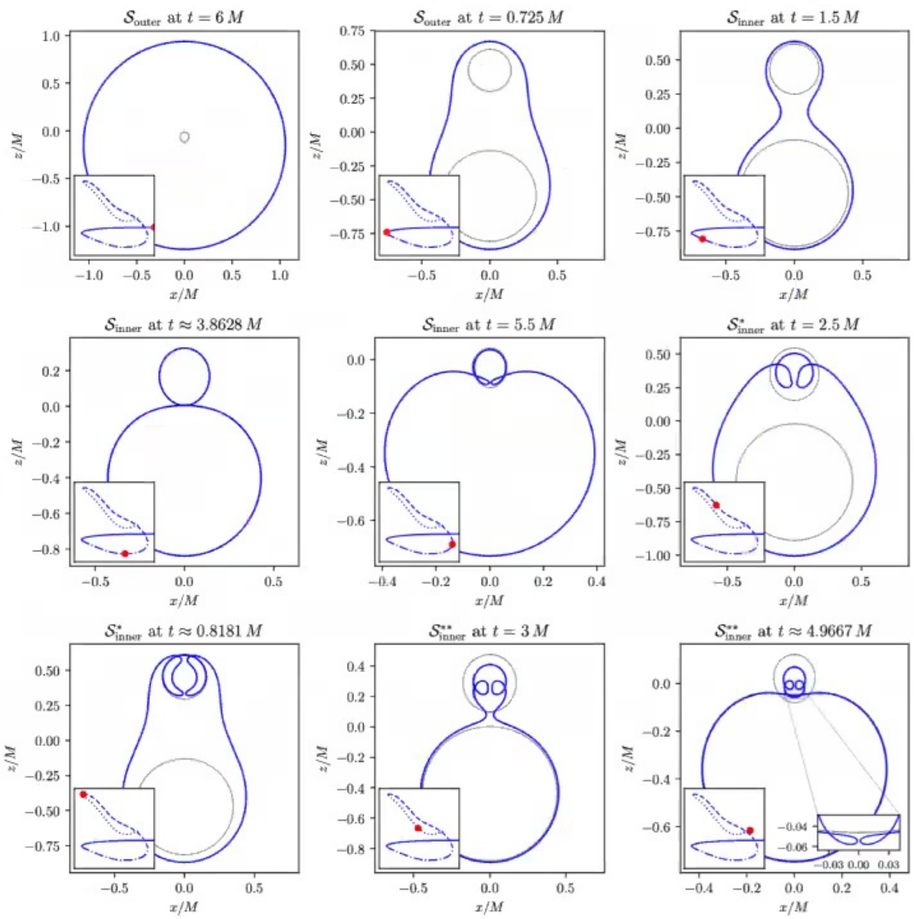


00:14 -00:05

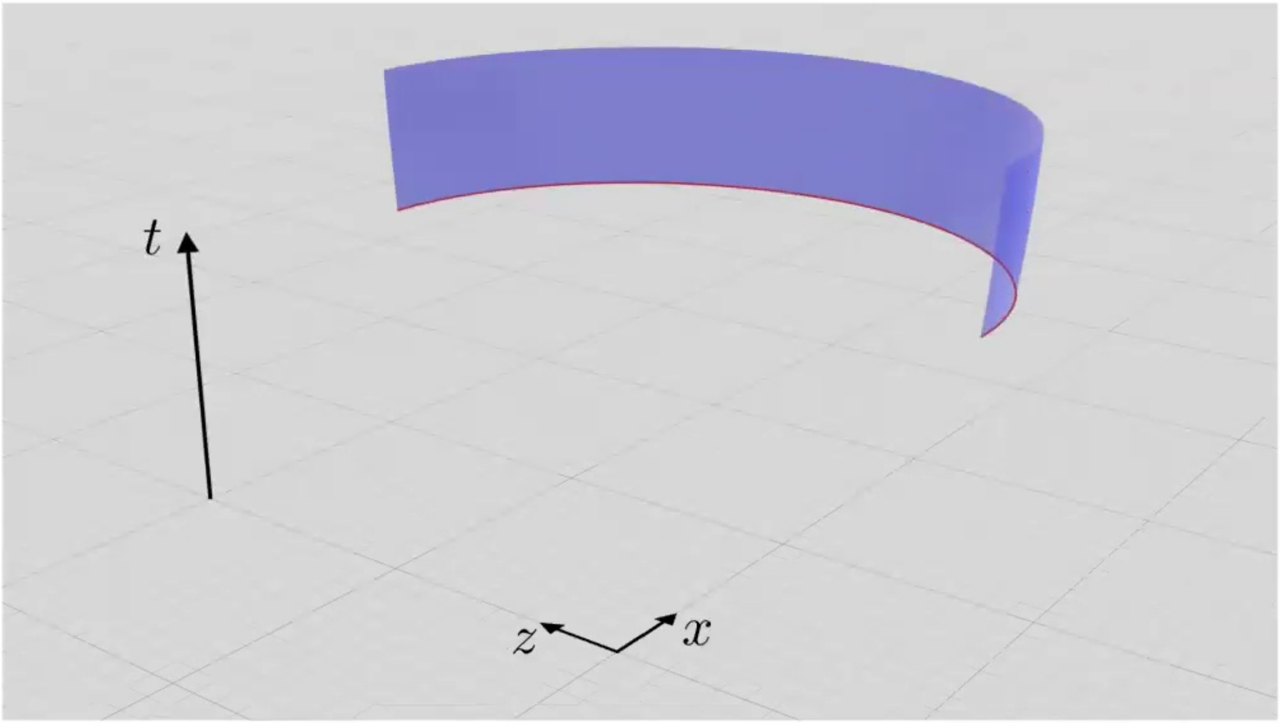
27


Evolution of Inner and Outer "Horizons"
MOTS that annihilates inner "horizon"
MOTSs at $t = 4.630010\ M$
area/M^2
$t_{\mathrm{bifurcate}}$
t_{touch}
t/M
z/M
x/M
End of Pook-Kolb, Birnholtz, Krishnan, Schnetter evolution
Inner "horizon"
Outer horizon
00:18
-00:01
27

Outer/inner split: one of many

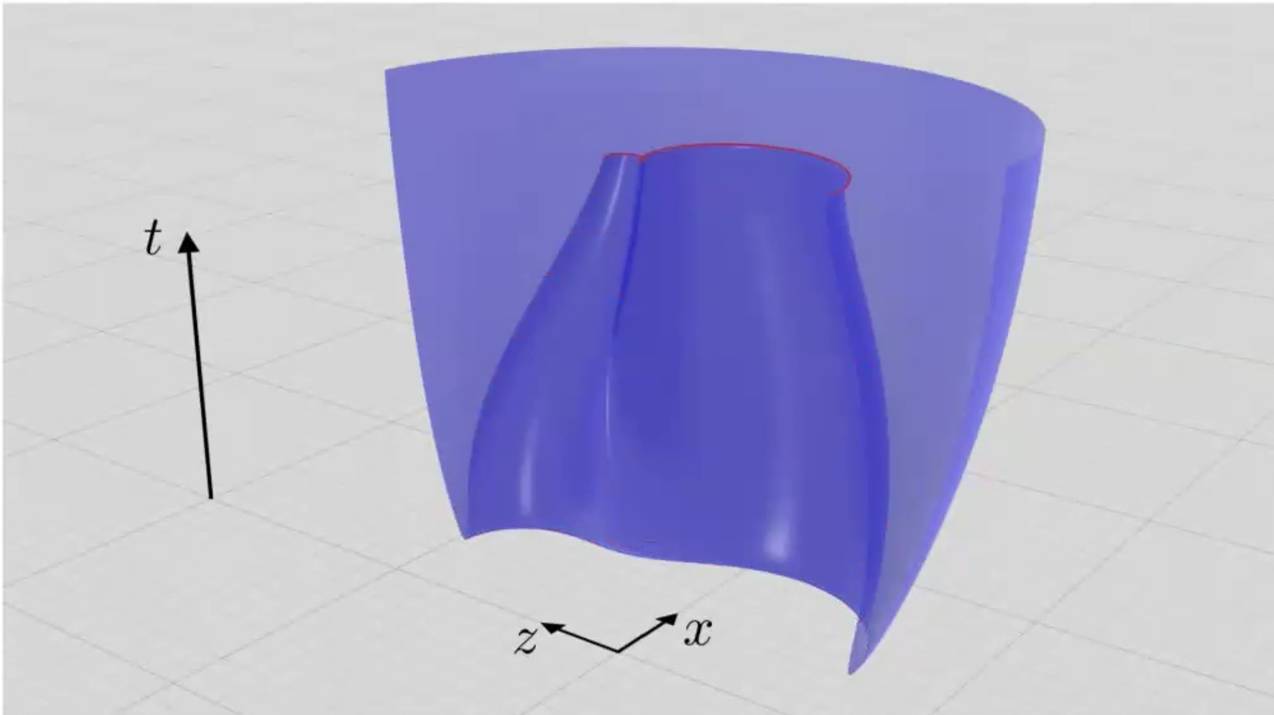


Part of the AH "Ball Gown"



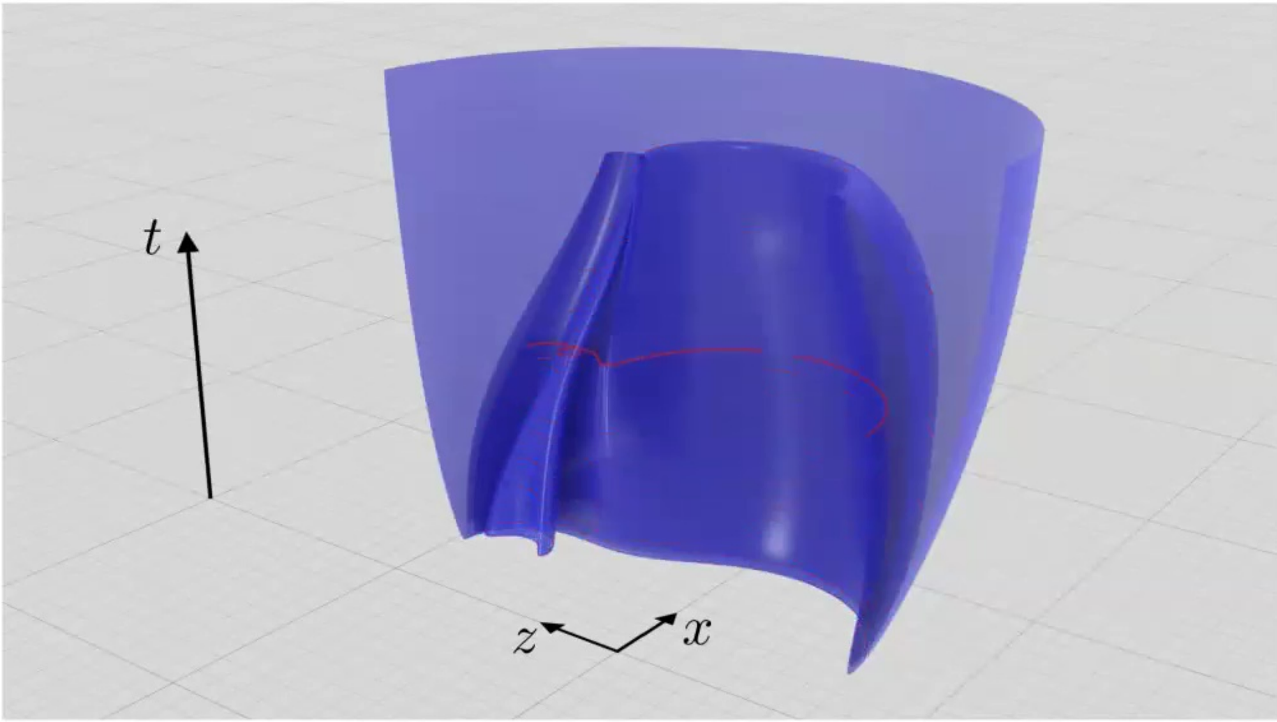
29

Part of the AH "Ball Gown"



29

Part of the AH "Ball Gown"



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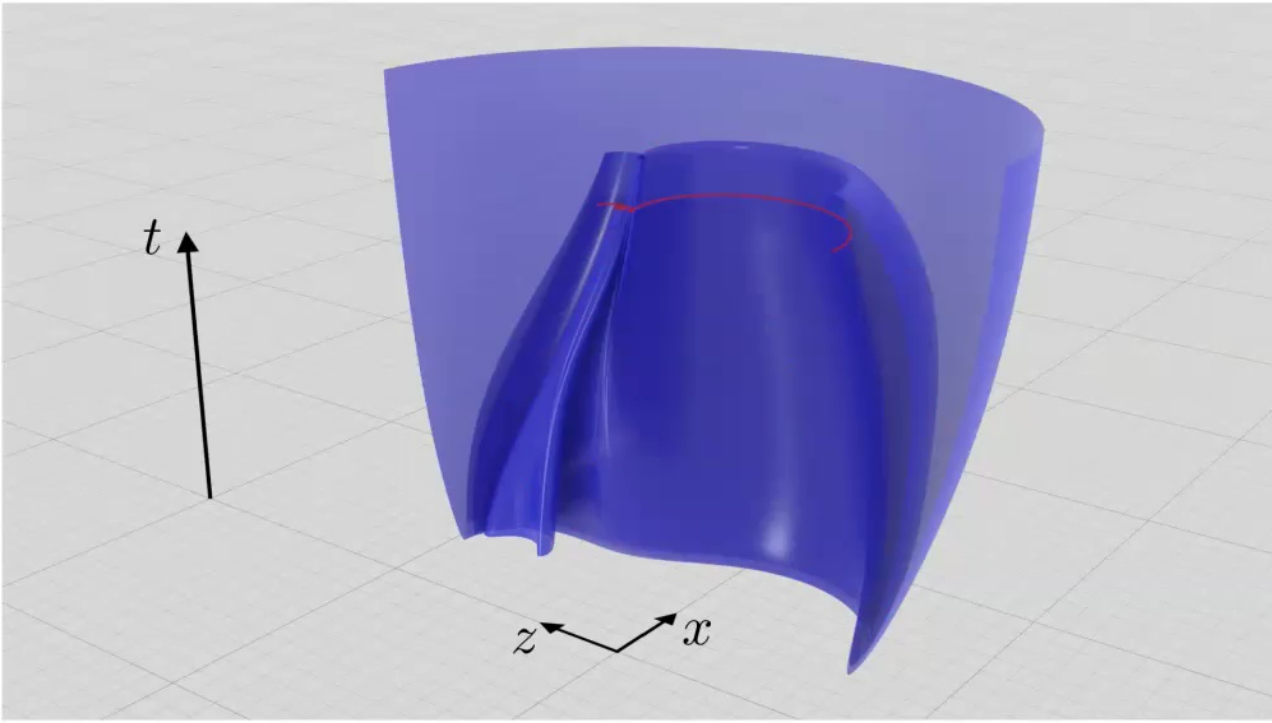
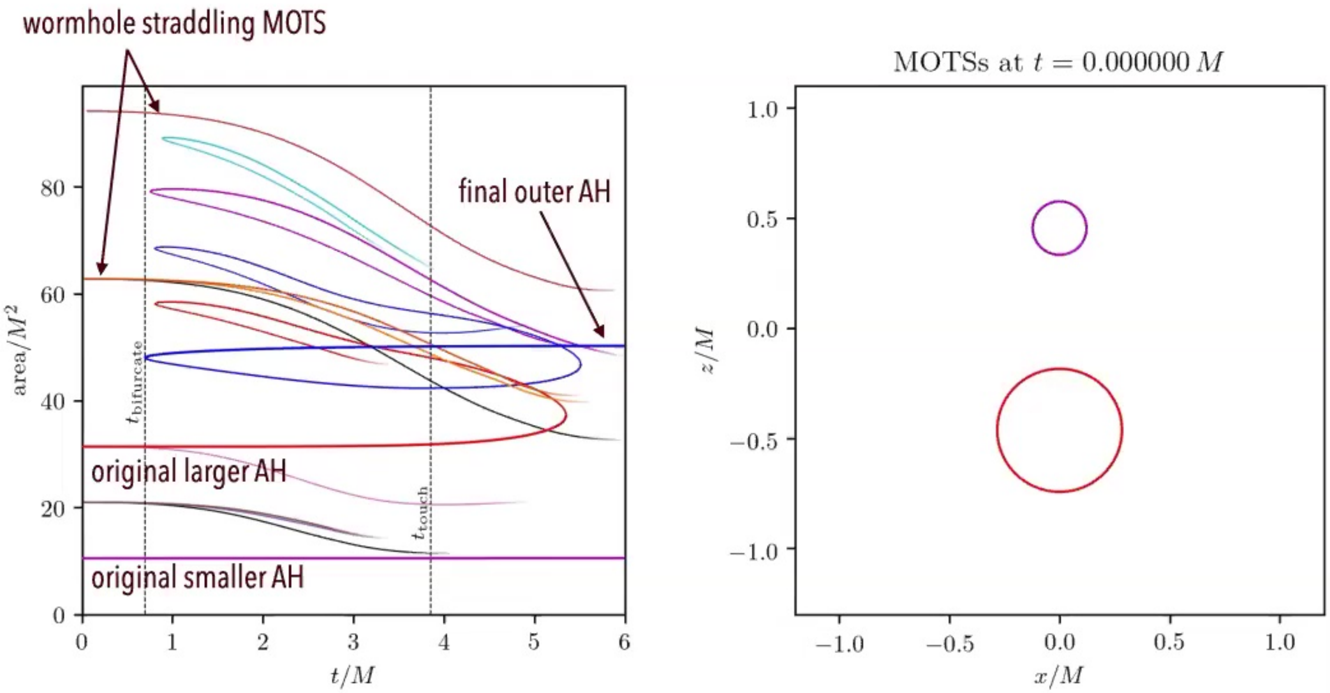
Part of the AH "Ball Gown"



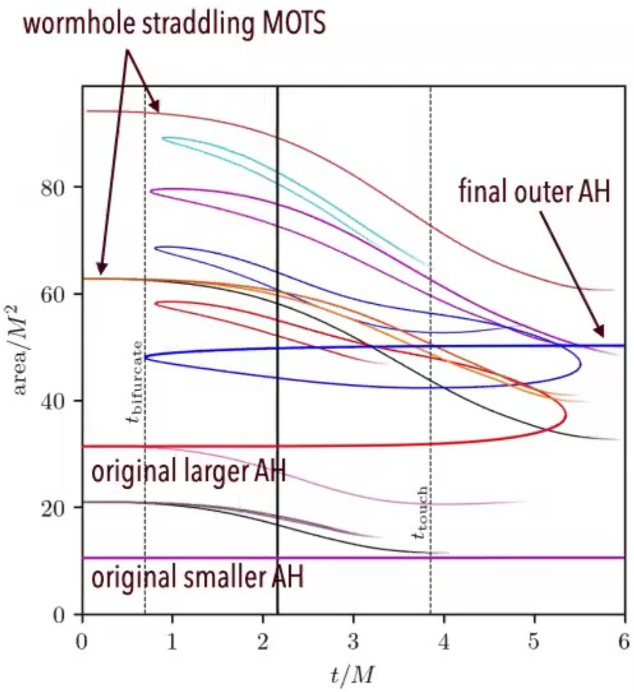
29

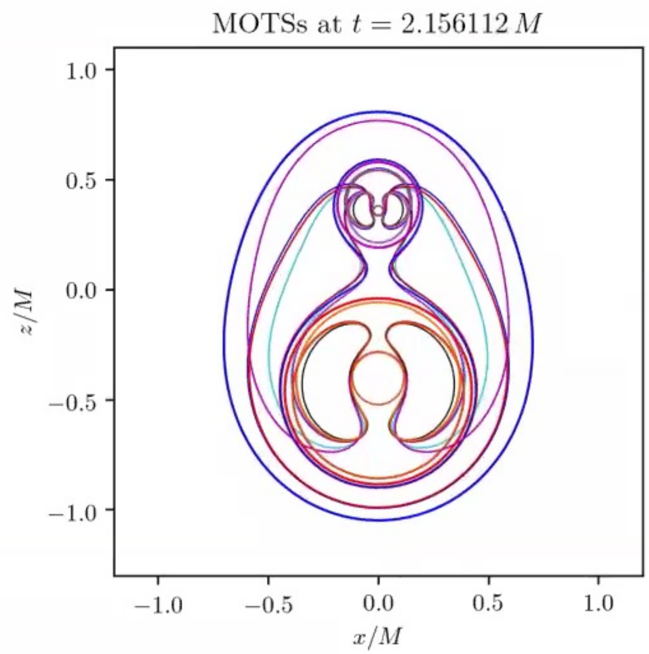
But there are many, many, many more MOTS…

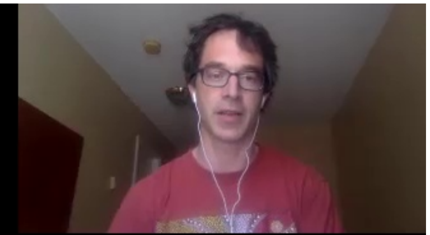


30

But there are many, many, many more MOTS…







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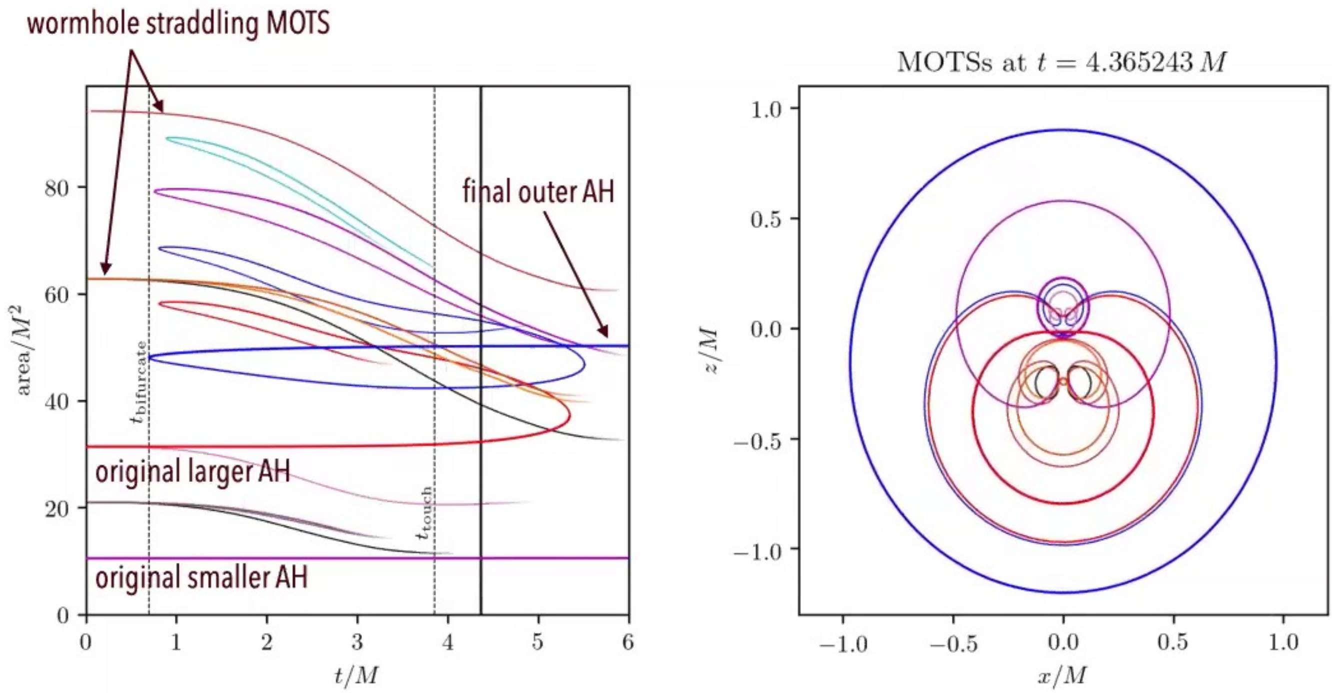
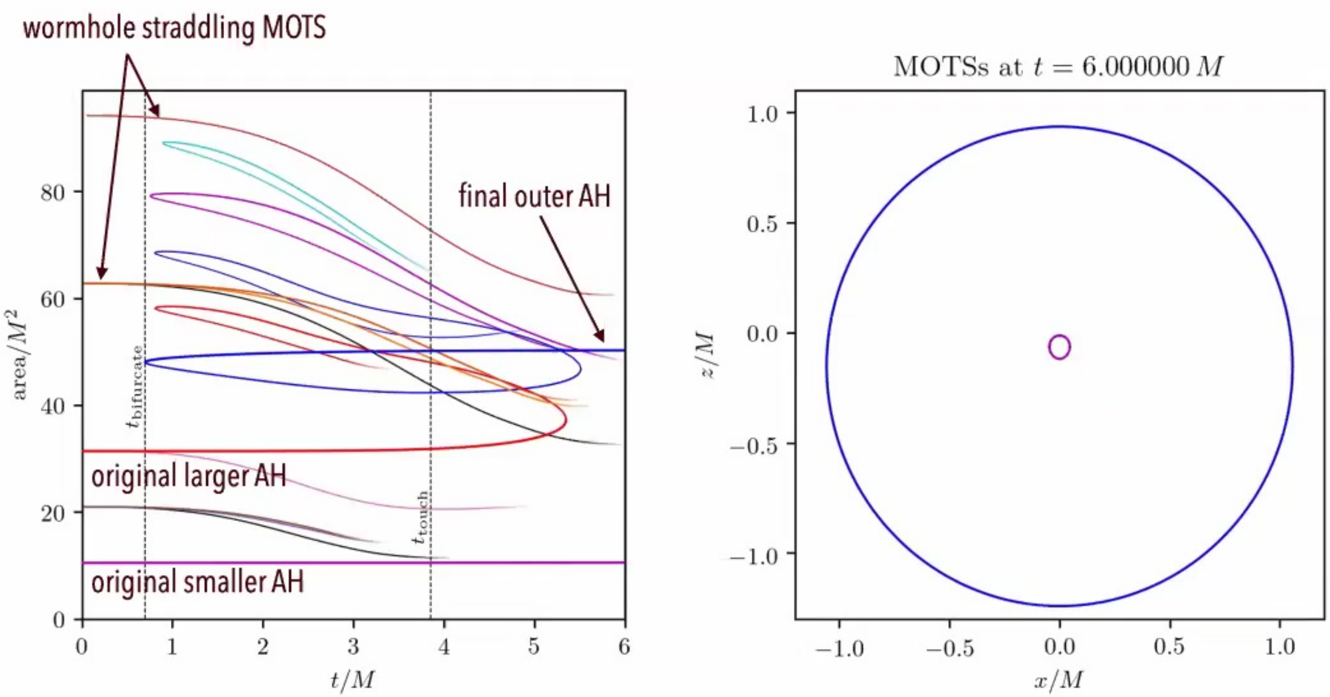


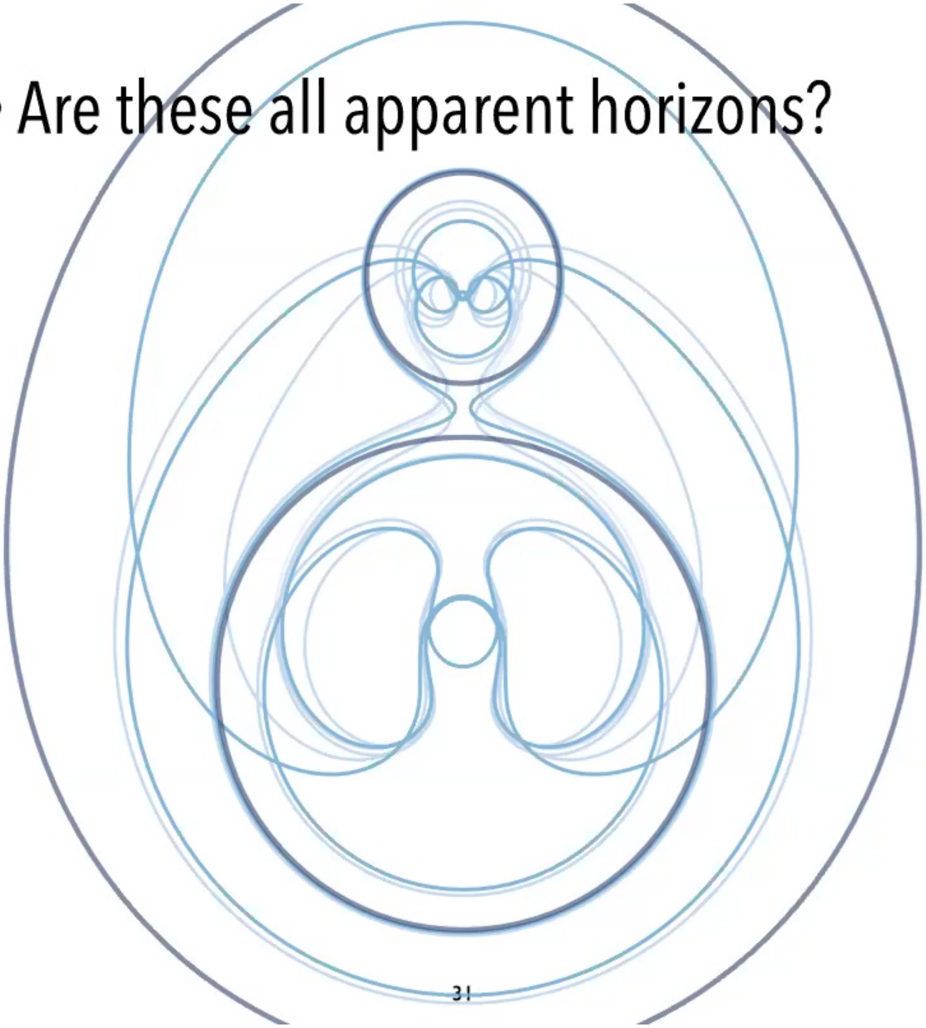
30

But there are many, many, many more MOTS…



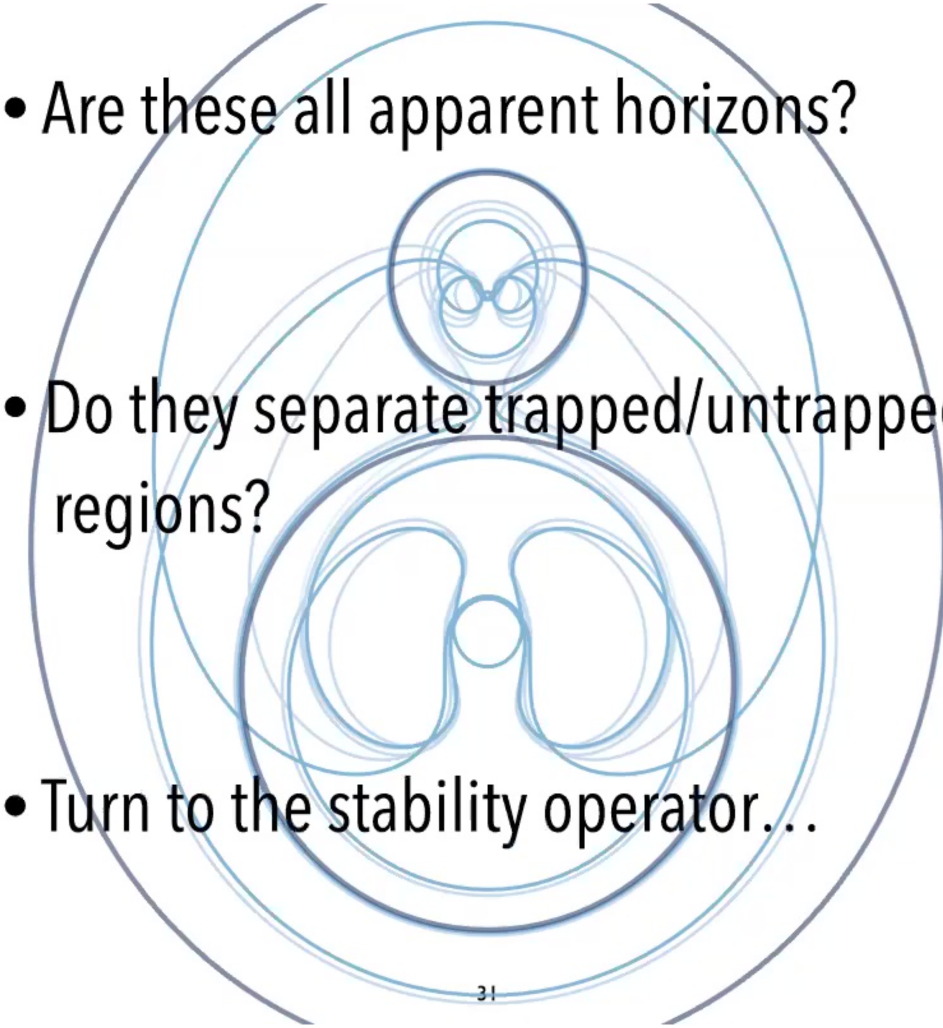
00:27 -00:00

- Are these all apparent horizons?

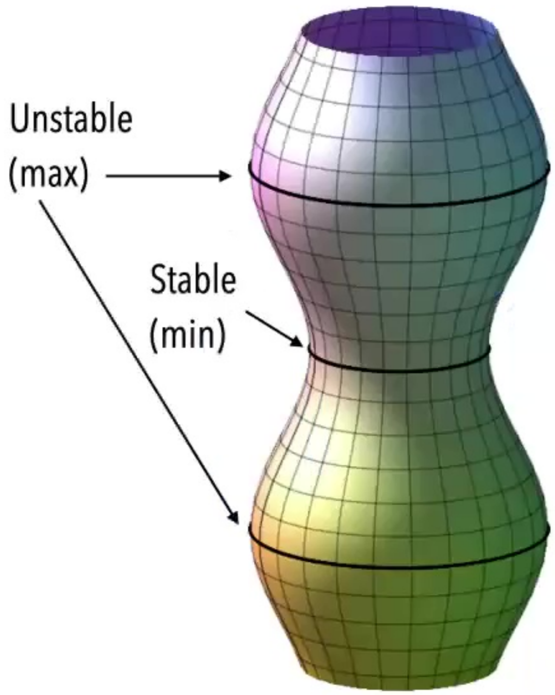


31

- Are these all apparent horizons?
- Do they separate trapped/untrapped regions?
- Turn to the stability operator…

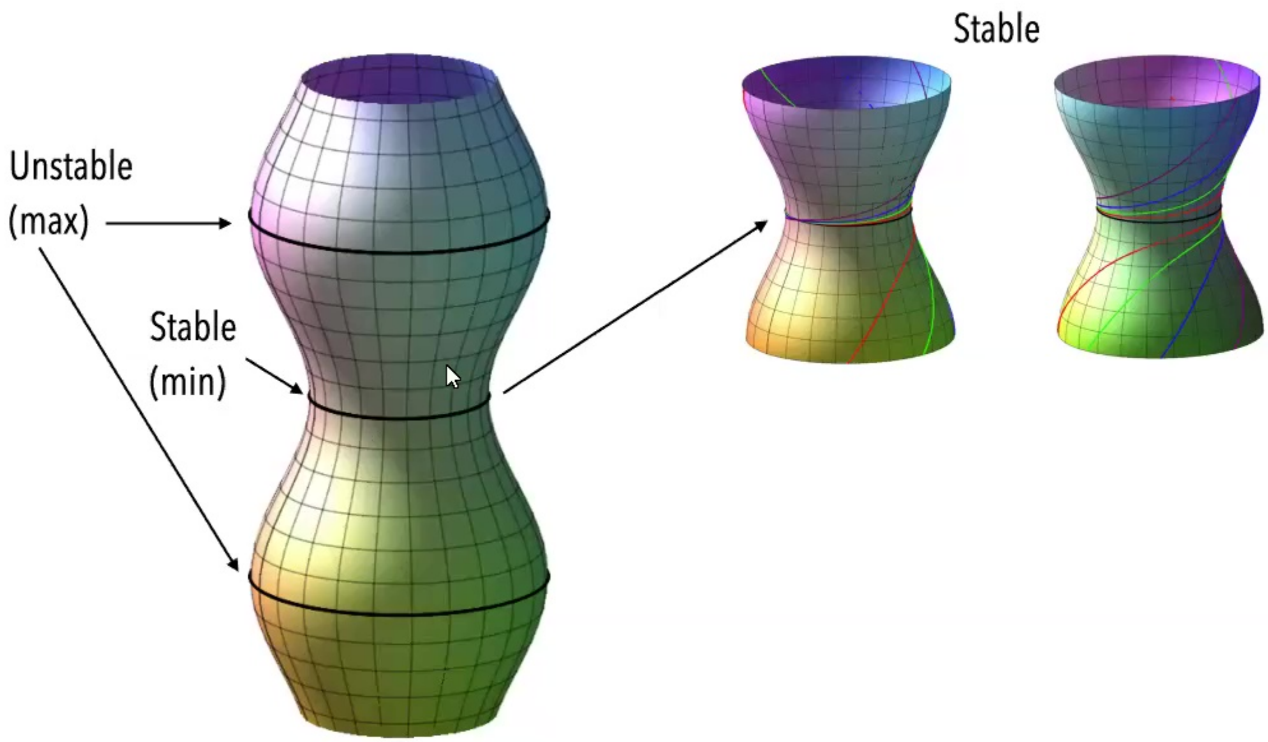


Stability Operator (Geodesics)



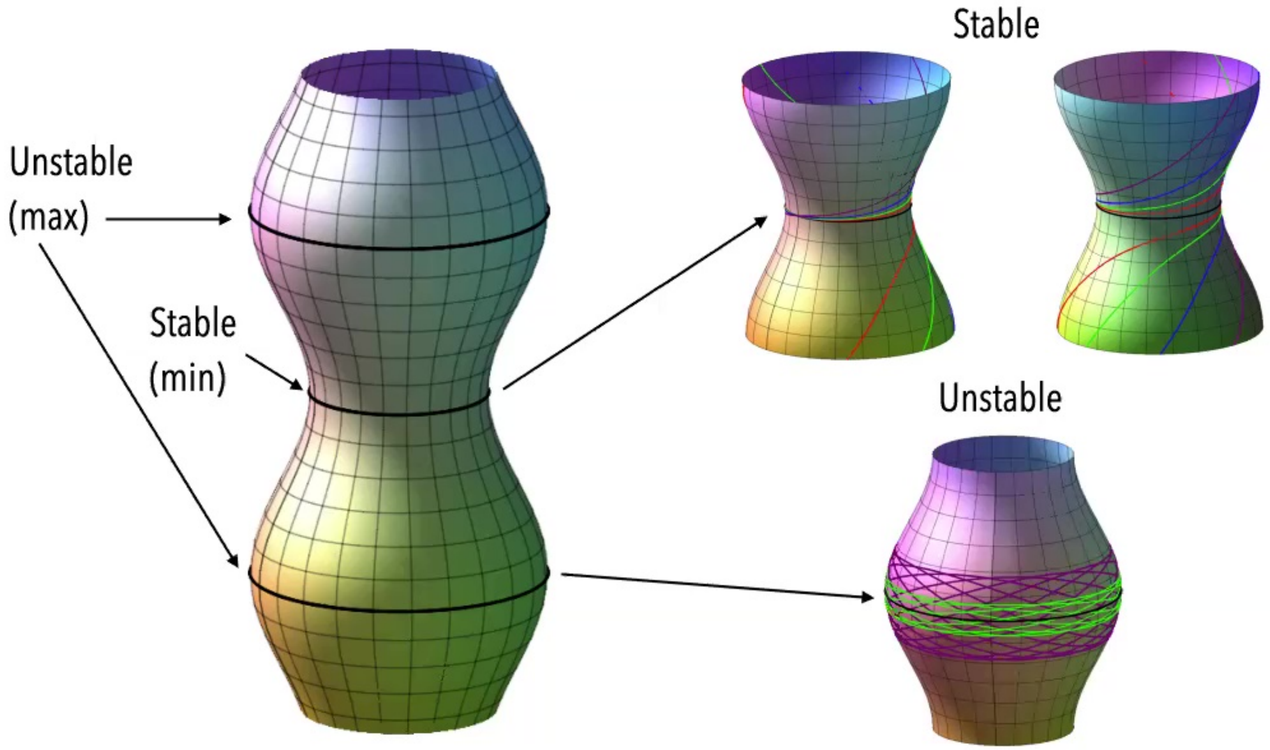
32

Stability Operator (Geodesics)



32

Stability Operator (Geodesics)



32

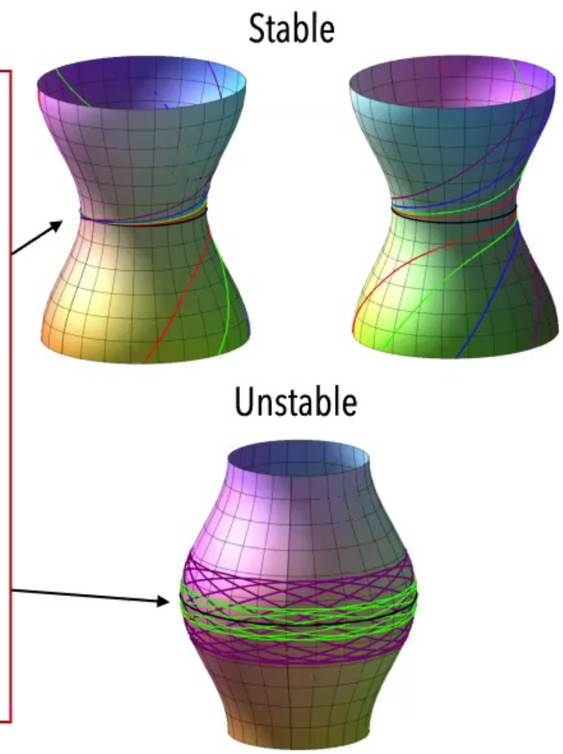
Stability Operator (Geodesics)

Governed by the second variation of length

$$\delta^2_{\psi N} L = \delta_{\psi N} k_N = -\int_{s_1}^{s_2} \psi(\mathscr{L}_\gamma \psi)\mathrm{d}s$$

where the Jacobi/stability operator is

$$\mathscr{L}_\gamma \psi = \left(\frac{\mathrm{d}^2}{\mathrm{d}s^2} + K\right)\psi$$



32

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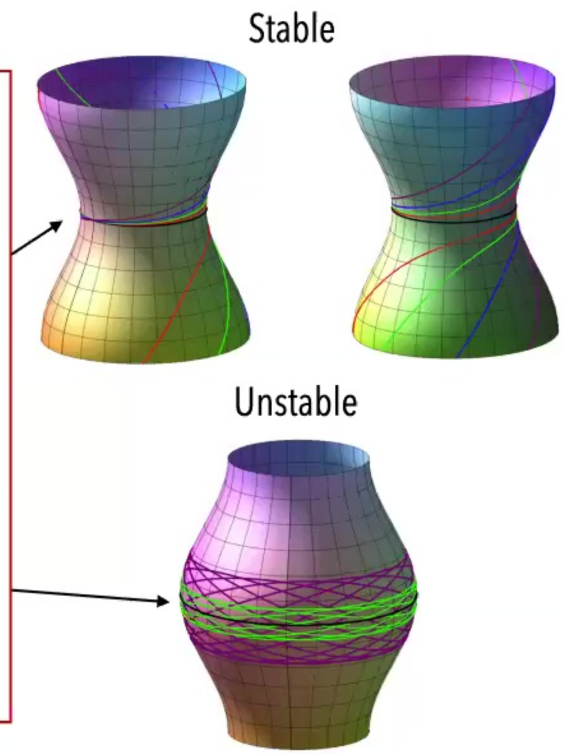
Eigenvalue spectrum of $\mathscr{L}_\gamma$ determines stability:

$\lambda_n > 0 \implies$ stable (minimum length)



32

Stability Operator (Geodesics)

Governed by the second variation of length

$$\delta^2_{\psi N} L = \delta_{\psi N} k_N = -\int_{s_1}^{s_2} \psi(\mathcal{L}_\gamma \psi)\,\mathrm{d}s$$

where the Jacobi/stability operator is

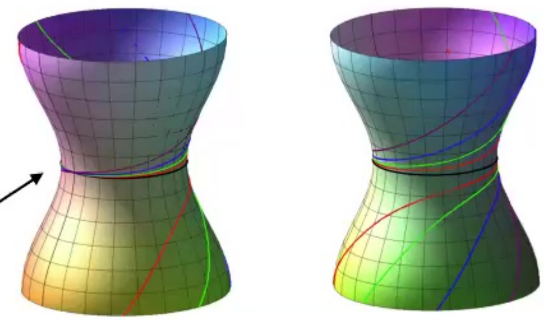
$$\mathcal{L}_\gamma \psi = \left(\frac{\mathrm{d}^2}{\mathrm{d}s^2} + K \right) \psi$$

Eigenvalue spectrum of $\mathcal{L}_\gamma$ determines stability:

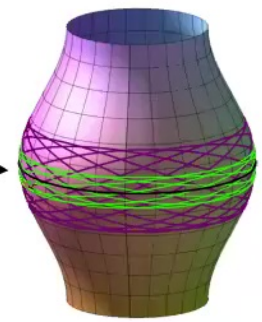
$\lambda_n > 0 \implies$ stable (minimum length)

$\exists \lambda_n < 0 \implies$ unstable, conjugate points for nearby geodesics

Stable



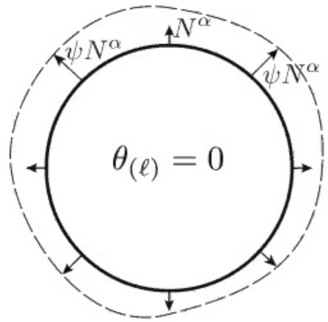
Unstable



32

Stability Operator for (axisymmetric) MOTS

- MOTS can be classified by the <u>stability operator</u>



2D surface Laplacian — 2D Ricci scalar — square of null shear

$$\delta_{\psi R}\theta_{(\ell)} = L_\Sigma \psi = -\triangle\psi + \left(\frac{1}{2}\mathcal{R} - 2\|\sigma_{(\ell)}\|^2\right)\psi$$

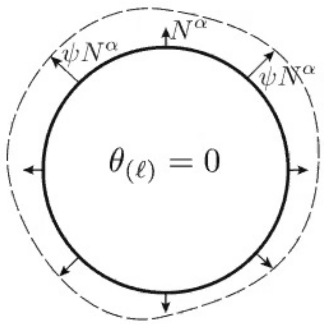
(Andersson, Mars, Simon, 2005, PRL 111102)

- Again information from its (real) eigenvalue spectrum $\{\lambda_n\}$

- $\lambda_n \neq 0$: MOTS smoothly evolves forward in time
- $\lambda_n \geq 0$: MOTS is ***stable***
  separates trapped from untrapped regions,
  expansion is spacelike (AH, DH, FOTH)
- Some $\lambda_n < 0$: MOTS is ***unstable*** and
  has ``conjugate'' points

Stability Operator for (axisymmetric) MOTS

- MOTS can be classified by the stability operator



2D surface Laplacian — 2D Ricci scalar — square of null shear

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(Andersson, Mars, Simon, 2005, PRL 111102)

- Again information from its (real) eigenvalue spectrum $\{\lambda_n\}$

- $\lambda_n \neq 0$: MOTS smoothly evolves forward in time
- $\lambda_n \geq 0$: MOTS is ***stable*** separates trapped from untrapped regions, expansion is spacelike (AH, DH, FOTH) $\Leftarrow$ apparent horizon / black hole boundary
- Some $\lambda_n < 0$: MOTS is ***unstable*** and has ``conjugate'' points $\Leftarrow$ not an apparent horizon

33

Ivan Booth: Ultimate Fate of Apparent Horizons

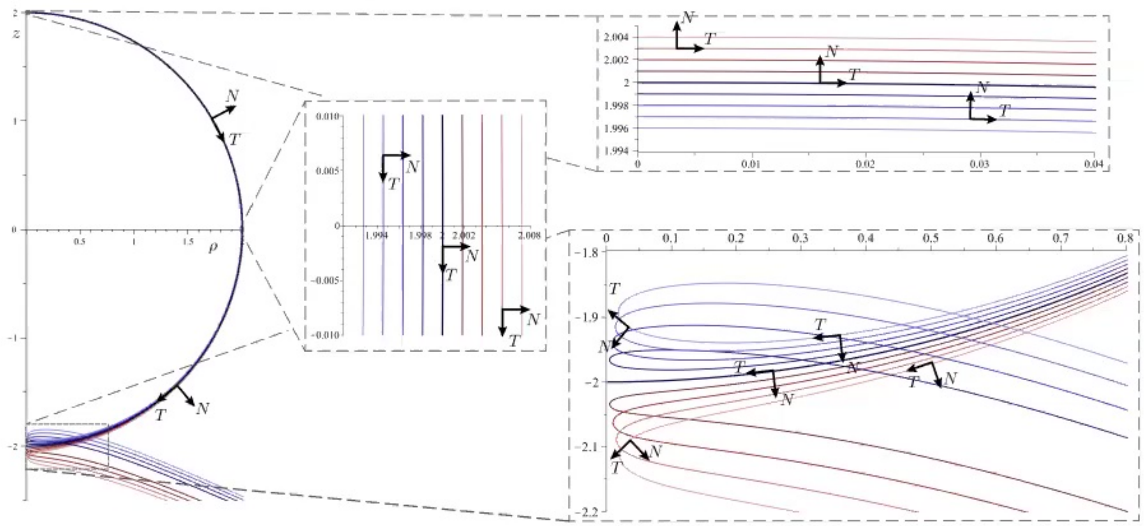
Stability for spherical MOTS (Schwarzschild)

$$\delta_{\psi R}\theta_{(\ell)} = L_\Sigma \psi = \left(-\triangle + \frac{1}{r_H^2}\right)\psi \quad \Rightarrow \psi_{lm} = \text{spherical harmonics}$$

$$\frac{1}{r_H^2} + l(l+1) = \text{eigenvalue spectrum}$$

Laplacian on sphere

Schwarzschild horizon: all eigenvalues positive $\Longrightarrow$ stable = apparent horizon



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Stability for spherical MOTS (Reissner-Nordström)

$$\delta_{\psi R}\theta_{(\ell)} = L_\Sigma \psi = \left(-\triangle + \frac{1}{r_H^2} - \frac{q^2}{r_H^4} \right) \psi$$

$$\Downarrow$$

still spherical harmonics

Outer MOTS eigenvalues: $r_{OH} = m + \sqrt{m^2 - q^2}$

$$\lambda_{lm} = l(l+1) + \frac{1}{r_{OH}^2} - \frac{q^2}{r_{OH}^4} > 0$$

$\Rightarrow$ stable = apparent horizon

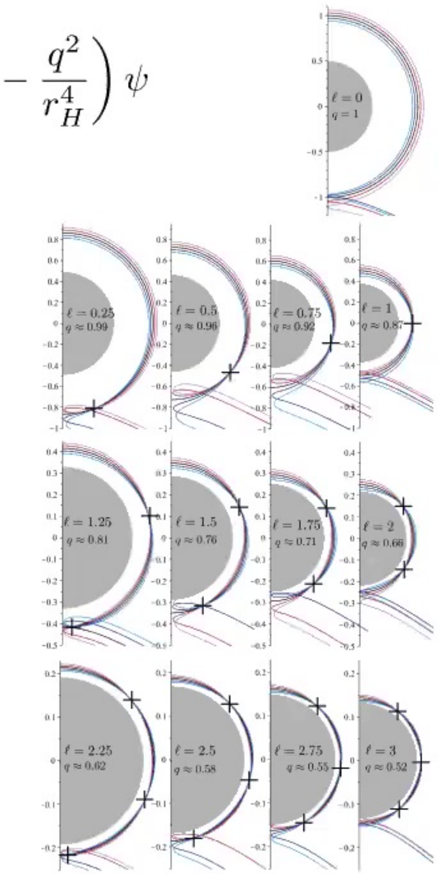
Inner MOTS eigenvalues: $r_{IH} = m - \sqrt{m^2 - q^2}$

$$\lambda_{lm} = l(l+1) + \frac{1}{r_{IH}^2} - \frac{q^2}{r_{IH}^4}$$

$\Rightarrow$ principle eigenvalue ≤ 0

$\Rightarrow q$ determines number of eigenvalues

$\Rightarrow$ not an apparent horizon



$\lambda_{00} = 0$

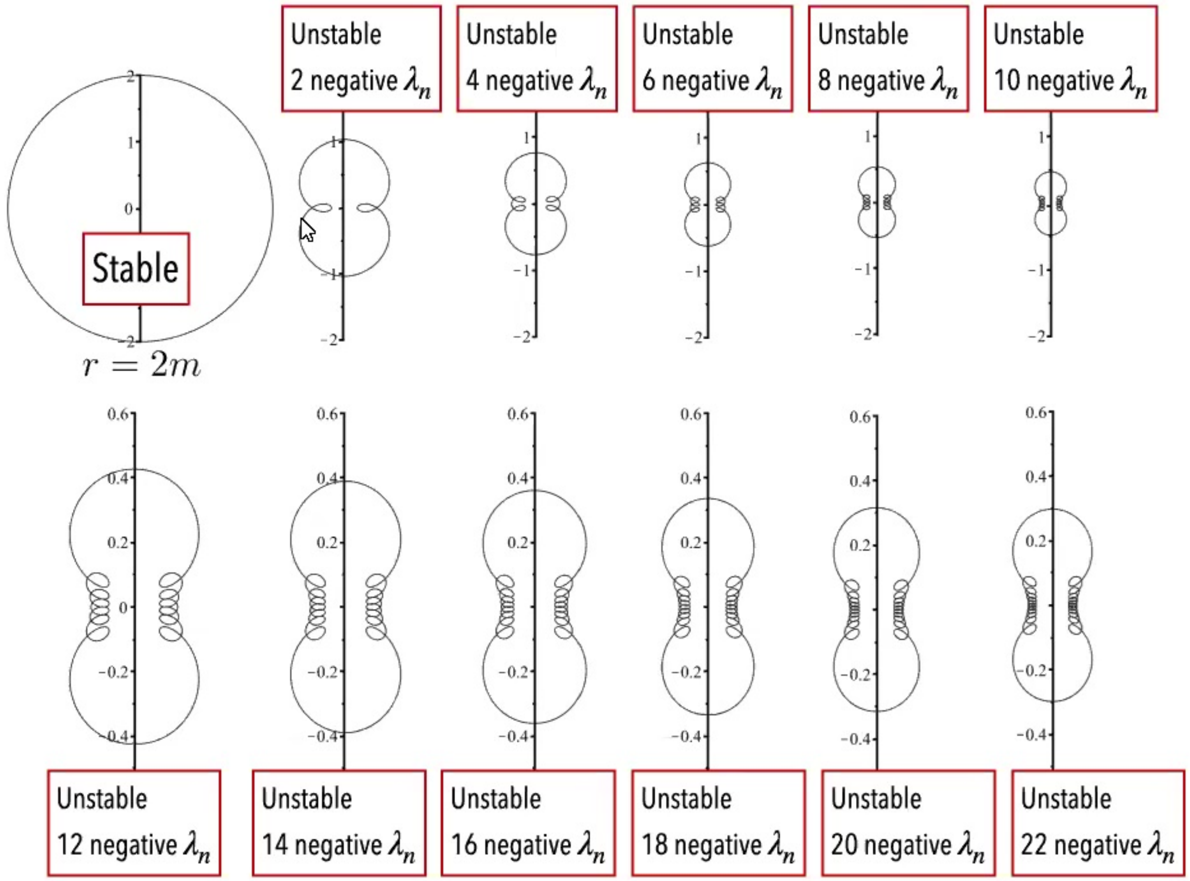
$\lambda_{1m} < 0$

$\lambda_{2m} < 0$

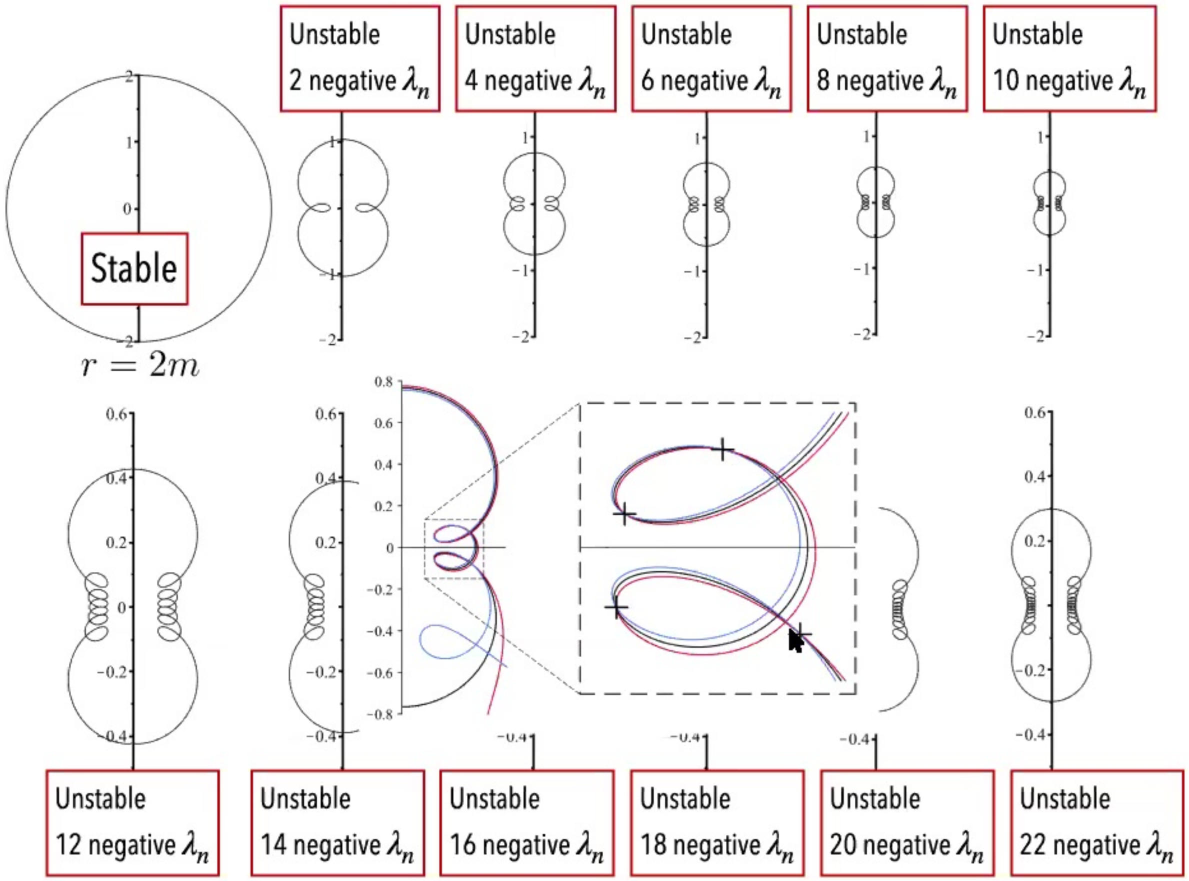
$\lambda_{3m} < 0$

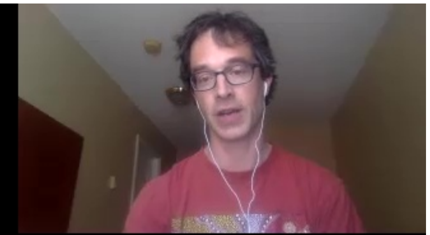
35

Stability for axisymmetric MOTS (Schwarzschild)

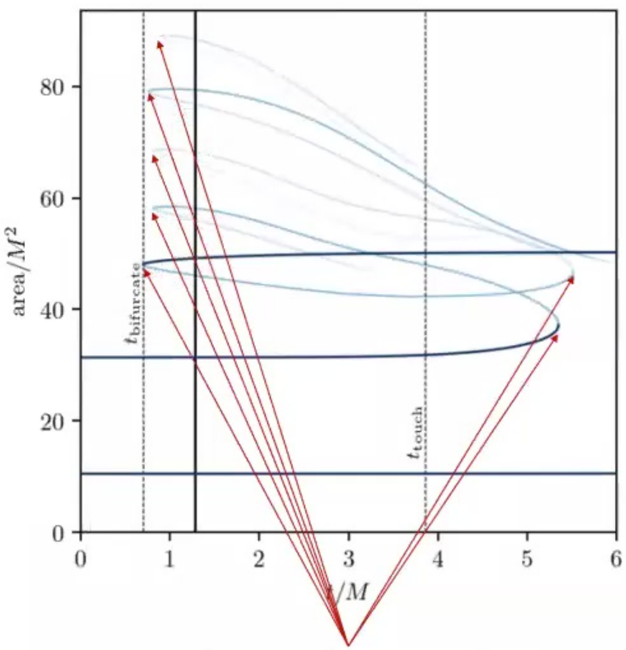


Stability for axisymmetric MOTS (Schwarzschild)

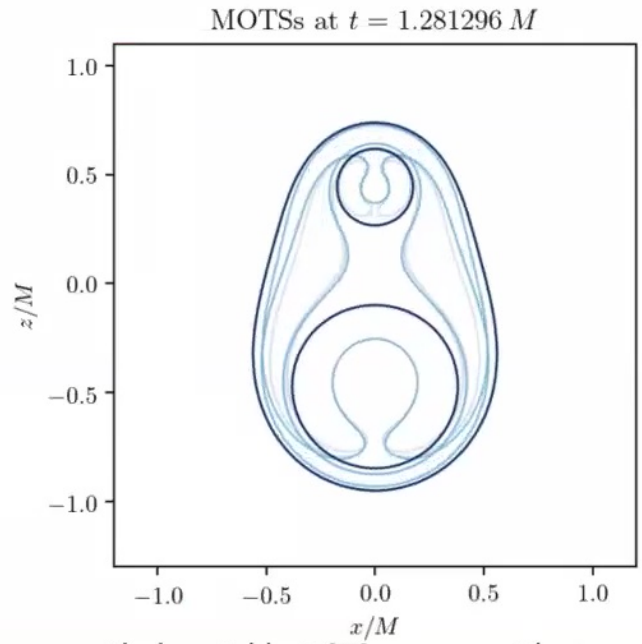




Many MOTS during a merger…



L_Σ has a vanishing eigenvalue at creation and annihilation points



Black = stable MOTS = apparent horizons

Blue = unstable MOTS

Fading = lost for numerical reasons

(Pook-Kolb, Booth, Hennigar 2021)

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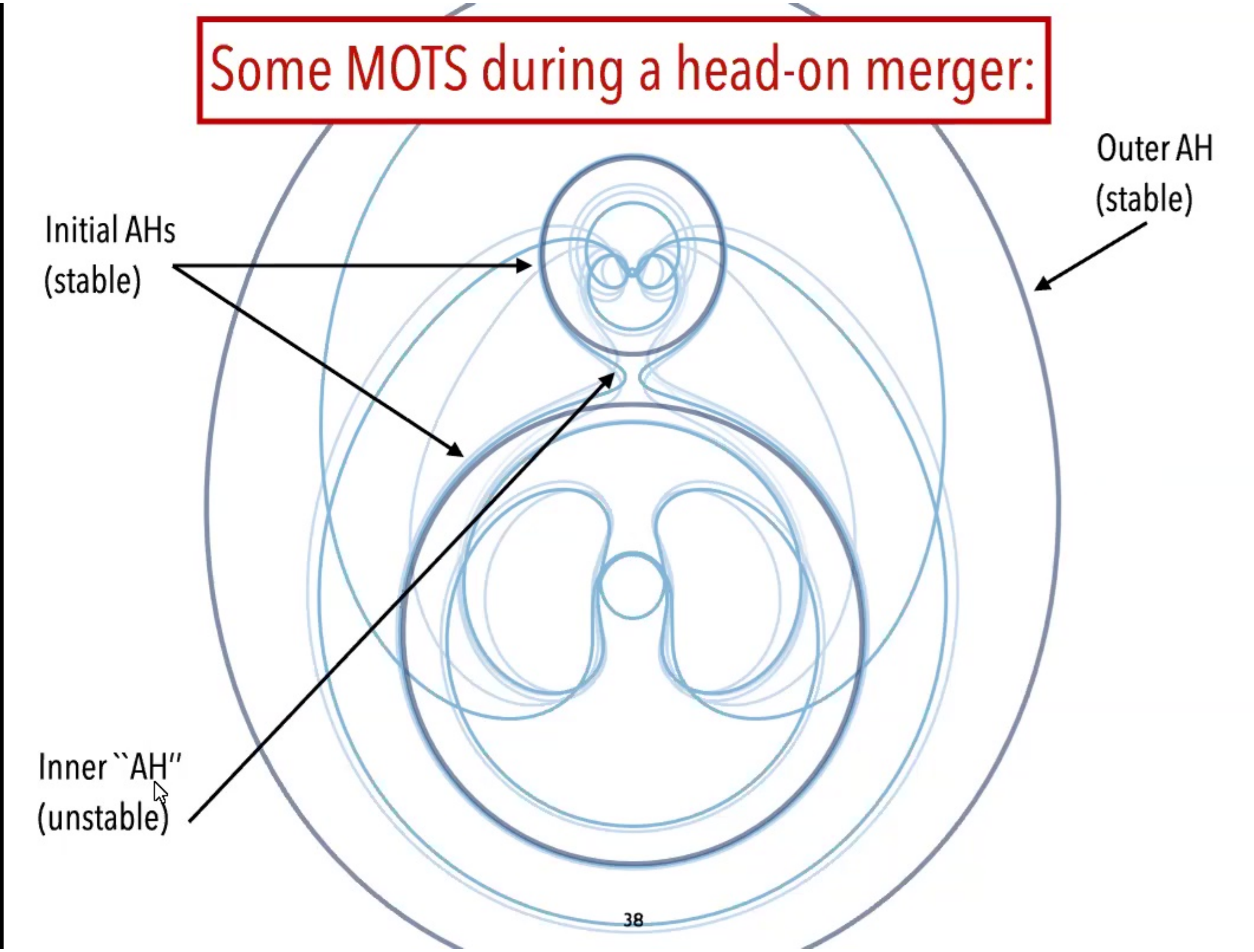
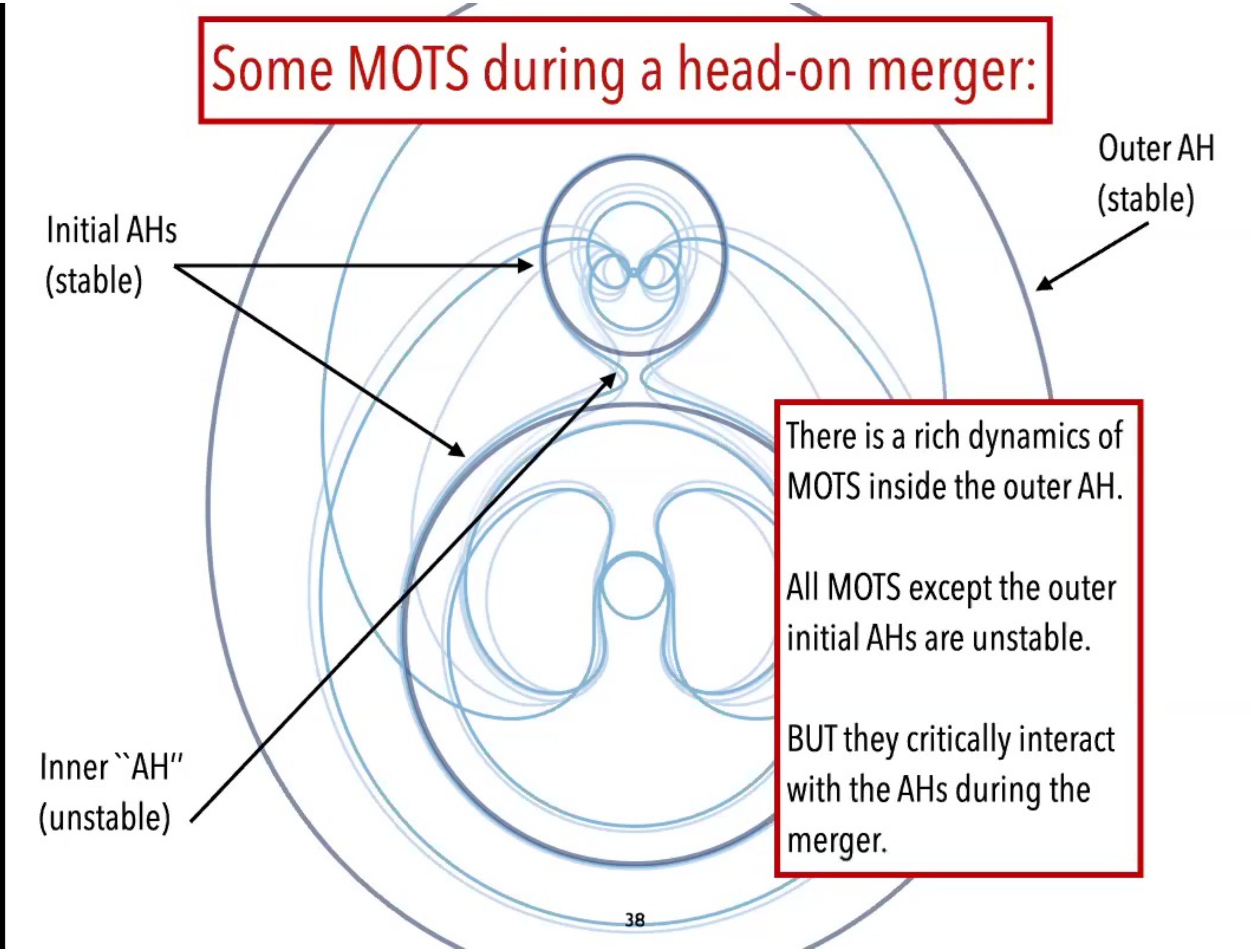
Some MOTS during a head-on merger:




Some MOTS during a head-on merger:
Initial AHs
(stable)
Outer AH
(stable)
Inner ``AH''
(unstable)
There is a rich dynamics of MOTS inside the outer AH.
All MOTS except the outer initial AHs are unstable.
BUT they critically interact with the AHs during the merger.
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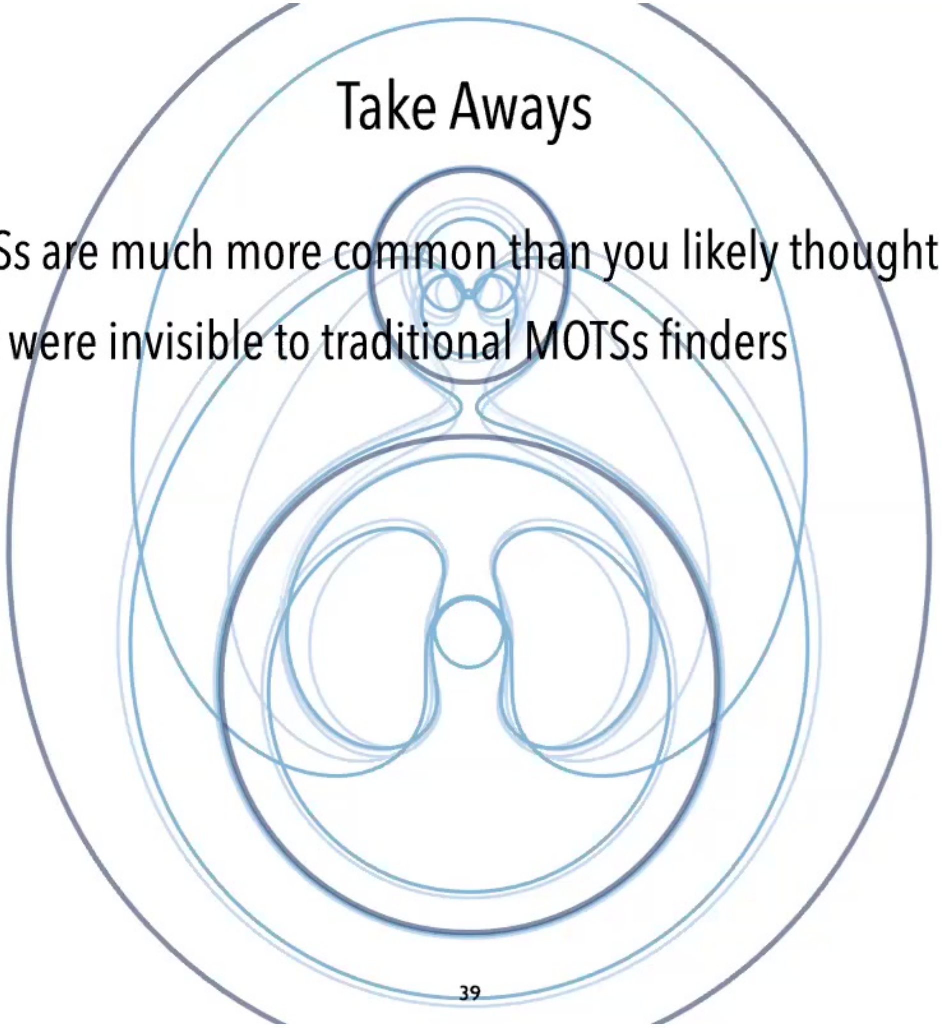
Take Aways



- MOTSs are much more common than you likely thought
- Most were invisible to traditional MOTSs finders

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Take Aways

- MOTSs are much more common than you likely thought
- Most were invisible to traditional MOTSs finders
- Exotic MOTS are important part of black hole mergers
- AHs are much rarer and distinguished by the stability operator
- The stability operator provides a lot of geometric information
- Our (non-rotating) MOTS finder codes are publically available

github.com/daniel-dpk/distorted-motsfinder-public/

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